Principles of Communication Systems - Part II
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Lecture – 36
Differential Entropy, Example for Uniform Probability Density function

Hello, welcome to another module in this massive open online course. So, we looking at information theory we have looked at Shannon's landmark result that is the channel coding theorem alright which is basically gives result for the fundamental rate at which the information can be transmitted across a channel, alright.

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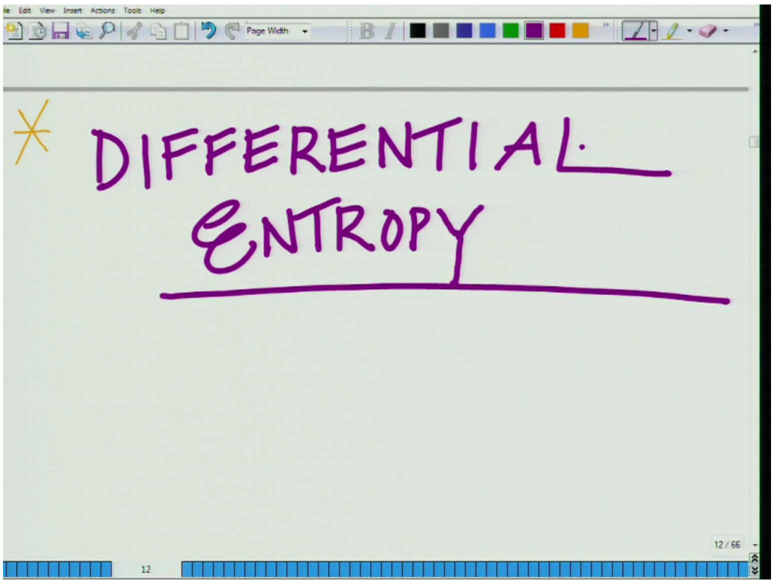

In this module let us start looking at a different concept and that is of differential entropy. So, we want to start looking at a new concept the concept of differential the concept of differential entropy and this differential entropy alright. So, far we have seen basically if you look at this.

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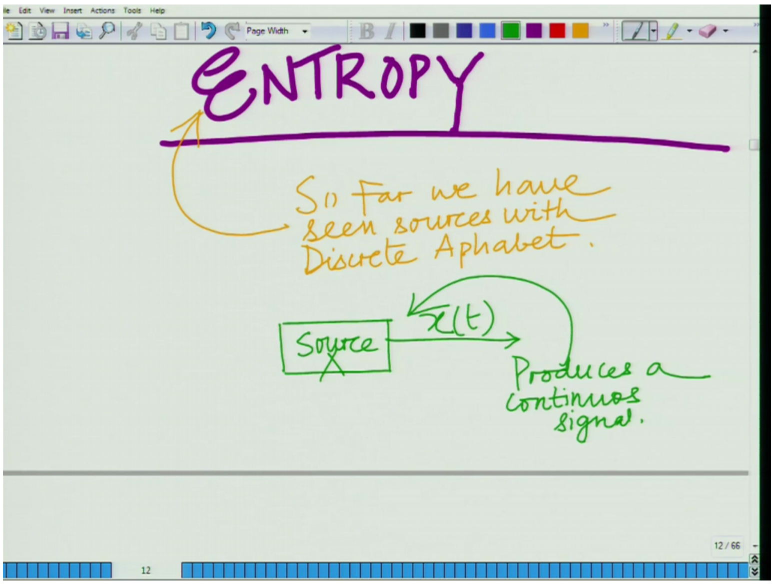

So, far we have seen discrete sources with a discrete alphabet. So, so far with discrete alphabet that is discrete sources, but what about a continuous source that is a source; source X this is the source X and this produces a continuous signal this source X produces a continuous signal that is a source which produces a continuous signal which can take values.

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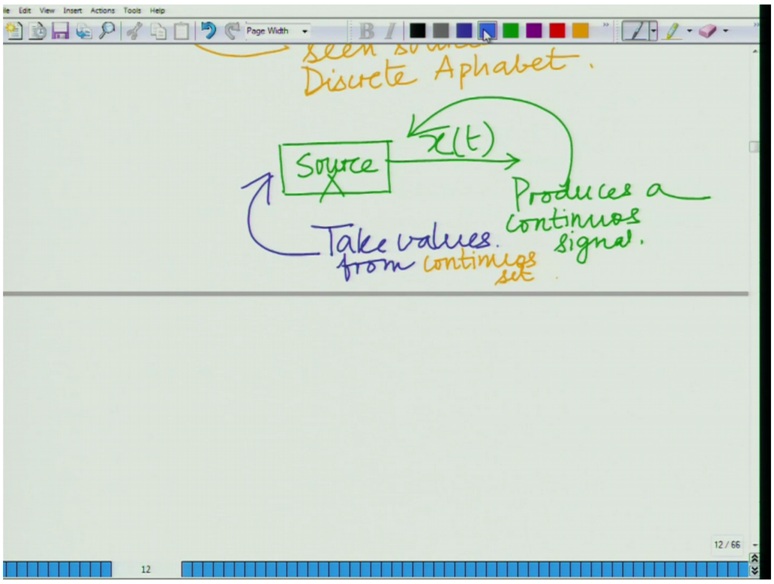

Which can take continuous values right we can take values from a continuum that is take values take continuous set take values from a continuous set for instance 0 to infinity we can have a continuous signal which can take values as against previous sources which are discrete in nature or like this alphabet the set the source alphabet is discrete consists of a finite number of symbols.

Now, as against that one can also consider source we generates continuous signals continuous or continuous signals which can take values which can take values from a continuous set which can take continuous set of values. So, naturally the probability distribution is characterized by a probability density function for such sources.

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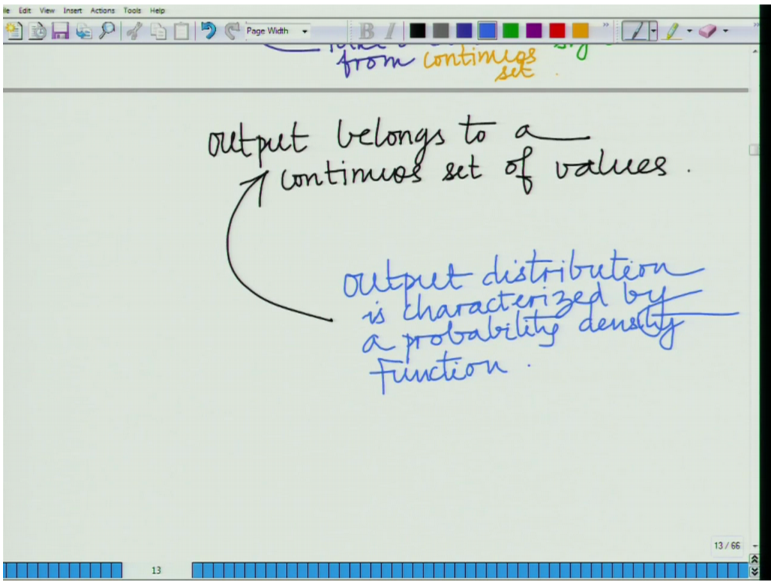

So, basically we are talking about the entropy the probability distribution output belongs to a continuous set of values. So, naturally the output belongs to a continuous and characterized by a even takes it can take continuous set of values and this is characterized by a the distribution is characterized output distribution is characterized by a. So, far we have seen a probability mass function. Now we are going to because output can take from a continuous set of values this is characterized by a probability distribution probability density function. So, this is the pdf; probability density function or you can say this is the output pdf of source X.

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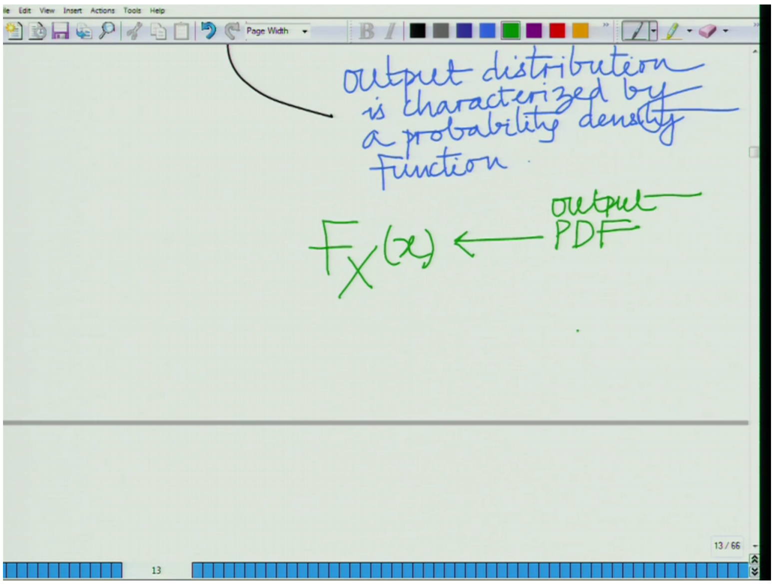

For instance F of X; that is output pdf; pdf of the output generated by the source is a probability density function for instance one very one very popular or one of the most commonly occurring random variables right to characterize the output of this continuous such continuous is the Gaussian probability density function.

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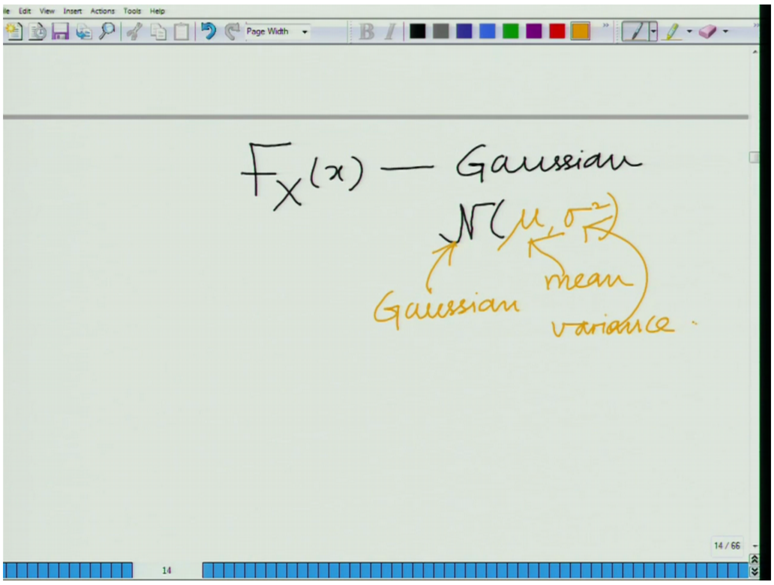

So, the source probability density function F of X of x can be Gaussian and from the fundamental knowledge of the fundamental course or probability we know that Gaussian can be represented as for instance mean mu having Gaussian. So, this shows a Gaussian random variable this is the mean sigma square is the variance.

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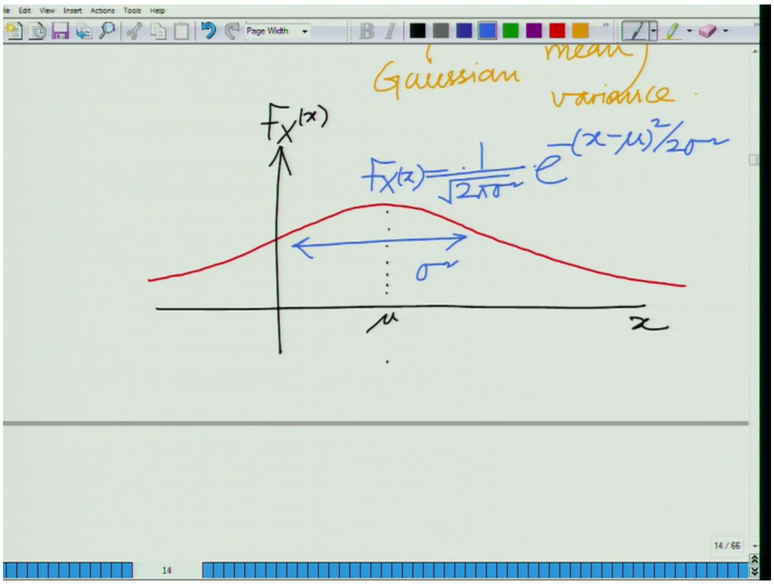

And the Gaussian probability density function looks as follows peak at the mean let us say the mean is nonzero. So, this is your F X of x; this is your X the peak is at the mean mu the spread of the Gaussian is proportional to its variance sigma square and the probability density function is 1 over square root 2 pi sigma square e raise to minus X minus mu whole square by 2 sigma square this is the probability pdf of the Gaussian correct.

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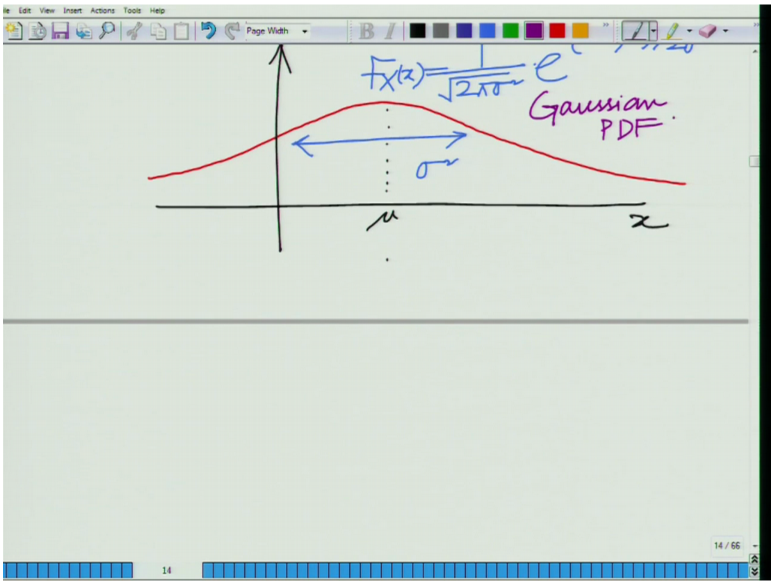

You already know this or you can have an exponential source the output; pdf is exponential correct.

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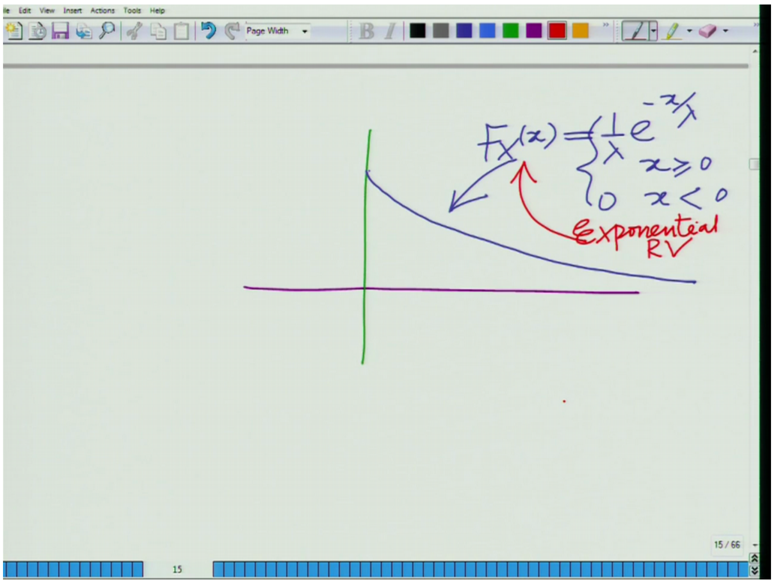

So, this can be something like. So, this is exponential pdf F of X; x equals 1 over lambda e power minus X over lambda for X greater than equal to 0 0 for X less than 0. So, this is your exponential; exponential random variable.

So, the source alphabet is continuous and has an exponential probability density function. So, the output is continuous takes values from a continuous set it is characterized by a probability density function. Now how do we define the differential entropy for such a source the differential entropy the average information the differential entropy let us not say average information for a particular reason so, the differential entropy.

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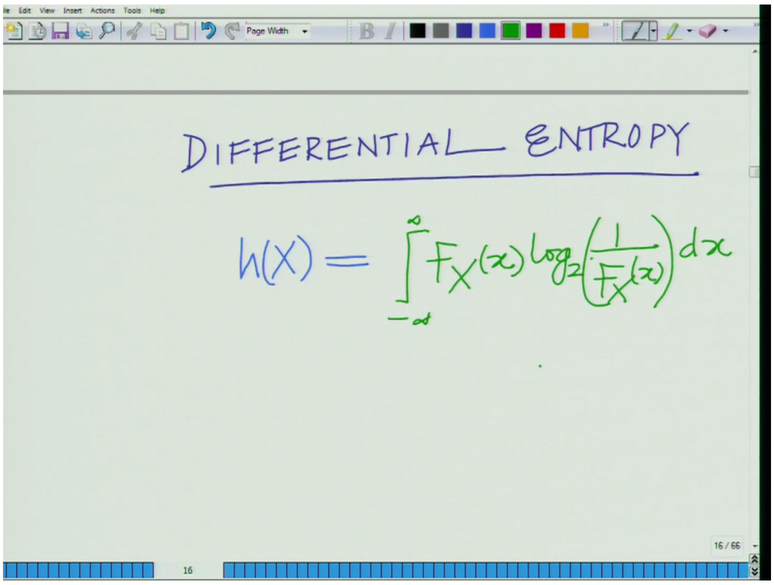

So, we are going to first let me first define the differential entropy and we are going to see something unique about this the differential entropy of such a source the differential entropy is defined as h of X equals minus infinity to infinity F X of X log 2 to the base 1 over F X of X d x. So, this is termed as differential entropy and remember this is the definition of differential entropy.

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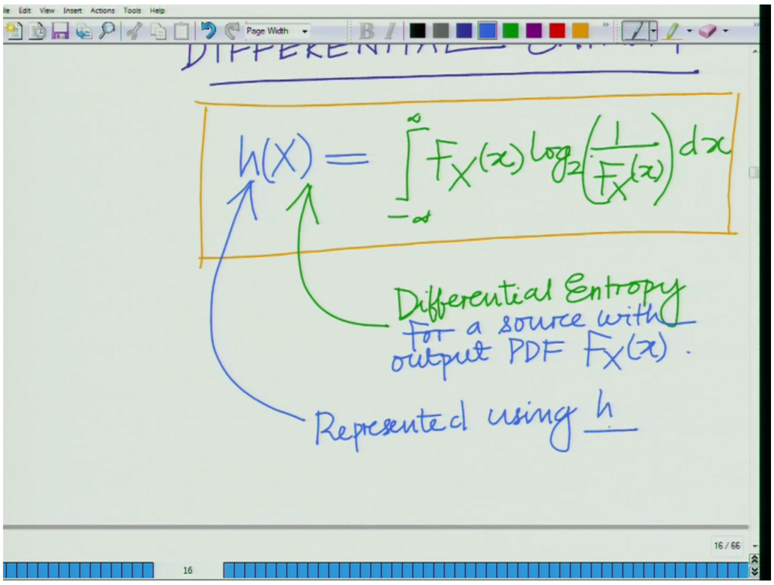

So, let me just draw this is the differential entropy for source with output pdf and note that as against the entropy of a discrete source this is represented using small h represented using h. So, this is something that you can note as against the entropy of a source with the discrete alphabet.

So, this is the differential entropy and from this you can see that its very similar to the definition of the entropy where we represents the probabilities correct in the log we have replace the probabilities right probabilities by the probability density function alright and the summation right the discrete sum has been approximated has been replaced by the integral which you can think of as evaluating a continuous summation.

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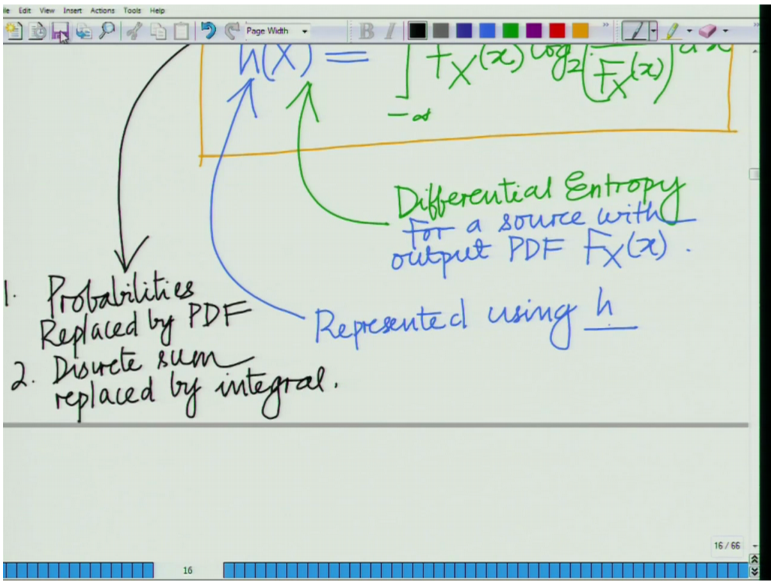

So, there are 2 major changes 1 is 2 major changes. So, its similar except for the probabilities replaced by pdf 2 the discrete sum has been replaced by the discrete sum has been replaced by the integral let us look at an example for this to understand this better.

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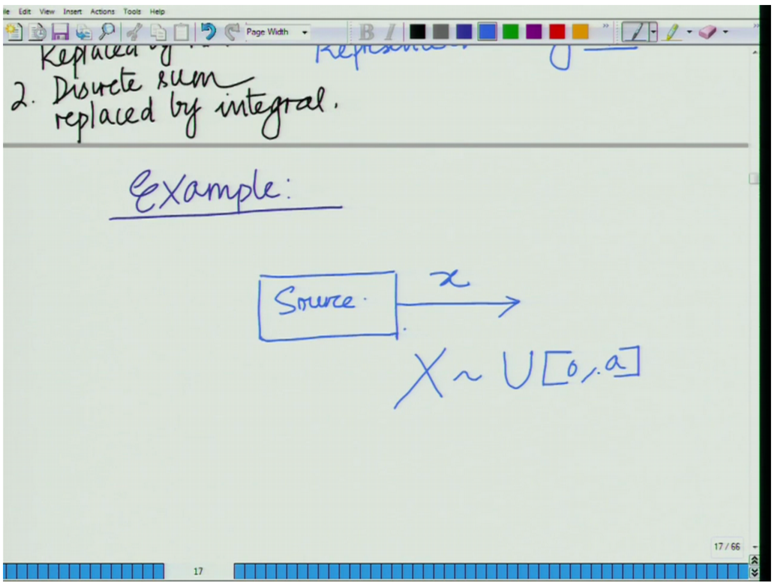

Let us look at an example for instance let us look at an example where I have a source where I have a source. So, this is my source; source generating the symbol X and the source X the pdf is uniform in 0 to a.

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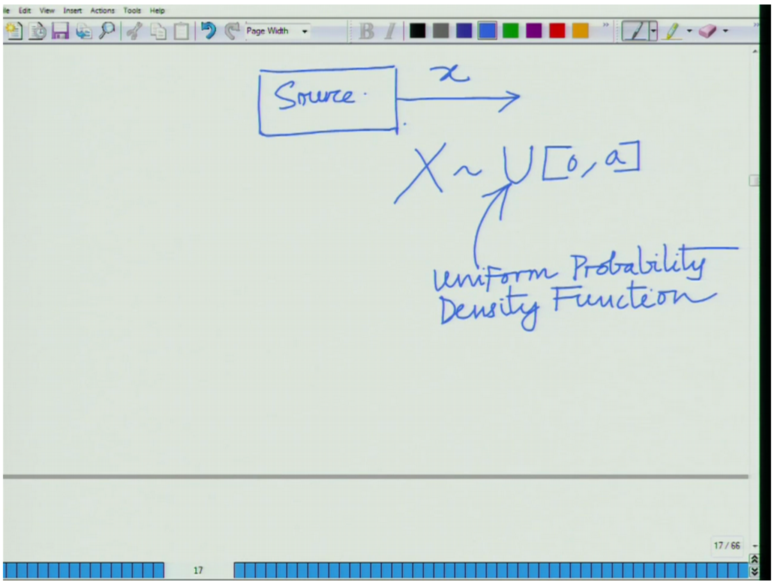

This denotes uniform probability density function and the uniform probability density function remember uniform probability density function over the range has a flat probability density function if you look at the uniform probability density function.

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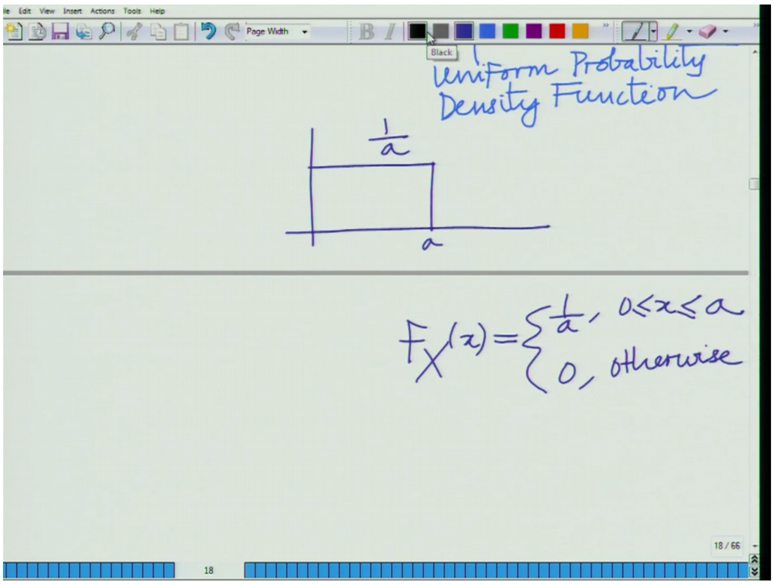

For instance this is a; this is 1 over a. So, this is uniform. So, the uniform probability density function is basically given as 1 over in 0 to a is 1 over a for 0 less than equal to X less than equal to a and 0 otherwise. So, this is the probability density this is the uniform probability density function 0 to X 1 over a 0 to a and 0 otherwise and you can see this satisfies all the properties of the probability density function because it is no negative and also the area under the probability density function is 1.

And now the entropy of a source was output is basically follows; they has the uniform pdf, alright.

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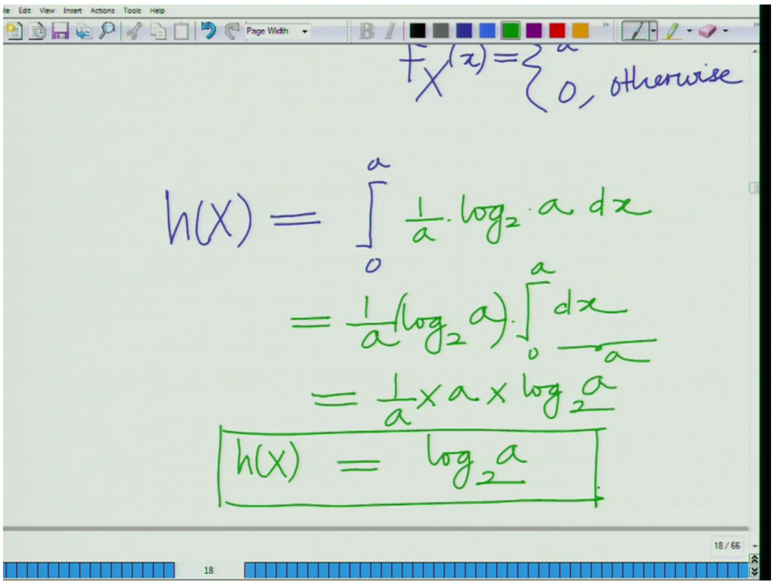

In 0 to a the source output is characterized by uniform probability density function the differential entropy that is h of X was such a source is given as well integral 0 to a F of X that is over a log to the base 2 1 over F of X a d X this is the constant. So, 1 over a log a to the base 2 will come out 0 to a d X which is 1 over a times a, this integral is a times a times log a to the base 2 which is log a to the base 2. So, you have evaluated the differential entropy of a simple source corresponding to the uniform probability density function that is F of X of x.

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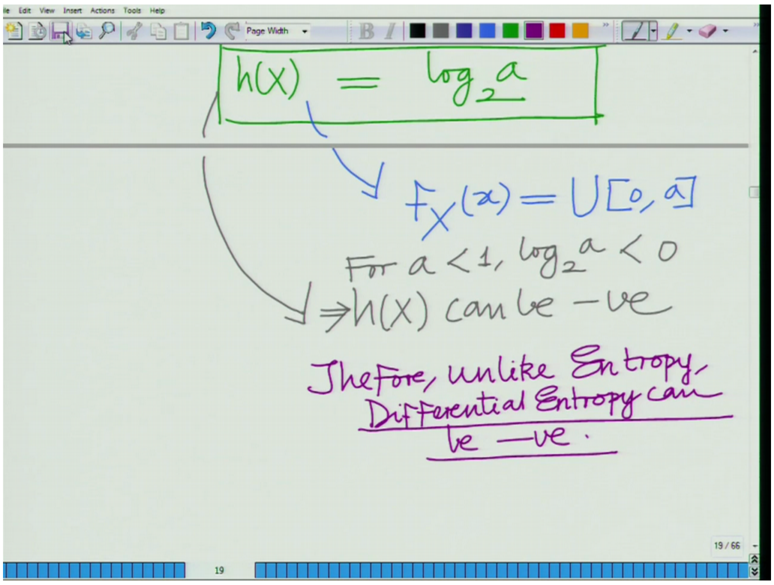

Now, you will observe something interesting that is we have evaluated the differential entropy which is relatively straight forward, but if you look at this differential entropy you will notice that if a is less than 0 the differential entropy is negative which is something that is very interesting which do not happen with entropy. Remember the entropy was always non negative that it 0 or greater than or equal it is greater than or equal to 0 how are the differential entropy can be negative and this is a very important difference.

So, you can see that h of X can be negative for instance for a less than 1 log a to the base 2 is basically less than 0 this implies h X can be negative therefore, unlike entropy therefore, unlike entropy the differential entropy and this is an important property of the differential entropy on like negative unlike entropy the differential entropy can be negative unlike the entropy this can be negative. So, this is an important difference one has to keep in mind while dealing with the differential entropy that is the entropy of entropy definition for sources which can a for sources with source alphabet with source alphabet that can take continue source alphabet there is not discrete that take a continuous set of values. So, the corresponding in definition is differential entropy and the important thing to realize is this quantity can be negative unlike entropy. We will stop with this and we look at other aspects in the subsequent modules.

Thank you very much.