Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 15
Impedance matching techniques: Part 1

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Hello and welcome to the NPTEL mooke on applied electromagnetic for engineers. In this module we will talk about an very important topic the topic that comes quite often at higher frequencies at microwave frequency or at RA frequencies infects so much so that most RF designers spend about 50 to 75% of their time for forming this particular function. What is this magic function that most people want to do at high frequencies incidentally most people at low frequencies do not even bother about this topic.

This topic is that what is called as impedance matching, what is impedance matching? Now from your first year circuit courses you might know very well that if I have a source with the fixed internal resistance so whatever the resistance of that will be, if I consider a source with the fixed internal resistance, then what value of the load resistance should I collect across the source terminals such that maximum power is delivered to the load.

Now this problem we know can be easily solved, you know you can either solve it by simple physical arguments showing that RL=0 does not give you power because the voltage across RL will be equal to 0 in that case or when RL=infinity that is short circuit or open circuit at cases do not give you maximum power transfer, you will get maximum power transferred or maximum power will be dissipated across the load resistor RL when that RL is equal to RG, where RG is the internal resistance of the source.

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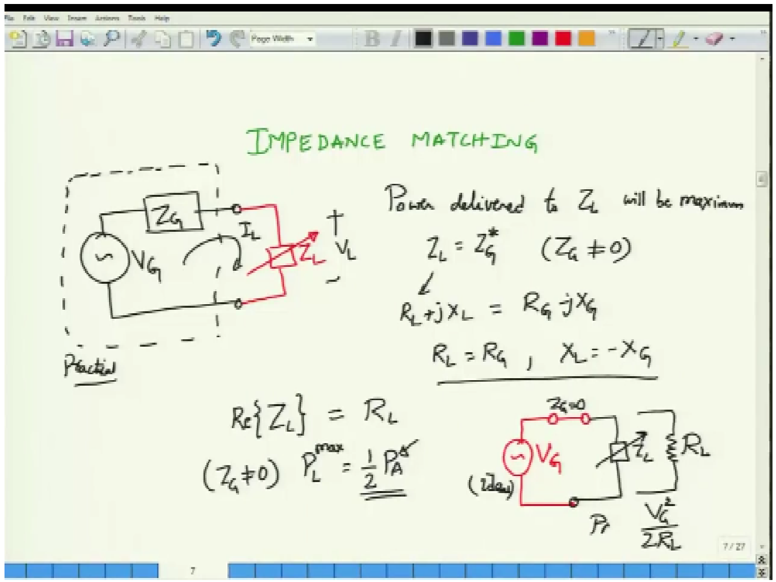

Now if we consider high frequencies we of course also not only at high frequency even at low frequencies we know that the generator impedance or the generator will not necessarily always have an impedance of there, it can also have a reactive component which means that we will be having the complex impedance as an internal impedance something that is not accessible to you. Please remember that this model which I have put inside this dotted lines is actually something of a idealization.

So we are assuming based on our measurements that we perform at the output relevance of the source that one possible way in which the source can be modeled is to have some source amplitude and the internal resistance ZG. Of course if you had an access to ZG then you could have add access to the power dissipated across ZG which of course you do not have right. So this one that is, so this is called as an internal impedance and this is idealized by the fact that we have drawn dashed lines around this source.

So this is not a perfect or an ideal source this is imperfect or a practical source and which has an impedance ZG which is internal to this source. Let us also assume that the source amplitude is some VG of course in the thing that we have been talking about over the last few modules mVG happens to be a phasor okay. But for our discussion not really important to know what the form of VG is.

So given a complex load ZG right, if I were to attach to this imperfect or the practical source a variable impedance ZL and I can change this ZL and I can of course go from ZL=0 to infinity and ZL is also complex, so I can change both the real part of ZL as well as the imaginary part of ZL. And the question now becomes what value of ZL that I should choose in order to maximize the power delivered across ZL.

So power delivered or power transferred to ZL there are complex load will be maximum, so I can deliver maximum power to the load only when ZL is equal to ZG complex conjugate. What does that mean in terms of VL and imaginary components? ZL if you were to write this as RL+JXL where RL is the real part or the resistive part of the load and XL is the reactive part of the load, then this must be equal to RG-JXG okay.

So clearly telling you that RL must be equal to RG this is not very surprising, because this is the same case that you are going to obtain when the load as well as the internal resistance are both real quantities, that is if both ZG and ZL consist only of resistances then we know that maximum power can be delivered across the load or transfer to the load when RL=RG. In terms of the reactance's when the load becomes complex then XL must be equal to –XG.

So if the source has a reactance which is capacity in nature then you need to connect at the load side an inductive reactance so that this inductive reactance can tune out the capacitor and thereby cancel the reactance's of both the internal resistance as well as the load leaving only the real of resistive cases behind. When we do this and then we take RL=RG and XL=-XG how much power is actually delivered across the load please remember that if I have a load which is complex, then the power will be delivered only to the resistive part of the complex load.

Because we know ideal capacitors or inductors do not dissipate energy. So when you have a complex load it is only across the real part of ZL or RL I know which will actually dissipate the power okay. So if you substitute the conditions of RL=RG and XL=-XG calculate what is the current and what is the voltage across this load okay, current through the loop is the load I mean is the current IL and voltage across the load is VL.

And if you calculate what is the power that is actually delivered to this, you will see that the power delivered will be half of maximum power that is available from the source. What is the, so let me write this as so power delivered under the max condition or the maximum power delivered okay, when ZG is not equal to 0 is equal to half the power that is available. What is the meaning of half the power available.

Does this quantity available power in our definition is slight of a idealization, why is that so supposing I had the same voltage source with VG as the amplitude and by some miracle of nature by internal impedance was equal to 0. So ZG in this particular case was equal to 0. Now if I ask you what is the load impedance that we need to connect such that I obtain maximum power then most of you would try to tell me that this know reflect should be RL=0 and there should not be any complex power to this, because that G is already equal to 0.

But clearly RL=0 is not the correct answer simply because connecting RL=0 short circuits the load terminals and you might get maximum current, but the voltage across the short circuit will be equal to 0, and therefore no power will be dissipated. So clearly this is not the condition, so this is where it is important to realize what is the maximum power transfer concept is telling you. Maximum power concept when ZG conjugate is equal to ZL applies only when ZG is not equal to 0 and usually fixed.

So for a fixed source impedance the load impedance must be conjugate of the source impedance in order to give you maximum power across the load okay. So as if I am saying by some miracle if the internal impedance were to crop to 0 or internal impedance is equal to 0. In that case no matter what load you connect you will always end up having the power dissipated right. And so that will actually give you the maximum power that can be extracted out of the source. Of course this source is an ideal source here simply because ZG is equal to 0, this is more of a practical source okay.

And when you look at the power that is dissipated in the ideal case, so here when ZG=0 the ideal load that I would like to connect would be any value of the load RL. For now if I just take the same value of RL in both cases okay, I can do so, because load is under my control I am varying the load, I can take whatever value of RL I want. And if I fix the value of RL to be the same for the imperfect of the practical case, and the same for the ideal case I do obtain a certain power.

What would be the power that I obtain in the ideal case? I will obtain $VD^2/2RL$ the factor is ½ because I am assuming VG to be the peak amplitude and as you know the power would actually be given in terms of the RMS value. So for me to do the RMS value of the voltage which is given by VG/ √2, so RMS voltage square divided by RL will give you the power. So this is what I would call as the power that is available. Then you actually look at the practical scenario when ZG is not equal to 0, you can show that the power that is delivered which could be the maximum power that would be delivered under the condition that the source impedance is 0 is only $VG^2/4RL$ okay.

So you can see that when you have this scenario you are only able to get about 50% of the efficiency that you would have obtained okay. So you are able to get only 50% efficiency simply because ZG is not equal to 0 okay. Because of the fact what is important to take away from the last 10 minutes is that, when I have the source impedance which is nonzero and fixed of a practical voltage source them maximum power can be extracted or maximum power can be delivered to the load provided the load impedance will be the complex conjugate of the source impedance.

But this theorem is very nice to talk about and do all this simple calculations. It seems tom address an important topic, that important topic is in practice you have the practical source okay, with some internal impedance and you have a load which is usually not of your choice also. For example, if I buy a Samsung television or some other television the corresponding load that the electronic circuit sees when I connect the cable would be actually fixed right. I can change the source of the source have its own internal impedance and even in the more older cases where you have an antenna you could buy one antenna and replace one antenna from another antenna.

And in that process the loads could not be exactly the same, the source impedance could not be exactly the same, so this minor mismatches would actually mean that you are most of the times not delivering maximum power to the load okay. So something has to be done in order to maximize the power that is delivered. This problem becomes even more acute when you are designing your own amplifier especially as high frequencies.

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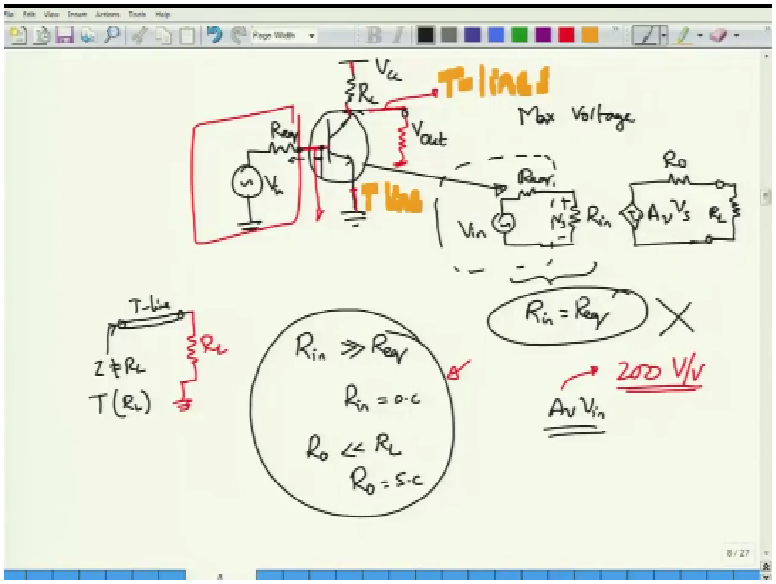

The problem at high frequencies is that in a low frequency condition you might write a simple BJT transistor first not the BJT amplifier you will have some base resistance and some voltage which is what you are amplifying. And then there will be some collector resistance across which you are taking the output right. So this is the VCC for this one and this is the equivalent base resistance that I am writing okay.

So for a single BJT amplifier like this you do not really worry about impedance mismatching fat what you want is in order to maximize the voltage that is given to the circuit or delivered to the circuit at the output, you do not even aim for a maximum power you aim for maximum voltage right. So if I replace this transistor with its equivalent circuit with the little bit of an idealization then I know that the condition that I need to have in order to maximize if I think of this simple amplifier then I know that the input impedance of the amplifier to which I am now connecting the equivalent source and the voltage to which I am amplifying.

So this is my input voltage, we can think of this as a practical voltage that we are connecting right. And then there will be some voltage gain let us call that voltage gain as some AV times nVS where VS would be the voltage developed across rpm, and then you will have some output impedance of the amplifier at R0, wit this you will connect the load RL right. And your goal is not to actually maximize the power transfer, because if that was your condition you would have tried making Rin=R equivalent.

But in practice you do not do this, what is in fact required is that Rin must be must, must larger than R equivalent ideally Rin must be open circuited. So when Rin is open circuited so we have Vs=Vin and when R0=0 so in the ideal case R0 must be very small compared to the load that you are connecting. So in ideal case of R0 equal to short circuit or equal to 0, you will see that the voltage across the load that you connect will be maximum and it could be something like AV times Vin, where AV is the voltage gain right.

So you have deliberately chosen to ignore maximum power transfer theorem and actually make a mismatch your efficiency might improve okay, but your power would not be the same as that, when I say efficiency I mean the voltage efficiency looking on to power efficiency. So what is going I mean why is it that low frequency people do not talk of impedance matching whereas high frequency and RF circuit are always talking of impedance matching okay.

The reason is in the low frequency phenomena, in a low frequency case this lead which you have connected to which you are going to connect the load resistance RL this lead can be considered to do the wire okay. Similarly, the lead that goes from the equivalent resistance or the voltage source that you have considered all the way to the base of the transistor this can also be considered to be a wire.

The ground to this actual chaffy ground or whatever the analog ground that you are considering this can also be considered as a wire which connects the emitted terminal of the transistor all the way to the ground. So this could also be a wire, this could also be a wire everything can be considered to be a wire, and when it is a wire it is a low frequency wire which means it is not a transmission line, so if what is the part of having the non transmission line wire. If the wire is just a wire please remember the voltage at one point of the wire will be the same as the other point of the wire okay.

So because of this case, because of the large voltage case that are involved typical values of AB will be some stage would be about 200volt or more than that. So you are really bothered about the loss of power that actually comes because of the finite length of the wires. So there is an issue of power, but you are not really thinking of maximizing the power, because your voltage gains of so high that you can afford to lose some power okay. if you get full power calculations you will see that under this ideal conditions you will not actually achieve maximum power.

But that is alright, because your goal is to go only to the maximum voltage on efficiency for the voltage not really for the power. But unfortunately when you go to high frequencies right all these lines which I drew and then happily call them as wires or no longer wires they are all transmission lines okay, you see this thick line that I am using okay to indicate that these are all transmission lines okay.

So these transmission lines will, what is the property of the transmission line, you might think that you have connected a certain load here okay or some RL value, but unfortunately at what you have connected to the connected terminals not really wire, but the transmission line okay. So what happens here is that the impedance seen looking at this point will not be equal to RL, but it could be some transformation right. So some RL that has been transformed because of the transmission line effect.

So it is same at every point, so whatever the impedance that you see here will not be the same as the impedance that you expect to see at this point. So the impedance seen after a transmission line effect is the transformed impedance and when there is a transformed impedance and if you do not control you do not know what is the impedance is, then you will essentially end up having reflections at every point okay.

What will the reflections do, they will create standing waves and what will the effect of standing waves they will actually not allow maximum power to be transferred. So there is some power that this reflected voltages hold up or the sum of the instant effect of the standing has hold up which prevents maximum power to be delivered. And if RF frequency the power is a very precious resource you want maximum power to be transferred if you do not transfer maximum power or do not transfer maximum energy then you are losing out that energy in the form of a standing wave.

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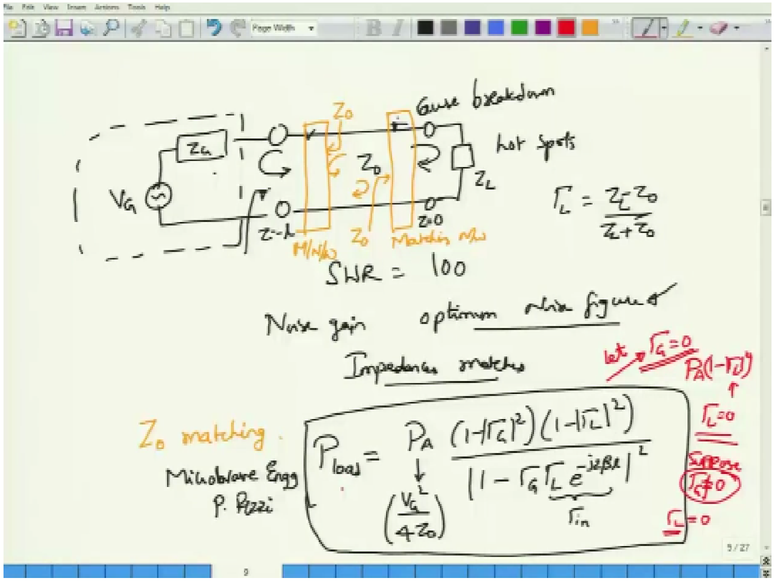

And the standing waves can get pretty nasty, you have a transmission line okay and then your SWR let us say it happens to be about 100, what it means is that the wire of the transmission line is subjected to about a voltage which is 100 times larger than the minimum voltage okay. If that happens the material of which the transmission line is made out of and the structure that the transmission line exist can actually cause break down or this particular large voltages can cause breakdown or voltage hotspots right.

So there will be voltage hotspot at which the voltage is maximum then there will be voltage minimum, voltage minimum is reasonably okay, I mean they are okay with the voltage minima, but this voltage maxima is pretty bad for us these are all the hot spots that I have okay. Moreover when we have a transmission line effect in the form of a connection between the node and the source okay, then if you do not match these two ends of the circuit, so you have a generator voltage using the internal impedance of ZG and the transmission line can be modeled as a line with the characteristic impedance that 0 let us consider an ideal loseless transmission line.

And then you have a load ZL here which is connected and if you do not match this or if you do not perform impedance matching what happens is that you would have created certain voltage reflections okay, simply because ZL will not be equal to Z0 and you remember that ΓL is given by ZL-ZL/ZL+Z0 right so you would have created a reflection here and when this voltage comes back we have not really talked about it, but you can easily estimate by switching this VG on to this side and realize that if ZG is not equal to or match to Z0 or equal to Z0 there will be one more reflection.

So in fact this multiple reflections in the study state what is causing the standing waves and these standing waves are simply we should not have them because they create voltage hotspots, because they do not allow maximum power to be transferred. And more super even when you are designing a high frequency transistor amplifier so high frequency amplifiers there is another concept called as noise gain okay or optimum noise figure.

So optimum noise figure is the maximum that you can no optimum noise figure that you can obtain for a given amplifier and you will obtain this optimum noise figure only when your impedances are matched okay. So only when you are able to transfer maximum power by implementing an impedance matching network then your noise immunity increases because your noise figure will actually be optimized under this particular condition.

So what exactly this impedance matching well if I go back to this transmission line circuit that we have talked about okay, we can very well see that the impedance here on the transmission line is Z0 which is not equal to the impedance of the loads in ZL. So in that case what we do is you put up an extra circuit okay, which is usually composed of loseless materials and this network is called as matching network okay.

The objective of the matching network is that if you look from the matching network point of view from left to right in this plane this impedance seem looking into this matching network will be equal to Z0 okay. So what is the importance of this when Z0 encounters Z0 in the transmission line encounters an impedance of Z0 clearly there would not be any reflections. Similarly you put up one more matching network at this stage and look at the impedance in here.

Although the impedance to the left is ZG after putting up a matching network okay your impedance seems to be equal to Z0 again when the waves reach the source end of the generator N there will not be any reflection. So you are actually eliminated reflections by performing what is called as Z0 matching or Z0 matching. If you go and the little bit of a mathematics you will also be able to find an expression for the available power, I will highly recommend the excellent book called microwave, engineering by Peter Rezzi okay.

Then I believe it in the chapter 2 or chapter 3 he derives this equation for the maximum power and this is a simple derivation you can do it in many ways. So this I like, because it deriving it in a very simple manner. And he actually shows that not the available power sorry, he shows that the power that is delivered to the load is given by the power that is available. So this is the maximum power that could be available and this is given by $VG^2/4Z0$.

Again this maximum power that is available is in the practical scenario okay. So you are connecting the transmission line and then you have a ZG which is the internal impedance of the source and whatever that power that you obtain while it comes to the load side on the lossless transmission line will be reduced. What are these factors which reduce, there is a mismatch at the generator and the transmission line, then there is a mismatch at the load and the transmission line.

So 1 minus magnitude ΓL^2 is the fraction of the amount that is actually transmitted, because ΓL magnitude square times the incident power is actually reflected back. Similarly, 1 minus magnitude of G^2 is the one that is actually transmitted for other transmitted the remaining part is actually reflected and goes into a standing wave there. And in addition to these two you will also see that because of this multiple reflection and propagation back and forth there will be an additional case of 1 minus magnitude G and magnitude for other $\Gamma L\ e^-$ and the minus sign simply because I have taken Z=0 here and therefore the source will get Z=-L this will be –J2βL, this magnitude square.

So we can think of this $\Gamma Le^{-j2\beta L}$ as the input reflection coefficient something that you would see at this point okay. So you will see that the total power that is delivered to the load is reduced from the available power by these factors. Now observe a very important thing, suppose ΓG=0. So in this expression suppose ΓG=0 what will happen to the load that is having load power that is delivered to the load, this would be available power times 1 minus magnitude ΓL^2 so this is the average power that is delivered to the load which is only less by a factor of ΓL.

And we can make this equal to PA provided you make ΓL=0 so when you make ΓG=0 this is the case of what we call as generator matched, then if you match the load automatically the power that you obtain will be maximized okay. Now suppose you are unable to do the generator match okay, so ΓG is not equal to 0, so suppose ΓG is not equal to 0 in which case no matter what you do with ΓL that is even when you make ΓL=0 no reflections on the load side you still end up having the load voltage of the load power to be less than the maximum available power.

So it is important not only to match something at the load side, but it is also equally important to match at the generator side in order to maximize the power that is delivered to the load. And there are various methods of doing this.

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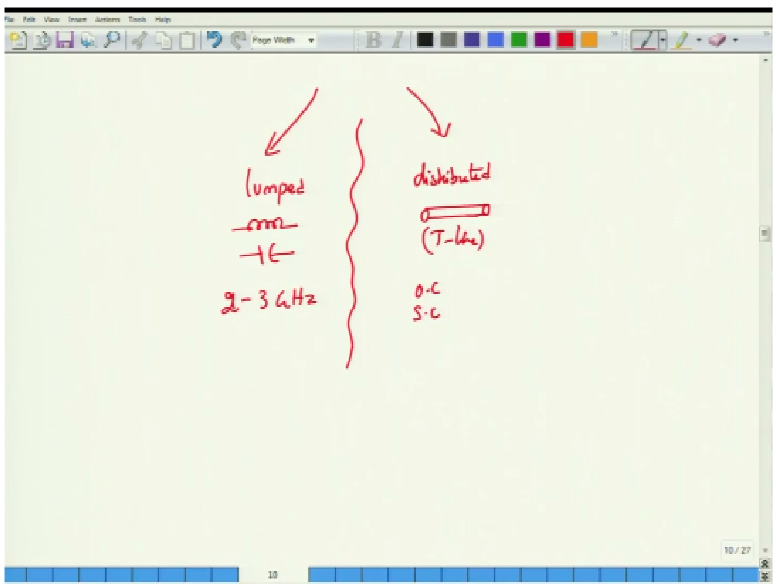

Most matching networks fall into two major criteria one called as the lumped matching networks and the other called as the distributed matching networks. The lumped matching networks utilize the lossless L and C and connect them in appropriate fashion in order to form a matching network at very, very low frequency one can use the transformer for the distributed circuits or for distributed matching networks you actually use the transmission line itself okay.

Various combinations of transmission line with transmission lines being open or closed all this combinations are used okay. Usually a lumped elements are okay until up to say 2 to 3 Giga hertz beyond this the dimensions that are required to realize an inductor or a capacitor becomes so prohibitive that you simply go to the distributed or the transmission line approach and here you can either use an open circuited transmission line or a short circuited transmission line, you can have a discrete scenario in which you actually have an open circuited stuff propagating through the varied along the transmission line or it could be a printed circuit for in which you create a micro strip line.

The micro strip line will act like a transmission line that micro strip line could be open circuited or close circuit I mean short circuited by bending a wire and connecting it to a ground plane. So these are the major types of matching networks, when you design a matching network what are the qualities that you are looking for your matching network should be simple, simple in the sense that the network should be easily tunable and you should not be too complicated to implement.

So you can have very un physical effects if you have un physical values that cannot be implemented and your matching network would be pretty bad okay. the matching network could be the narrow band or broad band, if you are designing filters you try a narrow band matching network, if you are designing amplifiers broad band amplifiers then you want a broad band amplifiers. You can also need or you also want narrow band that multi band simultaneously.

So you want one matching network at 2.4 the same matching network should also match at 4.8 at another frequency and so on okay. So these are multi band networks. And when you design this matching networks it is also important that you minimize the laws by the matching network itself. Why is the loss present, because no matter what ideal components you take, but in practice those components has some amount of loss and your goal as a matching network design that would be 2 minimize space.

Finally matching network should not cost much, they should be relatively cheaper or in expensive so that they do not become the major cost component in your microwave circuit. We will see how to perform this matching network or how to design these matching networks in the next module until then, thank you very much.

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