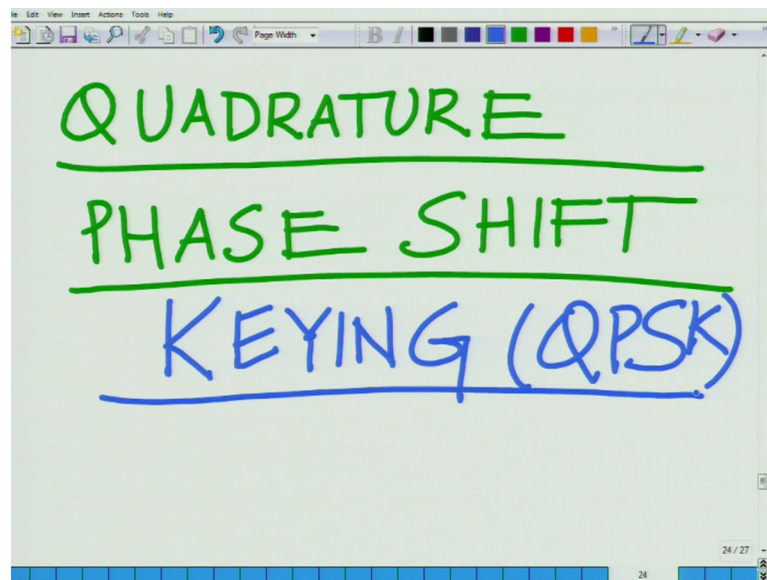


Principles of Communication Systems – Part II
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Lecture - 17
Introduction to Quadrature Phase Shift Keying (QPSK)

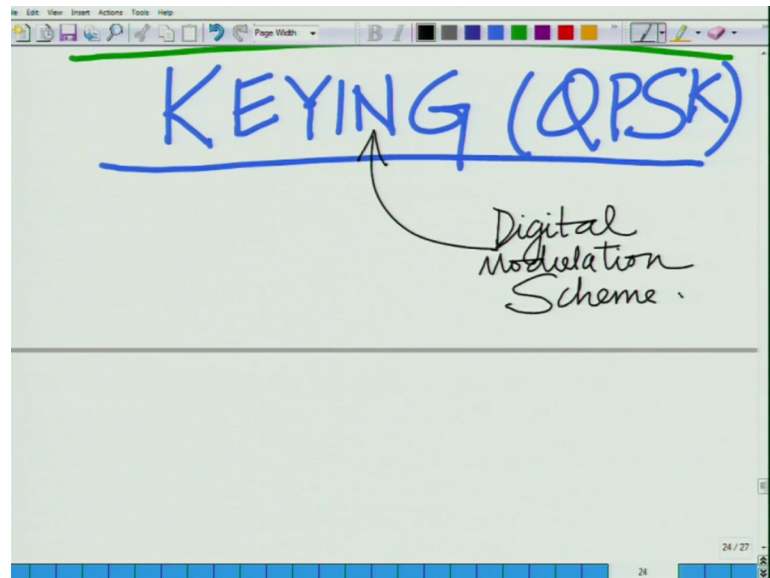
Hello. Welcome to another module in this massive open online course. So, in the previous module, we have looked at different digital modulation schemes. In this module we will start looking at another digital modulation scheme that is quadrature phase shift key.

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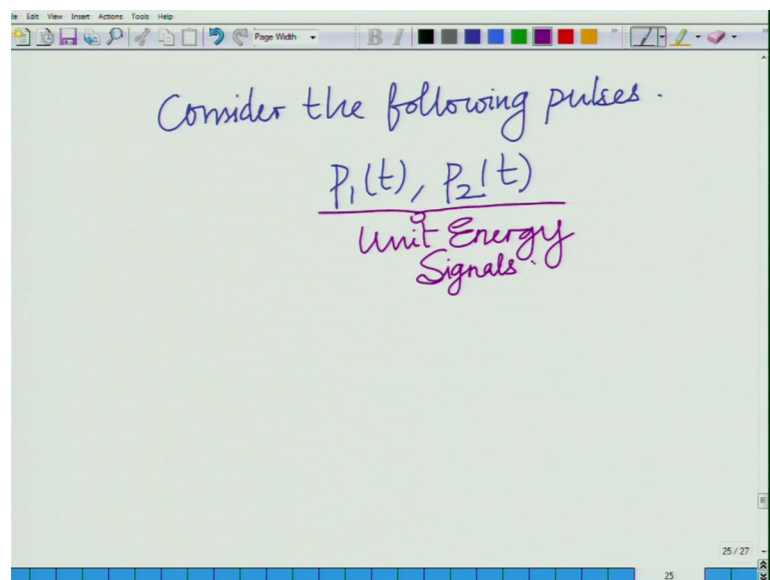
So, let us start looking at in this module, let us start looking at quadrature, quadrature phase shift key also abbreviated as QPSK. So, this stands for quadrature phase shift key.

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So, as we have again said several times before this is another digital modulation scheme. This is another digital modulation scheme consider the following basis functions.

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So, let us consider again $p_1(t)$ $p_2(t)$, these are again unit energy signals.

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The image shows a digital whiteboard with handwritten mathematical expressions. The first expression is $P_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$. The second expression is $P_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$. Below these, the condition $0 \leq t \leq T$ is written. Further down, the equation $T = \frac{k}{f_c}$ is written, with a green arrow pointing from the word 'duration' (written in purple) to the variable T . Below this, it says 'integer # of cycles' in purple. The whiteboard interface includes a menu bar at the top with 'File', 'Edit', 'View', 'Insert', 'Actions', and 'Tools'. A toolbar with various drawing tools is also visible. The bottom right corner shows '25 / 27' and a small '25'.

$$P_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$
$$P_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$
$$0 \leq t \leq T$$
$$T = \frac{k}{f_c}$$

duration \rightarrow integer # of cycles.

Now unlike FSK We have $P_1(t)$ equals square root of 2 over T , cosine $2\pi f_c t$ and $P_2(t)$, equals square root of 2 over T sin $2\pi f_c t$ both for $0 \leq t \leq T$. And T equals k over f_c that is the duration spans an integer number of cycles. This is we have seen several times before that is duration equals and integer number of cycles.

So, now if you look at this remember in a frequency shift keying the pulses were cosine $2\pi f_1 t$ cosine $2\pi f_2 t$ they were frequency shifted versions of each other. Now here we have cosine $2\pi f_c t$ sin $2\pi f_c t$.

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$$\sqrt{\frac{2}{T}} \sin(2\pi f_c t + \pi/2) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

Observe $P_2(t)$ is a phase shifted version of $P_1(t)$.

$\pi/2$ = integer # of cycles

Now, again once again, we have if you look at this if you look at this square root of 2 over t , cosine $2\pi f_c t$, let us say $\sin 2\pi f_c t + \pi/2$ this is equal to square root of 2 over t cosine $2\pi f_c t$; so these 2 pulses. So, cosine $2\pi f_c t$ is obtained by $\sin 2\pi f_c t$ by shifting the phase by $\pi/2$. So, the 2 pulses observe that P_1 or $P_2(t)$ is a phase shifted version of $P_1(t)$ it is a phase shifted version of $P_1(t)$, I have I am shifting the phase of $\sin 2\pi f_c t$ by $\pi/2$, I get cosine $2\pi f_c t$. So, these pulse $P_1(t)$ and $P_2(t)$. So, $P_2(t)$ is obtained by from $P_1(t)$ by phase shifting by $\pi/2$. Similarly, $P_1(t)$ can be obtained by $P_2(t)$ by phase shifting by minus $\pi/2$.

So, the basic point is at these 2 pulses are phase shifted versions of each other, and once again you can see both the pulses also have unit energy alright we already seen square root of 2 vertical cosine $2\pi f_c t$ this has unit energy.

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phase shift version
 $P_1(t)$

$$\int_{-\infty}^{\infty} P_2(t) dt = \frac{2}{T} \int_0^T \sin^2(\pi f_c t) dt$$

In fact, if you look at $\int_{-\infty}^{\infty} t^2 dt$ that is integral minus infinity to infinity $t^2 dt$ that is equal to integral minus infinity to infinity $\sin^2 2\pi f c t dt$, I am sorry integral from well this is from 0 to this is from we can 0 to t .

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$$= \frac{2}{T} \int_0^T \frac{1 - \cos 4\pi f_c t}{2} dt$$
$$= \frac{2}{T} \cdot \frac{T}{2} - \frac{1}{T} \frac{\sin(4\pi f_c t)}{4\pi f_c} \Big|_0^T$$
$$\boxed{\epsilon_p = 1}$$

This is equal to $\frac{2}{t} \int_0^t (1 - \cos(\sin^2 \theta)) d\theta$, which is again if you can look at it $\frac{2}{t} \left[\theta - \frac{\sin(2\theta)}{2} \right]_0^t$. This is of course, again $\frac{2}{t} \left[t - \frac{\sin(2t)}{2} \right]$. This is not $\frac{2}{t}$ apologies this is $4\pi f c t$ and by $4\pi f c$ between 0 to capital t

and this part this is 0. So, this is going to be again $\frac{dt}{T}$ over T by 2 this is equal to 1; so e p .

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Handwritten notes on a digital whiteboard:

- $p_1(t), p_2(t)$
- Unit Energy Signals.
- Energy of both $= 1$
- $p_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$
- $p_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$
- $0 \leq t \leq T$
- $T = \frac{K}{f_c}$
- duration = integer # of cycles

So, what I am trying to show is that the energy of both the pulses, that is if you look at both these pulses again that were without the same from symmetry energy of both these pulses of both is unity. That is we have e p equal to 1. Further if you look at the inner product.

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Handwritten notes on a digital whiteboard:

- Inner product,
- $\int_{-\infty}^{\infty} p_1(t) \cdot p_2(t) dt$

Now, coming to the inner product now coming to the inner product, we have integral minus infinity to infinity $P_1(t) P_2(t) dt$.

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$$\begin{aligned}
 &= \frac{2}{T} \int_0^T \sin(2\pi f_c t) \cos(2\pi f_c t) dt \\
 &= \frac{1}{T} \int_0^T \sin(4\pi f_c t) dt \\
 &= \frac{1}{T} \cdot \frac{\cos 4\pi f_c t}{4\pi f_c} \Big|_0^T
 \end{aligned}$$

Which is equal to integral 0 to T over t , well $\sin 2\pi f_c t$ into cosine $2\pi f_c t$, into dt equals 1 over T integral 0 to T $\sin \theta \cos \theta$ is $\sin 2\theta$. So, this is $\sin 4\pi f_c t$ dt which is equal to if you integrate, this is again one over T $\sin 4\pi f_c t$ integral is cosine $4\pi f_c t$ divided by $4\pi f_c$ between the limit 0 to T .

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$$\begin{aligned}
 &= \frac{1}{T} \cdot \frac{\cos 4\pi f_c T - 1}{4\pi f_c} \\
 &= 0
 \end{aligned}$$

Inner product $= 0$
 \Rightarrow Pulses are orthogonal.

And now again of course, let me just substitute the limit is this is cosine 4π now f_c into t is k . So, $4\pi k$ minus cosine 0 is 1 divided by $4\pi f_c$ cosine $4\pi k$ this is equal to 1 . So, this is basically equal to 0 ; so inner product of the 2 pulses.

So, if you look at the inner product between pulse $P_1(t)$ and $P_2(t)$ that inner product is 0 . So, both these pulses are basically orthogonal. So, inner product of pulses, inner products of the pulses is 0 . So, this implies the pulses are orthogonal.

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Inner product
 \Rightarrow Pulses are orthogonal.

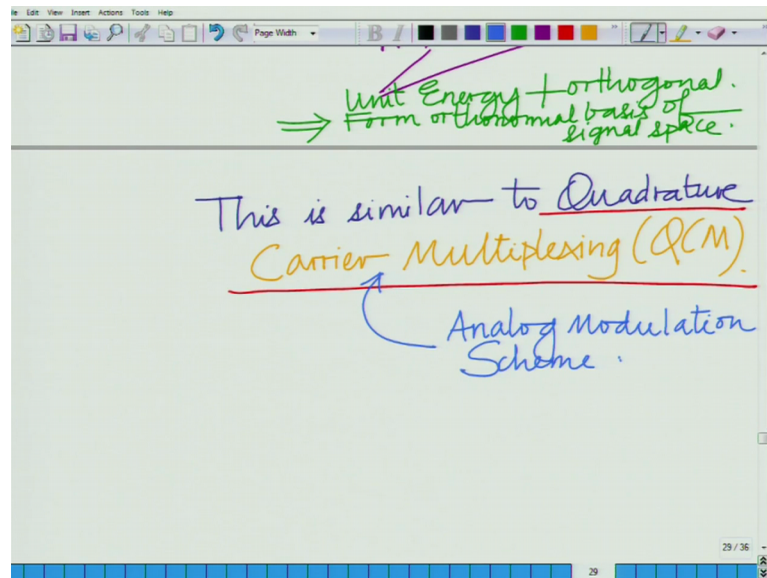
$$\sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

$P_1(t)$ $P_2(t)$

Unit Energy + orthogonal.

Therefore, $P_1(t)$; so square root if you look at $P_1(t) P_2(t)$ square root of 2 over t cosine $2\pi f_c t$, which is your $P_1(t)$ and $P_2(t)$ which is square root of 2 over t cosine or other $\sin 2\pi f_c t$, these 2 pulses $P_1(t) P_2(t)$ these are both unit nor unit energy plus orthogonal implies the they form the orthonormal basis a.

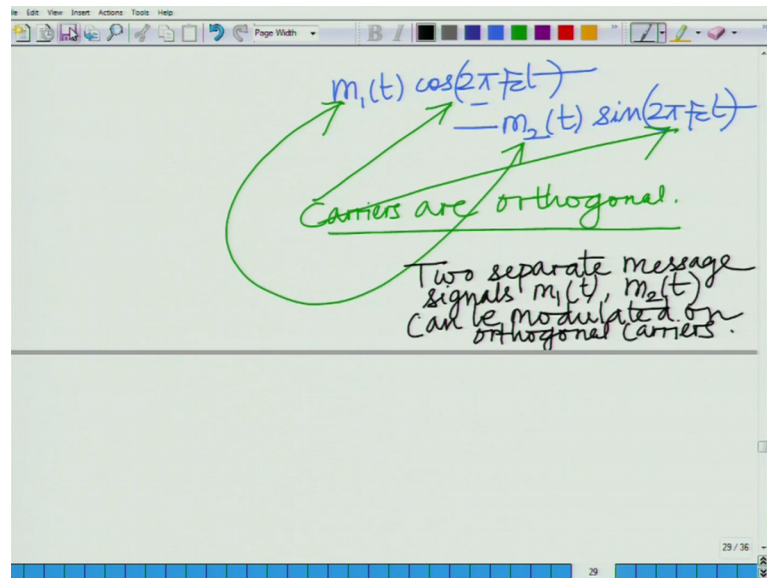
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For a signal space for orthonormal basis of they form orthonormal basis of a signals base and this is similar to if you look at this this is similar to quadrature amplitude modulation.

If you remember some of you go down your previous course is similar to quadrature not quadrature amplitude modulation, I am sorry this is similar to quadrature carrier multiplexing well in quadrature carrier multiplexing, which is an analogue modulation scheme remember quadrature carrier modulation is an analogue modulation scheme, in quadrature carrier multiplexing, what we do is we modulate a message signal $m_1(t)$ on to $\cos(2\pi f_c t)$ minus $m_2(t)$.

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So, 2 different message signals can be modulated one on cosine and one on sin because these 2 are orthogonal. So, these 2 carriers are orthogonal we have seen in quadrature carrier multiplexing that these 2 carriers are orthogonal therefore, you can modulate 2 separate signals 2 separate signals message signals $m_1(t)$, $m_2(t)$ can be modulated on.

So, one can modulate. So, what we have seen in quadrature carrier multiplexing is we have cosine 2 consider for a carrier frequency f_c , we have $\cos(2\pi f_c t)$ $\sin(2\pi f_c t)$ we have shown that these 2 carriers are orthogonal. And therefore, one can modulate 2 separate signals $m_1(t)$ $m_2(t)$ on each carrier $m_1(t)$ on the cosine channel $m_2(t)$ on the sin channel. In fact, in fact these are known as the in phase and quadrature channels. So, that is important this is an analogue modulate quadrature carrier multiplexing is an analogue modulation scheme.

Similarly, in digital communication we have 2 pulses $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ both of these are orthogonal both of these are phase shifted versions of each other. And we can similarly modulate 2 separate information symbols digital information symbols on each of the sub carriers this is nothing, but quadrature phase shift key. So, what I am going to do.

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Handwritten slide content:

$$x(t) = a_1 P_1(t) + a_2 P_2(t)$$

Two symbols can be modulated on $P_1(t)$, $P_2(t)$.

To begin with, each can carry a single bit of information.

Now, is basically I am going to generate my signal $x(t)$ as a_1 times $P_1(t)$ plus a_2 times $P_2(t)$. Now, note is that I am modulating not one symbol, but 2 symbols a_1 and a_2 . So, 2 symbols or 2 bit is can be modulated on $P_1(t)$ and $P_2(t)$. For instance, each can carry one bit of information they can more also, but to begin with let us say each carries to begin with let us say, to begin with let us say each can carry a single bit of information.

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Handwritten slide content:

To begin with, each can carry a single bit of information.

EX: $a_1 = \pm A$ ← Each carries 1 bit of information
 $a_2 = \pm A$ ←

Average bit Energy = E_b
 $\Rightarrow A = \sqrt{E_b}$

For instance, example a_1 equals plus or minus A a_2 equals plus or minus A and for bit energy equals E_b average bit energy equals E_b , we have seen already from BPSK this

implies a equals square root of E_b . So, that one can ensure average bit energy. So, this is plus or minus square root of E_b plus or minus square root of E_b , and there for now each carries one bit of information a_1 carries one bit of information, a_2 carries one bit of. So, each carries one bit of information implies $x(t)$ equals $a_1 p_1(t) + a_2 p_2(t)$.

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Average bit energy $\Rightarrow A = \sqrt{E_b}$

$$\Rightarrow x(t) = a_1 p_1(t) + a_2 p_2(t)$$

Carries 2 bits of information

Since a_1 a_2 carry one bit of information each $x(t)$ in total carries 2 bit is of information $x(t)$ carries 2 bit is of information since each a_1 and a_2 carry one bit of information.

So, similar to quadrature carrier multiplexing separate bit is of information in quadrature carrier multiplexing it is an analogue modulation scheme. So, separate message signals $m_1(t)$ and $m_2(t)$ can be modulated on the orthogonal carriers here we are transmitting separate information symbols a_1 and a_2 on the 2 orthogonal pulses that is $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, these are 2 orthogonal pulses unit not or pulses orthogonal they form the basis the orthonormal basis of a signals space which generated by linear combinations of these pulses

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$$x(t) = \begin{cases} A_1(t) + A_2(t) \\ A_1(t) - A_2(t) \\ -A_1(t) + A_2(t) \\ -A_1(t) - A_2(t) \end{cases}$$

Set of 4 possible Transmit Signals $x(t)$.
Forms the basis for QPSK or Quadrature Phase Shift Keying.

Now, therefore, the total number of possible signals we have, each a 1 a 2 can be plus a or minus A. So, the possible set signals is a 1 equals a.

So, let us write the possible set of signals. So, this is going to be a times P 1 t plus a times P 2 t a times P 1 t minus A times that is a times P 2 t then a 2 equals minus A minus A times P 1 t plus a times P 2 t. And finally, naturally minus A times P 1 t minus A times P 2 t this is the set of 4 possible signals in the scheme set of 4 possible signals since each can be plus or minus A sign. So, we have term 4 possible transmit signals x t we have 4 possible transit signals x t and this forms the basis of quadrature phase shift keying we are going to explore this further this forms the basis for QPSK or basically quadrature phase shift keying this forms the basis for quadrature phase shift keying.

So, that is what we have. So, what we have seen in this module is we seen in production to a new 2 digital modulation scheme and. In fact, very important digital modulation scheme this is quadrature phase shift keying we are not seen such a scheme before because. So, far the schemes that we have seen that is BPSK ba binary phase shift keying amplitude phase amplitude shift keying and frequency shift keying all carry one bit of information per each duration.

Now, here we are using a scheme which is using 2 pulses which are orthogonal to each other and transmitting independent bit is of information on each pulse. So, therefore, variable to convey 2 bit is of information per duration of a single symbol per single

symbol duration that is quadrature. So, we have seen the possible we have seen the basic signals that is $\sqrt{2} \cos(2\pi f_c t)$ and $\sqrt{2} \sin(2\pi f_c t)$. These are both unit energy, orthogonal and now we are generating a signal by combination of the that is $a_1 \sqrt{2} \cos(2\pi f_c t) + a_2 \sqrt{2} \sin(2\pi f_c t)$ that is modulating a_1 on $\sqrt{2} \cos(2\pi f_c t)$ and a_2 on $\sqrt{2} \sin(2\pi f_c t)$. Each a_i can be plus or minus A . So, we are able to transmit 2 bits per each symbol. And we have possible of 4 possible transmit signals by generated by all possible combinations of a_1 and a_2 and this forms the basis of quadrature phase shift key.

So, we will stop here and will continue to explore this in the subsequent module.

Thank you very much.