Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 13
Further applications of Smith chart: Part 1

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Hello and welcome to the NPTEL MOOC on applied electromagnetic for engineers. In this module we will begin with a few applications of Smith chart, the use of Smith chart to solve transmission line problems. We will continue this in the next few module and what I would suggest as I suggested in the previous module is that you have a Smith chart access, you know you either have a Smith chart printed or downloaded copy with you and while I you know catch the points and solve the problem you try to replicate the same thing when you are actually you know when you are viewing their video lectures.

So that way you will be able to understand how to use nature to solve the problems that we are going to discuss. So we begin with you know basic use of nature we have already discussed, so what we will do now is to see what type of problems that Smith chart allows us to solve, so one of the typical.

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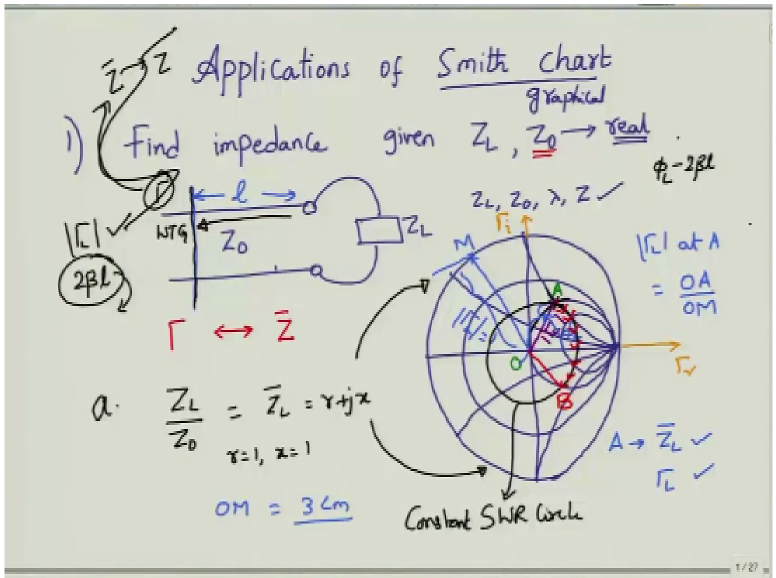

Application of Smith chart you know very basic application of Smith chart if you will think is to find the impedance, right of a node that is connected to the transmission line but at the position away from the load. So what we are trying to find out is the line impedance, the impedance measured on the transmission line at any point on the transmission line, given that node is that ZL okay. So remember how to solve this problem.

You can solve this problem without using Smith chart because given the value of Z_L and the corresponding Z_0 of the transmission line, if these 2 quantities are given to you at any point on the transmission line you can use this impedance transforming formula, that we discuss in 1 of the previous modules and find out what would be the impedance at that particular point on the transmission line. So the line impedance can be easily found out by transforming the impedance that M through the distance at which you are seeking the impedance.

So if I were to sketch the problem for us the problem that we are trying to address is that I have a transmission line, okay and to this transmission line we have connected a load and we give you the value of the loads at L. This is of course at a particular frequency that we are considering and the transmission line itself has the characteristic impedance of Z_0 in all of the applications that we are going to consider, we will assume that this Z_0 to be a real quantity okay, so which means that that characteristic impedance of the transmission line is purely real the line is completely loss less okay.

At any point on the transmission line I can find out what would be the impedance, right so how do I find out what is the impedance? We will call this as input impedance although this is not really input here it is just a line impedance that we are calling, so this impedance seen at any point that on the transmission line can be obtained by some complicated formula it is reasonably complicated that I do not normally remember that 1 other unless start deriving that.

That I know that this complicated formula expects us to provide with Z_L which is the load impedance at which the transmission line is terminated. The characteristic impedance Z_0 as well as the point at which you are trying to find the impedance okay, if you know all this and of course the wavelength λ remember that all the transmission line problems on length are normalized with respect to what they have linked right.

So given these four parameters you will be able to find out the impedance okay. However if instead of asking you the impedance at this point if I ask you for the impedance at this other point you will have to use this complicated formula which will have sine cos or tan equivalently and recalculate it. So this is where Smith chart will help you it will provide you with a graphical way of solving the same problem.

You do not have to use a calculator, all that you need to do is to have a Smith chart with you a pencil or a pen depending on what you are using, a scale a protractor a compass these are all the things that geometrical things that are needed in order to solve this problem geometrically. So that is why we call Smith chart as a graphical aid okay, it provides you with a graphical or with a geometrical visualization of the transmission line problems.

So how does it help in this scenario let me raise this entire complicated formula thing and tell you how easy it is to use the chart in order to find input impedance or the line impedance at any point okay. Of course you still need to know what is Z_L you need to know what is that Z_0 λ you need to know and the distance at which you are trying to find the impedance you need to know. So all these quantities are needed but on the Smith chart we have a very interesting way of finding this very interesting in a very simple they are finding the impedance.

So first we start with the Smith chart itself okay and I will draw only a few circles and a few representative arcs, so this is just not to clutter up the entire Smith chart but in your possession you will have a Smith chart with you, so you can be more accurate I will be slightly inaccurate my goal is to try and show you the methods okay, not to give you the exact value. That exact value can be obtained once you have the Smith chart with you okay.

So this is the Smith chart please remember what is the chart Smith chart? Smith at any point is just a correspondence between the reflection coefficient γ and the normalized impedance $\bar{Z}$, right so it is the reflection coefficient and the normalized impedance $\bar{Z}$ which is what is actually plotted on that Smith chart right. I have already talked to you about the constant are circles constant X arc and any point on the transmission I mean on the Smith chart will have something simultaneously represent γ.

Which is the reflection coefficient as well as the normalized impedance and since it is normalized impedance that we need to talk about, we need to normalize every impedance that we come across in the transmission line problems so what is the normalization value or to which value should be normal it is to the characteristic impedance of the transmission line to which we need to normalize. So the first step if you were to cut off just a okay would be to normalize the load impedance.

Which is given with Z_0 and obtain the normalized load impedance $\bar{Z}_L$ okay, on the Smith chart I can easily locate where that $Z\bar{n}$ is because the den bar will be in some r + jx way this what is inductive reactance that the load possesses. Then this x will be positive and all those values will be in the upper hemisphere if x is negative. It indicates capacitive reactance in which case these r + jx values will be found in the lower hemisphere which we have already seen in the earlier module.

So how do I locate this $Z_{\bar{L}}$ well I need to find out the corresponding r and corresponding x? For this application let me just consider r = 1, x = 1 okay, so this clearly lies in the above or the upper hemisphere part, so I look at r=1 which is this circle right and then this is x = 1 arc the point that I am looking at happens to be here let me try and use a different color to this point. So this is the point at which we have the normalized load impedance.

But please remember whatever that you find out the load impedance at this same point you can also specify the reflection coefficient how? Because the axis that you have here is actually the real part of the reflection coefficient γ r and this axis y is the imaginary part of the reflection coefficient, so clearly whatever the point that we have made here let us call that point as point a okay.

At this point a if I were to you know make a line from the center let us call the center as o okay center of the chart as o and if I now draw a line o connecting o to a the length of this line will tell me the magnitude of the reflection coefficient at that particular point. Since we are just starting with the load here, so this is going to give me magnitude of the reflection coefficient okay, we also have seen that the largest circle on the Smith chart will have the maximum value of γl correct and that maximum value will be =1.

So for example if I am located on the outer periphery of the chart and then draw this line, this line will have the maximum value of γl which we will have to take it as 1. Of course if I were to actually bring this chart on a piece of paper this will be some number that is 0 and let us call this as some M okay. O M might be 3cm for example it may not be I am just giving you an example of 3cm okay.

In which case the magnitude γl at a can be found out by the ratio Oa / Om right, this is the narrowest magnitude that you are able to find which will have a γl of 1therefore this line Oa must be proportionately scaled up by effective for M value so normalize it and then what you find the ratio of Oa – Om will be magnitude γl okay. But this is just a knighthood of γ you can also find out the angle from the real axis and find what the corresponding value of π L. is

So effectively point has represented the normalized load impedance, it also has represented or given you load reflection coefficient γ L. These values are known at the load but let us say I want to find what would be the impedance at this particular plane, which happens to be some distance M away from the load okay. So at some distance and away from the load now remember what is the reflection coefficient magnitude behavior?

The magnitude of the reflection coefficient as you move away from the load towards the generator or towards the source will remain the same. Magnitude of γL will remind the same as you move from mode towards the generator, so as you move towards the generator the magnitude of γ L will remain same. But if you have travelled a certain distance M then the phase of this reflection coefficient angle will have changed by a value of 2 β x L and of course this has to happen in the clockwise direction as we have seen.

So there is 1 possible way of finding the new value of the reflection coefficient at this particular plane you can find out the new value of reflection coefficient which is complex reflection coefficient, let us call this as just γ not γ L because you have moved away from the load, from this γ you can find out the impedance. Normalized impedance from the normalized impedance you can find out what is the input or the impedance value impedance at that particular point.

So you can do this mathematically right once you have d1 the gra π cal ting or since every impedance that matters is already present in the split charge you do not have to do any of these things okay, after you have located the load okay move a distance of 2 β and or move an angle of 2 β and clockwise direction. For this you can either put your protractor and then measure the angle π L and then from π L - 2β L and then locate on this circle locate the new value of the reflection coefficient that point find out what would be the angle right. λ

So if I want to follow that method first I have to draw a circle right this circle on which γM will be constant will be called as constant γ L circle, that I also know that constant magnitude of γL will also help me calculate the value of the standing wave ratio and standing ratio would also be constant okay, so this circle therefore is called as constant VSWR circle. Here the circle has a certain radius and we will come back to finding the fact xwr value later on but this is called as the constant VSWR circle.

So on this circle you need to move you need to move clockwise, you see I moved here all the way so maybe I wanted to stop at this point because that would have given me to detail movement. I obtained on this circle a new point, so I start from 0 then Center this I mean fill to 0 as a center and this new point which we will call as point B I draw the line and at this point I read out what is the value of r and x.

In this case it turns out to be I mean very close to the value of r but you are looking at is about 1 and then x will be about some value, which would be less than 1 but please do not go by the exact number that I am writing here because that is not the correct value that I have obtained, this is just a procedure that I wanted to tell you. Let me do one thing let me first erase all this and tell you what exactly the procedure that we have followed.

So this was cluttering the entire you know discussion, so we do not need to clutter ourselves any more so let me solve the problem for you in the simpler way or rather give you the procedure remembers. What we wanted to find out was that we wanted to find out what is the impedance at a certain distance L.

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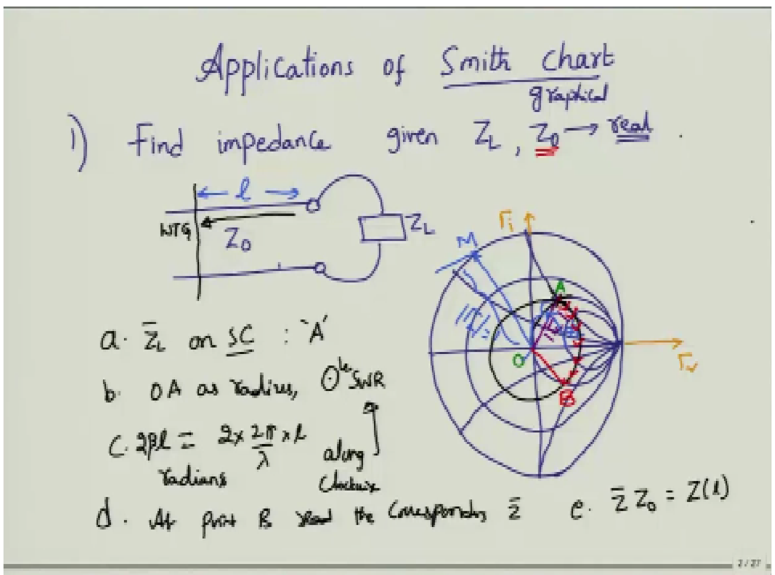

So the method that we started was start with the $Z_{\bar{L}}$ okay on the Smith chart okay, so I will use the word SC to denote Smith chart okay, so on the chart you mark this one this happens to be the point A okay on the chart this happens to be the point A, with OA as radius draw a constant SWR circle which is this black circle, so you can see this black circle that I have drawn that corresponds to the constant SWR circle.

Then determine how much you have to move you have to do and I am the left to β how do you find out data is already given to you, so 2 x 2 π by λ x L, please note that this is individuals if you want it will convert this Radian value in degrees as well so in which case you will be moving 2 x 360^0 x L/ λ. So it is just the way of whether you measure things in radians or degrees that matters okay, so you are moving 2 x 2π / λ into L radians along clockwise so what is the move along the constant SWR air circle clockwise direction okay to locate 2 new point.

So at point B read the corresponding complex impedance this will give you only the normalized complex impedance multiply this is the final step, I will write it here multiplies $\bar{Z}$ obtain the normalized impedance at the distance L away from the load, so this is bar by leg not a very simple 5-step procedure native okay. We can even simplify this procedure further I will tell you how to do that? Okay so let us solve the same problem that we solved earlier.

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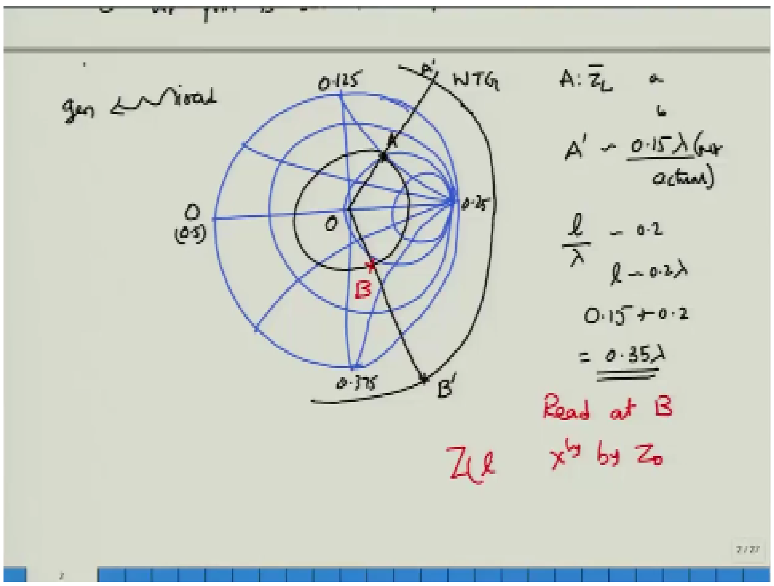

So we again begin with a simple Smith chart not trying to clutter your Smith chart for you, so I will draw just a few arcs as I did in the previous page and I will draw a few important circle there should be all okay. The first step is to again locate R = 1 and J = 1 in this case this happens to be the point here which is the point at his is the lower point P a is the normalized lower value with Center O and O a as the radius I draw the constant SWR circle, so I have completed stepA and completed step B.

Now what I do is I extend this line A outside the chart okay, if you examine your Smith chart closely or follow the last module you will see that there are 2 scales printed on the Smith chart, 1 scale is called as WTG scale it says wavelengths towards generator okay this is the way in which humans are they from the node and towards the generator so on that WTG scale not the point 8 prime okay.

So note down the value of a prime, so let us say a prime happens to be something like 0.15λ okay, this is not the actual value so please again I am cautioning you this is not the actual value you know then whatever is the WTG scale value, by the way this value will simply be used as a reference it has no meaning whatsoever that it is required okay. Next what you need to do is you know what is the value of L that is at what distance are those from the load you are trying to find out what distance away from the load you are trying to find out the impedance.

So you divide that L /λ so let us say after you have divided L/ λ it turns out to be 0.2 okay or in terms of L itself is 0.2 x λ okay, what you need to do is that from the WTG scale that you have noted at point a prime which happen to be 0.15 λ add 0.15 +0.2 okay this will give you 0.35λ, so on the WTG school you keep moving okay and you learn that somewhere over here this is 0.25 if you look at the picture this would be like this so this is point0.125 this is 0.25 this would be 0.375 and soon so 0.35 should come somewhere over here.

So let us cut this on this WTG scale I have moved a 0.2 λ which is I have actually landed on 0.35 λ value, so this scale is continuously marked in the increasing value of 0 okay. So clearly whatever is 0.25 will also be the same as 0.5 here sorry whatever is the 0 that would be the same as 0.5 okay and point by 0.5 corresponds to λ by 2 which is 1 complete rotation, so you go from a exchange the line to a prime find out the value that value is just for the reference that we have found I mean used so from back to that value you add 0.2 which is the distance that you have to actually move normalize with respect to λ.

So you need to add 0.2 λ to come up to 0.35 λ on the WTG scale once you have obtained that point call that as B prime, now draw as straight line from pole all the way to B prime okay now you see where you are intersecting the SWR circle you are intersecting the rest of your circle here call this point as point B and read the coordinates at B multiply that coordinates by the 0 in order to obtain the impedance write a distance of L away from this.

So I hope you really appreciate the simplicity all they have taken about 20 minutes to explain the concept I did this so that you really understand what is going on with this simple application, once you understand this application other applications are quite easy to follow. So let us consider a second application of Smith chart okay.

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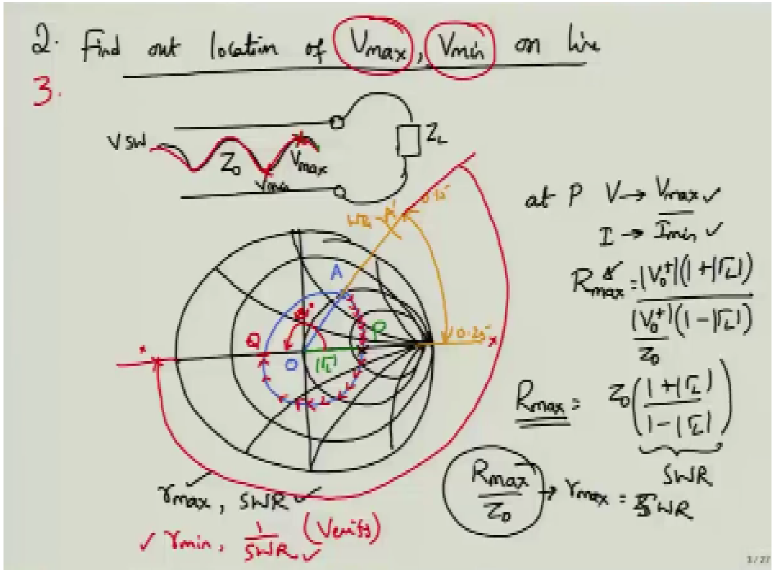

In this application we are trying to find out the value of the location of the Vmax and the location of Vmin on the line by Vmax and Vmin on the transmission line why should we have Vmax and minima whenever you have terminated a transmission line which is this line with a load which is different than the characteristic impedance of the transmission line, you will see that there will be a Vmax minima pattern, so this will be the standing wave voltage standing wave pattern this is not SWR but this is just the voltage standing wave pattern and you will see a series of Vmax, so this is V Max and this is we mean okay, so depending on the load you will have the locations of V Max and you need to be different okay.

How do I find out when we will hit the Vmax and the voltage minimum on the line? Well it is kind of very simple right, let us start again by the Smith chart okay, so this is the Smith chart right and then assume that we have been given a certain load impedance okay, so we have been given a certain load impedance to work with. So this is the Smith chart as before if Z_L is given find out what is the $Z_{\bar{L}}$ clearly the $Z_{\bar{L}}/Z_0$ we will again go to some arbitrary locations, so let us say this is the value of the $Z_{\bar{L}}$ okay.

This is 0 this is A and I need to draw a constant SWR circle, so this is usually the first step okay so you draw the constant SWR circle like this. Now imagine that you keep moving along this constant SWR circle shown by this arrow towards the generator right, so you keep moving and you reach this point okay this red point on the horizontal axis you reach that point. If I now ask you what is the value of γ L? Here it will tell me the value very easily because the value of γ L magnitude is simply given by this length right.

So I have reached a point that is called the surf point 2 okay so OP /OM please remember what is OM is from the previous application right, so OP / OM will give me the magnitude of γ L but at this point what is the phase angle of γ or rather I should not call this as γ and I should simply call it as but it does not matter it would be γ that the magnitude anyway does not change, so I can call it as mount mod γ or mod γ L does not matter. So you actually want your balance you started off at some point okay at maybe at this point and then as you move towards the node you have now reached this point Vmax.

Why? Because the phase angle here will be =0 all right so if the phase angle is 0 what would that indicate to you? So what is the voltage at that particular point the voltage along the line at the point where π M = 0 is simply given by magnitude, so since you are only looking at the magnitude this is the magnitude of V zero + x 1 + magnitude of γ L correct because this would be 1 + γ L magnitude into e to the power of π L or to the power of J 2 β J + π L and then as you know moved a certain distance Z = Z max at that particular point the total phase this is the total phase which would be the load phase angle + 2 β Z this entire thing =0 right.

Fill this entire thing is equal to zero γ there will be completely real and this would be the maximum value of the voltage that you are going to get and how much distance you had to move here, so this will give you the location of the Vmax how much distance you have to move will give you the location of the Vmax from the load point right. So you moved this particular distance as usual I mean as before you go and find out what is a prime on the WTG scale this is 0.25. Suppose you found out that this was 0.15 then clearly you must have moved 0.02 λ in order to land that at 0.25 rights.

So you have to move 0.02 those four from the load the position of load maxima atom set 0.5 λ this is not the only position on a loss less transmission line you will found the Maxima at 0.05 + λ / 2 right but I mean 0.25 λ + λ / 2 or 0.5 5 λ because every λ / 2 these positions will keep changing, so this is how you find out the position of maxima. Now a very interesting thing has happened over here, you have found out what is the line impedance and you found out what is the corresponding value of this location of the load Maxima, you can use the value of magnitude γ L to also find out what is the maximum value of the voltage?

Provided of course some1 has given this v-0 + to you okay if some1 specifies what is V0 + to you will be able to find out what exactly is the Vmax at. This point I have another interesting bit for you to find okay, let us look at this point right on the horizontal axis starting from the point A you landed at Point P right, on the horizontal axis what would be the normalized impedance value there? So at this point we also have 1 more interesting information in order to find out that information let us look at what is 0.3 representing.

So at Point P we have the voltage to be maximum that we also know that the current there will be minimum correct, so what is the ratio of the voltage maximum to current minimum that would be the maximum value of the impedance and at this point both of both quantities V Max and I mean are real quantities therefore, what you find is the real maximum value of the impedance. So the maximum impedance that you can find out the maximum resistance that you can find on the transmission line will be = Rmax and this fellow will = Vmax by I mean the ratio of these 2 will be that and what is V max V0 + magnitude x 1 + mod γ L and what is I mean is V0 + magnitude but / 0 because this is the current and 1 - magnitude of γ L correct.

So now Rmax = Z x the quantity which is 1 + γ L / 1 - mod γ L this quantity is nothing but standing the ratio and you can now see that if I divide this R max /Z_0 which will give me the normalized impedance R max that would be equal to SWR are directly, so what I find at P are 2 different values 1 I found out what is the normalized maximum impedance or resistance on the transmission line that is our max. I will also find out the corresponding value of SWR.

In fact at this point if you read out whatever the value that is there here it might be about 2.2 or something okay the 2.2 will simultaneously give you the standing wave ratio and also the normalized maximum resistance, if you then multiply this normalized maximum resistance by the characteristic impedance of the transmission line you will be able to find out what is the value of R max. Now there is another thing that you can find out from this right, so let us continue moving along the same circle okay.

As we continue to move along the same circle after a length of λ /4 sorry after a distance of λ / 4 I landed at the point Q. At Point Q the Vmax which was there must be converted to voltage minimum right, so I would converted this one into Vmin and then the voltage is going through the minima the current will be going through the Maxima. So what we find at Point Q are the normalized value of the minimum resistance that you are able to find on the transmission line and 1 by SWR.

I will leave this as simple exercises for you just have to invert this relationship and then find out that this would indeed give you Rmin and 1/ SWR, so we have actually covered 3,2 applications here 1 is we can find out the location of the Vmax and also this particular locations happens to be the Vmin as well, because the phase angle here will be 180^0 right, so with respect to the x-axis the phase angle will be equal to 180^0 and at that particular point γ L or γ becomes - magnitude of γ Land then what you get is a Vmin and whether the location of the Vmin starting from a prime you have moved a certain distance.

Here on the WTG scale right or you already know how much you move to find out v-max add to that λ / 4 and you will be find out finding out what is the location of the Vmin, as again the Vmin keeps repeating every λ/ 2 okay T reiber sinking, so at Point Q you found out the minimum value of the resistance you find out what is wonder SWR okay and you can also and we have also found out what is the minimum value of women.

So you know what is 1-mod γ L you can calculate that 1 and if you know what is V_0 + magnitude you can easily find out what is women okay, in fact knowing the Vmax and the mean values allows you to construct the voltage standing wave pattern, it will allow you to once you know what is Vmax and we mean it will allow you to construct this particular graph okay. So that is one other application in case you are interested. How to find out the standing wave pattern on the transmission line how to sketch that one? So you need to know the location of the maxima location of the minima what is the value of the v-max what is the value of women so all this information can be very easily extracted from the Smith chart thank you very much.

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