Principles of Communication Systems - Part II
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Lecture - 05
Additive White Gaussian Noise (AWGN) Properties, Gaussian Noise, White Noise

Hello, welcome to another module in this massive open online course. So, we are looking at an additive white Gaussian noise channel which we have said is the simplest one of the simplest models that can be use to represent a digital communication system or a digital communication channel or the effect of channel or which can be used to model the effect of the channel in a digital communication system.

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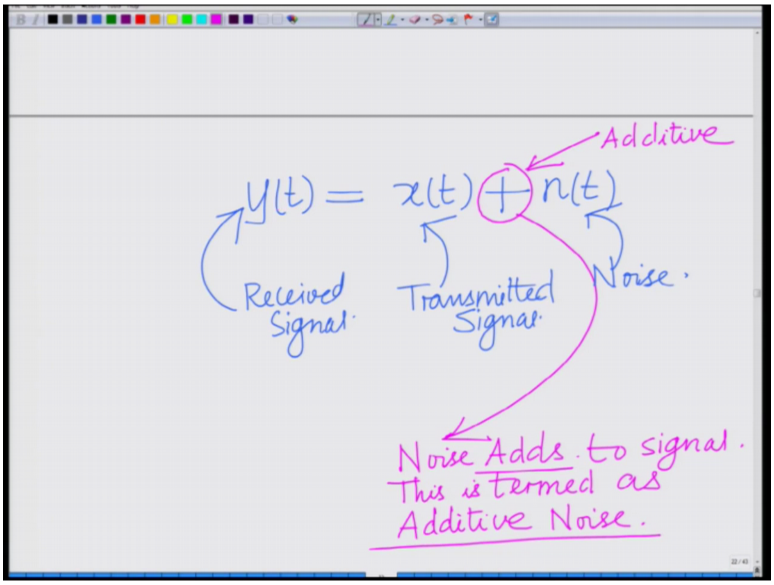

And we have said in that we have looked at a simple system in which y t the received signal y t is simply the transmitted signal x t plus the noise. So, the noise is adding to the transmitted signal x t this is termed as additive noise, alright, we have already seen that this noise which adds to the signal is termed as additive noise.

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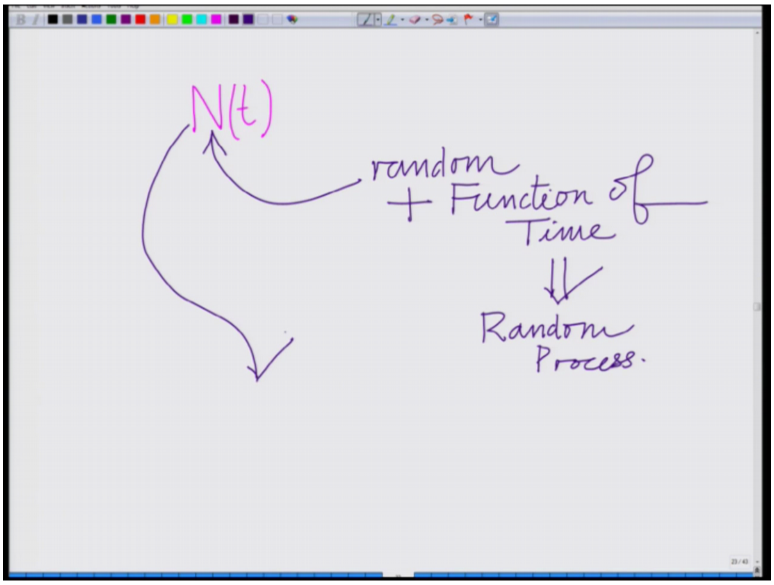

Now, the other important property; the other type of noise which is one of the most popular noise models in a typical communication system is to consider Gaussian noise that is noise N t noise process N t that is noise N t which is random and which is a function of time therefore, it is a random process. So, noise this is random plus it is a function of time, it implies this is a random process, correct.

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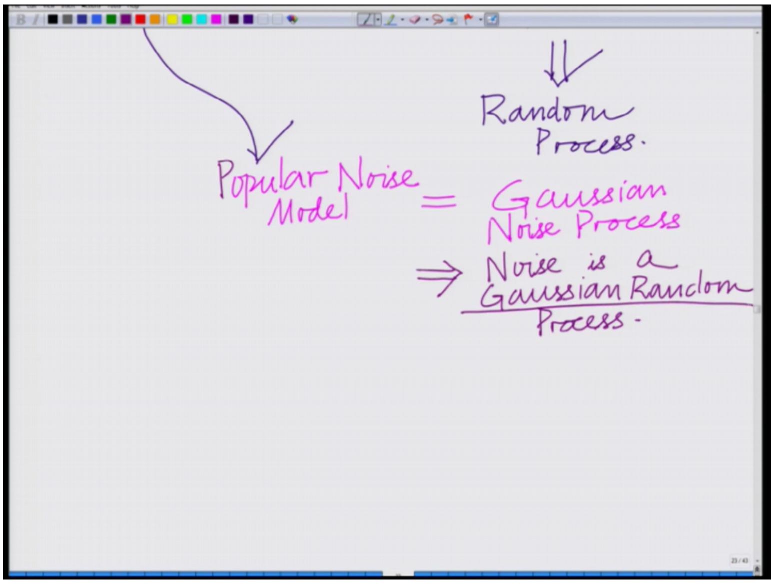

Now, another very popular model for noise is to consider this noise process N t as a Gaussian random process, another very popular model, since most naturally occurring random processes are Gaussian. So, popular noise model is a Gaussian noise or basically a Gaussian noise process which basically means it is a noise is a random process. So, this noise Gaussian noise basically means the noise is a Gaussian random process alright. So, this implies noise is a Gaussian random process. Now, we are going to see the properties of the Gaussian random process. So, this means that your noise is a Gaussian random process.

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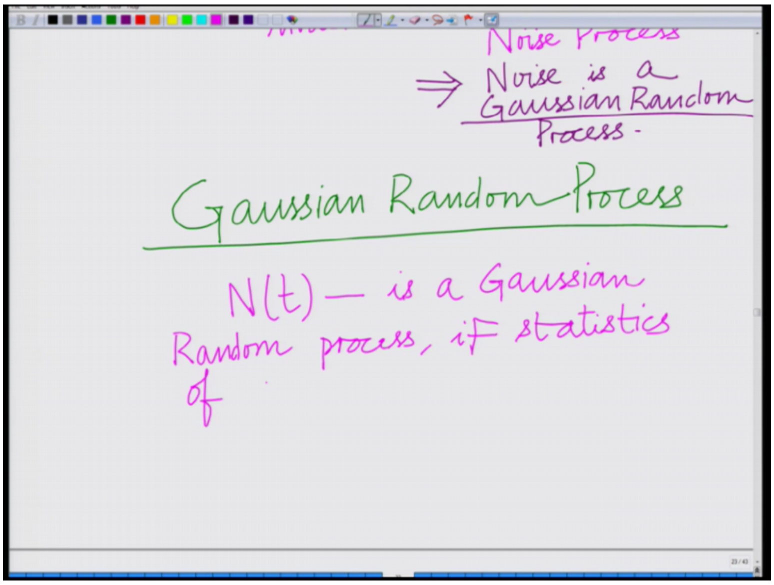

Now, what is the meaning of this Gaussian random process, random process we call the random process is a Gaussian random process. Now, let us characterize a Gaussian random process, a Gaussian let us characterize a Gaussian random process. Now, we call a random process N t is Gaussian random process, if the joint statistics of if the statistics of all orders are jointly Gaussian is Gaussian random process, if statistics and that this is important statistics of all orders not just a single orders statistics of all orders.

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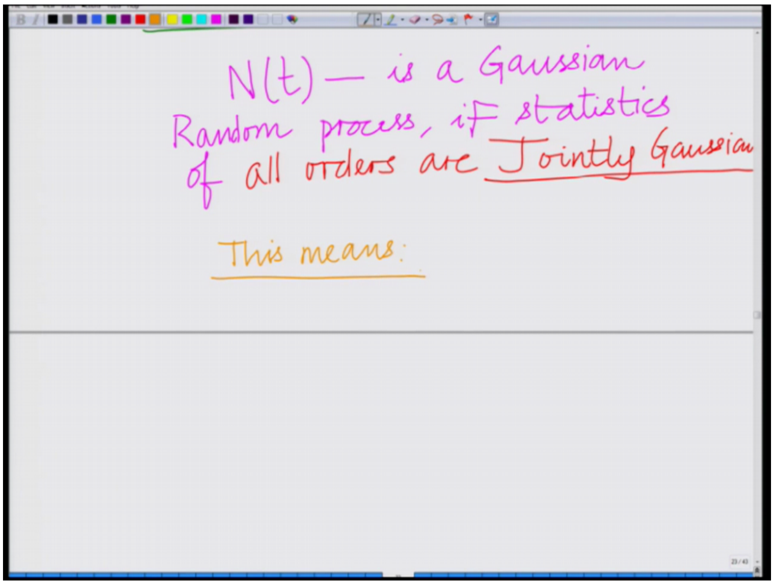

If statistics of all orders are jointly Gaussian, this implies this means this basically means that is if you look at noise samples N t 1.

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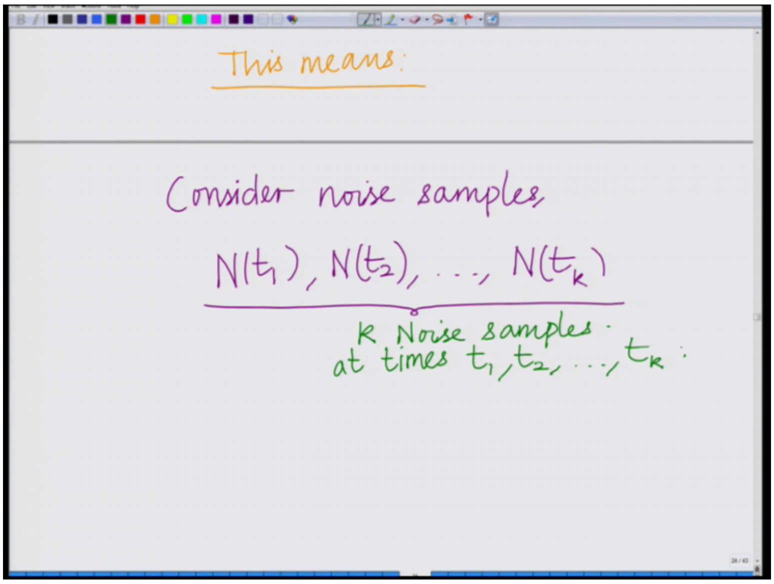

That is that is if you consider noise samples N t 1, N t 2 up to N t k these are basically you can see these are k noise samples. These are k noise samples at times t 1 that is what we are basically doing is we are taking this noise process N t. And we are considering k n noise samples at time instants t 1, t 2 up to t k this noise samples are given as N t 1 and t 2 up to N t k.

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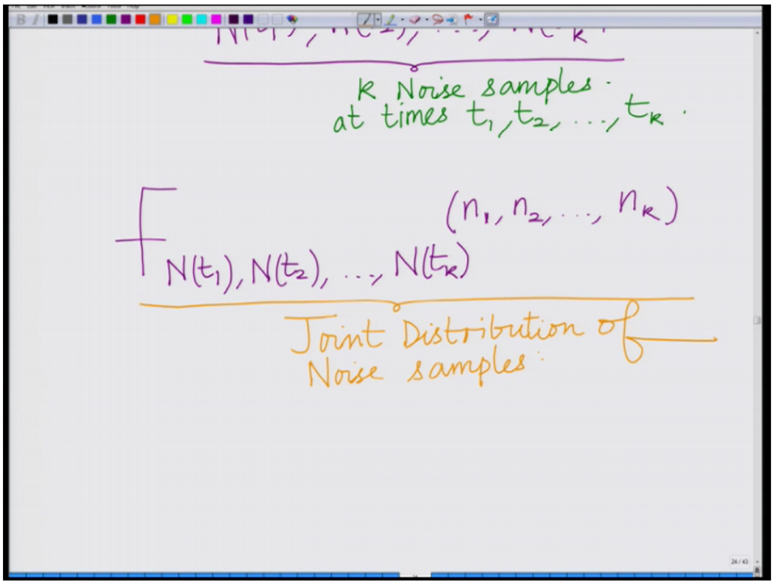

And you look at if you look at the joint distribution of these now k noise samples. If you look at the joint distribution of the k noise samples, let us denote this by F of N t 1 comma N t 2 comma and so on N t k n 1 n 2 up to n k, this is the joint distribution of the noise samples.

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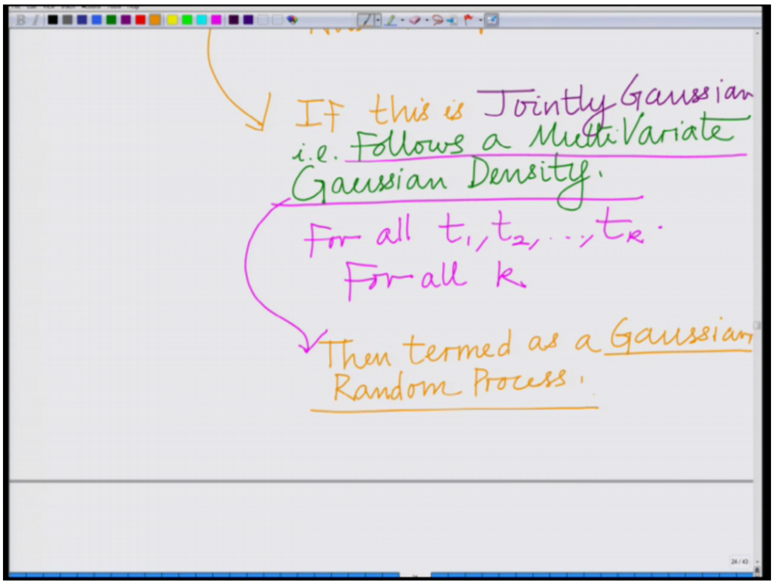

Now, if this joint distribution of noise samples, if this is jointly Gaussian alright. This means it has a multivariate Gaussian density. If this is similar to the scalar Gaussian density, there is a multivariate Gaussian density that is considering multiple random variables that is follows a multivariate follows the multivariate Gaussian density. And this has to be true for not any particular correction follows the multivariate Gaussian density for all choices of t 1, t 2, t k and more importantly and also not more importantly also for all k. That is if you choose any k points if this follows if there are N t 1 and N t 2, N t k for all such noise samples all such combinations of k noise samples and for all values of k.

If this follows a Gaussian density then it is termed as a Gaussian then as Gaussian then it is termed as a Gaussian random process or basically also your noise is a Gaussian. If it is a Gaussian random process noise plus Gaussian random process implies that is noise plus not Gaussian not simply Gaussian. But a Gaussian random process noise plus your Gaussian random process is basically your noise which follows a Gaussian random process is termed as a Gaussian noise.

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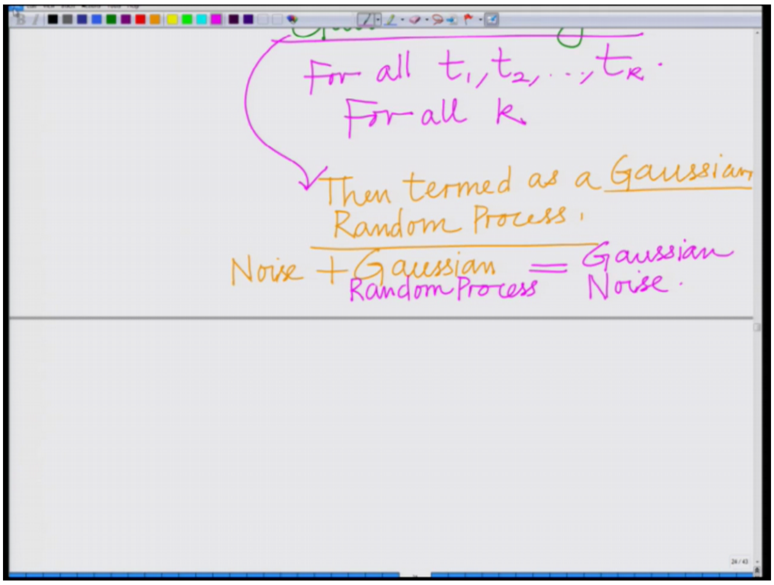

So, as you can see we term noise as Gaussian basically if you the statistics of all orders are jointly Gaussian what that means is if you look at the noise process N t consider k samples at t 1, t 2, t k, N t 1, N t 2, N t k. If the joint density probability density function of N t 1, N t 2, N t k is Gaussian that is it is jointly Gaussian for all such samples that is for all such instance t 1, t 2, t k. And for any possible value of k that is also important, it is not fixed it should not be it is not for a particular value of k, but for any such value of k that is for k equal to 1, k equal to 5, k equal to 10 or a 1000 that is we take a 1000 noise samples and the samples are arbitrarily chosen time instance. The joint probability density function should be a multivariate should be Gaussian that is it should follow a multivariate Gaussian density. And then such a random process is known as a Gaussian random process.

In particular if the noise correct if the noise follows is a Gaussian random process, it is known as Gaussian noise. And this as we have said is one of the most popular models all right because most naturally occurring probability are random variables follow Gaussian distribution or we can also say that the most naturally occurring random processes are Gaussian random process. Therefore, the Gaussian noise all right to model this thermal noise the circuits, the thermal noise in these circuits in this at the receiver, which arises from basically from the thermal noise in the circuits at the receiver is modeled as a Gaussian random process. And this is one of the most popular models popular noise models for a digital communication system. And this is employed almost throughout that is we will look at literature or digital communication a dominant fraction of literature would employ the Gaussian noise process.

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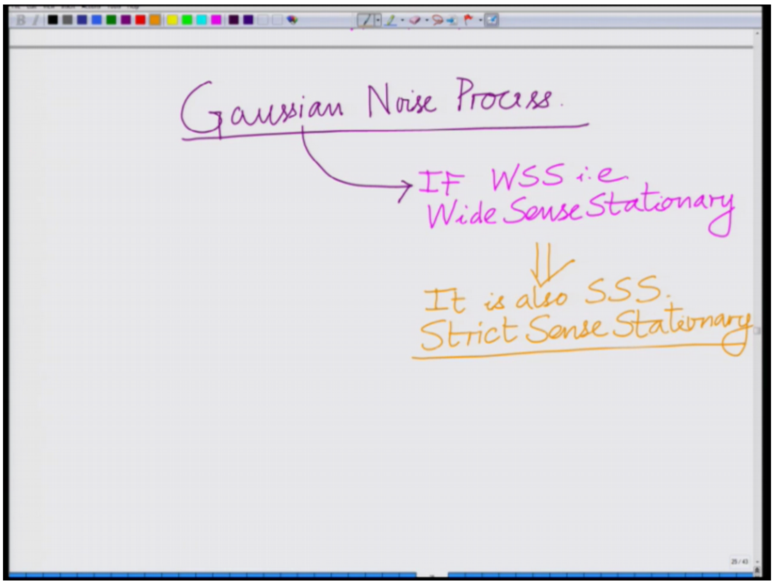

Now, in particular the Gaussian noise process is a very interesting property. If a Gaussian noise process is wide sense stationary, if it is WSS that is if a Gaussian noise process is wide sense stationary then it implies it is also strict sense stationary. Normally this is not true for a random process wide sense stationary random process that is a strict sense stationary process is a much more rigorous condition. So, strict sense stationary processes are all strict sense stationary processes are a wide sense stationary, but the other way round is not true that is all wide sense stationary processes are not strict sense stationary.

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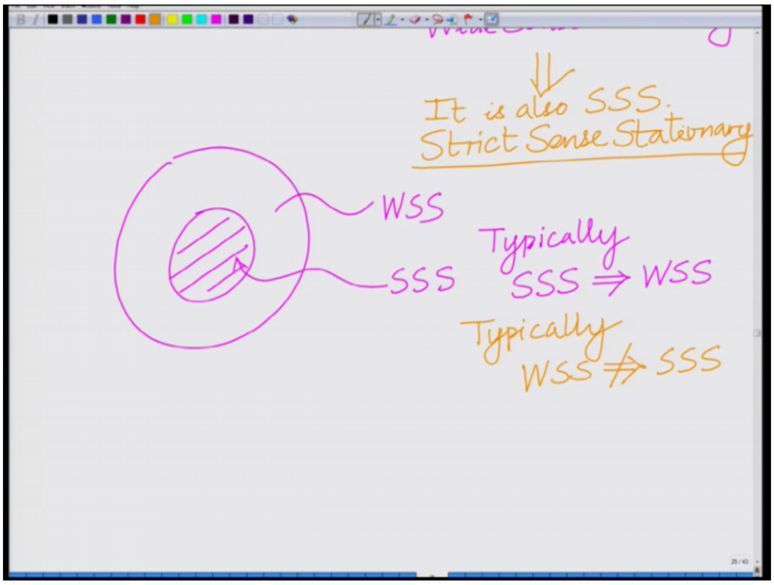

So, strict sense stationary processes are a strict subset of white sense stationary processes. So, if you look at the set of random processes, if we look at the set of random processes then there are several random processes that are wide sense stationary, but only a few of them are strict sense stationary. So, typically if it is strict sense stationary, it implies wide sense stationary. Typically wide sense stationarity does not because wide sense stationarity is a much more relaxed condition. So, typically wide sense stationarity does not imply strict sense stationary.

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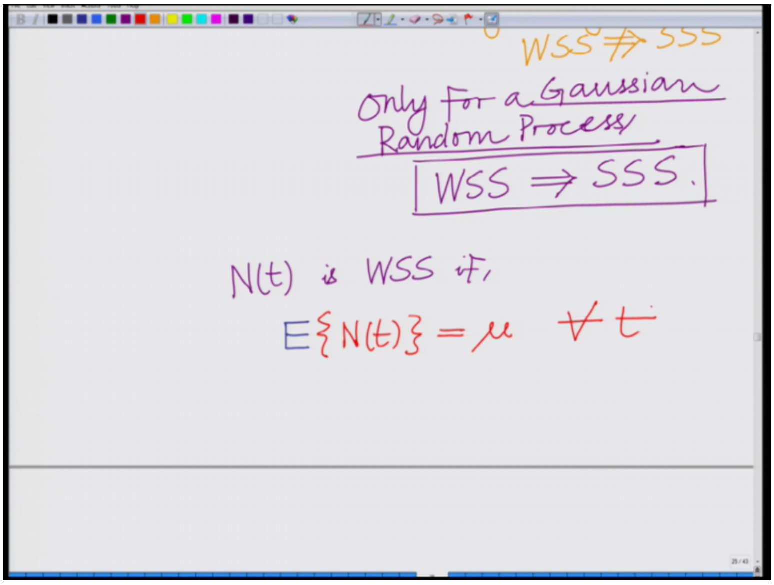

However for a Gaussian random process wide sense stationarity implies that is the interesting for a Gaussian only for a Gaussian random process wide sense stationarity implies. But this is important to realize that only this happens only for a Gaussian random process. It is not for any general random process only for the specific case of a Gaussian random process a wide sense stationary that is a Gaussian random process is wide sense stationary then it is also strict sense stationary. And it briefly revise, we can briefly see what are the conditions a wide sense Stationarity. Random process wide sense stationary that is N t is wide sense stationary if well expected value of N t equals mu for all t that is the mean is constant.

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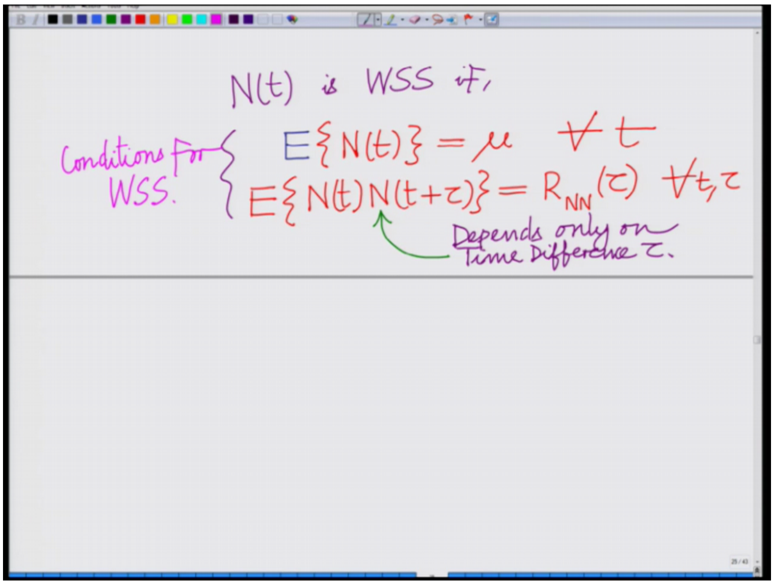

And expected value of N t that is the correlation or the cross correlation between two time instance t and t plus tau t and t plus tau the correlation depends only on R N N of tau once again for all t comma tau. This depends only on tau you can see that the correlation between N t and N t plus tau depends only on the time difference tau. So, these are the conditions for wide sense stationary. So, these are the conditions for wide sense stationary. And for a Gaussian random process what we have shown is that if these conditions are satisfied, if the Gaussian random process or the Gaussian noise process is wide sense stationary then it is also strict sense stationary. But we have to keep in one has to keep in mind that this is only for a special random process that is if the random process is Gaussian.

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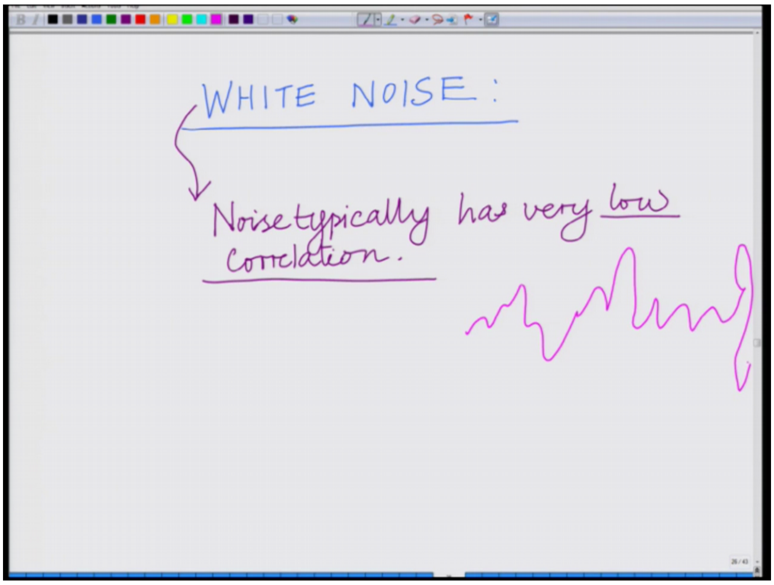

Now, let us look at the other property. So, we have seen additive. So, we have seen the following properties, we have seen the noise additive property of the noise Gaussian property when is the noise Gaussian. Now, we are going to see a different property which is the white whiteness or when do we call noise as white noise. So, we have to see the property of white noise. So, the other important thing is property of noise is white noise. Now, this property is motivated by the following observation noise typically has very low noise typically has very low correlation that is temporal correlation. If you look at the correlation that is if you look at a noise process then the noise process looks something which is very erratic very, very, very, erratic something that is unpredicted I mean if I have to draw it.

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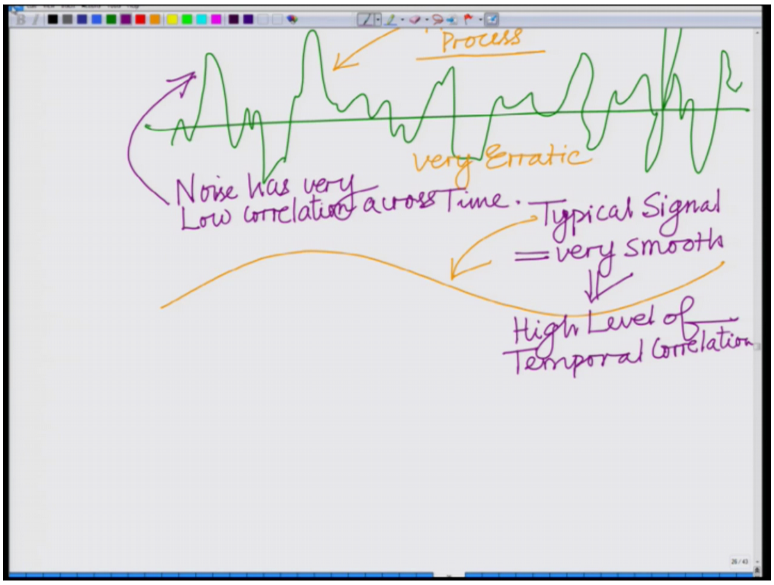

Let me just draw it appropriately that is the whole point the noise process is very erratic. So, noise process is very erratic. It has very low temporal unlike a signal which is very smooth for instance if you look at a signal which is smooth, so you have a noise process which is very erratic. And the signal typical signal which is very smooth because you have a high level of temporal correlation this has a high level of temporal correlation or correlation into highly correlated in time. On the other hand, noise has very low correlation; noise has very low correlation across time. And the signal the typical signal has very is smooth; it has a very high level of temporal correlation.

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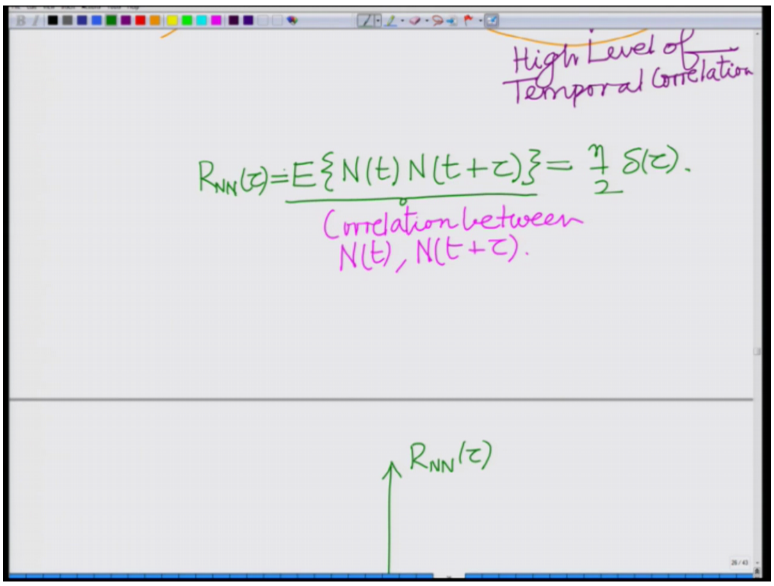

So, one way to model this very low temporal correlation is noise is to model the correlation that is if you look at any two time instants expected value of N t into N t plus tau equals eta by two times delta tau. And this looks like that is if you look at the correlation between this is this quantity is nothing but expected value of N t and N t plus tau this is a correlation between N t and N t plus tau. We are saying that this correlation between N t and N t plus tau is eta by 2 times delta tau, which means it looks something like this.

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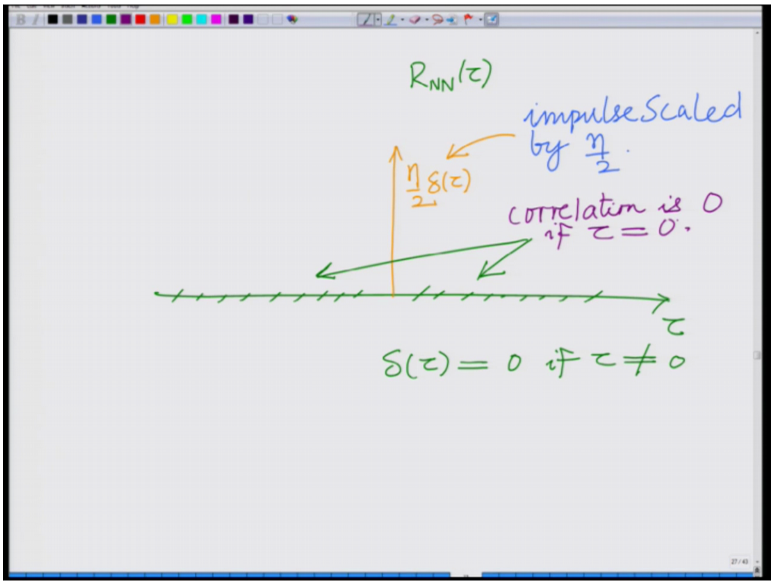

This is the axis; this is tau. And this is R N N of tau that is if we plot this correlation R N N of tau. Now, this correlation is simply an impulse function scaled by this is simply an impulse function that is it is simply that is if you look at this correlation it is simply an impulse scaled by impulse scaled by eta by 2 pulse scaled by eta by 2. So, this is simply an impulse this is simply an impulse scaled by eta by 2 which means if a tau is not 0, this is 0. You can observe that delta tau correct delta tau equal to 0, if tau naught equal to 0 implies which implies at all these points that is tau not equal to 0, correlation is 0 that is noise. That is if you look at this expected value N t that is if you look at this correlation expected value of N t N t plus tau this is equal to 0, if tau is not equal to 0.

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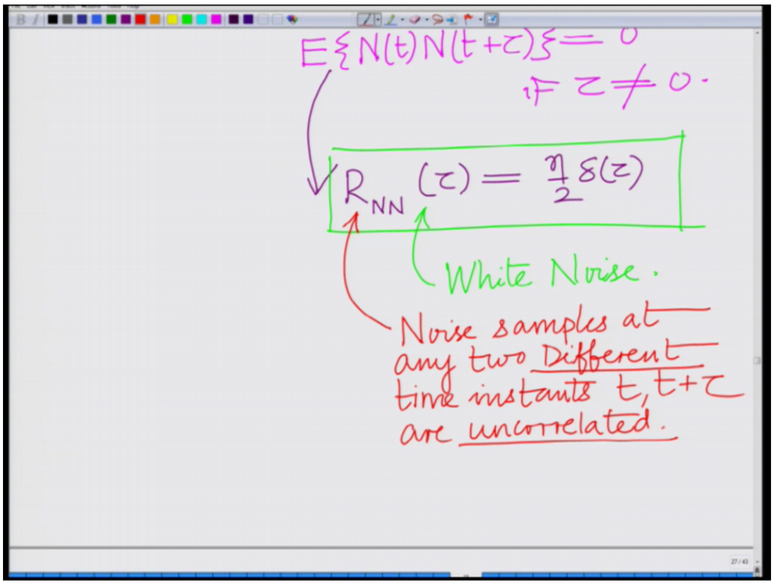

What this is saying is if you look at any two different time instants t and t plus tau that is t and t separated by a small time difference tau, the noise samples at these two incidents are uncorrelated. And of course, further we are not saying it is independent if the noise process is Gaussian then they are also independent. But anyway at this point for any general noise process which is white. If the noise the noise samples expected value of N t N t plus tau noise samples are two different instance any two different instance N t and N t plus tau is 0 that is they are uncorrelated. Such a noise is known as white noise that is expected value of N t into N t plus tau is eta by 2 times delta tau. If tau equal to zero it is delta zero eta by 2 times delta 0 if tau is not 0, then it is 0.

So, the noise correlation the noise correlation is given by the temporal correlation or you can look at the autocorrelation function of the noise is given by eta by 2 times delta tau such a noise is termed as white noise. This is termed as white noise, which means that noise at any two different time instance noise sample at any two different time instants are uncorrelated. The key operative word here is different any two different time instants t comma t plus tau are uncorrelated and it is known as white noise.

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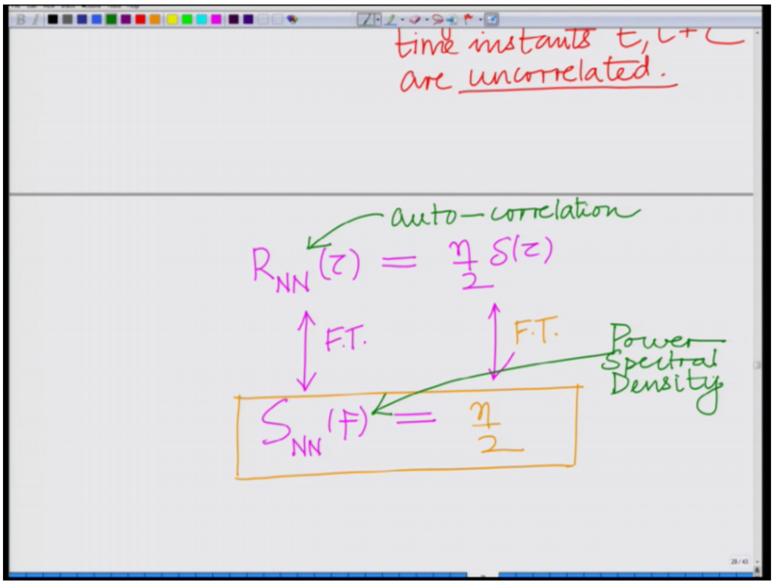

Because if you look at the power spectral density, remember we have R N N tau equals eta by 2 times delta tau. If you take the Fourier transform of this then you get the power spectral density that is S N N of F and that is basically the Fourier transform of eta by two delta tau and the Fourier transform of delta tau is nothing but unity. So, S N N by 2 f is eta by 2 simply for all frequency it is simply eta by 2. So, therefore, if you look at the power spectral density, remember this R N N tau, this is the autocorrelation function S N N F this is the power spectral density, this is the power spectral density that is the PSD.

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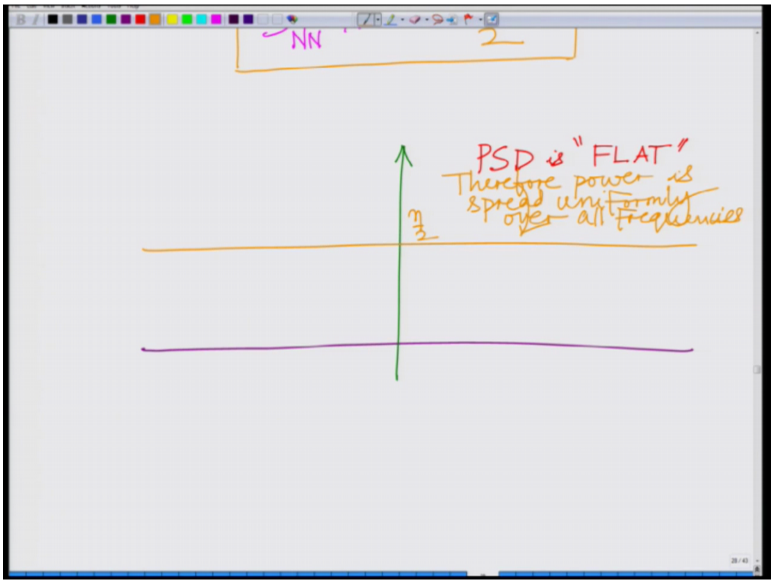

And if you look at the PSD, PSD of white noise is simply eta by 2 that is if you look at the PSD, PSD is simply eta by 2 that is it is uniform that is it is a uniform power spread over all frequencies that is power the power spectral density is flat. Therefore, power is spread uniformly over all frequencies. So, similar to white light which has power spread uniform they are all frequency over all visible light frequency components all right the power spectral density you observe for white noise has a power that is distributing. Remember we said the power spectral density characterizes the distribution of power of a random process in the frequency domain.

So, therefore, for this power spectral density; which is proportional to delta tau. If you take the Fourier transform, Fourier transform of delta tau is 1. So, Fourier transform of eta by 2 times delta tau is simply eta by 2. So, therefore, the power spectral density is flat over the entire frequency domain, which means that the power is spread equally over all the frequency components from minus infinity to infinity, hence this is termed as white light. Since, it is similar its behavior or its power spectral densities power distribution across the frequency is similar to that of white light which has uniform frequency uniform distribution of power across all components in the visible light spectrum. So, this is basically, so power we see PSD is flat power is uniformly distributed.

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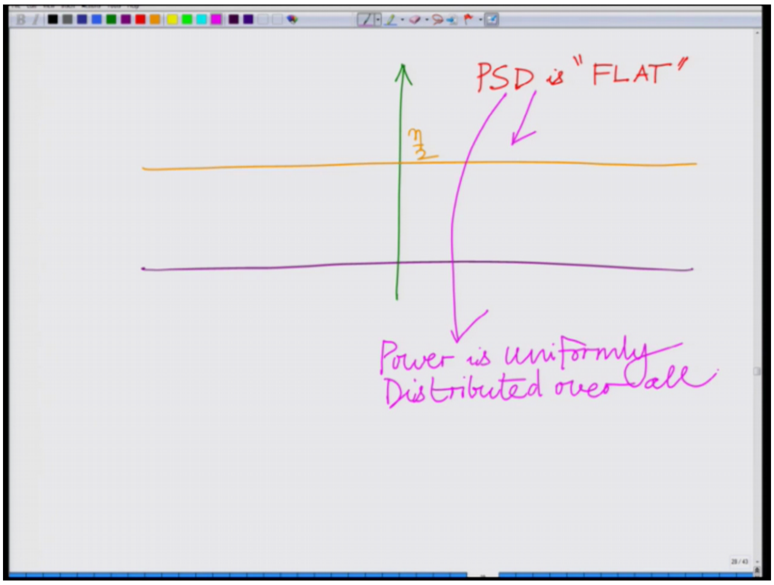

So, you see the PSD is flat power is uniformly distributed overall frequency components similar to white light.

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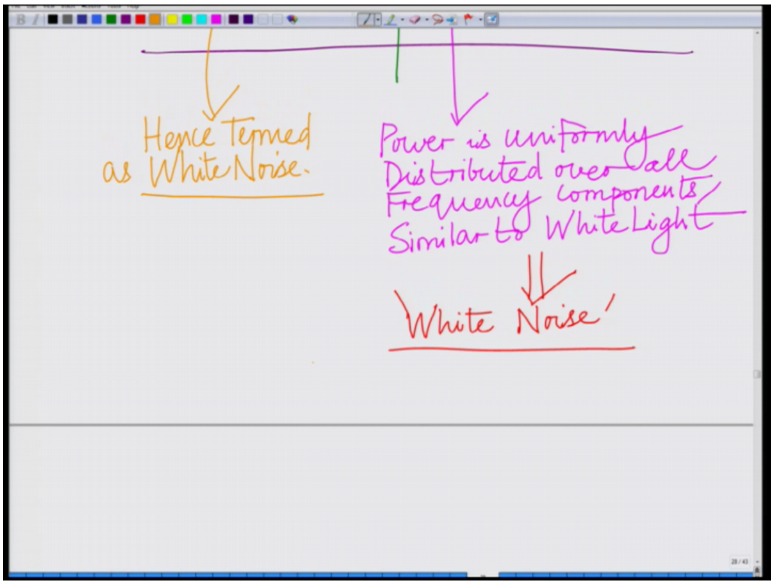

Remember white it is a combination of all the colors. So, let is us implies this is such noises hence the system dissimilar to white light hence termed as white noise hence termed as white noise. The PSD, which is flat over the frequency domain hence termed as white noise hence termed as white noise. This is termed as white noise that is the whole point.

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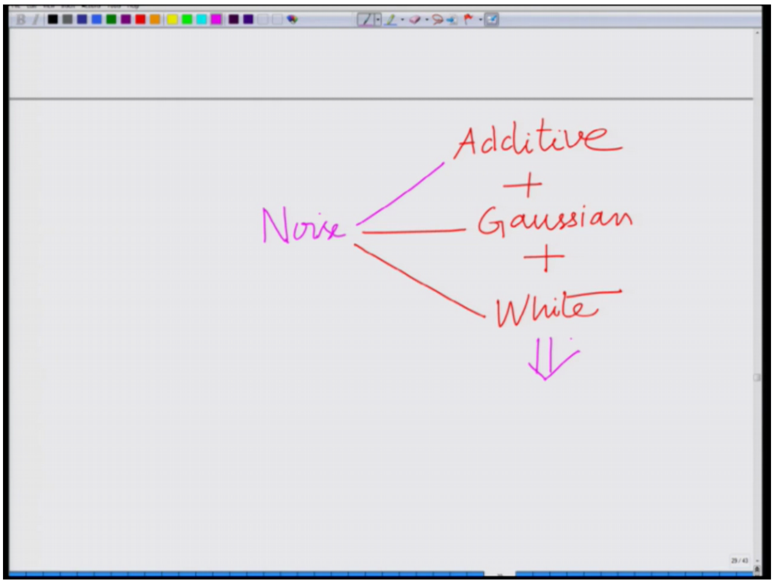

Therefore, now we can say that the noise process, now we have seen the three properties additive, Gaussian and white. So, now we can say noise which is additive, now these are the three different components. And notice that none implies other noise which is additive plus Gaussian plus white that is noise which is additive plus Gaussian plus white.

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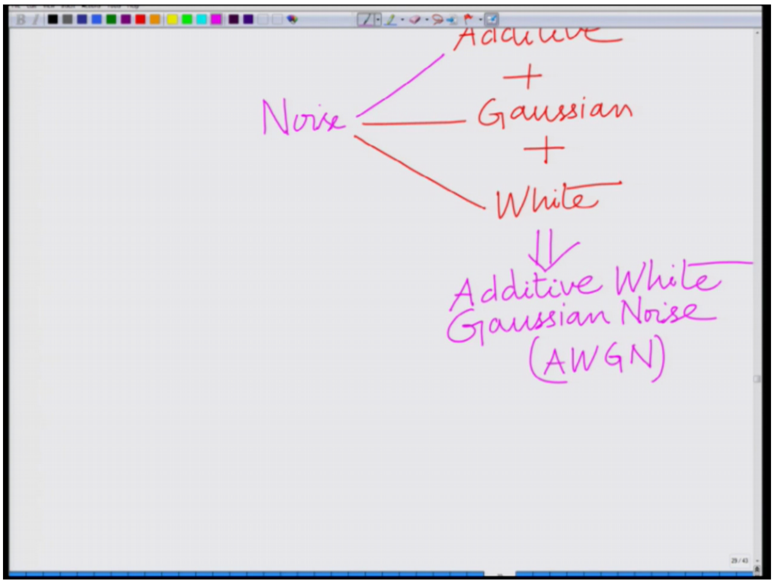

If the noise is three different things satisfies these three different properties, then the noise is additive if only then the noise is additive, white, Gaussian. And further realize that all these three conditions are very different none of them implies the other or no subset of them implies that. For instance the noise additive does not imply it is white or Gaussian noise is white does not necessarily mean the noise is Gaussian noise is Gaussian does not mean that the noise is white. So, all these three components are independent all these three different properties are separate all right none of them implies the others. Only if the noise satisfies these three separate criteria that is if a it is additive it is white and it is Gaussian such a noise is known as additive white Gaussian such a channel is known as an additive white Gaussian noise channel.

So, now, we go back to our digital communication channel model. This channel model where y equal to x plus n. Remember we have looked at this channel model y equal to x plus n, where we have y t equals x t plus n t such a channel is known as an additive white Gaussian.

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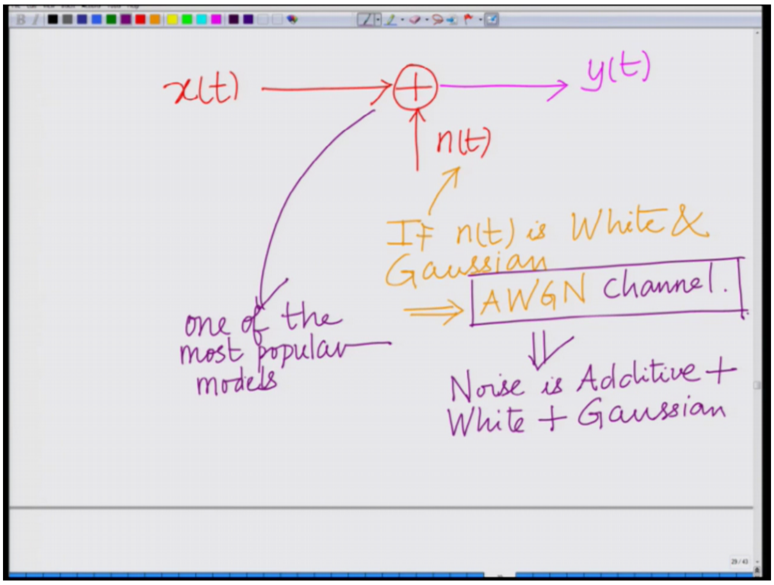

Let me just quickly draw a simple schematic. This is a very simple yet a very powerful channel model, which can be used to characterize a general digital communication channel. So, we have the transmitter signal, it is a very simple channel model as you can see, I have the transmitted signal x t, I have additive noise. So, I have additive noise n t and I have this thing. So, signal plus noise, noise is additive. If the noise if n t is white and Gaussian and of course, you can see from the figure that the noise is additive implies it is an AWGN, noise is additive and Gaussian, it is an AWGN channel. AWGN channel implies that you have an additive noise, AWGN channel implies that basically noise is additive.

Again I am reiterating the same thing because it is important plus white plus Gaussian. So, one of the most popular models one of the most popular models as well as one of the most practically applicable models plus you can also say that practically applicable for a digital communication system, one of the most popular models plus also practically applicable for a digital communication set. So, basically that summarizes the AWGN noise AWGN channel model which is basically a transmitted signal x t your noise n t is additive; in addition if the noise n t is white and Gaussian. So, noise n t is additive. So, x t plus n, it is a very simple model, x t signal transmitted signal plus noise n t gives rise to that receive signal y t in addition.

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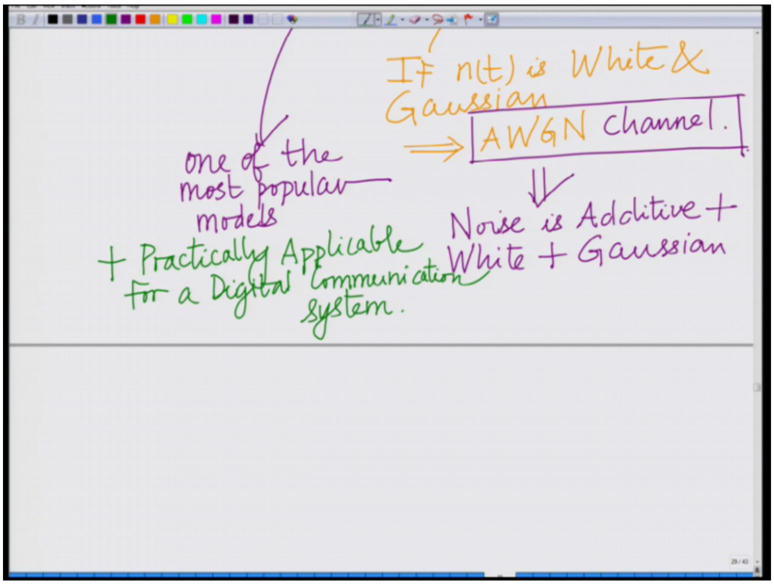

So, the noise is additive in addition if the noise is white and Gaussian this is known as an additive white Gaussian noise channel. One of the most simplest, one of the most frequently used and one of the most popular and also one of the most practically applicable channel models or models for a typical digital communication system.

So, we will stop here; and based on this model, we will analyze the performance look at optimal schemes for this digital communication system, for a digital communication system and also their performance in the subsequent modules.

Thank you very much.