Principles of Communication Systems – Part II

Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 24
M-ary PSK (Phase Shift Keying) – Part I, Introduction, Transmitted Waveform, Constellation Diagram

Hello. Welcome to another module in this massive open online course. So, today in this module we are going to look at a different modulation scheme that is M-ary PSK or M-ary phase shift keying, similar to M-ary QAM which is quadrature amplitude modulation and as M symbols. A M-ary PSK or phase shift keying has M symbols and it is a phase shift keying based digital modulation scheme.

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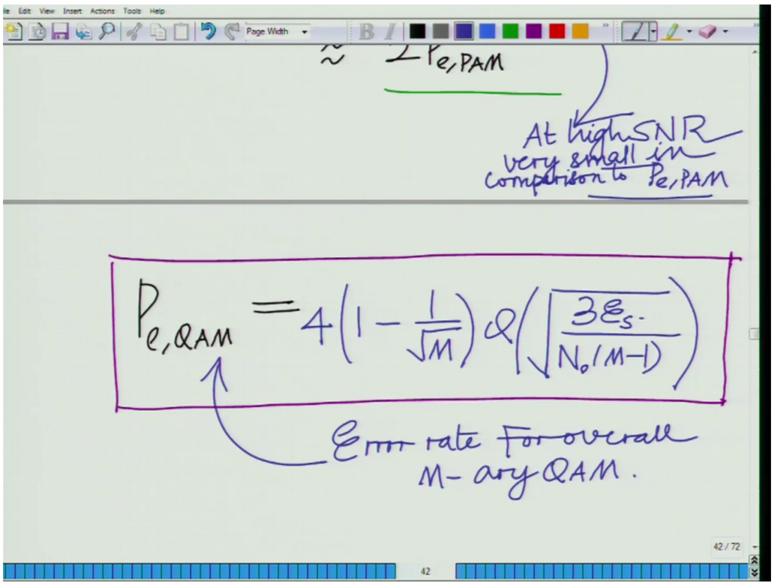

But, before we proceed further just want to make a minor correction in the overall probability of symbol error derived for M-ary QAM last time there has to be a factor of two. So, this twice as we have seen the problem overall probability of error is twice the probability of error approximately twice the probability of error for the constituent QAM; so for the constituent M-ary PAM. So, this has to be 4, because the probability of error for the constituent PAM is a factor of two. So, this is twice that it will have a factor of 4 this is 4.

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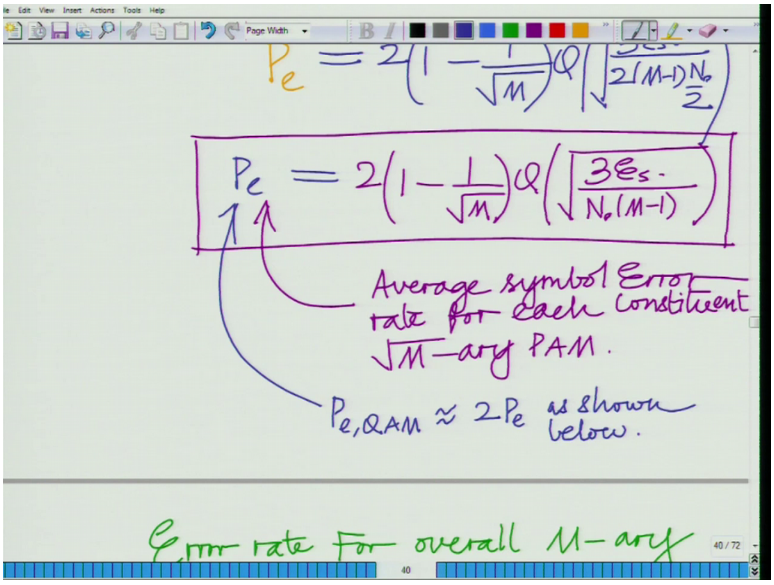

This is a factor of 4 this is twice the probability of probability error of PAM or Pe, QAM approximately twice Pe as shown below.

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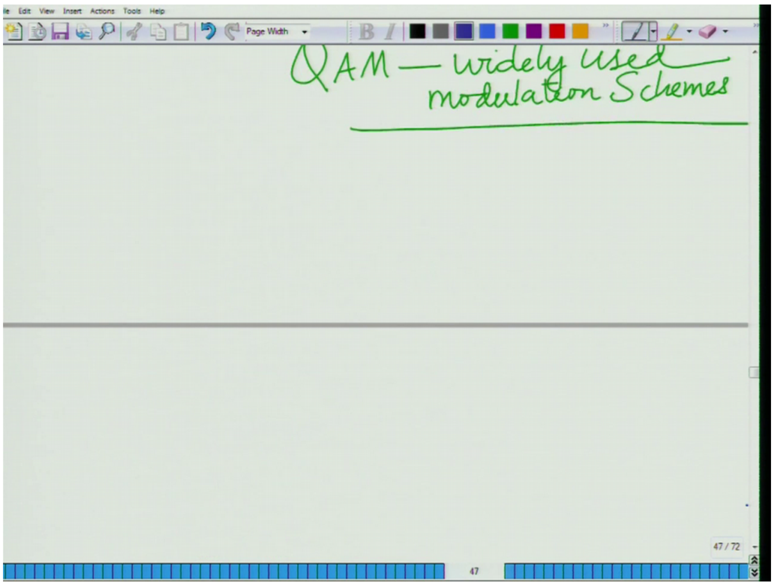

Anyway, let us move on to the modulation scheme under consideration which is yet another modulation scheme a digital modulation scheme which is termed as M-ary phase that is your M-ary PSK .

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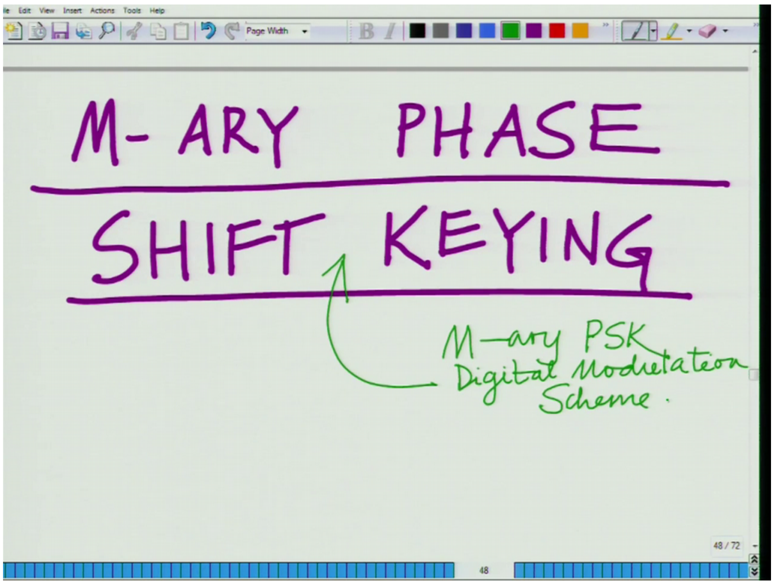

This is another digital modulation scheme and again this also generalizes the concept of binary phase shift keying which has 2 phases.

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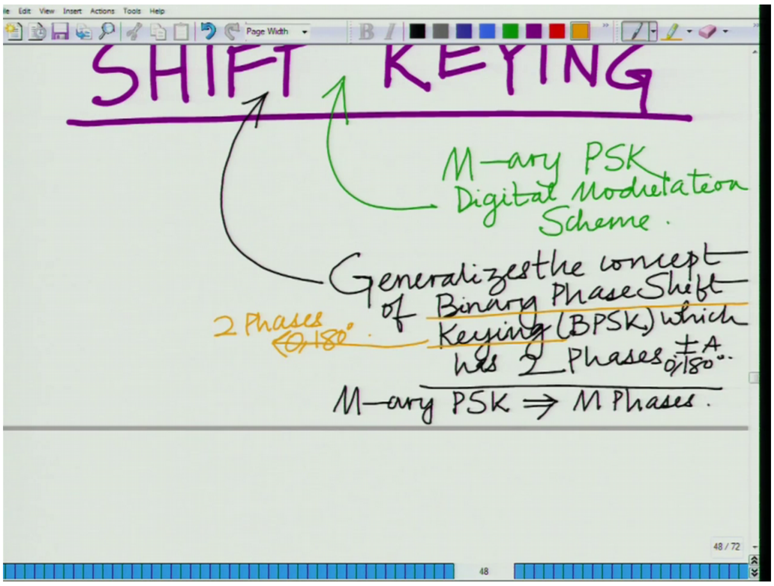

Generalizes the concept of binary phase shift keying BPSK, binary phase shift keying or BPSK which has 2 phases and it generalizes to M-ary phase shift keying. In M-ary phase shift keying basically there are M phases. So, in binary phase shift keying there are 2 phases plus or minus A that is 0 comma 180 degrees. So, if you look at binary phase shift keying there are only 2 phases which are 0 comma 180 degree.

So, M-ary phase shift keying generalizes concept of binary phase shift keying and allows for M phases, that is symbol can have one of M phases and this worth this as M symbols M possible symbols or M possible symbols in the constellation and therefore, once again similar to M-ary QAM one can communicate log M to the base 2 bits per symbol.

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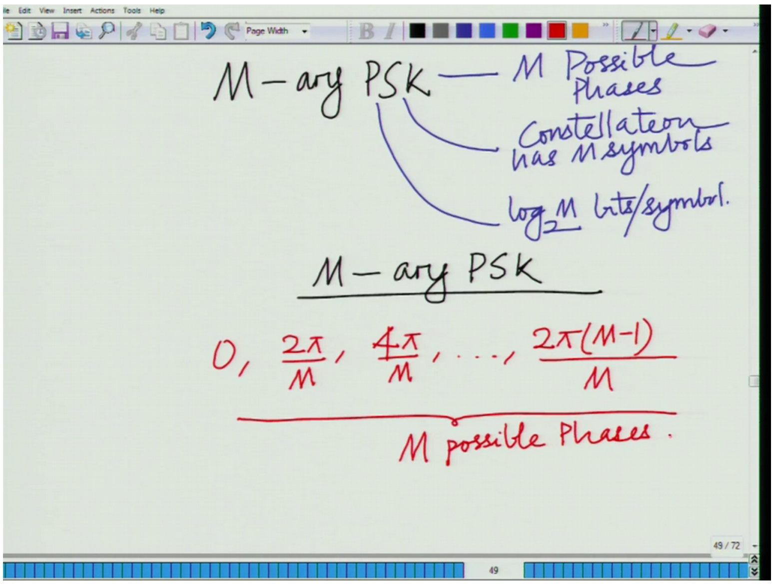

So, there are M possible phases, M-ary PSK M possible symbols implies the constellation has M symbols implies we have log M to the base 2 bits per symbol. We can communicate log M to the base 2 bits per symbol and the M possible phases in M-ary PSK the M possible phases are 0, 2 pi by M, 4 pi by M and so on so force until 2 pi M minus 1 by M. So, these are your M these are the M possible phases. Basically, the M a the i-th phase you can write it as 2 pi i by M where 0 less than or equal to i less than or equal to M minus 1.

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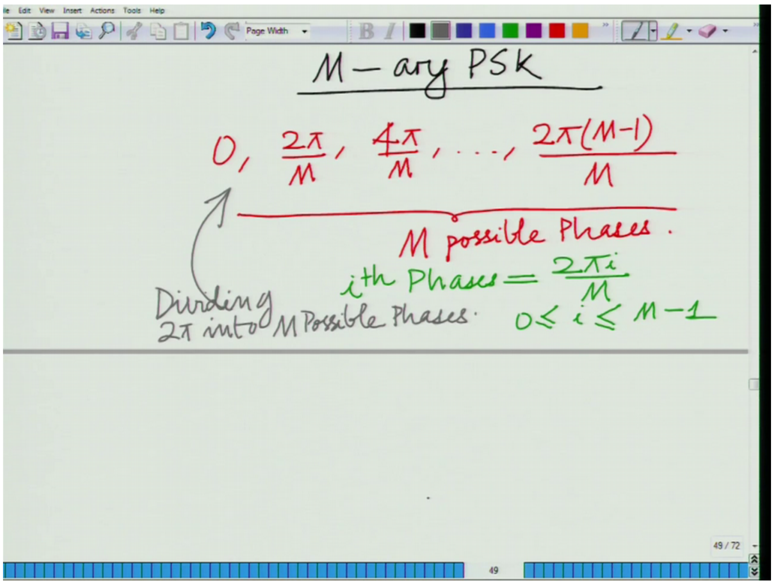

So, we are taking 2 pi 2 pi remember 2 pi corresponds to 360 degrees and we are dividing it into M possible phases. So, by basically what we are doing is dividing 2 pi into M possible phases dividing 2 pi into M possible phases.

Now, of course, is M equal to 2 again you see if M is equal to 2, for M equal to 2 the phases are 0 comma 2 pi i where i is equal to i divided by M, 2 pi divided by 2 equal to pi.

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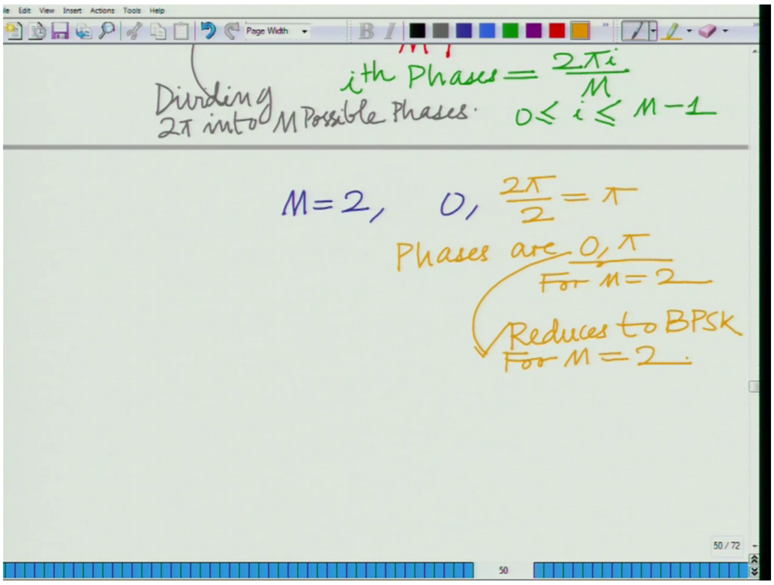

So, phases are 0 comma pi for M equal to 2, which means it reduces to binary phase shift keying. So, if you consider M equal to 2 the phases are again 0 and pi that is 0 180 degrees. So, for M equal to 2 this reduces to binary phase shift keying and in general you can have any M. So, that is M-ary phase shift keying. So, we are generalizing binary shape phase shifting which corresponds to M equal to 2 phases to M-ary phase shift keying which can which contains one of M possible phases that is symbols can have one of symbols can take one of M possible phases that is the whole idea. So, this reduces to binary phase shift keying, the idea is reduces to b PSK for M is equal to 2.

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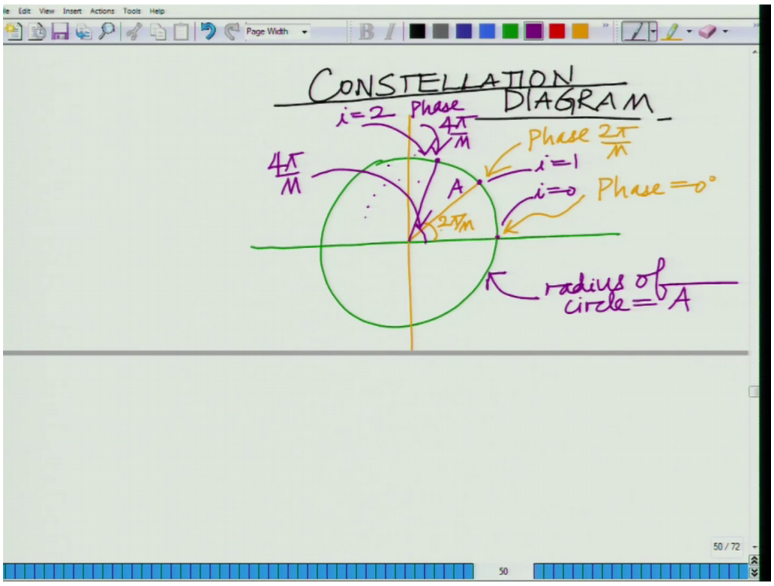

Now, let us look at the constellation diagram as I have said we are taking a phase of 2 pi we are taking the phase remember phase of 2 pi corresponds to 360 degree. So, you can represent it using a circle, now the first symbol is at phase 0 second symbol is at phase 2 pi by M this is let say 2 pi by M, third symbol is at phase right the phase difference between the symbols. So, third symbol is at phase 4 pi by M this is phase 4 pi by M and so on and the radius of the circle.

So, radius of circle equals A; the radius. So, these constellation point it is a constellation. So, this is another constellation point this is the 0 constellation point corresponding to i equal to 0 constellations. So, this corresponds to i equal to 0 this corresponds to i equal to 1, this corresponds to i equal to 2. So, the i-th point where i lies between 0 and M minus 1 is at a phase 2 pi i over M, on this circle of radius a, I can take values from 0 to M minus 1 therefore, there are total of M points each is phase shifted from the previous 1 by 2 pi by M. So, we are taking the phase total phase of 2 pi 360 degrees and dividing it into M phases and therefore, now if you look at the i-th point.

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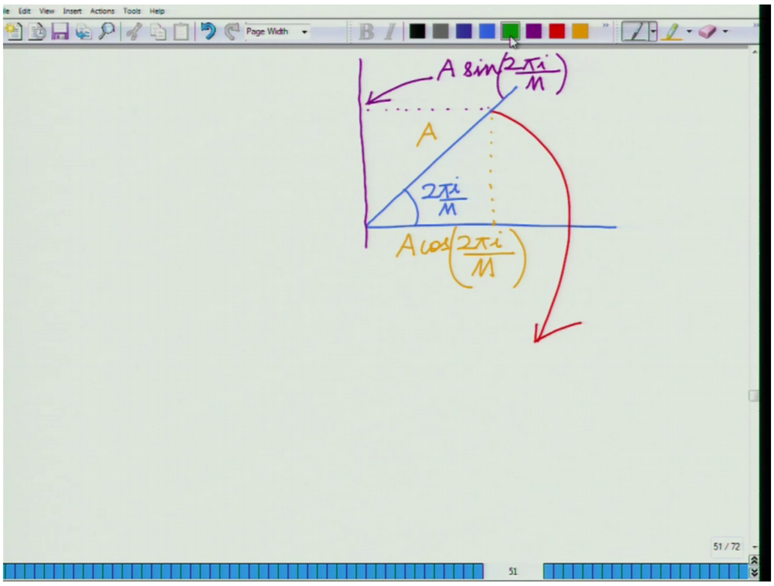

If you look at the i-th point corresponding to phase. So, if you look at the i-th point 2 pi i over M radius is A.

So, if you look at this the x coordinate is going to be A cosine 2 pi i over M, and if you look at the y coordinate that is going to be A sin 2 pi i by M. So, this can be represented as symbols 2 symbols the 1 is the x coordinate. So, each constellation point.

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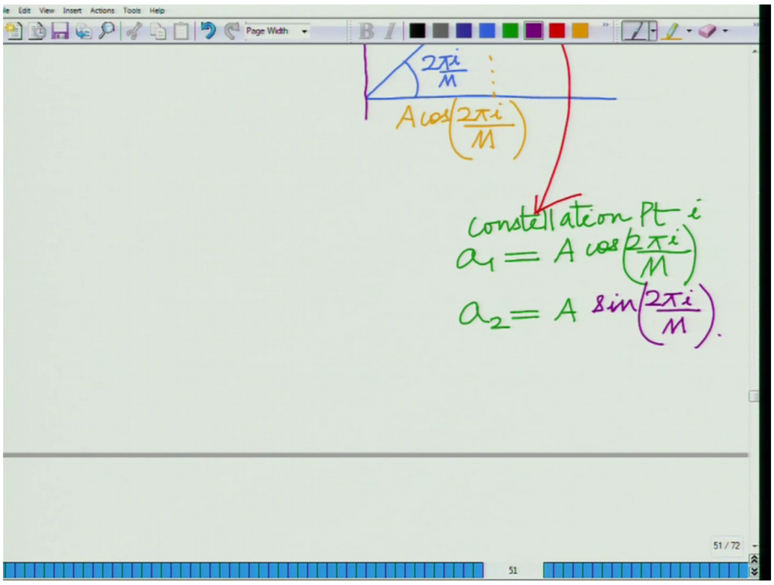

So, this is constellation point i. So, this can be represented as a combination of 2 symbols a 1 is the x coordinate which is A cosine 2 pi i divided by M, a 2 equals A sin 2 pi i divided by M. So, it is a combination of 2 coordinates a 1 and a 2 each correspond to, now you can say the each of these corresponds to basically on each of these is transmitted along, we have 2 symbols right similar to QAM is a combination of 2 symbols, each is transmitted along an orthonormal basis function of the signal space.

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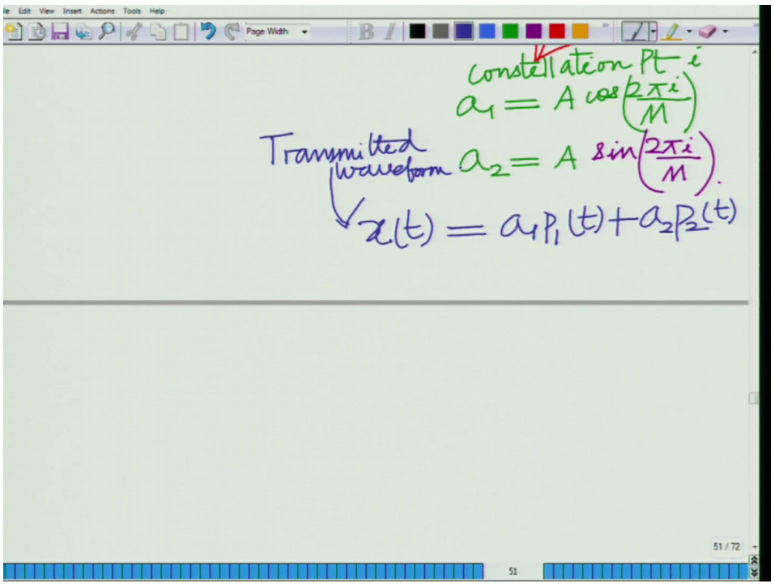

So, the net transmitted waveform x t can be x t equals a 1. So, this is similar to QAM. So, net waveform.

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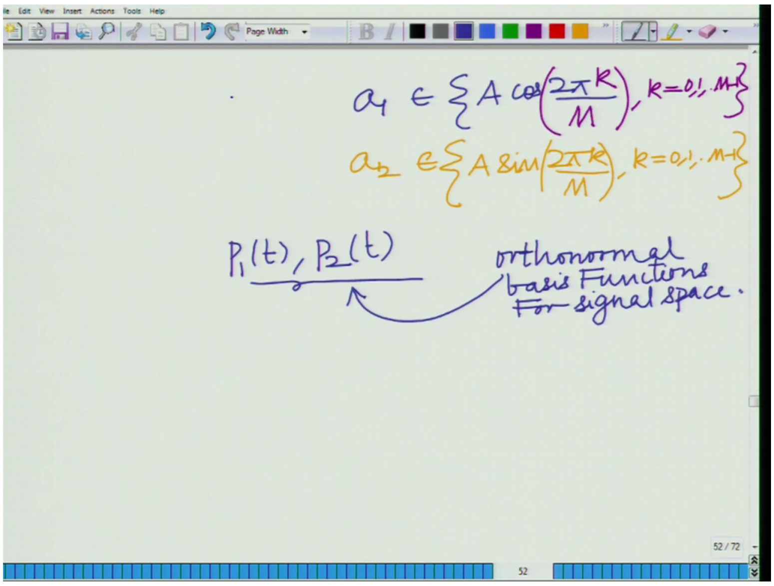

So, this is your transmitted waveform were a 1 belongs to the set A cosine or let us use the index k A cosine 2 pi k by M I am simply changing the index to k, k equal to 0 1 up to M minus 1 and a 1 or a 2 element belongs to A sin 2 pi k by M, k equal to 0 1 up to M minus 1. So, we are transmitting and of course, as we have seen before P 1 t and P 2 t these are orthonormal basis functions for the signal space.

These are the orthonormal basis functions for the of the signal space.

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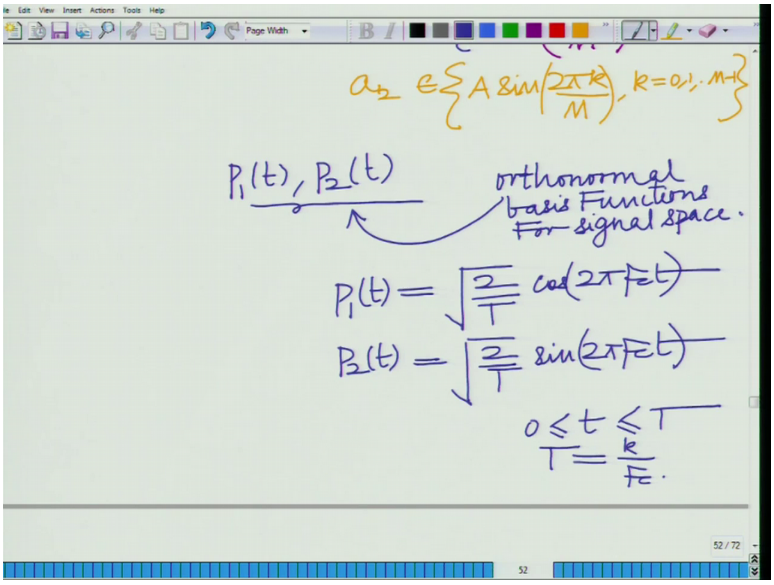

For instance we have said that is a P 1 t equals well again unit norm that is unit energy square root of 2 or T cosine 2 pi F c t, P 2 t is square root of 2 or T sin 2 pi F c t where 0 less than equal to t less than equal to capital T the symbol duration and T correct T equals k over F c or basically contains an integer number of cycles so, that is basically what we have seen before also.

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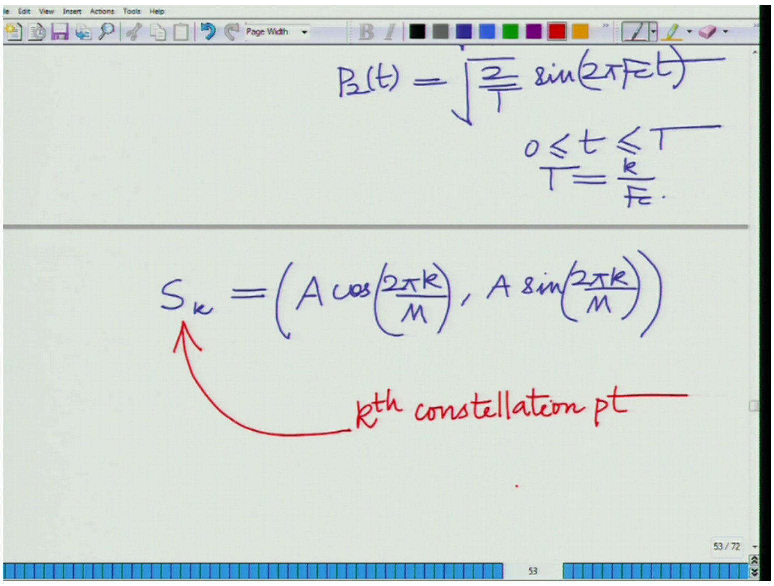

And the k-th symbol remember we have said k-th symbol is A well times cosine 2 pi k by M, A times k 2. So, the k-th symbol or basically the k-th constellation point is A cosine 2 pi k by M, A sin 2 pi k by M. So, this is your k-th constellation point this is the k-th constellation point and now if you look at.

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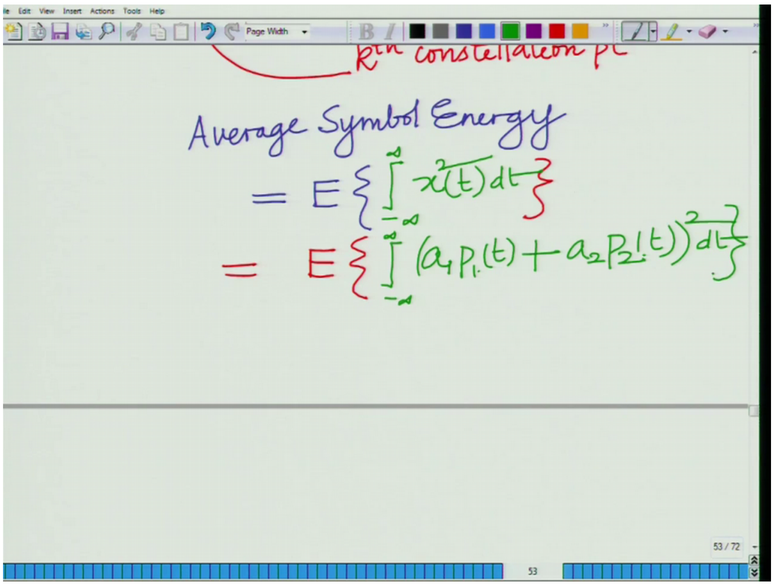

Now, if you look at the average energy average symbol energy that is equal to expected value of, well expected value of you can say average energy of the waveform expected value of x t square, expected value of or expected or energy of a t square or expected value of you can write this as expected value of integral minus infinity to infinity, just be a bit more elaborate minus infinity to infinity x square t d t which is equal to well let us write it this way minus infinity to infinity, a 1, P 1 t plus a 2 P 2 t whole square d t.

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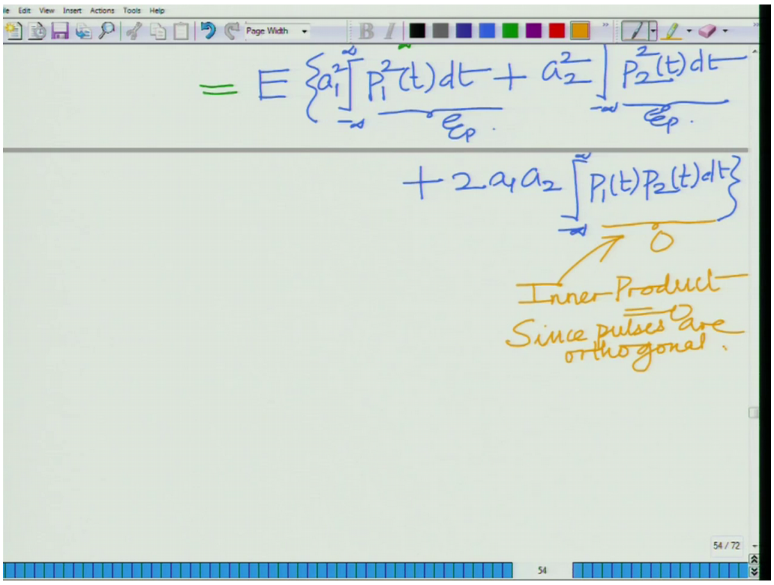

Well, which is equal to the expected value of minus infinity to infinity, a 1 square P 1 square t d t plus a 2 square integral minus infinity to infinity, P 2 square t d t plus twice a 1 a 2 integral minus infinity to infinity, P 1 t, P 2 t, d t of course this is nothing but energy of pulse E p, energy of pulse E p and this thing is nothing but integral minus infinity to infinity P 2 t, d t this is 0 this is nothing but the inner product of P 1 t, P 2 t pulses are orthogonal since pulses are orthogonal in fact, they are orthonormal.

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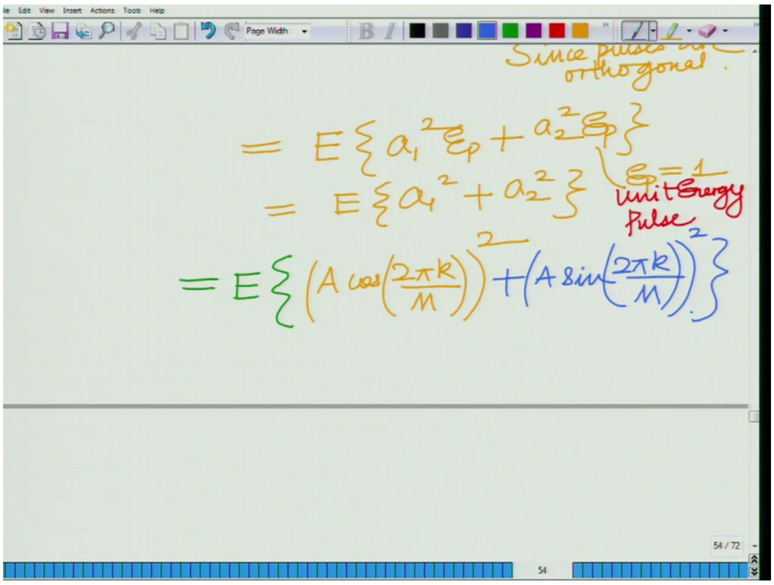

So, this is equal to expected value of and you will observe something very interesting over here this equal to expected value of a naught square E p, plus a 1 square E p, well E p equal to 1.

So, this is equal to expected value that energy of the pulse is equal to 1, a naught square plus a 1 square E p equal to 1 that is we are considering a unit energy pulse. Now this is equal to remember a naught I am sorry this is not a naught rather a 1 square plus a 2 square, and remember a 1 is A cosine 2 pi k by M whole square plus A sin 2 pi k by M whole square which is equal to of course, A square is a constant expected value of cosine square 2 pi k by M, plus sin square 2 pi k by M this is of course 1.

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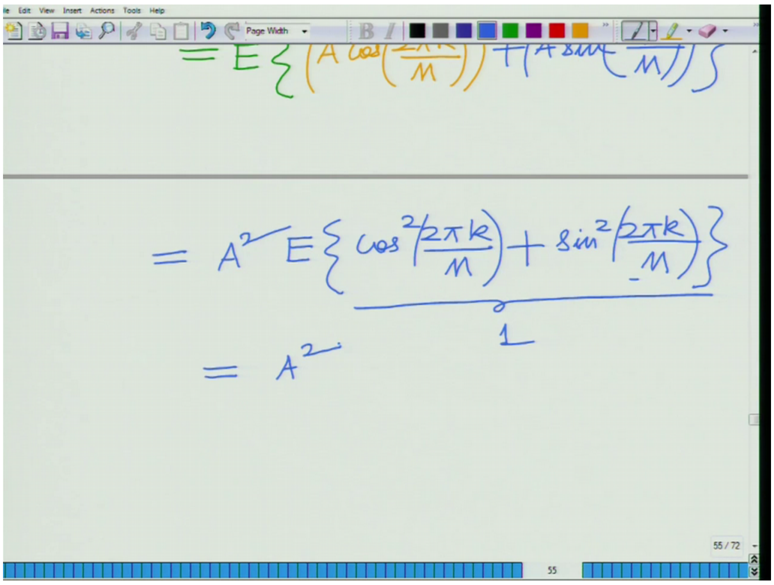

So, this is equal to a square.

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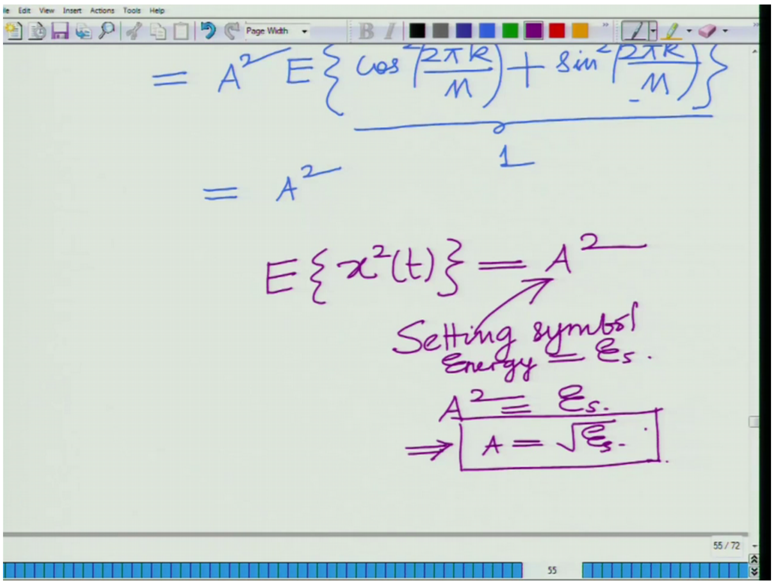

Basically, what we have is if you look at this, the expected value average symbol energy or expected value of x t square or expected value of x square equals A square setting the symbol energy equal to E s, we have a square equal to E s which implies A is equal to square root of E s.

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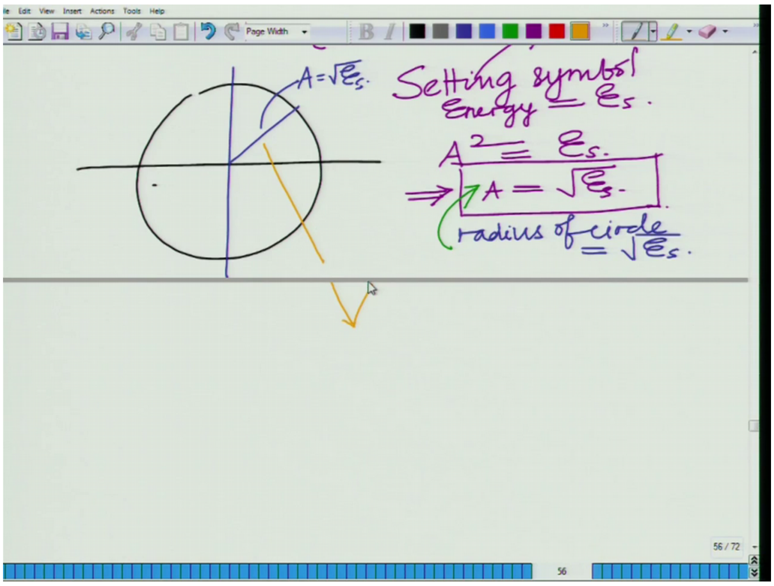

So, the radius of this circle; the radius of the circle of the constellation: so if you look at the constellation the circle of the constellation correct the constellation is a circle the radius of that circle is square root of E s. And in fact, all symbols have unlike QAM or per (Refer Time: 23:06) the average energy is he E s, here not only the average energy, but actually the energy of each symbol if you observe it because the radius is constant the energy of each symbol is E s. So, that is something interesting about the M-ary. So, if you look at the M-ary.

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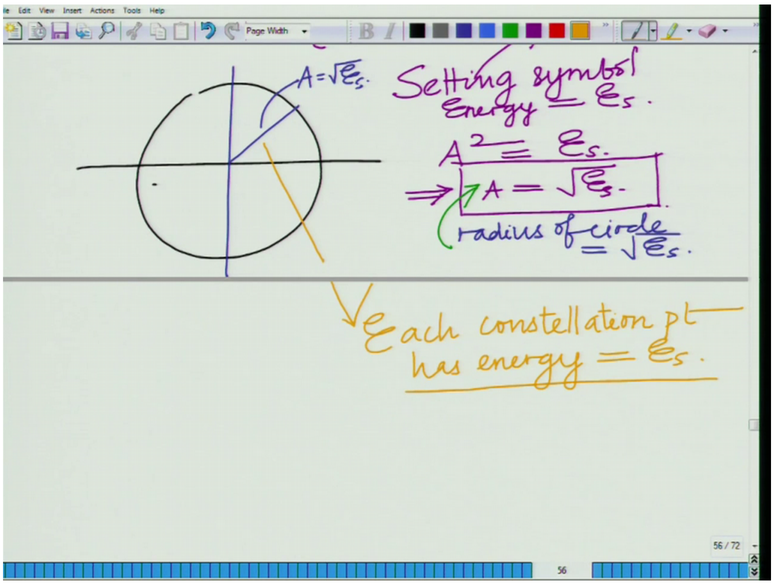

So, a equal to square root of E s that is radius of circle and you can observe that each point has ident each constellation point unlike PAM, each constellation point each constellation point has not only average energy, but each constellation point actually has energy E s. So, that is basically your M-ary PSK constellation.

So, M-ary PSK constellation consists of M points arranged around a circle of radius square root of E s and each constellation point differs from the previous 1 by phase shift of 2 pi by M, and the i-th point of the k-th point is given as a or square root of E s now we can say since a square root of E s with square root of E s cosine 2 pi k over M and a it is a combination of 2 combination of 2, basically coordinate that is a 1 which is square root of E s cosine 2 pi k over M and a 2 which is square root of E s cosine 2 pi k over.

So, will stop here and look at the receiver design, the detection and also the ensuing probability of error and other aspects is a subsequent modules.

Thank you very much.