**Principles of Communication Systems – Part II**
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**Lecture - 15**
# Introduction to Frequency Shift Keying (FSK)

Hello, welcome to another module in this massive open online course. In this module, let us look at another digital modulation technique that is frequency shift key.

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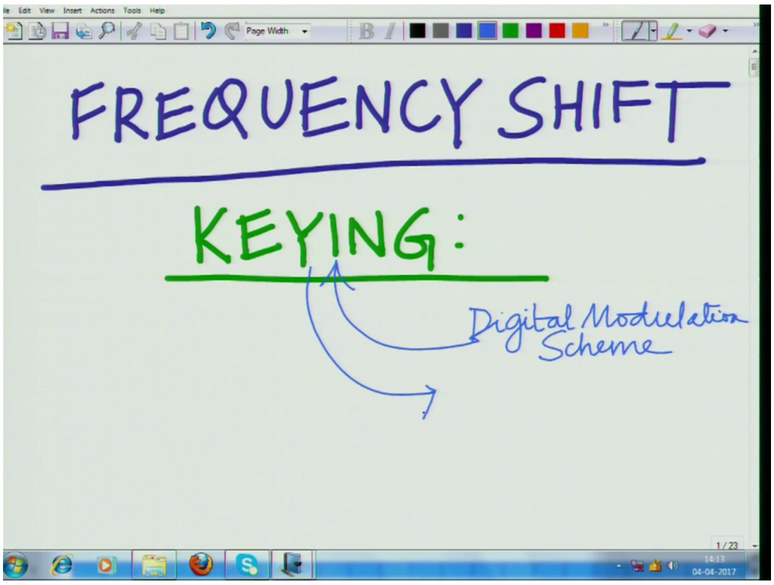


So, we will be looking at a new digital modulation scheme which is termed as frequency shift key. This is a digital modulation scheme as we have already said and it is based on the signal space concept that we have previously described.

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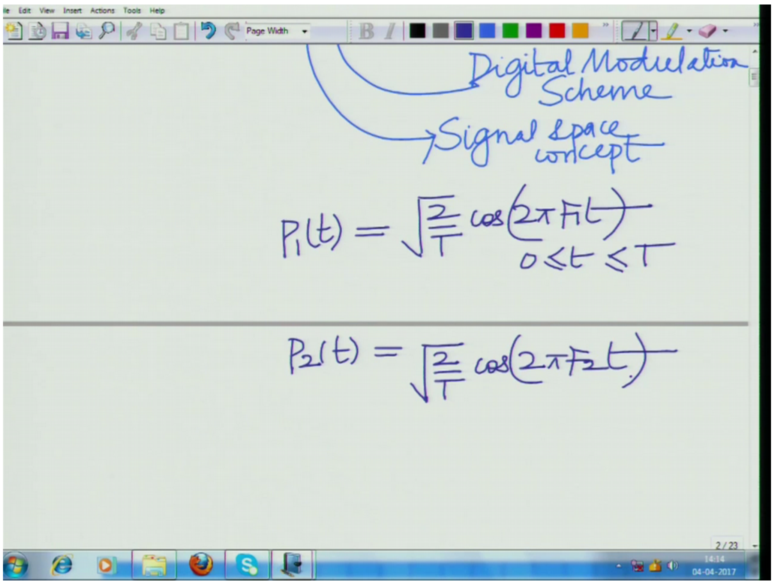


Where we have 2 pulses correct which form an orthonormal basis for the signal space. So, let us again consider our 2 pulses P 1 t equals square root of T cosine 2 pi F 1 t 0 less than equal to t less than or equal to T and we also have P 2 t equals square root of 2 over T cosine 2 pi F 2 t.

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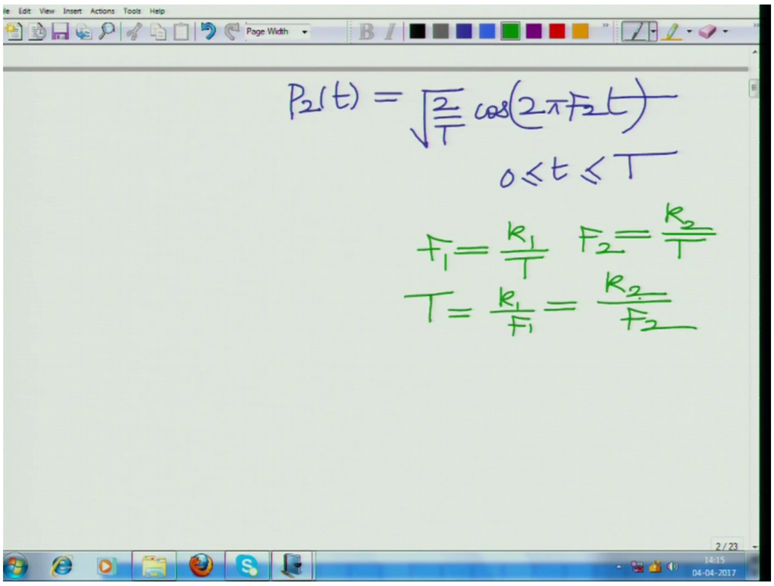


0 less than equal to t less than equal to T and we have F 1 or basically we have well T equals K 1 contains K 1 cycles or the wave F 1 equals over T and F 2 equals K 2 or T over T is basically K 1 over F 1 and K 2 equal over F 2 t equals K 1 over F 1 and this is

also equal to K 2 over F 2 contains K 1 cycles of P 1 that is a sinusoid for sinusoidal frequency F 1 and contains K 2 sine cycles of the other sinusoid at frequency F 2 and we have also seen that these 2 pulses are basically.

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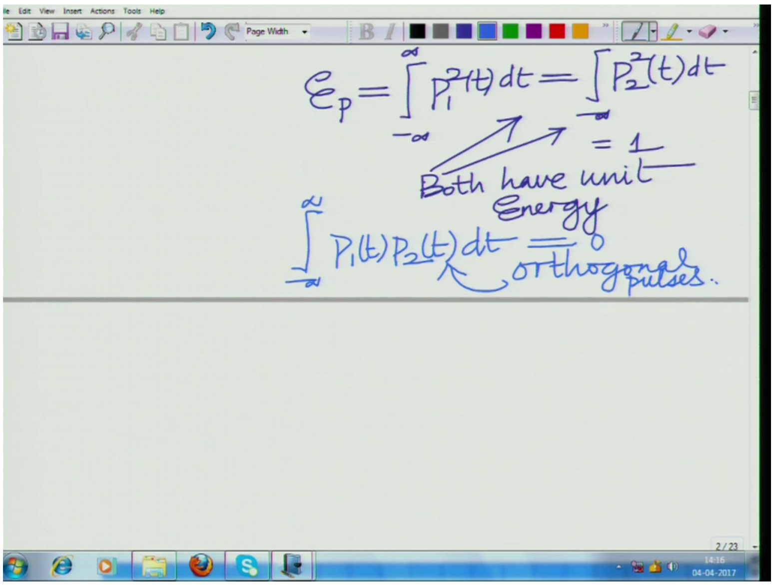


The first thing is that the energy of the pulses is E P equals P 1 squared d t equals minus infinity to infinity P 2 square equals one that is both pulses have unit energy and more importantly if you look at the inner product this is equal to 0 which means these are orthogonal pulses these are orthogonal pulses.

So, we have 2 pulses which are orthogonal to each other and also the pulses are basically both of them have unit energy all right these are orthonormal these form the basis orthonormal basis of the signal space.

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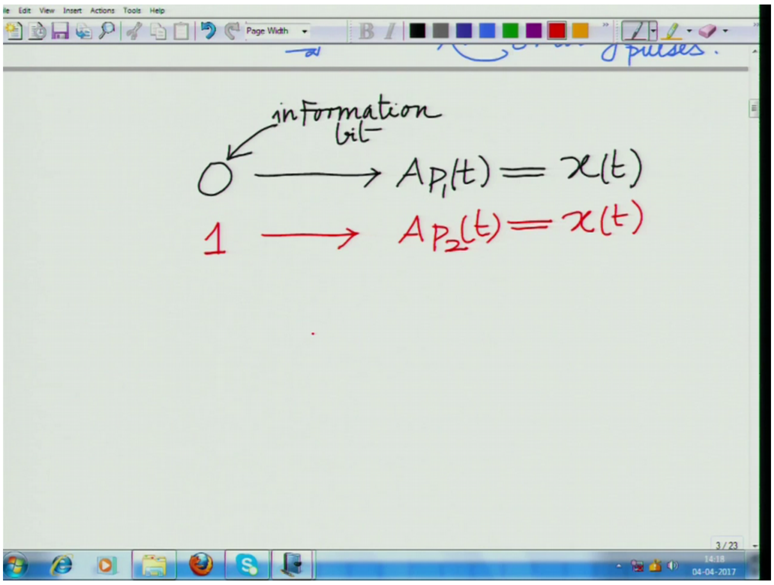


Now, the digital modulation signal is given as x t equals well A times P 1 t that is let us look at it this way that is corresponding to the information symbol 0, this is the information bit, 0 is mapped to A times P 1 t which is equal to x t and one the information symbol, 1 is mapped to A times P 2 t which is equal to x t. So, other words you have to write it clearly.

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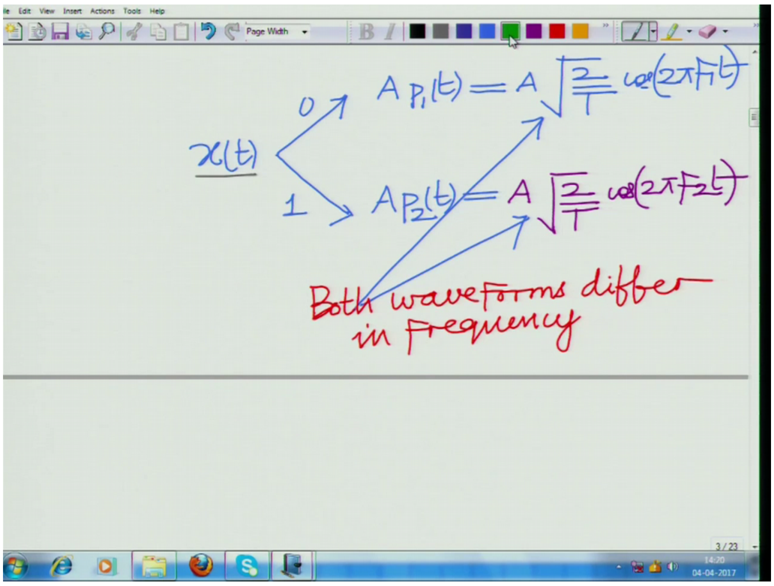


I have x, I have x t corresponding to information symbols 0, corresponding to information symbol 1, this is A times P 1 t which is A times square root of 2 by T cosine

2 pi F 1 t and corresponding to 1. This is A times P 2 t which is A times square root 2 pi cosine 2 pi F 2 t and now you can see that if you now look at this x t, correct, what you are doing is you are either mapping it to waveform (Refer Time: 06:51) A times square root of 2 over T cosine 2 pi F 1 t or you are mapping it for the information with one to the waveform A times square root of 2 over T cosine 2 pi F 2 t.

Now, if you can observe carefully these 2 waveforms cosine 2 pi F 1 t cosine 2 pi F 2 t differ in the frequency that is you are switching the frequency from F 1 to F 2 and F 2 to F 1, correct. So, to indicate A 1 you are transmitting waveform (Refer Time: 07:17) frequency F 1 to indicate to indicate A 0, you are transmitting the waveform at frequency F 1 to indicate A 1 you are transmitting a waveform with frequency F 1, correct and therefore, since these 2 waveforms are obtained from each other by shifting their frequencies this freq scheme this digital modulation scheme is termed as frequency shift key.

So, which is an important point? So, you can observe that both waveforms are waveforms differ in frequency that is one is obtained from the other by shifting in frequency.

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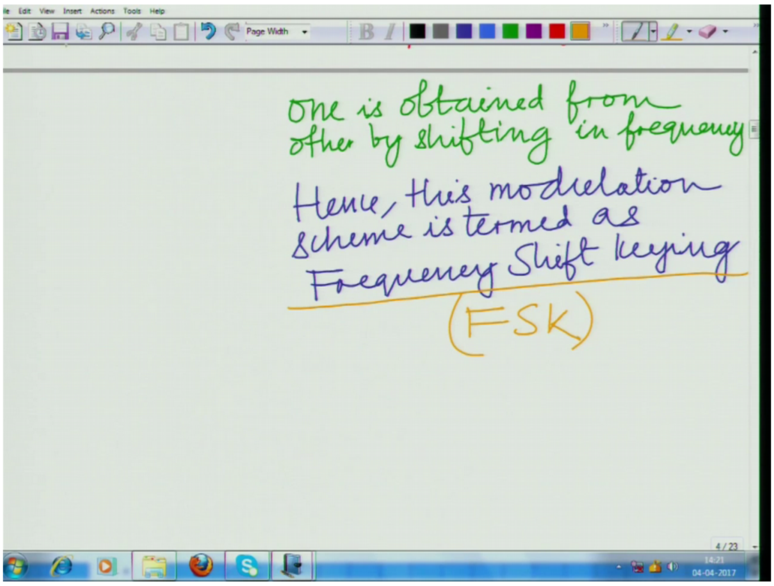


Hence this modulation scheme is termed as hence this modulation scheme is this is termed as frequency shift key or F S K, this is the waveforms are basically frequency shifted versions of each other.

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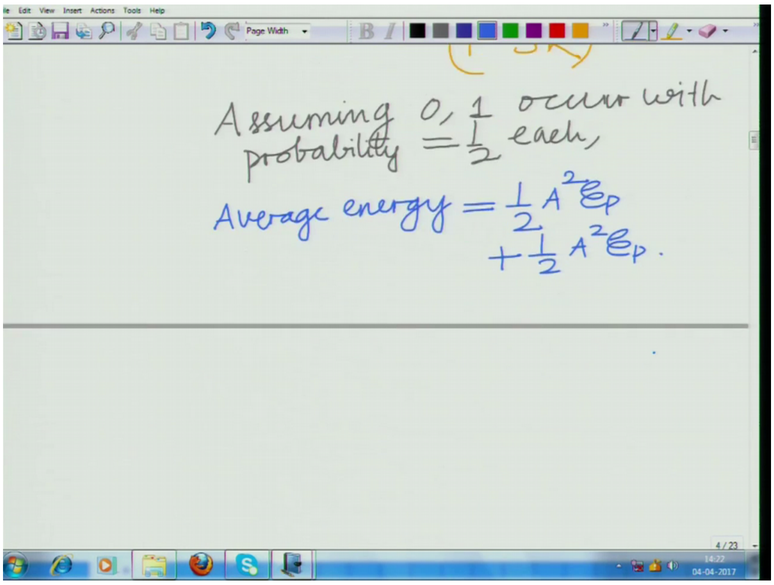


Further once again assuming that 0 and 1 occur with equal probabilities occur with probability equals half we have average energy equals half A square E P plus half A square E P.

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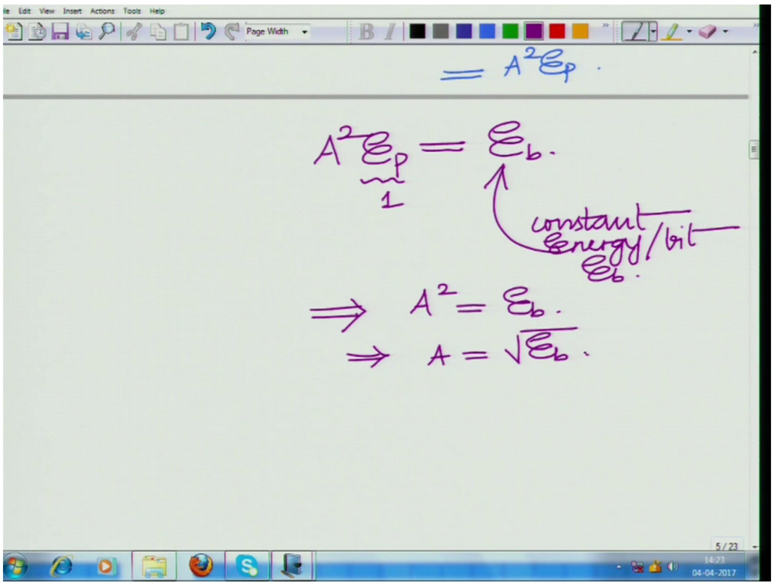


Which is equal to A square E P and similar to previous, if we want to set similar to the previous modulation schemes, if you want to set energy per bit is constant that is you want to set a constant energy per bit E b implies a square E P has to be E b and we know

E P equals 1 which implies a square equals E b which implies A equals square root of E b.

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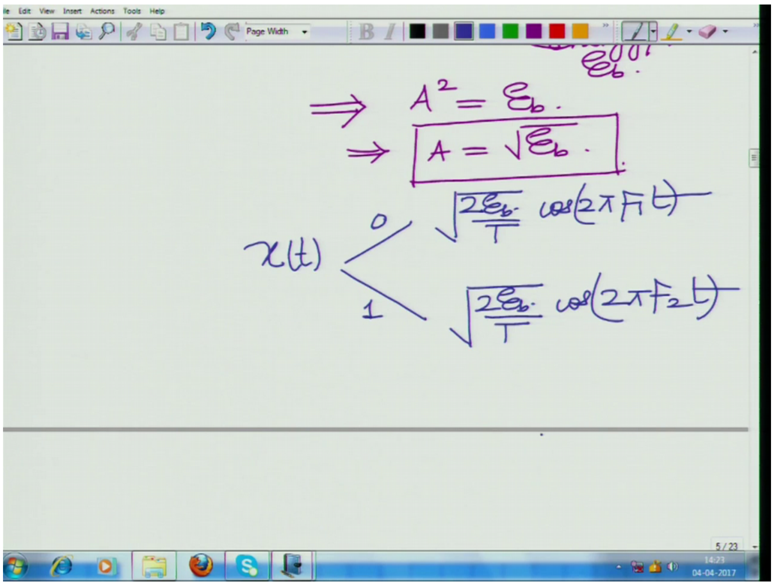

So, that is what we have and therefore, once again the transmitted waveforms are either A times square root of 2 over T that is remember we will get 2 square root of E b over T cosine 2 pi F 1 t or these this correspond to 0 correspond to one 2 square root of cosine 2 pi F 2 t corresponding to will corresponding to 0.

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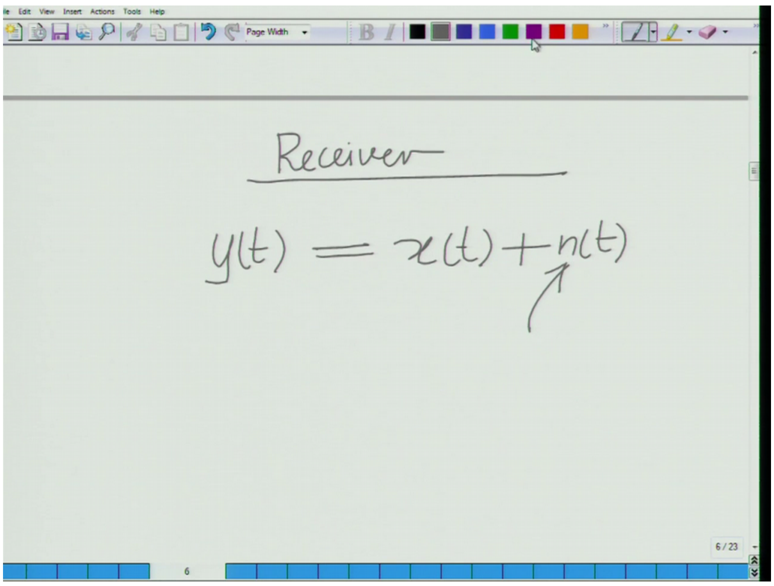

So, now, therefore, now consider what happens at the receiver at the receiver in the digital communication system once again we have y t equals x t plus n t where n t is additive white Gaussian noise.

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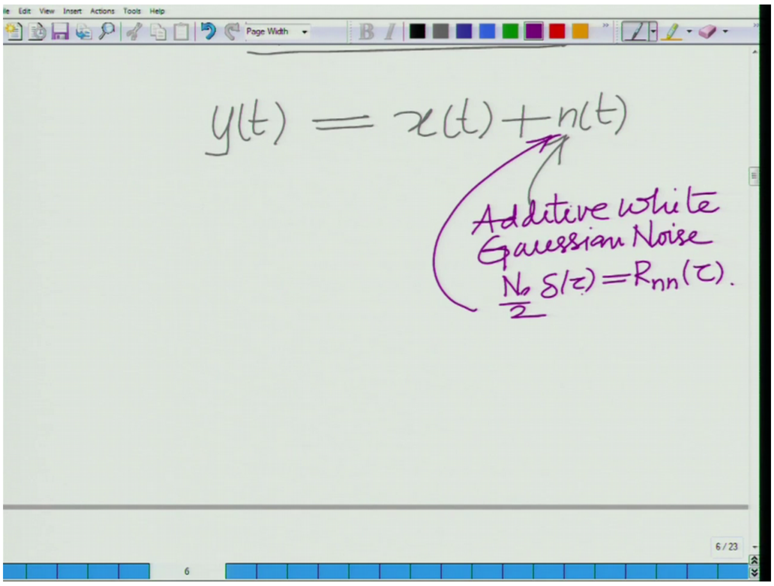


This is your standard which we have seen so many times. So, far this is your additive white Gaussian noise with P S D or with autocorrelation eta naught N naught by 2 delta tau this is the autocorrelation.

Now, corresponding to the transmission of 0 you have be remember there are 2 pulses A times P 1 t is corresponding to the transmission of 0 A times P 2 t is corresponding corresponds to the transmission of the information bit one therefore, corresponding to the transmission of 0 you will have the addition of noise A times P 1 t plus n t corresponding transmission of one you will have the addition of noise A times P 2 t plus n t.

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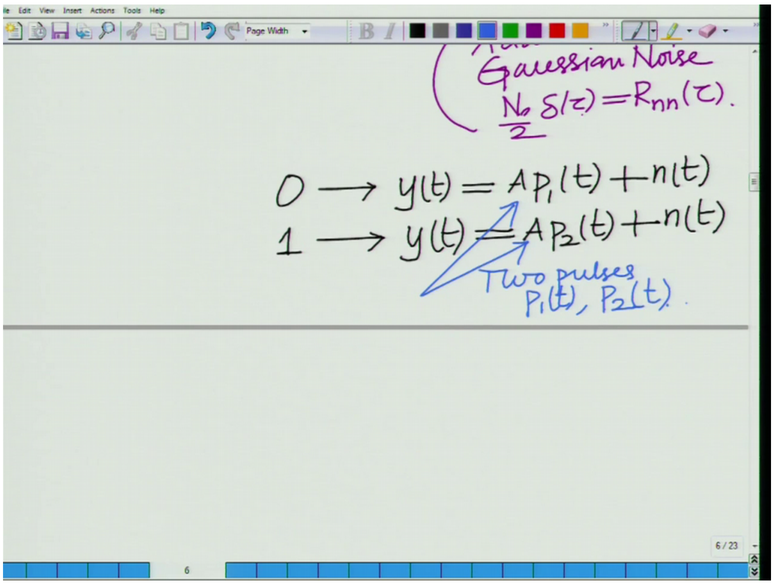


So, let us note that. So, corresponding to the transmission of 0 we have y t equals A times P 1 t plus n 2 t n t corresponding to the transmission of y of 1 or bit one we have A times P 2 t plus n t.

Now, we have an interesting conundrum or an interesting puzzle over here we have 2 pulses P 1 t and P 2 t.

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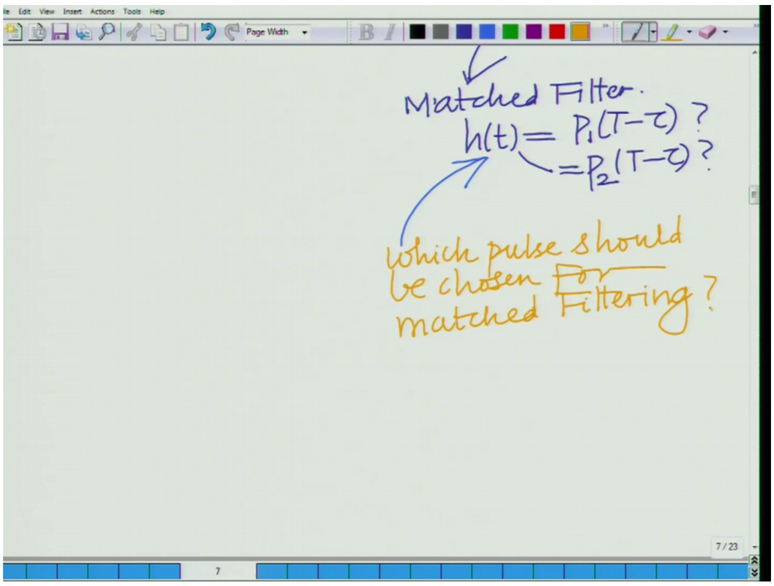


Now, what we need to determine is when we matched fill matched filter if you want to use we are not seen a situation like this before if you use a matched filter is h t equals P 1 t minus tau or is it equal to surprisingly is it equal to P to t, so which one should we choose? So, which pulse should be chosen for matched filtering that is the basic question that we are trying to address, which pulse which pulse should be chosen which pulse should be chosen for matched filtering and it is not immediately obvious which pulse should be chosen for matched filtering. So, let us try to address in a different way.

So, let us what we will do is we will try to modify this received signal y t a little and try to see if we can derive from it what the optimal matched filter should be.

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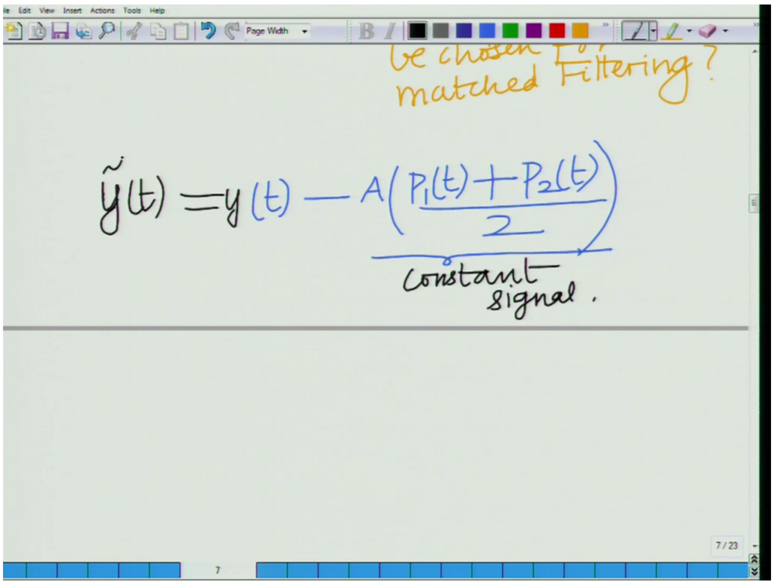


So, for that what I am going to do is from y t I am going to subtract A times P 1 t plus P 2 t divided by 2, now this is a constant signal. So, I can always do it constant signal and let us denote this as let us denote this by y tilde t.

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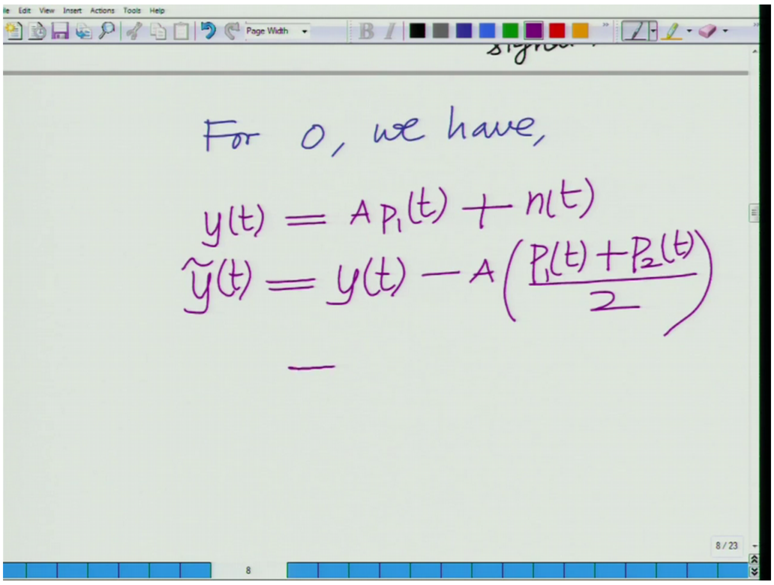


Now, therefore, corresponding to the transmission of 0 for 0 we have or for information bit 0 we have y t equals A times P 1 t plus n t y tilde t equals y t minus A times P 1 t plus P 2 t divided by 2.

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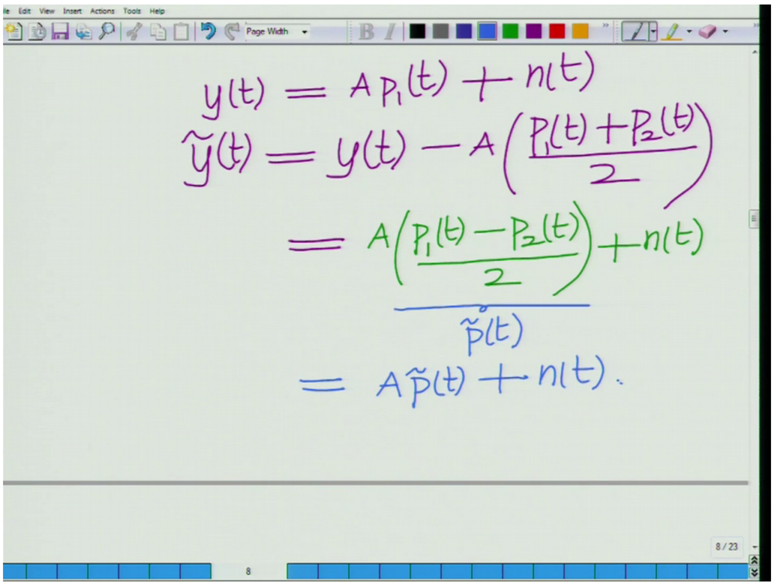


Which is equal to well A times P 1 t minus A times P 1 t it has P 1 t plus n t minus A times P 1 t plus P 2 t divided by 2 this is equal to A times P 1 t minus P 2 t divided by 2 plus n t.

Similarly, so let us call this as P tilde t that is your P 1 t minus P 2 t divided by 2 is P tilde t. So, that is A times P tilde t plus n t.

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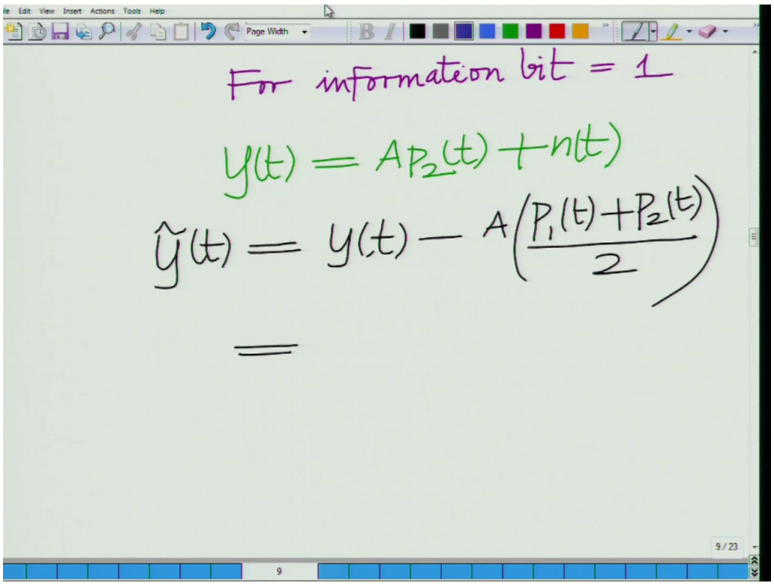


And further for one or for information bit one we have y t equals A times P 2 t plus n t and y tilde t equals y t minus A times P 1 t plus P 2 t divided by 2 which is equal to well it is P 2 t plus n t minus A times P 1 t plus P 2 t divided by 2.

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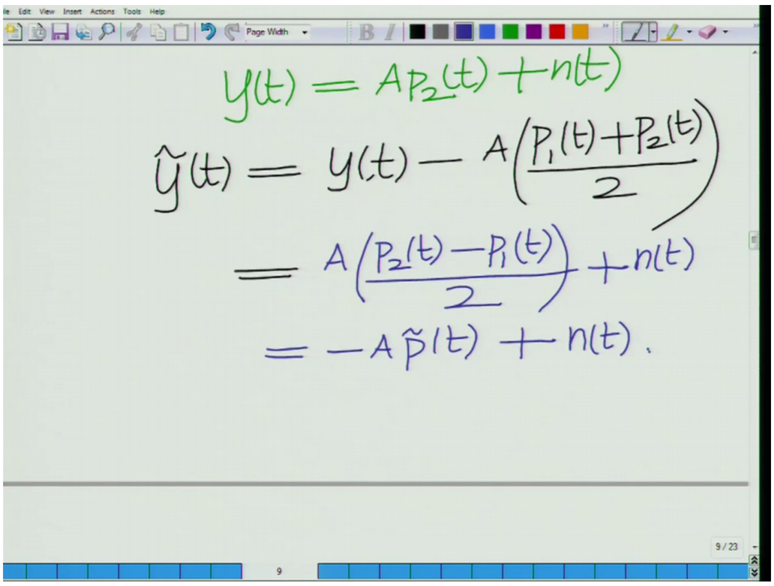


This is equal to A times P 2 t minus P 1 t divided by 2 plus n t. Now if you see; this is P 2 minus P 1 t divided by 2 which is minus P tilde. So, this is minus A P tilde t plus n t.

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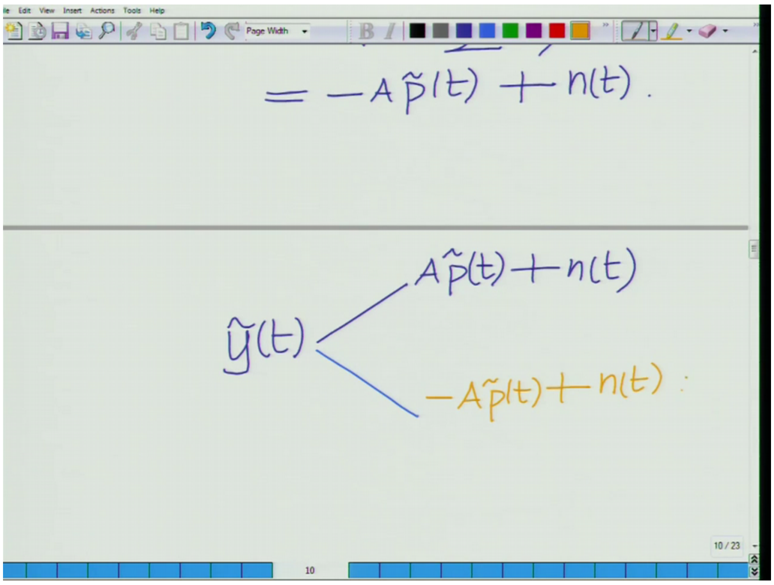


And now you have something very interesting if you look at y tilde t naught y t, but y tilde t, this reduces to A times P tilde t plus n t and in the other scenario this is minus A times P tilde t plus n t.

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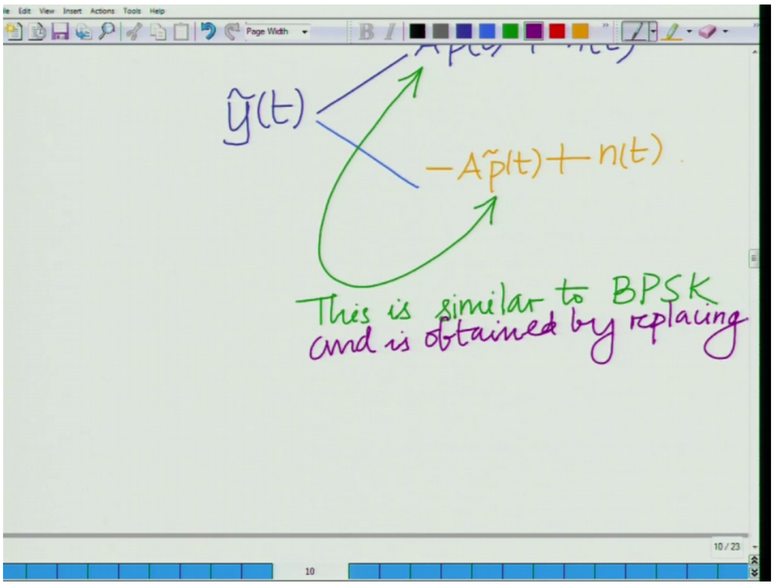


And if you look at this if you look at this; this is similar to binary phase shift key that is a type in binary phase shift keying we have A times P t and minus A times P t here we have A times P tilde t and minus A times P tilde t only difference is we are replacing P by P tilde in comparison to binary phase shift key.

Therefore you can see that the optimal matched filter will be P tilde t not P 1 t or P 2 t, but P tilde t and P tilde t is nothing, but P 1 t minus P 2 t divided by 2 the 2 is simply a scaling factor. So, basically the optimal matched filter will be proportional to P 1 t minus P 2 t neither P 1 t nor P 2 t, but basically P 1 t minus P 2 t that is the important point that one has to realize.

We will justify a in a movement, but this is element that we keep in mind. So, c over I reduction is the goal we said sectorization is a good one for us to hang now what are the methods can we do for reducing the reducing the interference. So, the there is another method that is use for reducing interference let me just highlight that for you. So, reducing c over I so here is a cell again we will use the same frequencies F 1 F 2 F 3 are available to me I have assigned F 1 to my user and let us assume that there is another co channel cell somewhere in the vicinity which also has a subscriber to whom F 1 has been assigned.

Now; obviously, there is interference between these 2 users. So, this particular user we could change the frequency from F 1 F 2 or so, this is similar. So, note that this is similar to binary phase shift key and is obtained by replacing and is obtained by replacing P t by P tilde t, P tilde t is basically your P 1 t minus P 2 t divided by 2 therefore, optimal matched filter.

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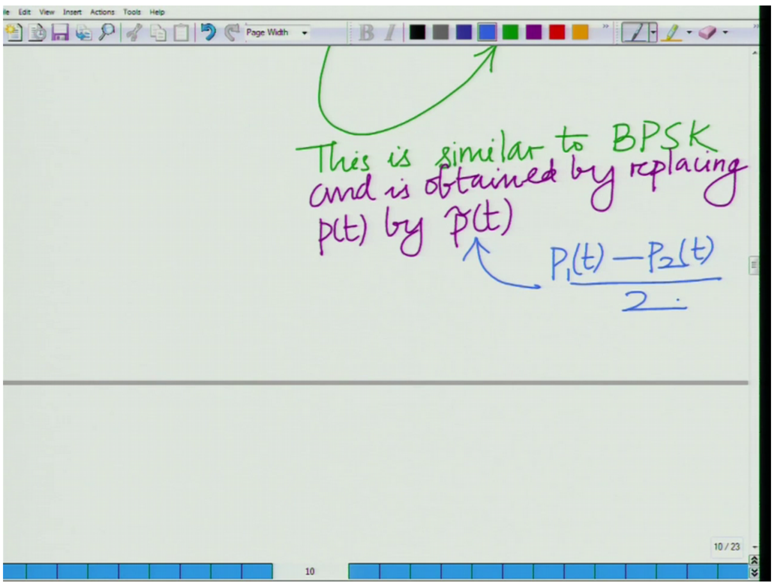

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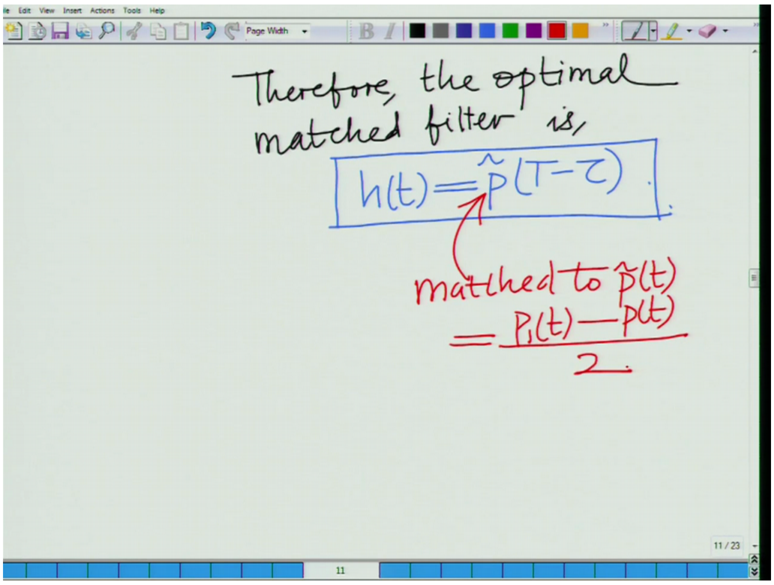


Therefore the optimal matched filter is h t equals P tilde t minus tau that is match to P tilde t P tilde t equals P 1 t minus P 2 t divided by 2.

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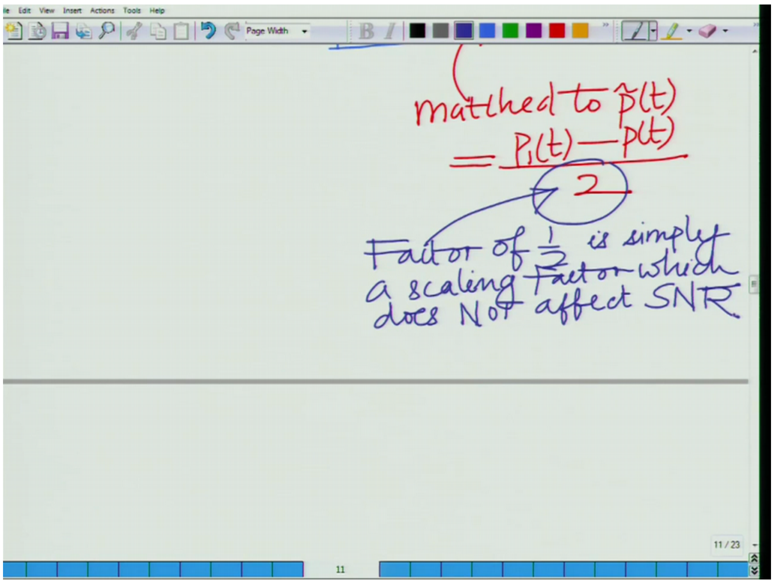


Finally note that this 2 this factor of half is simply a scaling factor it does not affect the S N R, hence which does not affect the S N R.

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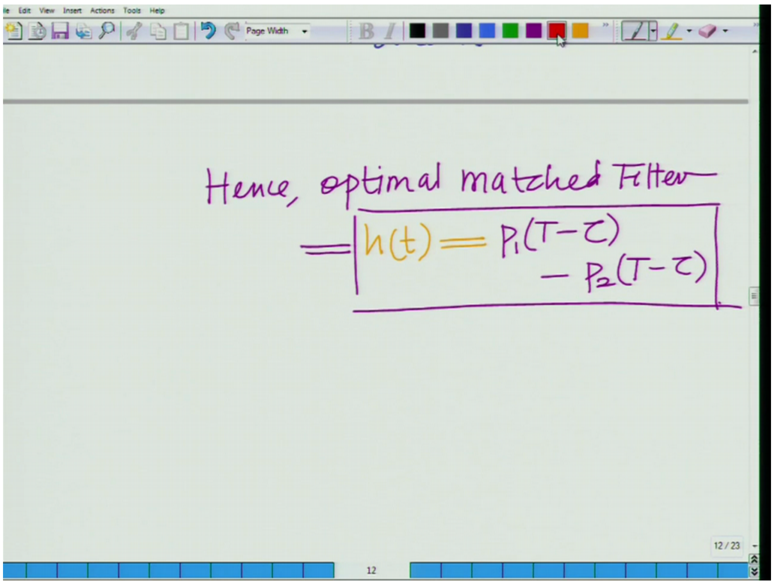


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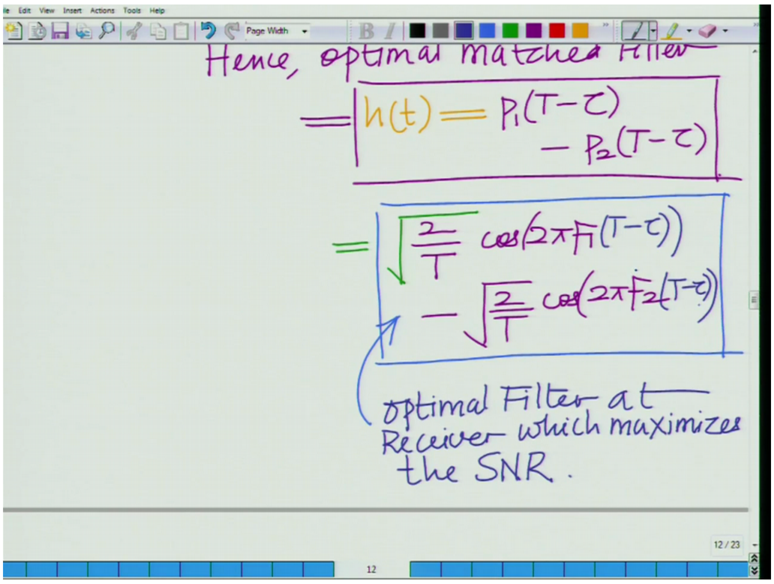


Hence optimal matched filter is simply you can say h t equals well P 1 t minus tau minus this is your optimal matched filter or basically which is basically square root of well you can square root of 2 over T cosine 2 pi F 1 t minus square root of 2 over T cosine 2 pi F 2 t this is the optimal this is the optimal or this is the matched filter this is the optimal filter which maximizes the S N R.

Optimal filter at the s n r which ma optimal filter at the receiver which maximizes the s n r. So, that is basically proportional to P 1 t minus P 2 t one can simply choose P 1 t minus

P 2 one can choose P 1 t minus P 2 t that is P 1 t minus P 2 t divided by 2 or basically simply P 1 t minus P 2 t therefore, the optimal matched filter will be P 1 capital t minus tau the and minus P 2 capital t minus tau. So, that is basically your matched filter I am sorry I have to simply change this, this is not t, but rather t minus tau similarly over here the optimal matched filter is F 2 t minus tau. So, this is P 1 t minus tau minus P 2 t. So, it is matched to P 1 t minus P 2 t which is P 1 t minus tau minus P 2 t minus tau that is the optimal matched filter.

So, employing the matched filter one can again carry out the analysis the receiver. So, employing this matched filter that we have derived correct employing this matched filter one can again carry out the analysis the rest of the analysis at the receiver derive what is the signal to noise power ratio and also what is the corresponding probability of bit error which we will do sub in the subsequent module.

Thank you very much.