Principles of Communication Systems - Part II
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Lecture - 38
Joint Conditional/ Differential Entropies, Mutual Information

Hello. Welcome to another module in this massive open online course. So, we are looking at the differential entropy of a continuous source and in that on the same lines, let us extend also the concept similar to the entropy. We had the concept of joint entropy and conditional entropy. So, let us extend the differential entropy to also the other aspects. So, there is the joint and the conditional entropies, ok.

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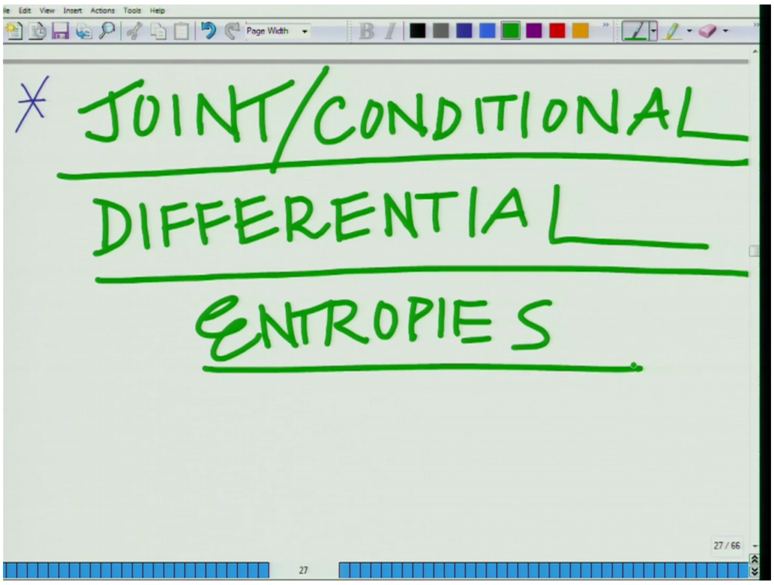

So, we are going to look at the joint and conditional. I should say differential entropies and the definitions are rather similar, ok.

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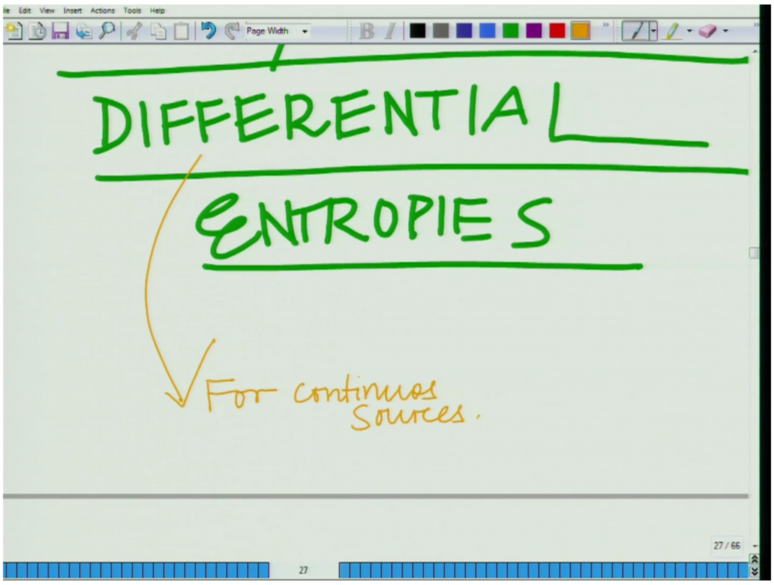

Remember this is for continuous sources, correct continuous sources which distributions are characterized by probability density functions, ok and the definitions are similar.

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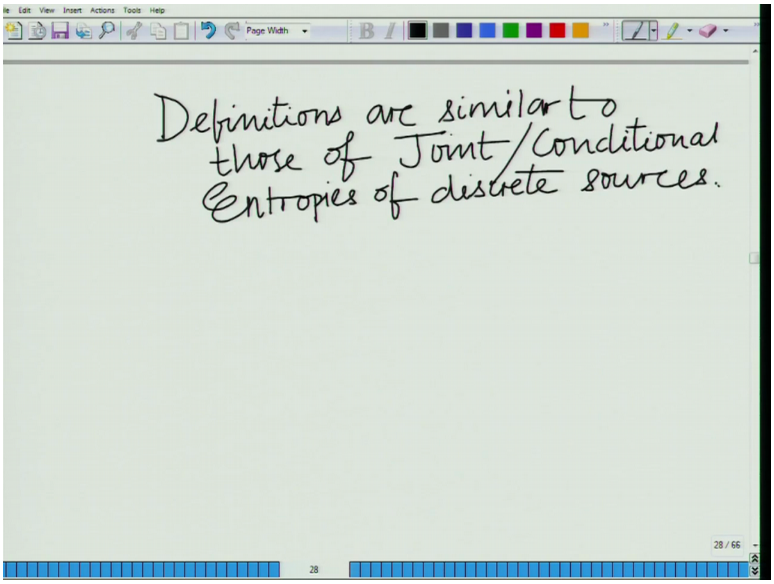

So, the definitions are similar to those of the joint and conditional entropies where discrete sources. Of course, we have seen the differential entropy. Similar to that of the entropy with the probability is replaced by the probability density function and the summation replaced by the integral which is also can be viewed as a continuous summation and therefore, the joint entropy and the conditional and joint and conditional differential entropies are also similar for instance.

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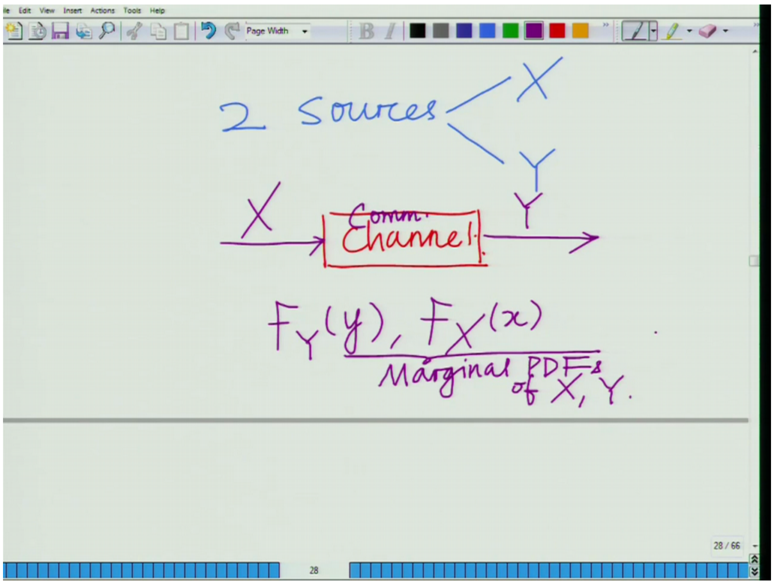

Let us say we have two sources continuous sources X and Y or rather these can also be in the context of a communication channel which is going to be relevant moreover discussion later. So, you can have a channel X denotes the transmitted symbols, Y denotes the received symbols in the context of a communication system. So, this can be a communication channel.

So, basically Y is another source represented by the symbols represented characterized by probability density function now. So, F Y of y, we have several quantities F X of f, these are what are known as the marginal PDF s marginal probability density functions of X, Y also. Now, we have the joint probability density function of X similar to the joint probabilities X and Y.

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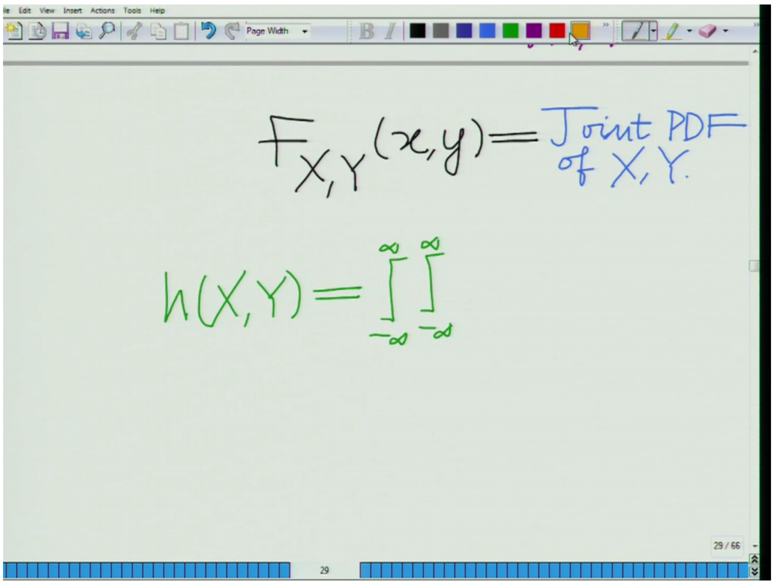

We have the joint probability density function of X and Y. So, we have the joint probability. So, this is basically your joint PDF of X, Y and when you have the joint PDF, you have h of X, Y. The joint differential entropy is similarly defined as minus infinity to infinity. The double sum, the continuous double sum will become the integral, the double integral correct.

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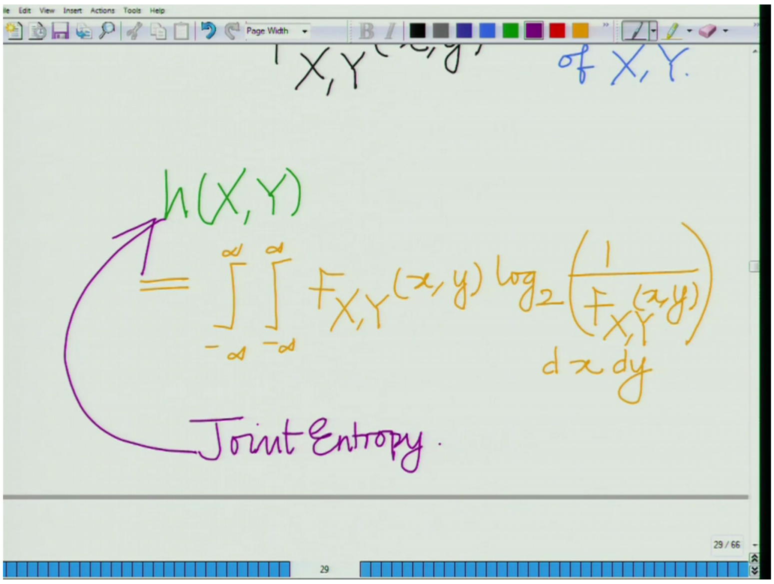

So, this can be defined as this is equal to minus infinity to infinity minus infinity to infinity F of X, Y log to the base two 1 over F of X, Y d x d y. So, this is your joint differential entropy or you can simply call this as a joint entropy. In this context, this is in the source X and Y are continuous, ok.

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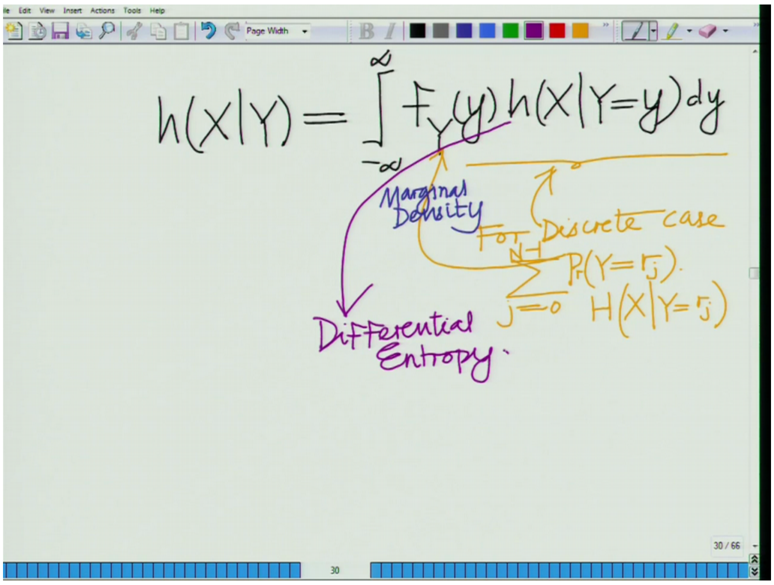

Now, similarly one can define the conditional entropy for continuous sources that is h of X given Y. Remember that is you take the marginal probability density function of Y and then, average the conditional entropy of X given Y equal to Y times d y.

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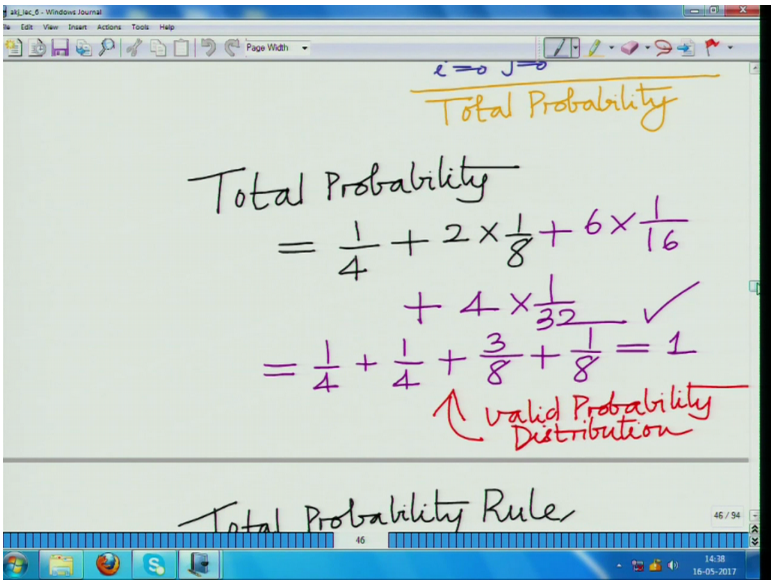

In case now you might recall that for instance, for our this definition, you might recall that in the case of our, in the case of the discrete symbols, it is given as follows.

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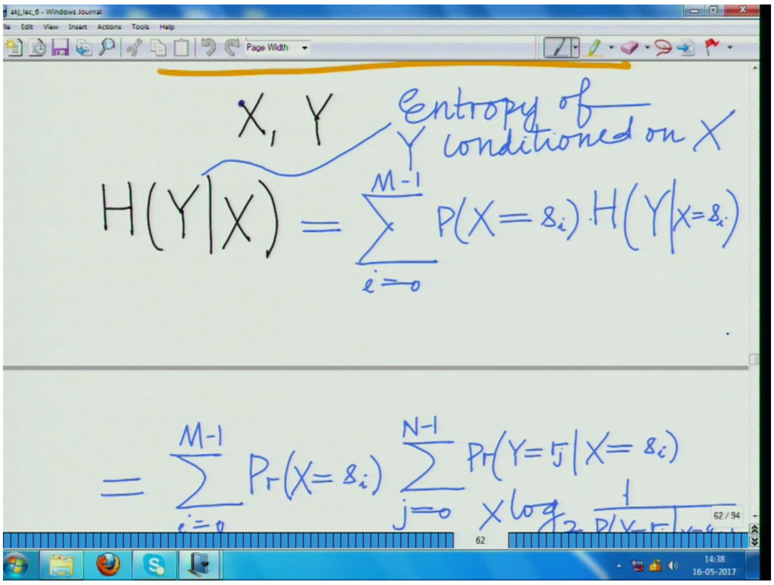

You might recall this definition. It is probability X equal to s i. This is of course the conditional entropy of Y given X, similarly for X given Y, ok.

So, this is you can write this as if you want to recall the definition for the discrete case. The equivalent definition would be summation probability Y equal to r j i. Think we are using j equal to 0 to n minus 1 times h of x given y equal to r j. So, that would be the equivalent definition for the discrete case. We are replacing the probability by the marginal density correct and of course, similarly the entropy by the differential entropy correct h of x given this is the differential entropy. So, you can see the definition is very similar to that what we have for the discrete sources that replaced the probability replace the probability by the probability density function. That is the marginal probability density function and entropy for the discrete case by the differential differential entropy for the continuous source X, ok.

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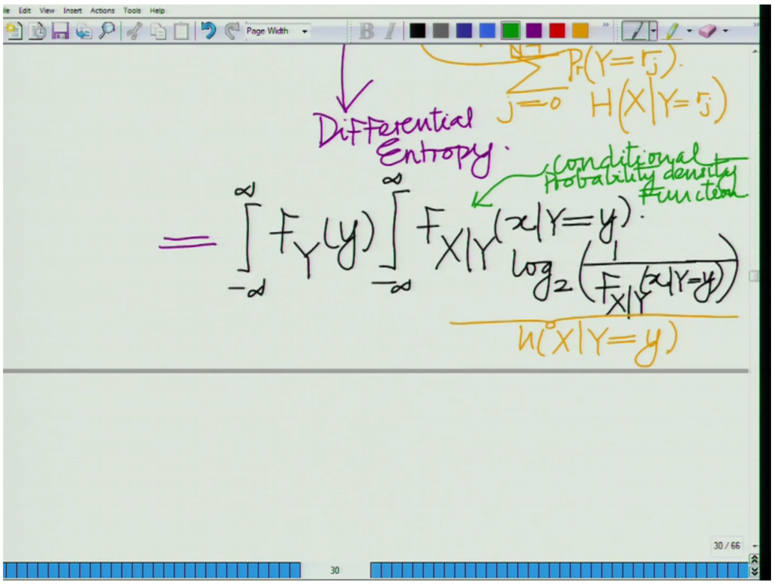

Of course, I can now simplify this as follows. This will be integral minus infinity to infinity F of Y that is the marginal probability density and the differential entropy that is F of X given Y times X given Y equals Y times log 2 to the base 1 over F of X given Y X given Y equal to Y. This is basically your h X given Y equal to Y. What we have here is the conditional probability density function. So, all these concepts you must be familiar from probability and random process. This is the conditional probability density function.

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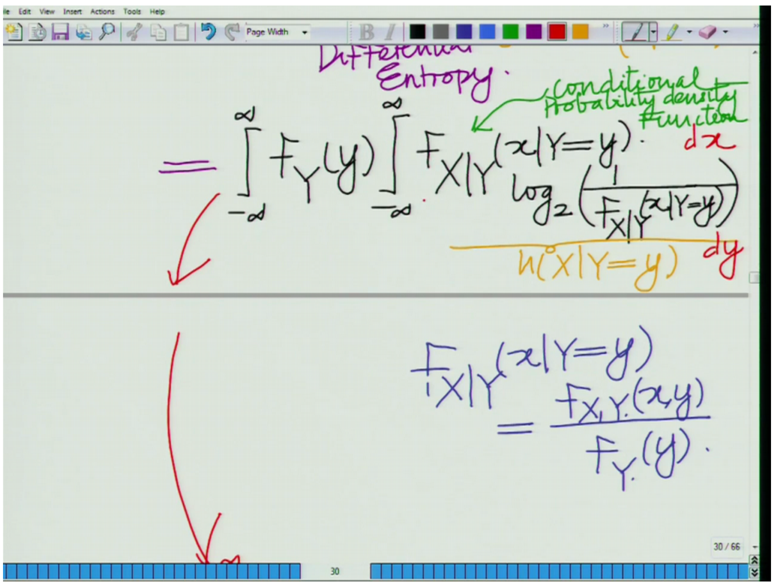

The conditional probability density function can be defined as follows that is F of X given Y of X given Y. We just write the definition for your convenience F of X given Y X given Y equals Y equals F of X, Y divided by F of F of X. So, this is the conditional probability density function and now if you look at, you can write this again as the following thing. You can basically write this as follows. Of course, there has to be an integral, sorry there has to be an integral with respect to here the integral with respect to X followed by another integral with respect to Y.

Now, if we look at this F of Y, the probability density function of Y into F of X, Y given F of X given Y is nothing, but the joint probability density function F of X, Y.

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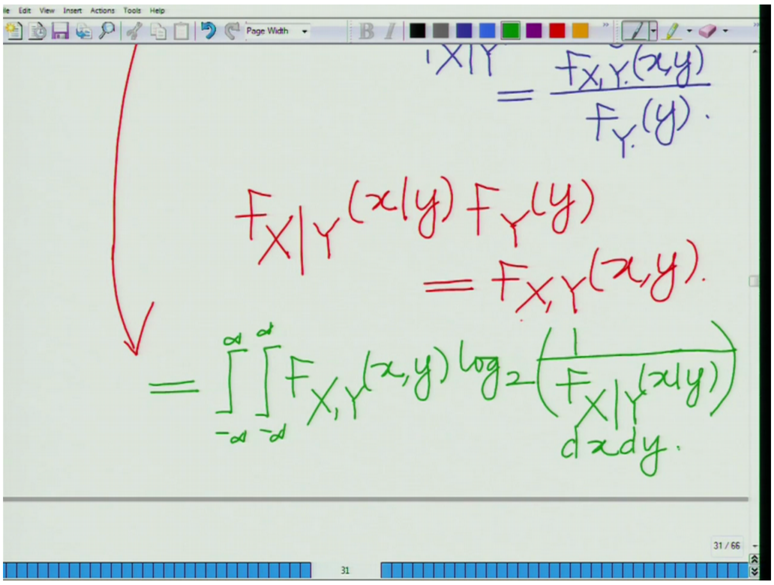

So, therefore, you will have combining these two that is if we use the property that where we use the property that F of X F of X F of X given Y times, this is equal to the joint probability density function. So, F of X is the conditional probability density function, F of X given y times the probability density function equal to the joint probability density function.

Therefore if you look at this joint therefore, if you look at this conditional entropy, the definition will boil down to minus infinity to infinity minus infinity to infinity F of X, Y correct X, Y times log to the base two 2 over F of X, Y. I am sorry F of X, this will still be F of X given Y X given Y times dx dy, ok.

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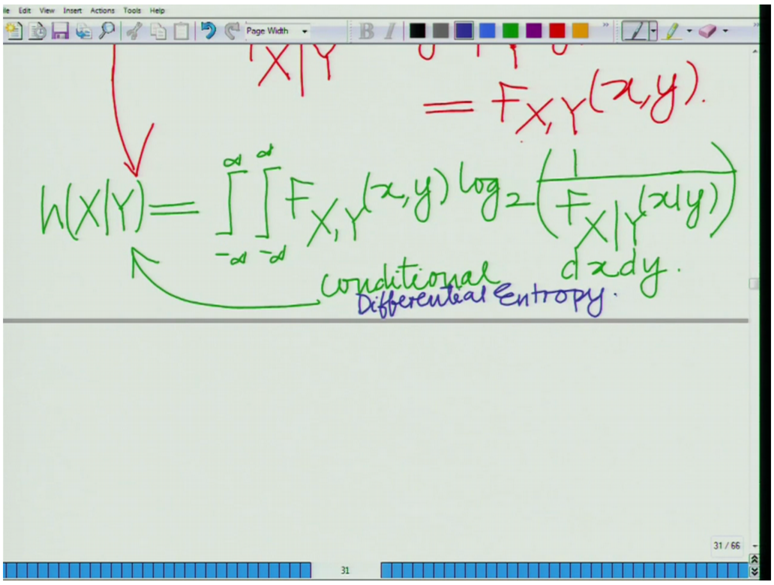

So, this is basically your h of X given Y. This is the conditional entropy or rather your conditional differential entropy and finally, one can also define the mutual information between these two continuous sources.

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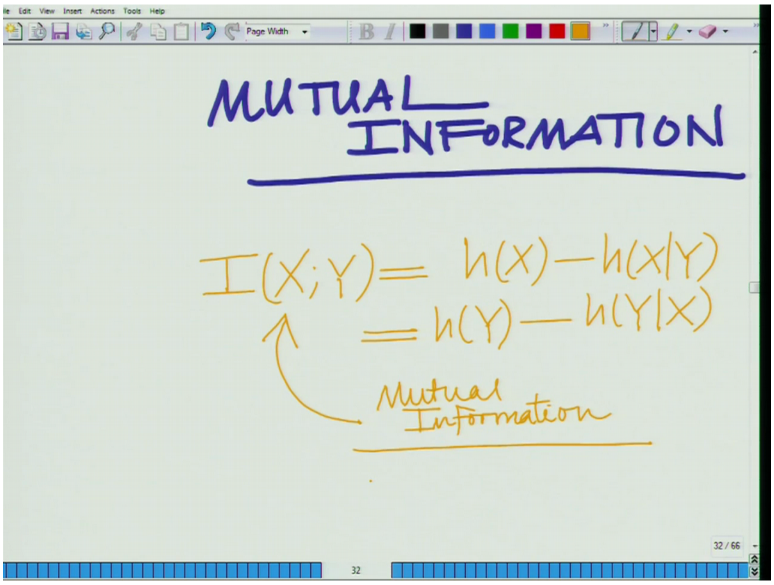

The definition has again once again similar to what we have for the discrete sources, that is you have the mutual information that is I of X, Y equals h X. Of course, now in terms of the differential entropies h X minus h given Y equals h Y minus h Y given X and this is basically your mutual information basically.

So, in this module what we have seen is basically we have extended the definitions of the joint entropy, the conditional entropy and the mutual information to continuous sources and we have seen that these definitions can be obtained similar to the discrete, similar to the scenario with discrete sources as parallel by replacing the entropies with the differential entropies, the probabilities with the corresponding probability density functions correct and the definitions are similar for the continuous case.

So, we will stop here and look at other aspects in subsequent modules.

Thank you very much.