Principles of Communication Systems – Part II
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Lecture - 16
Optimal Decision Rule for FSK, Bit-Error Rate (BER) and Comparison with BPSK, ASK

Hello. Welcome to another module in this massive open online course. So, we are looking at frequency shift keying all right. And in frequency shift keying well we are considering 2 different wave forms, to represent the information bit 0 and one which is shifted in frequency correct.

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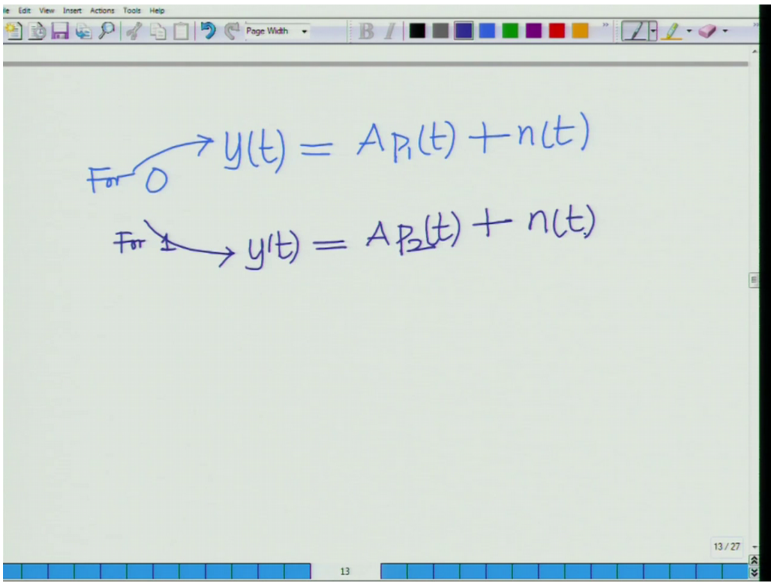

So, we are considering 2 different frequencies f 1 and f 2 since they are wave forms are shifted versions of each other with frequencies this is known as frequency shift key all right. And what we have seen in this is that my y t corresponding to 0 is a times P 1 T plus n t. That is, this is for information bit 0 for 1, we have y t equals a times P 2 T plus n t.

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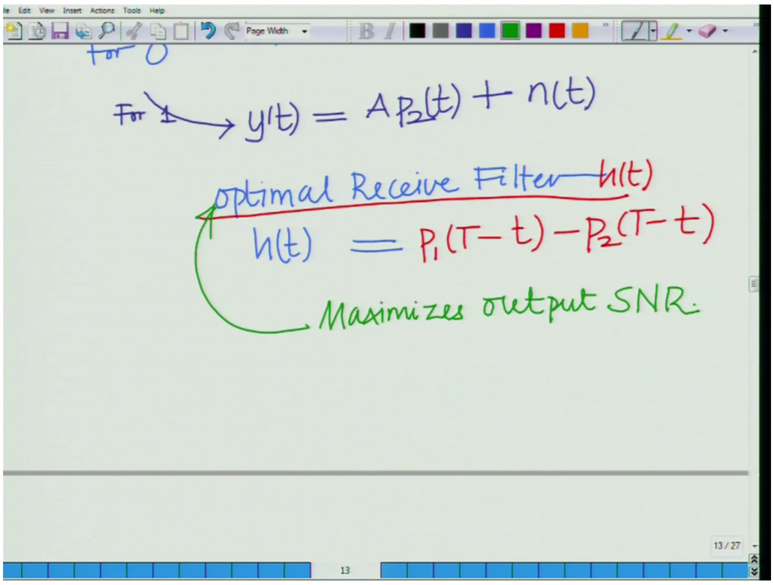

We have seen the matched filter optimal received filter, optimal receive filter a is h t equals P 1 well T minus t minus P 2 T minus P 2 T minus t. So, this is the optimal receive filter, optimal receive filter in the sense we must be well familiar by now optimal in the sense this maximizes the output SNR or signal donates power ratio.

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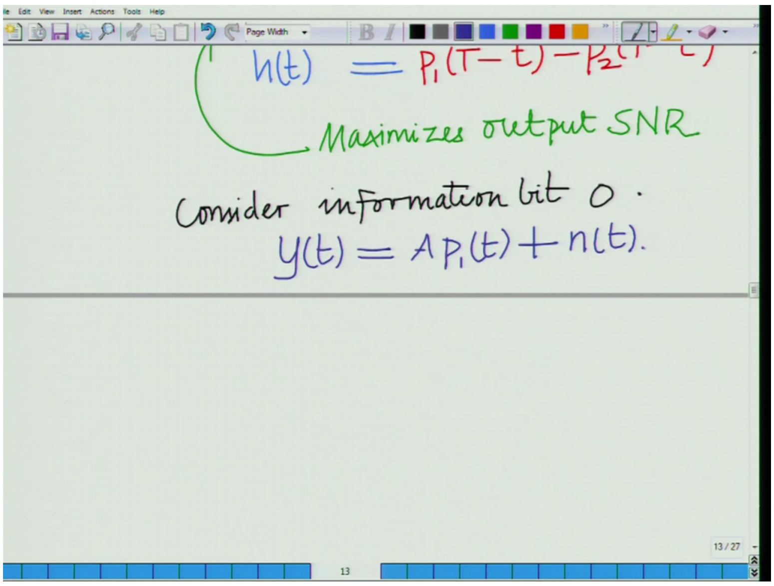

Now, consider the transmission of information bit 0, consider or consider information bit 0. We have for this y t equals A times P 1 T plus n t.

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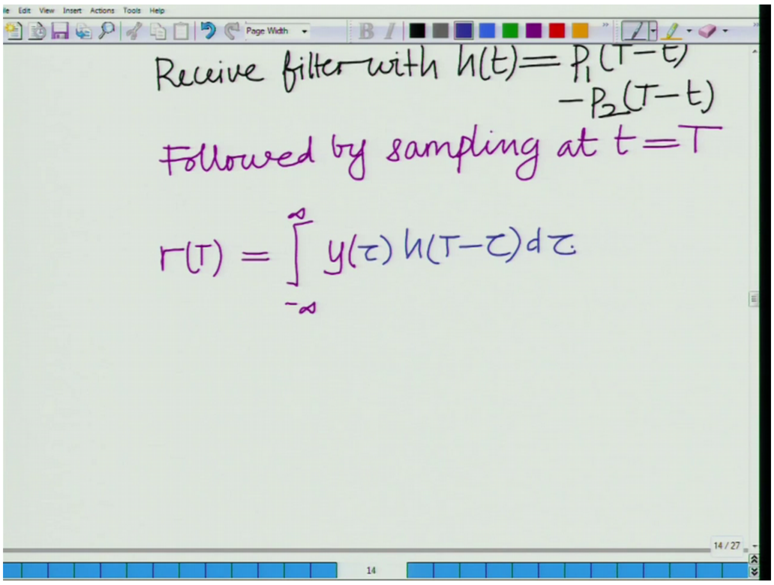

Now, matched filter or receive filter with h t, h t equals P 1 T minus t minus P 2 T minus t followed by sampling at followed by sampling at t equal to T. Now that gives us if you can look at it, we have r of t equals well integral minus infinity to infinity, y of well y of tau times h T minus tau d tau substituting for h of t.

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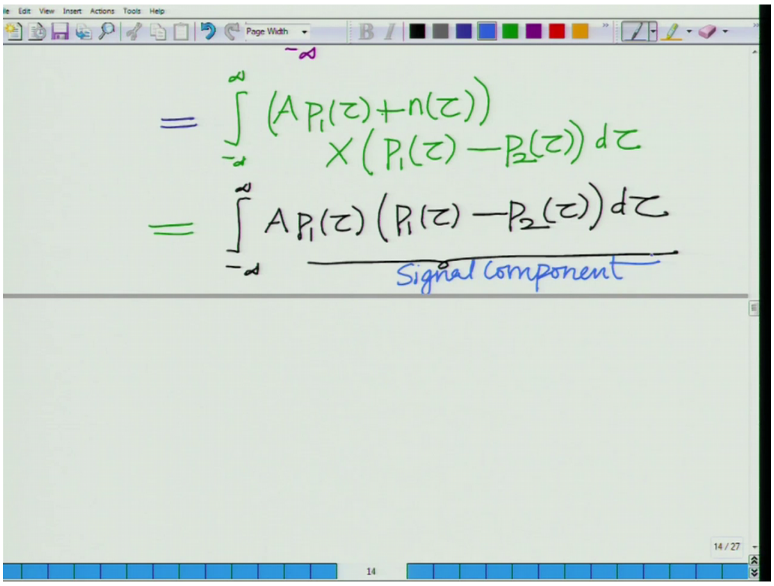

This gives us integral this gives us integral minus infinity to infinity y of t is basically you are well this is a P 1 times P 1 of tau a P 1 of tau, of course, there is noise also a P 1 of tau plus n tau times now the filter h t capital H capital T minus tau P 1 which is you can see is simply P 1 tau minus P 2 tau d tau, and that gives us well that can be split naturally into 2 terms, A P 1 tau into P 1 tau minus P 2 tau d tau. This is your signal component, plus integral minus infinity to infinity n tau, P 1 tau minus P 2 tau d tau and this is your noise component.

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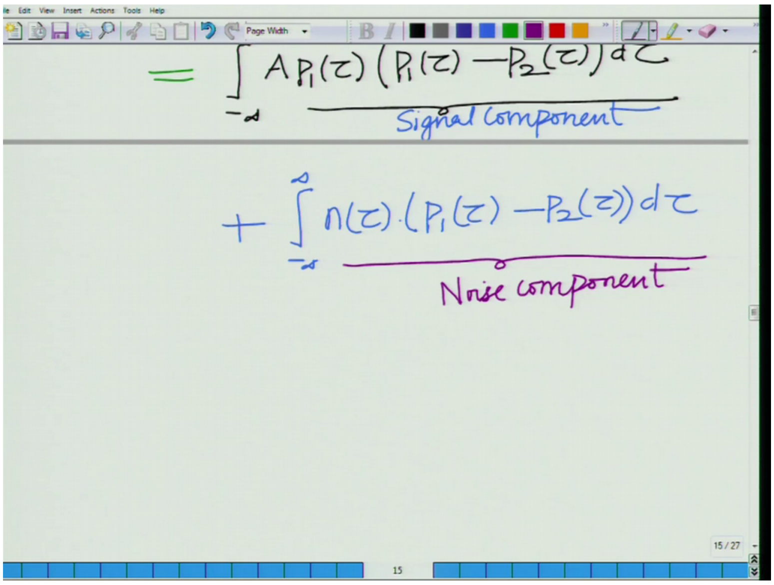

So, corresponding to the transmission of information signal the inform corresponding to information bit 0, we have computed what is the output after filtering with h t followed by sampling at t equal to capital T where capital T is the symbol duration these are in the signal component and the noise component.

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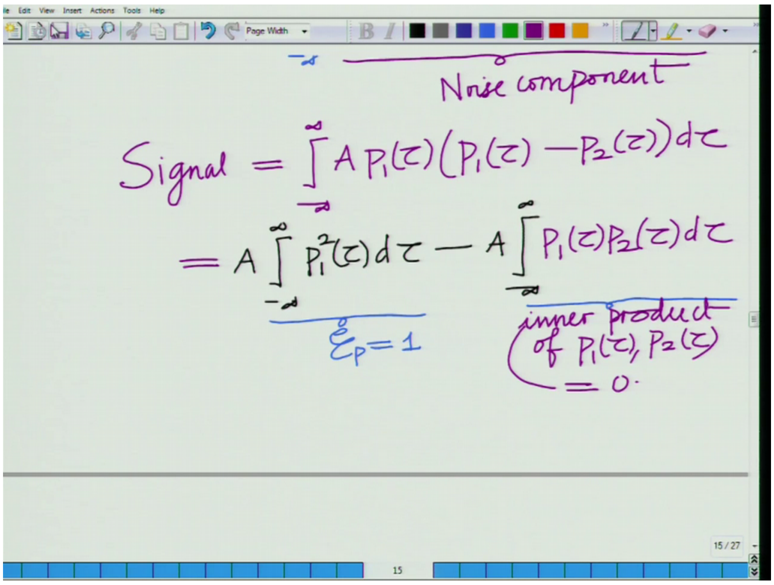

Now, let us simplify both these components individually now let us come to the signal component. Now the signal equals minus infinity to infinity a times P 1 tau into P 1 tau minus P 2 tau d tau, which is equal to integral minus infinity to infinity a times P 1 square tau d tau minus A times integral minus infinity to infinity P 1 tau P 2 tau d tau. Now you can see this is nothing, but the energy of the pulse this is equal to 1 and this integral minus infinity to infinity P 1 tau P 2 tau d tau, we have seen this is the inner product of P 1 tau comma P 2 tau which is equal to 0. So, this is my product is 0 because P 1 tau and P 2 tau we have said are orthonormal basis functions of the signal space.

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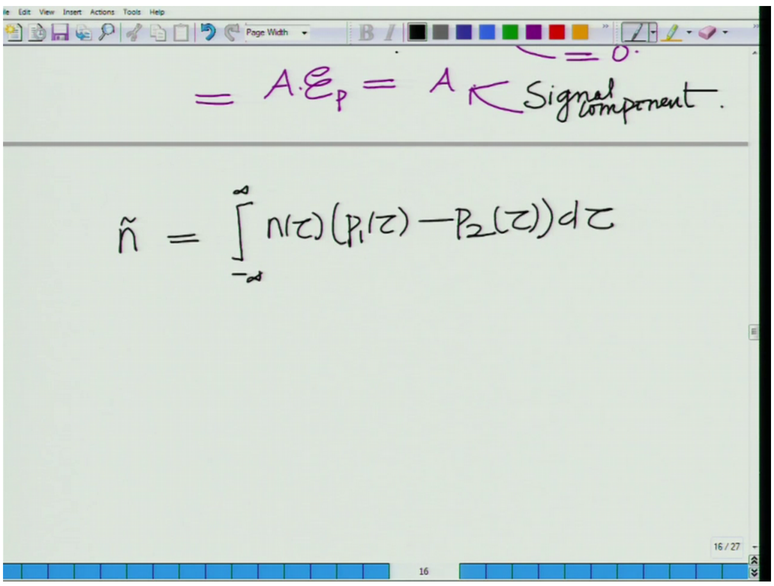

Therefore all that is left is a times E p substituting E p equal to 1, this is remember the signal component after matched filtering. Now if you look at the noise component n tilde, which is equal to minus infinity to infinity n tau P 1 tau minus P 2 tau d tau.

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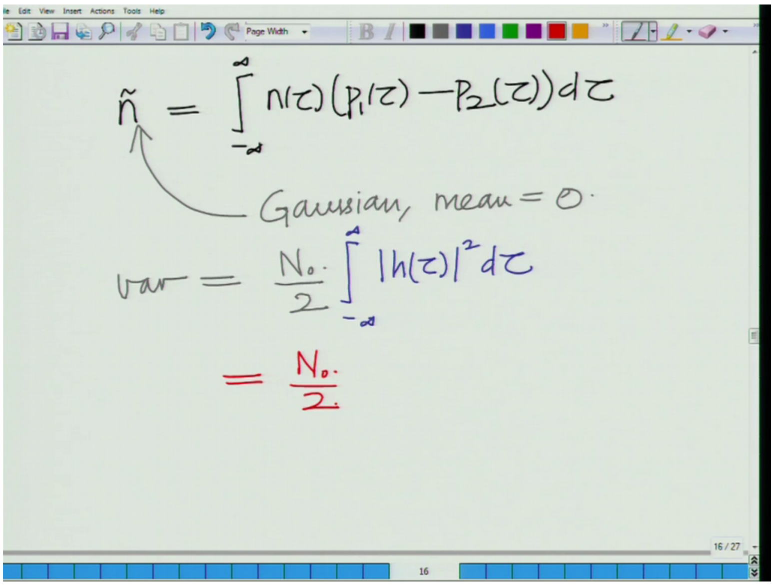

Now, this noise we have previously see in from an analysis of the noise output at the noise at the output of a matched filter we are said the this is the input noise is a Gaussian noise process the output noise process also Gaussian once you sample it you get a Gaussian random variable, if the input noise process is 0, mean the output noise process is also 0 mean. So, n tilde is Gaussian mean equal to 0. Now the variance is eta naught by 2 and this also we have seen integral minus infinity to infinity magnitude h tau square d tau or h T minus tau square d tau, which is equal to n naught by 2, integral minus infinity to infinity well P 1 tau or.

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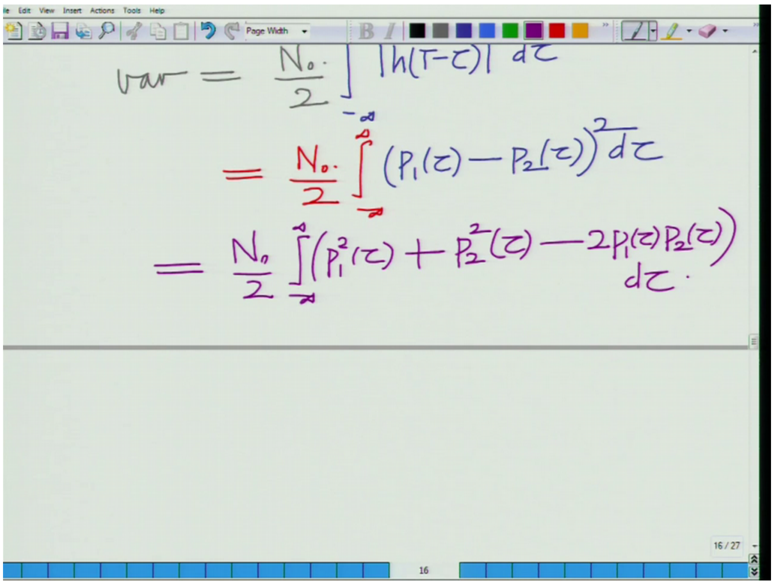

Let me write it for the sake of just being clear let me write this as T minus P 1 tau minus P 2 tau square, d tau. Now this is now expand this P 1 tau minus P 2 tau whole square, what you have is P 1 square tau, plus P 2 square tau minus, twice P 1 tau P 2 tau d tau.

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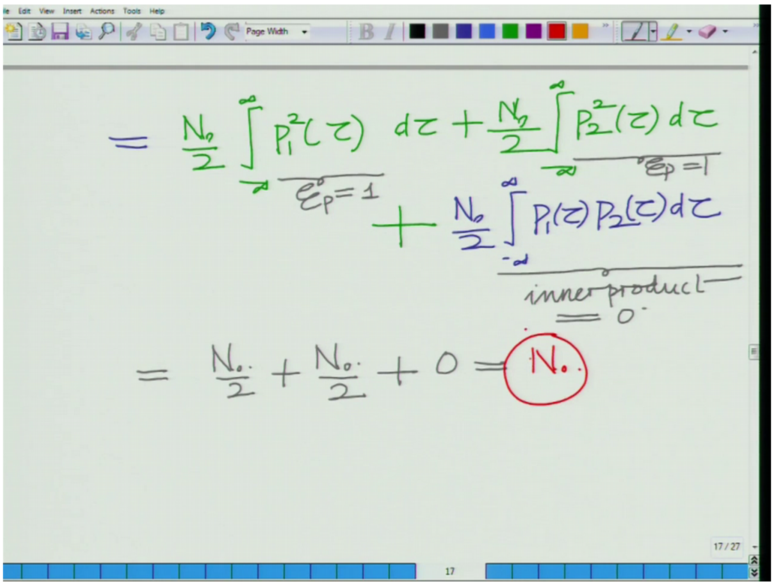

Now, is split this into 2 terms or split this into other 3 terms. This is equal to n naught by 2, integral minus infinity to infinity, P 1 square T minus tau d tau plus n naught by 2 integral minus infinity to infinity P 2 square not T minus tau.

P 2 square tau d tau plus the third term, which is n naught by 2 integral minus infinity to infinity P 1 tau P 2 tau d tau. And now you can see once again this is integral minus infinity to infinity P 1 square tau d tau that is E p the energy of the pulse which is 1, integral minus infinity P 2 square tau d tau this is also E p which is equal to 1. This is once again this is inner product between P 1 and P 2, which is equal to 0. So, what is remaining is basically if you can see the noise power or the noise variance that is n naught by 2, plus n naught by 2 we are setting E p equal to 1 plus 0, which is equal to n naught n naught by 2 plus n naught which is equal to 0 this is a noise power at the output after sampling.

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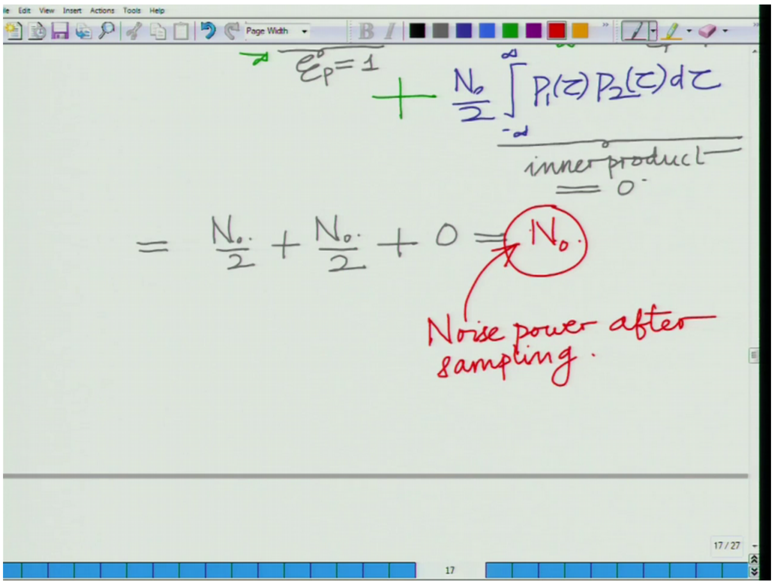

So, this is the noise power. Previously we had always noise power n naught by 2.

But now we have noise power after sampling equals after sampling equals n naught. So, observe that this is basically once again slightly different with respect to what we had for amplitude shift keying and also binary phase shift keying as noise power, after match filtering and sampling was n naught by 2 now we have noise power n naught.

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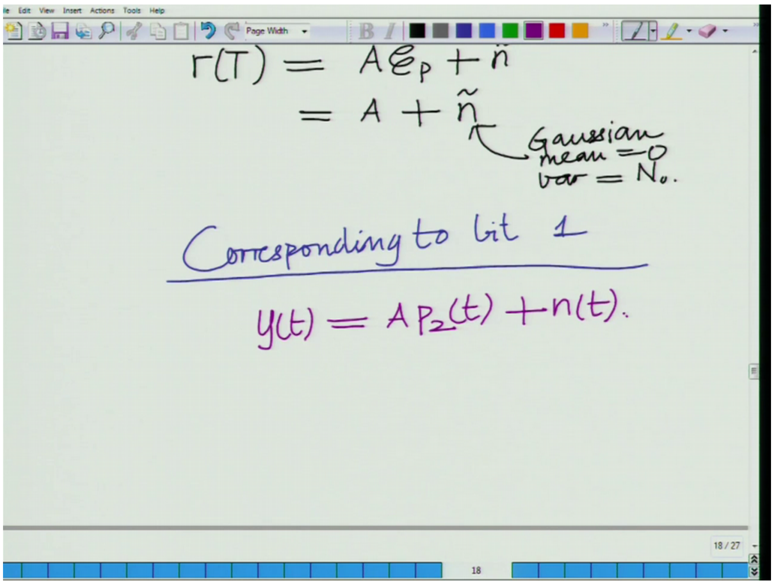

So, basically the output is r T equals well A E p plus n tilde setting E p equal to 1 this is a times writing E p again because it is. So, that the expression is general and can be valid also for scenarios where E p is not necessarily equal to 1 n tilde this is Gaussian with mean equal to 0, and variance is equal to n naught. Now similarly corresponding to the transmission of information bit corresponding to bit 1, well corresponding to bit one we have y t equals a times P 2 T plus n t.

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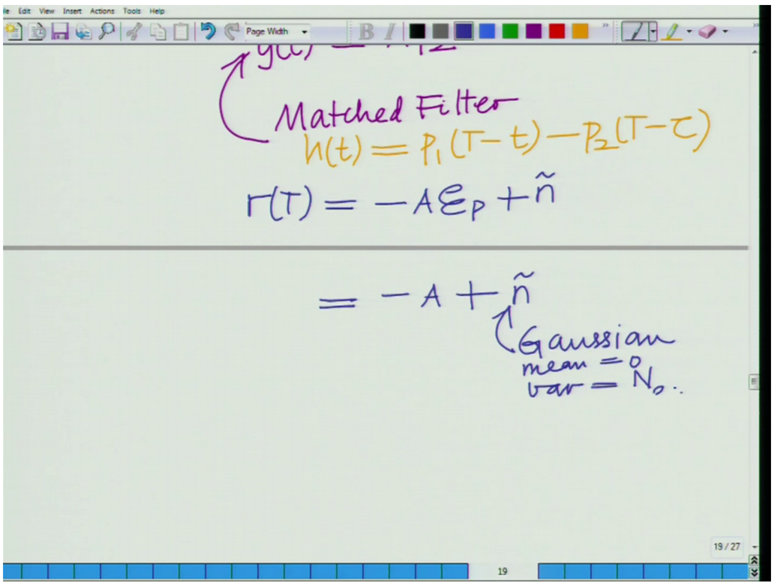

Now, when we matched filter this, or receive filter this, with once again h of t equals P 1 T minus t minus, P 2 T minus t the output will be well, you can verify this r T is minus A E p plus n tilde again setting equal to 1, this is equal to minus A E p or minus simply a minus A plus n tilde, where n tilde is Gaussian. Once again is a same thing the mean equal 0 the variance is equal to n naught.

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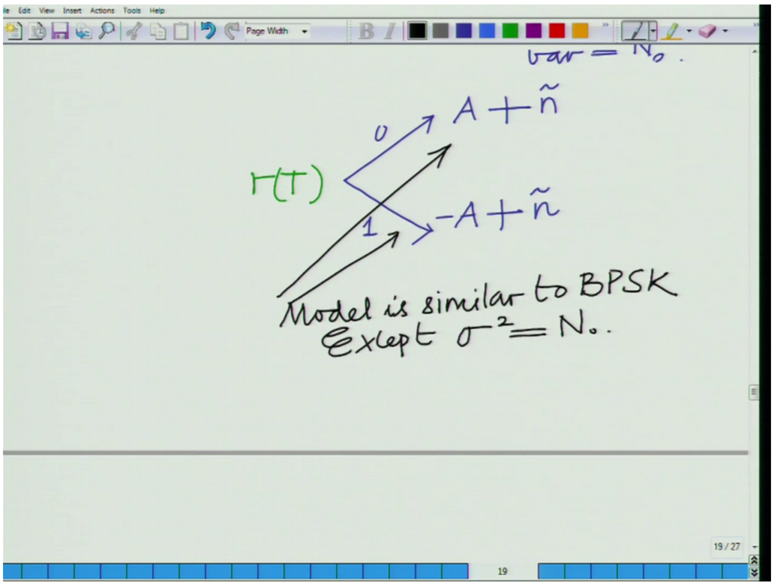

Therefore, now if you can look, if you look at this team comprehensively I can summarise this as the output after filtering and sampling is basically given as corresponding to 0 and 1, this is A plus n tilde this is minus A plus n tilde. Now you can see this is similar to BPSK this model, if you can observe closely, model is similar to BPSK that is binary phase shift keying except noise variance is sigma square equals n naught, naught n naught by 2 right. In BPSK in the noise variance what we are computed n tilde that is noise variance after filtering and sampling was n naught by 2, here it is n naught.

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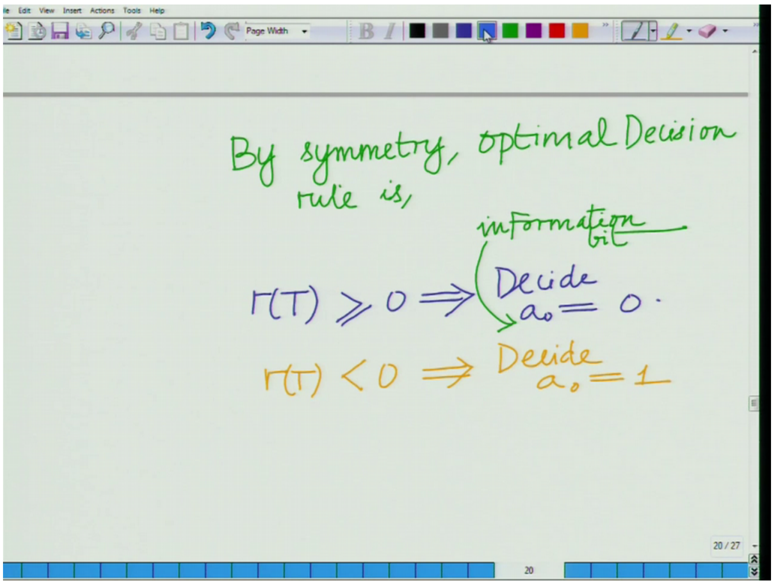

Therefore, now the threshold again by symmetry, look at it the mean is shifted for 0 the mean shifted to a for 1, the mean is shifted to minus A, symmetry by symmetry again the optimal detection rule is to compare the sample r T with 0. So, once again by symmetry by symmetry, the optimal decision rule, optimal decision rule is the optimal decision rule is once again that is your r T greater than equal to 0, implies decide well decide a naught equal 0 that is this is the information bit. On the other hand if r T is less than 0 that implies decide a naught is equal to 1. So, therefore, that summarises your decision rule.

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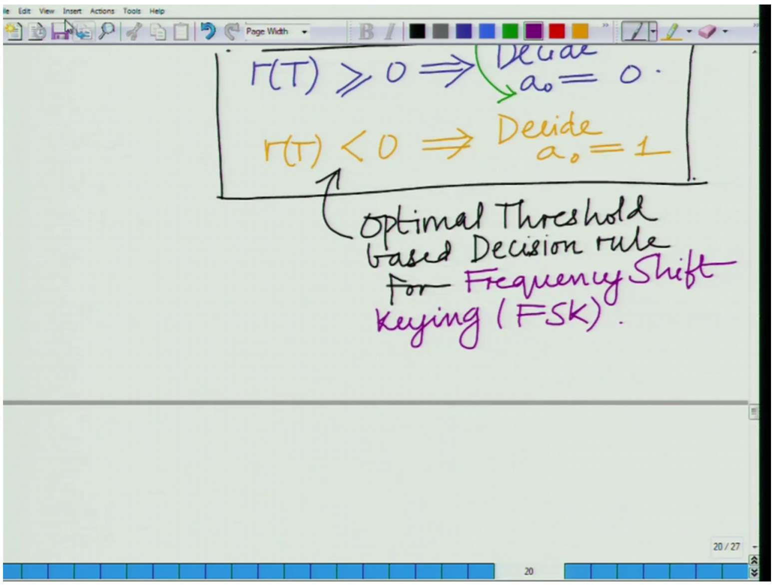

This is the optimal decision of FSK is not very different from that. In fact, very similar to that of b s k this is the optimal. In fact, we can say threshold based decision rule for frequency shift keying FSK or frequency shift keying, that is f frequency shift keying FSK.

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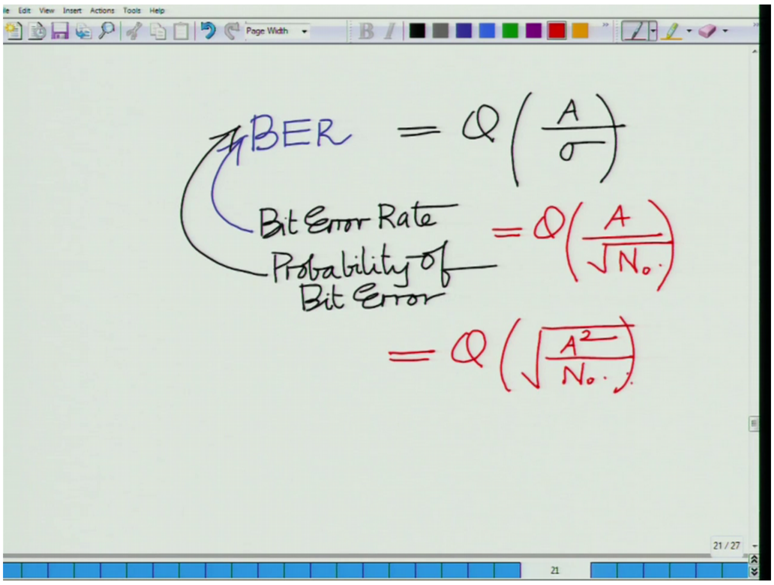

And the corresponding bit error rate again we can see, corresponding bit error rate there is if you look at this bit error rate, which is either which is again another thing that you should be familiar very familiar with, by now this is the bit error rate bit error probability or this known by many terms or the probability, this is equal to Q times A divided by sigma which is equal to Q times A divided by square root of n naught, which is equal to Q times A square divided by n naught.

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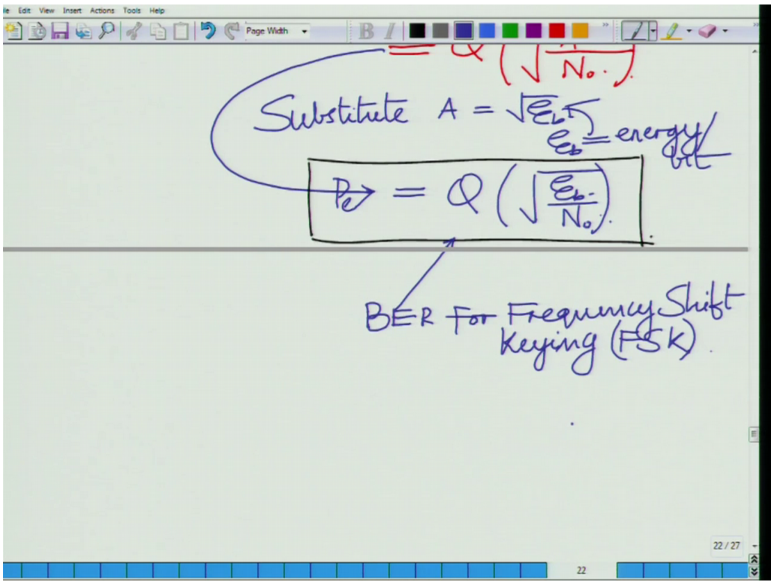

Now, substitute A equal to Eb substitute A equal to square root of e b, as where Eb as we have seen before equals the energy per bit. So, bit error this becomes Q square root of Eb over n naught this you can say is the probability of bit error or bit error rate. So, this is the bit error rate for frequency shift keying, let us note that. This is the bit error rate this is the bit error rate for frequency shift keying that is FSK. So, this is the bit error rate for frequency shift keying, and again you can see if you compare it with bit error rate now bit error rate for.

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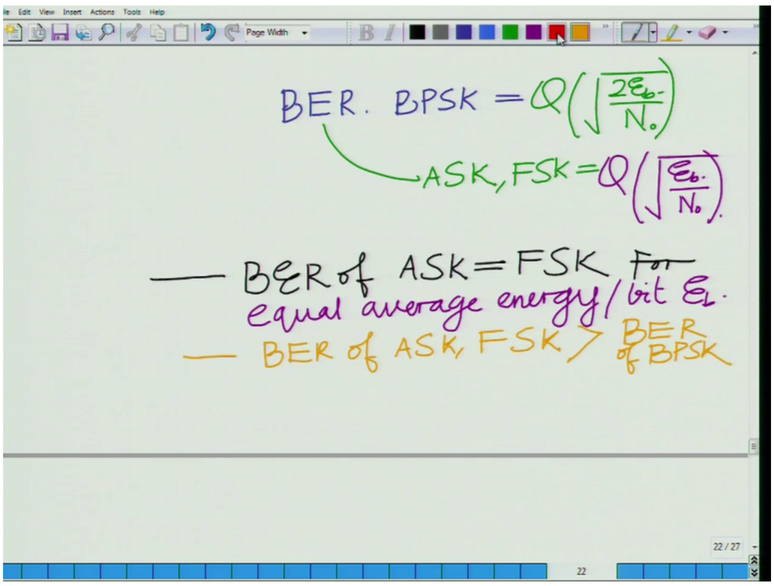

Now, if you look at bit error rate for BPSK that is basically Q square root 2 Eb over n naught. And bit error rate for both amplitude shift keying comma frequency shift keying is Q square root Eb over n naught. And if you remember or discussion our previous discussion the Q function is the yield probability of a Gaussian random of the standard Gaussian random variable therefore, the Q function is decreasing in it is argument and therefore, this implies that the bit error rate of both frequency shift keying and amplitude shift keying for an average bit energy of Eb is lower the bit error, I am sorry bit error rate is higher the bit error rate performance is worst because remember the Q function is decreasing therefore, Q of square root of 2 Eb by n naught is less than Q of square root Eb over n naught.

So, the bit error rate of both amplitude shift keying and frequency shift keying is higher and how much and how in efficient our amplitude shift keying and frequency shift keying in comparison to binary phase shift keying we have seen previously that they are 3 dB worse in comparison to binary phase shift keying the reason being you need twice the average bit energy that is you need twice the average bit energy Eb to achieve the same bit error rate performance in amplitude shift keying and frequency shift keying as that of BPSK. So, let us also note that once again all though that should be plenty clear. So, the first point is BER of ASK is identical to that of FSK; obviously, the comparison has to be fair for equal for equal average energy per bit Eb. Further the BER of ASK comma FSK is greater than BER of BPSK.

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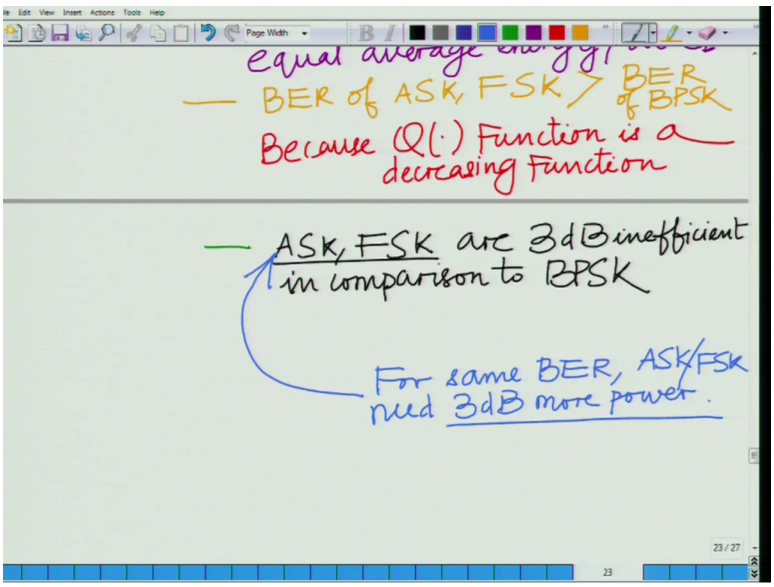

This is because remember Q function is decreasing in it is argument. Gaussian tale probability is a decreasing a monotonically decreasing function. And more importantly we also have the fact that how in efficient are there ASK comma FSK amplitude shift keying and frequency shift keying are 3 dB in efficient or 3 dB worse. Which basically means for same bit error rate to achieve the same bit error rate you are ASK amplitude shift keying slash frequency shift keying need 3 dB they need 3 dB more power.

So, they are basically 3 dB to 3 dB worse amplitude shift keying and frequency shift keying at 3 dB worse that binary phase shift keying. So, basically you can say binary phase shift keying for the same average energy per bit Eb achieves the lowest bit error rate amongst BPSK, ASK and frequency shift keying all right. So, BPSK is the amongst these 3 BPSK binary phase shift keying is the most efficient modulations scheme and the reason for that if you explore it will be because binary phase shift keying for the same average bit error rate uses antipodal signalling that is it uses signals plus A and minus A all right which maximises the distance between the constipation points.

You can note this in formally all though we have not shown this regressly it maximises the distance between the conciliation points for the same average bit error rate. While both amplitude shift keying and frequency shift keying have do not do this all right. So, they have a poor performance. So, their performance is poor in comparison to the in comparison to binary phase shift keying which has which we have shown to be in a very efficient digital modulation scheme. So, that completes our discussion on frequency shift keying based on the based on this frame work remember of the signal space the concept of signal space where we are now implying not one, but 2 pulse wave forms which are both of unit energy normalised to unit energy all right and also no orthogonal to each other.

So, they constitute and orthonormal bases for this signals space we are representing the information bit 0 using a times P 1 T that is along the signal P 1 T bases function P 1 T, we are representing the information bit one by a times P 2 T which is along the direction in the signals space or which is along the basis function P 2 T and therefore, we are now using a 2 dimensional signal space in frequency shift keying that is the interesting aspect about frequency shift keying all right.

So, we will stop here.