Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 16
Impedance matching techniques: Part 2

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Hello and welcome to the NPTEL mook on applied electromagnetic for engineers in this module we continue the discussion on impedance matching we first consider the case with the impudence matching using transmission lines these are the distributed circuit try matching networks that I talked about in the previous module okay let us right into impedance matching with the transmission line.

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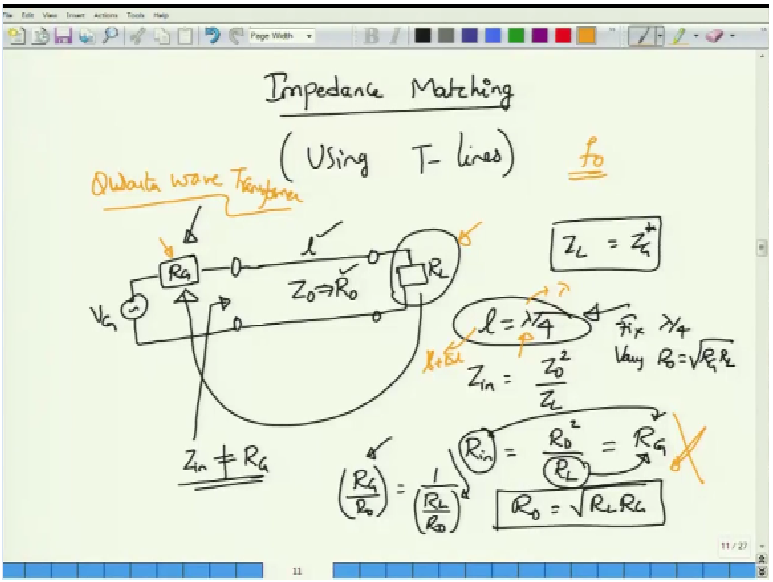

Impedance Matching

(Using T- lines)

$\underline{f_0}$

Quarter wave Transformer

$\boxed{Z_L = Z_G^*}$

$\ell = \lambda/4$

Fix $\lambda/4$

Vary $R_0 = \sqrt{R_G R_L}$

$Z_{in} = \frac{Z_0^2}{Z_L}$

$\underline{Z_{in} \neq R_G}$

$R_{in} = \frac{R_0^2}{R_L} = R_G$

$\left(\frac{R_G}{R_0}\right) = \frac{1}{\left(\frac{R_L}{R_0}\right)}$

$\boxed{R_0 = \sqrt{R_L R_G}}$

Suppose we have a transmission line okay with a characteristic impedance we will assume again loss less transmission lines okay one of the reasons why we assume loss less transmission line is because the matching network itself needs to loss less and then connected to a certain load ZL you might have a source whose impedances is ZG connected to a phase voltage VG as I said the goal of the transmission line in the previous line module as I said goal of the matching network would be have matched then impedance of ZL to ZG okay it is not the goal to match that L TO Z or Z0 or Zo to ZG while that is also very important thing.

That by itself does not give you maximum power transfer but goal is that you match ZL with ZG again to match ZL with ZG I need to know that ZL or I need to make the condition equivalent to ZL complex conjugate or ZL conjugate to be equal to ZG complex conjugate right so this is the condition that I am trying to achieve, for now let me consider only real cases that is I consider both ZL and ZG to be composed of two resistances, RG and RL while we know the answer RL should be = GL for maximum power transfer unfortunately what you can see is that there is a transmission line in-between right.

So because there is a transmission line with a characteristic impedance of Z0 or in the loss less case this simply becomes = R0 and there is a certain length L the impedance seemed looking into this point the impedance will be complex in general this impedance seem looking at the input terminals of the transmission line will be equal to RG.

So therefore there is now maximum power transfer that would take place in this condition however we are designing the transmission line so we are very well free to choose whatever length of the transmission line that we want and whatever the value of R0 that we want let us see what we can do by choosing these two parameters in order to make the input impedance seen here at the input terminals of the transmission line to be = to RL okay we can do that very easily by considering L = λ/ 4.

Because when take L = λ/ 4 the input impedance or the impedance seen looking into the transmission line of a length λ/4 will be equal to $Z0^2$ /ZL okay or for the real case that we are considering this would mean Rin the real part of Zin will be = real in this case Rin = $R0^2$ / RL, but please note that this is only Rin the impedance seen looking into the transmission line and the load okay if we somehow make this = to RG then we have achieved our operation why because in the normalized case that you look at right.

Or in the case that we have looked at Rin = RG or if divide this R0 and to this side I see that the normalized load RG / R0 must be = 1/ RL / R0 right, so the normalized impedances source impedance must be equal to 1/ normalized load impendence okay or load resistance, but 1/ normalized load resistance is nothing but admittance and how do I go from impendence to admittance I need to use a quarter wave transformer or I need to use quarter wave length transmission lines.

So this is the reason why this called as quarter wave transformer because it transforms RL x 1/ RL and when you choose R0 = √RL x RG then you are done then you are essentially ensuring that this RN = RG but RN = RG simply means that RL will now be equal to RG why we can substitute $R0^2$ with RL RG here RL cancels and RL will be = RG^2 in other words if I use a λ/ 4 transmission line then no matter what resistive load RL that I connect I can transform that into see into look into RG by suitably varying Z0 so you fix the transmission line length okay you fix the length to λ/ 4 then vary R0 such that if you are matching RG and RL you simply have to take R0 = √RG x RL.

This why this is called as quarter wave transformers so let us write this one this is a quarter wave transformer or QWT you might object to this quarter wave transformer because no load will be usually completely restive and it would be = ZG as well so there is a version in which you can also replace ZL, RL/ ZL and RG / ZG but that is not very important for us to go into now but what you would be objecting to is that even if the impedances happen to be real they will be real only at a particular frequency f0.

More importantly suppose we cannot manufacture length of λ/ 4 either because λ is large or because there is an imperfection in λ so λ ends up being λ+ Δλ so if the impedances if the length is not exactly = λ/4 then this either transformation rule does not work okay, it works only when L = λ/4 in case your L happens to be some L + ΔL so this is not λ+Δλ this as to be L + ΔL where you go beyond the design λ / 4 wave length and how bad would your system suffer, and you can actually do that there is a small calculation that is involved but I am going to skip the calculation and show you graphically what happens okay.

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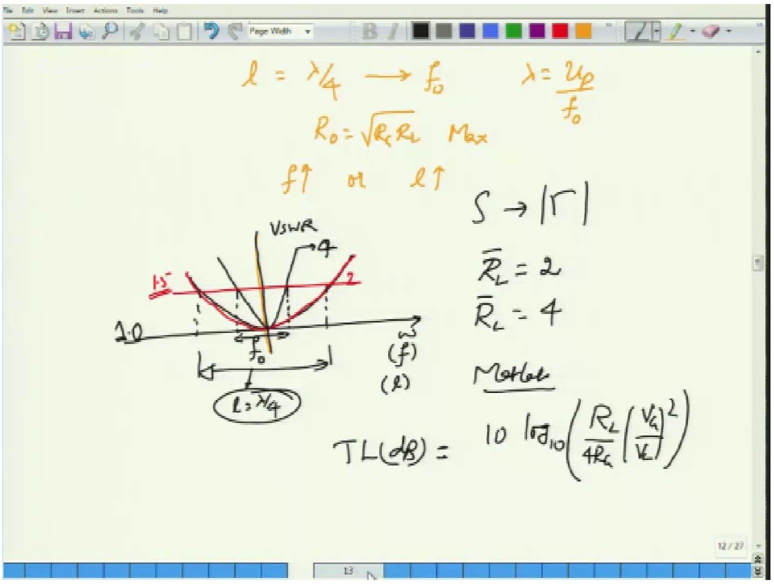

So corresponding to a length l = λ/4 will be a certain frequency f0 because λ is related to frequency λ is Up where the Up is the phase velocity divided by frequency will give λ so at that particular operating frequency if the line as length of λ/ 4 then we have seen that perfect matching can be archived provided L – λ/4, so in that case I will have R0 = √RG RL and I would have had maximum power transfer but if we either change frequency or we change L due to manufacturing constrains or just because the operating frequency as changed and I cannot change R0 every time or length of a transmission line every time on printed circuit board if I have fixed that track length track width, track height and everything then the track is actually fixed.

I cannot keep changing that one right or even I mean there is a solution in the discrete case where ever you can move the tuner as we would call but on a printed circuit board with a track length I cannot do anything much about that so in that case if you move away from L or away from f then you actually suffer pretty badly in terms of your VSWR please remember that VSWR will give information about the miss match because VSWR is directly related to ΓL magnitude or Γ magnitude and Γ is giving you the information about how much is the incident and the reflected wave are related.

So if Γ is = 0 there would not be reflections, so if you look at this case look at this one for two case let us look at for two cases when first we assume that the normalized impedance is twice the character impedance that is 2:1 ratio in which you are trying to match or you might be trying to match say 4:1 ratio right, so if the normalized load happens to be about 4 then you see that for the two cases the VSWR starts to version okay.

And this is along the frequency or ω(f) okay the frequency which I am plotting and this happens to be the center frequency f(0) for which the length will be = λ/4 so if I have chosen this one and even I am increasing frequency or it could be increasing L so both qualitatively give you the same answer insert quantitatively they also they give you the same answer, so this case which is the one that we are return is for the case of 4 and for the case of 2 :1 impedance matching ratio this could be the scenario.

Okay so this is for the case of RL = or RL bar = 2 so you can see that the larger the impendence miss match that you are trying to match smaller will be the corresponding band width what is band width if I move slightly away from ω0 or f(0) or if move slightly away from the design length L either by increasing the length or by decreasing length due to manufacturing constrain my VSWR shots up any VSWR above 1.5 is typically considered to be very bad okay I will give you as an exercise in the sheer you can actually calculate at what frequencies these VSWR occurs hit 1.5.

So starting from the minimum value of 1 at the design wave length they will quickly hit 1.5 and you see that the larger is the impedance ratio the smaller is band width the bandwidth is slightly better when your impendence is are of the same order in which you are trying to match, so this is how bad the quarter wave transforms it is as you move away and you will have an opportunity in the tutorial sheet to actually to know perform all this calculation using mat lab and plot this VSWR as a function of the operating frequency f.

Despite the factor VSWR increases we still use quarter wave transformer not in a single section quarter wave transformer but multi section quarter wave transformers the reason why we do that if because if you calculate what is called as the transistors loss in deadly I will not derive this expression, so if we calculate the transistors laws again you can write a short mat lab code in order to do this thing you will see that no matter what the value that I am trying to match if I get the transistors laws write I am actually okay even with the power transformer.

Because if I do not use any transformer the transmission laws will be much more than I am using so even a poor quarter wave transformer poorly designed poorly implemented quarter wave transformer will still be some sort of an improvement over using no impendence matching at all okay.

If this does not mean that you should go and design a poor quarter wave transformer it simply means that even if by some problem the quarter wave transformer is not very good transfer to be poor there is still merit in using this quarter wave transformer for impedance matching we now go another technique of impendence matching in which we are going to use a transmission line and this time the design will be done by using a graphical means that as that is smite chart okay.

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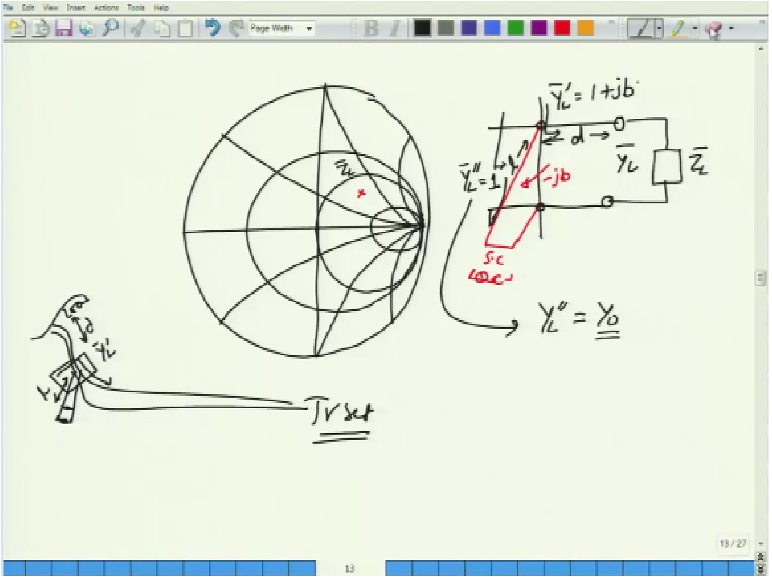

Let us first go back to the so this is the smith chart that we are familiar with we will draw only a few arcs again it would help in case your able to get hold of a smith chart and perform this calculations as I am performing the calculation so you will learn a lot when you do that okay so let us say this our compressed smith chart that we have consider and what we are trying to do her is that suppose is start with a load not some particular point so this is the load or maybe I can try a different load.

Let me re draw that arc which did not put in so this is the unit circle okay so that is the unit circle that I have for this smith chart and suppose I am located at this particular point. So this is normalized load that I am looking at so this is the normalized load so going back to the circuit itself I am at the transmission line terminals connecting to the load. And I know that characteristic impedance of a transmission line which I have already design to be = Z0 okay.

And I normalizes this loads do that normalized load is ZL/ Z0 okay now watch what happens as we move towards the generator that is away from the load what happens on the transmission line my impedance keeps changing at every point remember by impedance at any point on the transmission line is the line impendence and this is some Z0, ZL + JZ0 something that we have already talked about right.

So the impendence continuously changes while it is continuously changes at some particular point okay the impedance might be in such a way that you would hit 1+ JX when you hit 1+ JX then you can actually stop you can cut this transmission line replace this transmission line position on to this left hand side by a reactive you know transmission line which is terminated with something and then give you a reactance of – JX such that when you move past that plane the total impedance seen here will be = 1.

And of course this is normalized therefore you should remember that this is normalized impendence = 1 which means that I have matched my ZL to Z0 I do the same operation at the transmitter and also the only problem that you would see with this is that you have to cut your transmission lines and that is not always a easier option to cut nor that is something that you would like to do on printed circuit board for example if you are implementing this even or otherwise it is actually little difficult to cut something and then replace something in the series.

So this solution although is very nice we do not normally look at that insisted what we do is we recognize that smith chart is as good as an impedance chart as good as it a admittance chart right so what we do now is we simply realize that I can work with admittances so I instead of talking about ZL bar I talk about YL bar at the load side and then I move by L bar, so at some point at some plane when I land I would have obtained YL bar’ that is the transformed admittance which will have a real part = 1 but some imaginary part = JB okay.

Where B is the substance, so I have actually moved in such a way that I am landed on the transformed admittance of 1+JB so when I do that right I can at this point connect a stub because the stub admit and seen looking at this admittance if I am able to manage that to be = - Jb the stub could be either short circuited stub or could be an open circuited stub okay does not matter whichever it is depending on the application you might chose one over the other okay.

So if my stub provides an admittance of – Jb pure reactive admittance – Jb that the total admittance seen after including this stub which is connected in the this is called as stub, stub being a very small transmission line which I have connected in parallel okay so because of this parallel thing I am adding admittances, in the series the stubs add impedances but in parallel they add admittance, so when I add admittances of – Jb that transformed normalized admittance will simply be = 1.

That is you have achieved a condition that YL bar is = Y0 the impedance seen looking at this point on the transmission line is equal to the admittance Y0 the characteristic admittance of a transmission line and most importantly you did not have to cut any transmission line you just had to talk this point and this point and connect a short stub and you can do this very well even on a printed circuit board so on a printed circuit board let us say this is the track that is going around then you can actually talk a track and then create a wire of a appropriate length or create a track of appropriate length and then do in this.

Of course in pay you can even since you are doing it not on the dynamic case you can even do the series step but on antenna tuner for example types of the full suppose say this is my antenna and these are the antenna line that are connected may be to the TV set not in the this one so here I can put a tuner okay which will connect from this two lines a short circuited stub or a local circuited stub and you can simply move the tuner okay why do I need to move the turner I need to know what is a distance over which I have to move from the load to the point where I reach YL.

So if I move from the load this is the load if I move distance of D then I would have reached YL bar which is 1+ Jb then if I adjust the length of this one there are adjustable shorts available that is why you normally see this one with the short circuited stubs, so you are this adjusted stub shorts will give you a length or a length of the stub let us say is about L, okay that L should be choose such that you get a admittance of – Jb okay.

If b happens to be positive here then you need a – Jb if b happens to be negative then you need a + Jb okay let us see how this entire thing works.

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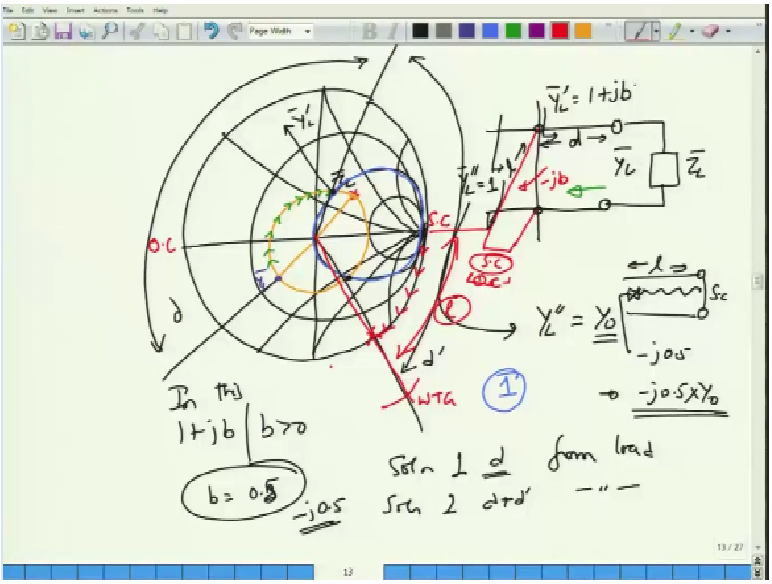

So we started off with a load her and what do we do in order to find out the admittance in order to find out the admittance we need to first draw the SWR circle so this is the SWR circle that I have drawn and then start moving towards to the generator right and away from the load and moving towards the generator so I keep moving before moving I should first find the admittance so to find the admittance I need to after drawing the SWR circle I need to locate a point opposite okay so that I start with the admittance over here okay so this is my YL bar this was ZL bar now is start moving not from here so let me draw this once more okay.

So let me draw this once more we need couple of minutes so you can see that between drawing the basic idea is you start with load and then you transform the load okay so you transform the load into a admittance load so this is one more circle okay now we are good to go you start again with a load here and then correspondingly draw the SWR circle does not matter what the load value is we will give the numbers later for you to practice but start with SWR circle you cam clearly see that this is a circle that I have drawn and from there locate the diametrically opposite point which will give you the admittance.

So this is your admittance YL bar okay and this your point where you start of the load ZL bar after you have come to admittance you start moving towards the generator that is you keep moving away from the load at some point you will actually end up meting this unit circle, right so on the unit circle what is your admittance the on the unit circle that admittance is actually equal to one the real part of the admittance is equal to 1, so if you look at the intersection of this SWR circle with the unit circle let us mark that point as say may be a black point over here.

So this point right is the point where the impendence is YL time okay and the distance that you had to move in order to reach this point is simply that distance d on the transmission line that you have moved so you started of with YL bar you reached YL' which is given by the intersection of the SWR circle with the unit circle okay of course if you continue your journey you would reach one more point here in which case you would have consider you can consider this as a second solution this as first solution okay.

First solution you have at a distance of d for second solution you would have to cover an extra distance okay and extra distance of say d' so that solution 1 is at a distance d from the load okay where as solution 2 will be at a distance of d + d' from the load okay we will not consider the second solution it is more or less that whenever you want to place a stub you want to place the stub as close as possible to the load therefore you usually or okay with the first position itself so once you have reached this position okay which is YL' right now all that is less is to read to what is the value of admittance here and that would be some 1+ Jb here b is positive in this case so in other case or in this illustrative case b is positive right.

So let us say that b is above point 3 or may be here it is because inside it is occurring inside the it is about point 3 let us say or 0.5 let us say what you now need to do is to connect either a short circuited stub let us assume that we are looking at the short circuit stub of certain length L such that the reactance here will be – J. 5 please note that this again the normalized reactance the actual substance is obtained by un normalizing this 1 time multiplying it with Y0 okay.

So this would be the admittance and or substance and then 1/ substance will give you the impendence correspond or the reactance corresponding to this any we will not worry about that, so what we need to know or what you need to do is to reach – J .5 form the short circuit right so either you can actually start from the short circuit point and then moves towards the generator such that you reach – j.5 okay.

That is actually quite simple on the smith chart when you consider it as an admittance chart this point will become the short circuit okay and you know have to move along the outer periphery because there is no imaginary part for the admittance the imagery part of the admittance is 0 so which happens to be outer circle so you keep moving from the short circuit point on the varying reactance circle that is on the out circle such that you hit about – j. 5 so let us say that happens about at this point.

Okay from here you draw a circle and note down on the WTG scale what is the value here and this difference this is the distance which you have covered from the short circuit towards the generator which will give you the length of transmission line that is necessary in order to add a substance of – j. 5 or – Jb so that once you have found the length as well as the distance at which the stub is to be placed in this example we consider the stub to be short circuited but you could consider an open circuited stub in which case you will actually start on this side and then move towards the generator okay.

Right so that is what you would do and then you do that you will end up with the solution which is correct for the open circuited stub okay so depending on what kind of stub that you employee you can obtain the appropriate length of the stub which will allow you to add the required amount of substance okay that is one small catch here suppose someone tell you that the characteristic impedance of the stub as to be different from the characteristic impedance of the main transmission line that is the stub as say Y0’

When the main line as some Y0 in that case what you do at the point where you find out what is the reactance right so is the amount of substance that you need to add by the stub right so if it was = Jb you would have added – Jb x Y0 that would-be the un normalized substance that need to be added so this un normalized substance you divide by Y0’ which is the stub line characteristic admittance.

So when you divide this you will end up with a new normalized B so b’ that would not be equal to the original b but it would be Y0 divided by Y0’ where this ratio will tell you what should be the value at which you have to scale the + J . 5 term right the 0.5 be the b value which was a residual substance if you think of how much you should scale this 0.5 of B value is given by ratio of the character of the main line to the secondary transmission line or the stub line this completes our impendence matching with stub or with front stub as we would call we have used only one step in the particular in olden days.

You cloud still use you could use multiple step so there where two steps 3 steps tuner in a 5 step tuner was available for impendence matching using this stub case these are all parallel or front stub matching cases you can even have stubs in the series but the problem is you have to cut the transmission line. So far we have considered impendence matching with districted or transmission lines. In the next module we will consider impudence matching with a numbed element circuit thank you very much.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy

Co – ordinator, NPTEL IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

Badal Pradhan

Tapobrata Das

Ram Chandra

Dilip Tripathi

Manoj Shrivastava

Padam Shukla

Sanjay Mishra

Shubham Rawat

Shikha Gupta

K.K Mishra

Aradhana Singh

Sweta

Ashutosh Gairola

Dilip Katiyar

Sharwan

Hari Ram

Bhadra Rao

Puneet Kumar Bajpai

Lalty Dutta

Ajay Kanaujia

Shivendra Kumar Tiwari

an IIT Kanpur Production