Principles of Communication Systems - Part II
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Lecture - 13
Optimal Decision Rule for Amplitude Shift Keying (ASK), Bit-Error Rate (BER) and Comparison with Binary Phase Shift Keying (BPSK) Modulation

Hello, welcome to another module in this massive open online course. So, we are looking at amplitude shift keying and we said in amplitude shift keying, either, we are transmitting the amplitude level A or the amplitude level 0, alright and the 2 waveforms are distinguished by the amplitude level. And we have looked at what is the; we have looked at the optimal matched filtering at the receiver and the optimal statistic after sampling that is r of T.

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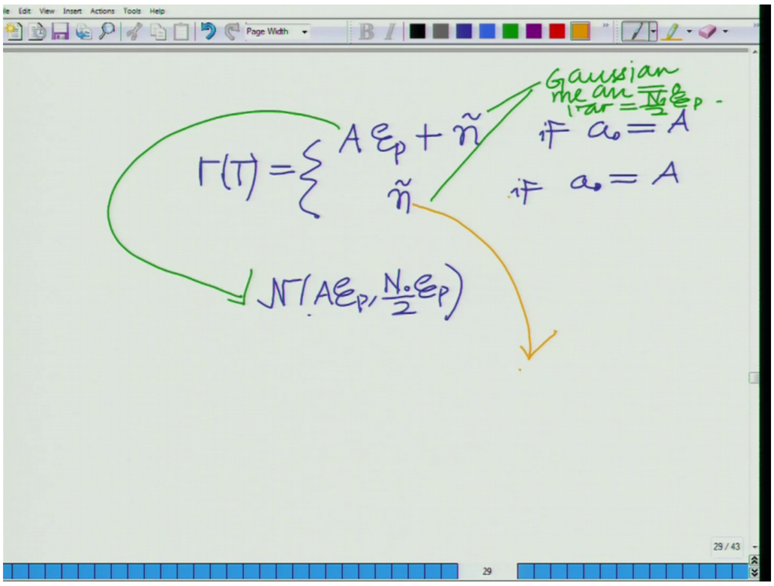

We have shown is given as r of T, this is either A E p plus n tilde if a naught equals A or simply n tilde if a naught equals 0 and we have said in this scenario r T equals A E p plus a tilde. Well this is a Gaussian, so we have seen first n tilde in both cases n tilde is Gaussian mean equal 0 variance equals n naught by 2 times E p.

So, in this case r T is Gaussian with mean A E p that is a Gaussian shifted to A E p and variance remains unchanged that is n naught by 2 times E p and in this case where you have a naught equals a in this case it is simply Gaussian with mean equals 0 and variance equals n naught by 2 times E p.

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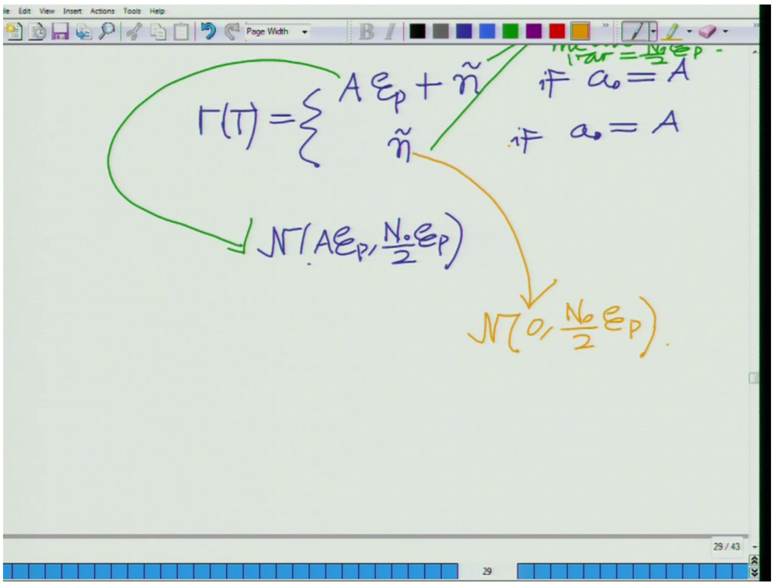

So, the probability density functions corresponding to a naught equals a naught equals, a and a naught equals 0 or still Gaussian in nature and similar to BPSK they are also shifted.

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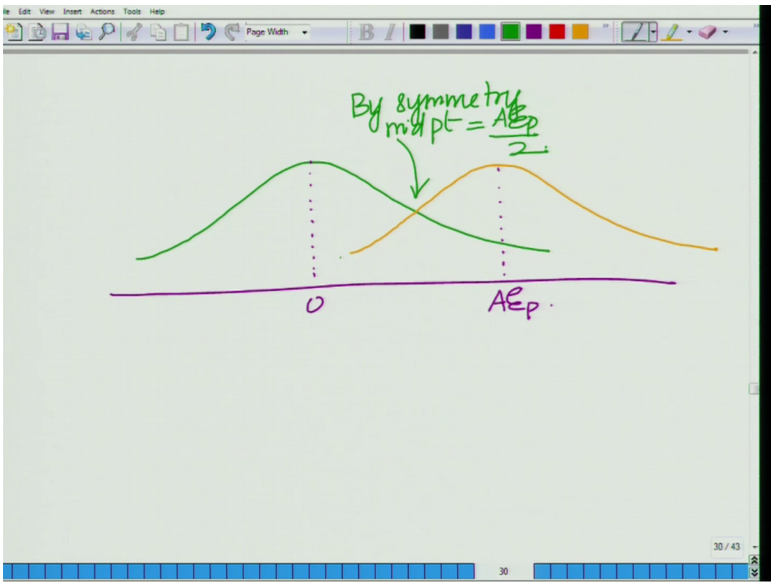

But in amplitude shift keying corresponding to a naught equal 0 the mean is still 0 correct and corresponding to this amplitude corresponding to a naught equals A the amplitude is A E p. This point is A E p and by symmetry midpoint is A E p divided by 2 therefore, one can decide a naught equals well A, if r T is greater than or equal to A E p by 2 and on the other hand decide a naught equals 0 if r T is less than A E p by 2.

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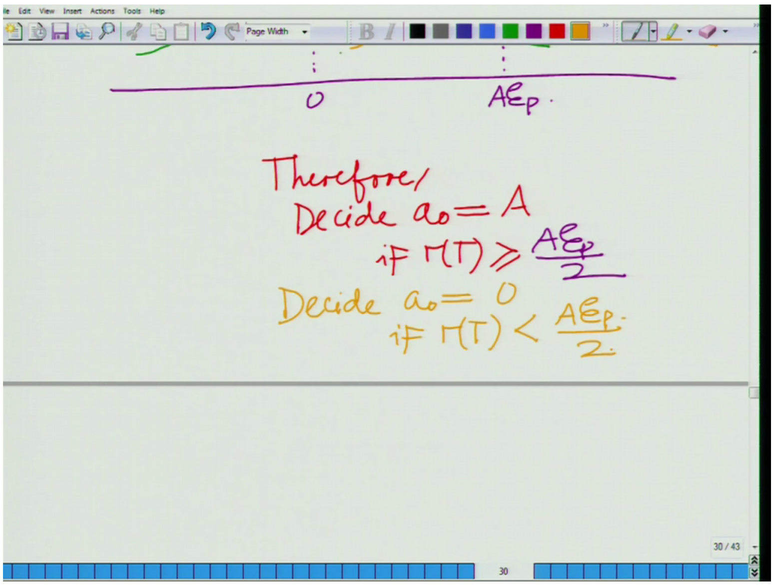

Now, one other way to look at this is we have remember r T equals A E p plus n tilde and here we have this is corresponding to a naught equals A and this is corresponding to a naught equals 0.

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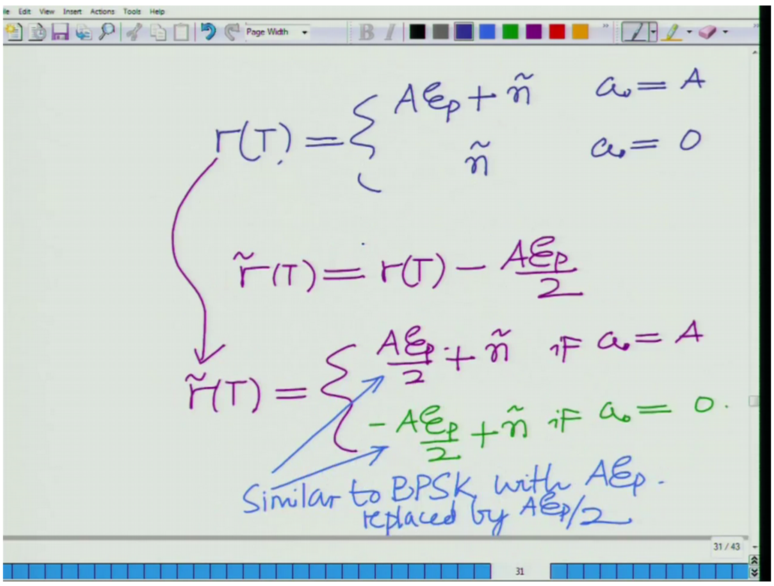

Now, if you consider r tilde T equals r T minus A E p by 2, now if we subtract A E p by 2 then we have well r tilde T will be A E p minus A E p by 2 that is A E p by 2 plus n tilde if a naught equals A and this will become minus A E p by 2 plus n tilde if a naught equals 0.

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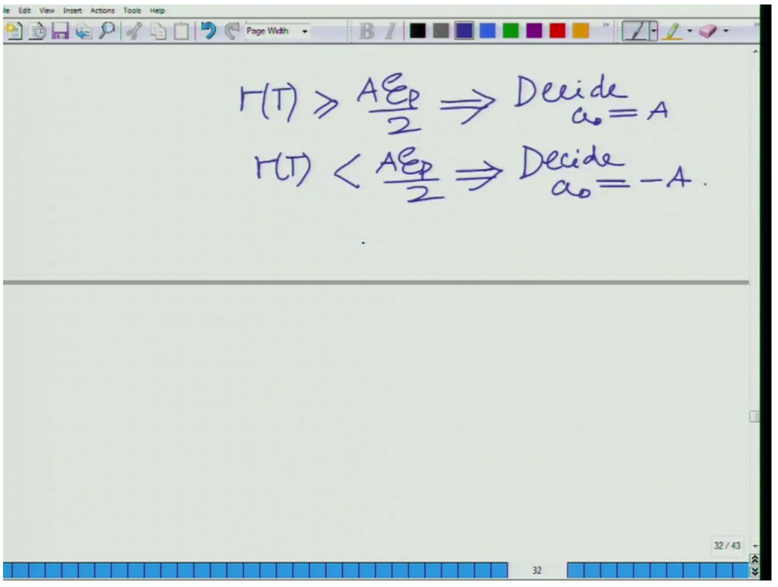

Now you can see there is symmetry, it is symmetric about 0. So, now, you can see this is similar to BPSK; similar to BPSK with A E p replaced by A E p by this is similar to BPSK with A E p replaced by A E p by 2. Therefore, again now we have something that similar to BPSK that is A E p by 2 plus n tilde minus A E p by 2 plus n tilde. Remember for BPSK binary phase shift keying we had A E p by A E p plus n tilde minus A E p plus n tilde therefore, in that sense this is similar to binary phase shift keying with A E p replaced by A E p divided by 2.

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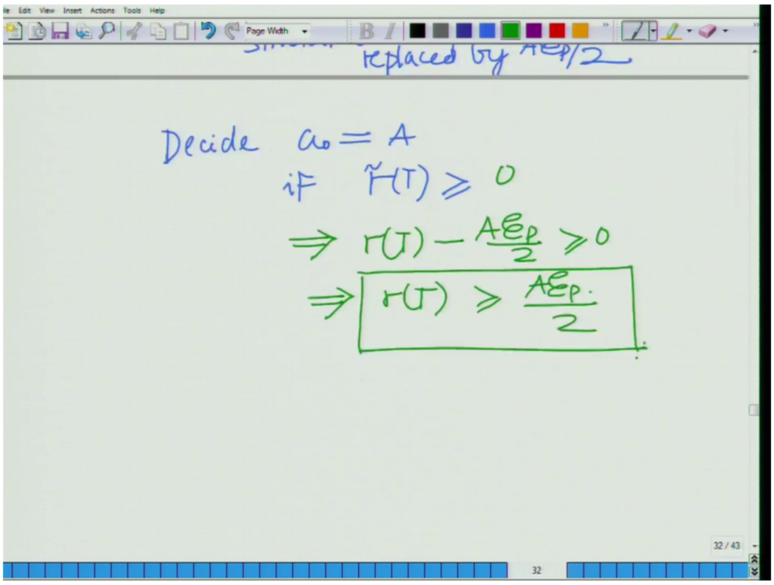

Hence the optimal detector again can be to decide a naught equals A if r tilde T greater than or equal to well 0 implies r T minus A E p by 2 greater than equal to 0 implies r T greater than or equal to A E p by 2.

So, decide, so the decision rule is decide again we get back the same decision rule r T which is r T greater than or equal to A E p by 2 implies decide a naught equals A, r T less than A E p by 2 implies decide a naught is equal to; decide a naught is equal to minus A. So, this is the decision rule that we have.

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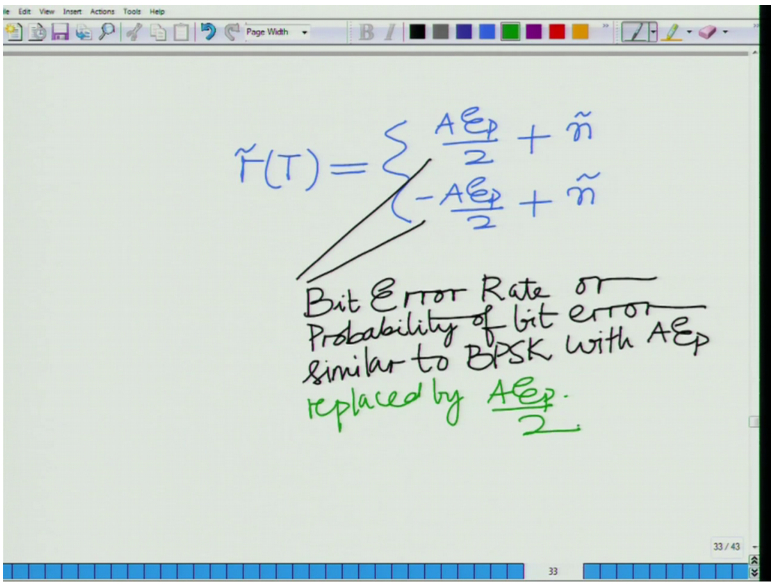

Now, if you look at the probability of bit error, again the probability of bit error can now be derived easily now you see this is similar to BPSK if you look at r tilde T, this is equal to well A E p by 2 plus n tilde A E p by 2 minus A E p by 2 plus tilde. So, bit error rate is similar to that of BPSK with A E p replaced by, a bit error rate or probability of bit error this is the same thing both mu 1 and the same is similar to BPSK with A E p replaced by A E p by 2.

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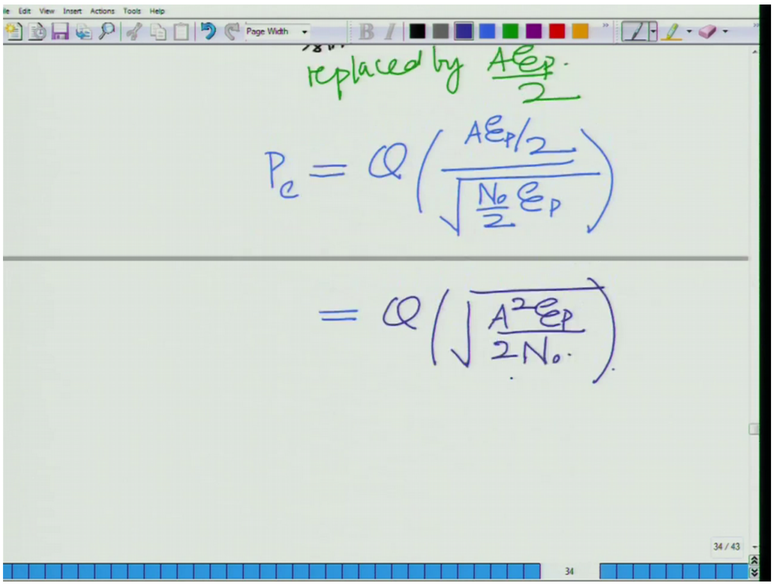

Therefore, probability of error is Q A E p divided by 2 A E p divided by 2, divided by square root of n naught by 2 times E p, now, we are going to substitute which is equal to well let me first simplify this, which is equal to Q square root of A square E p by N naught which is equal to Q A square E p by 2 N naught.

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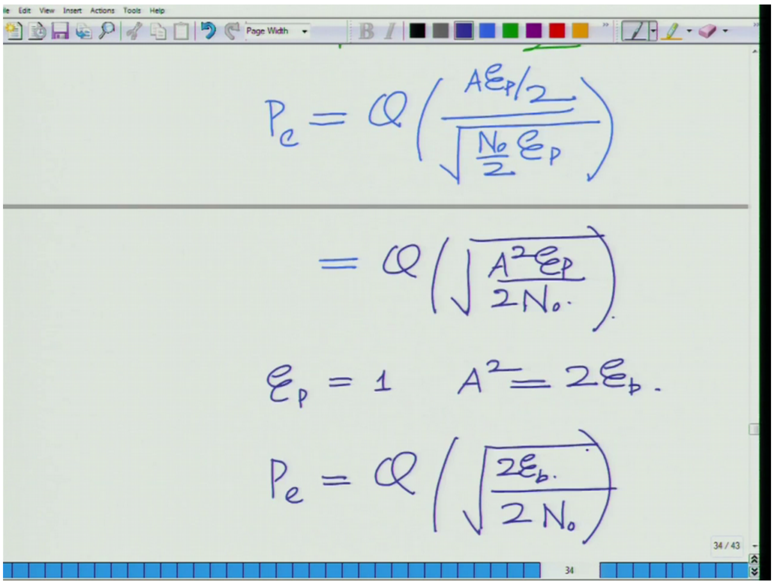

Now, substitute E p equal to 1 that is pulse normalized to unit energy and remember we have calculated in amplitude shift keying we have said that for average bit energy E b A square must be equal to 2 E b. So, substituting a square equals 2 E b we get P e equals Q square root of A square is 2 E b in to E p is 1 divided by 2 n naught divided by 2 n naught which is equal to Q square root of E b over N naught that is your probability of error.

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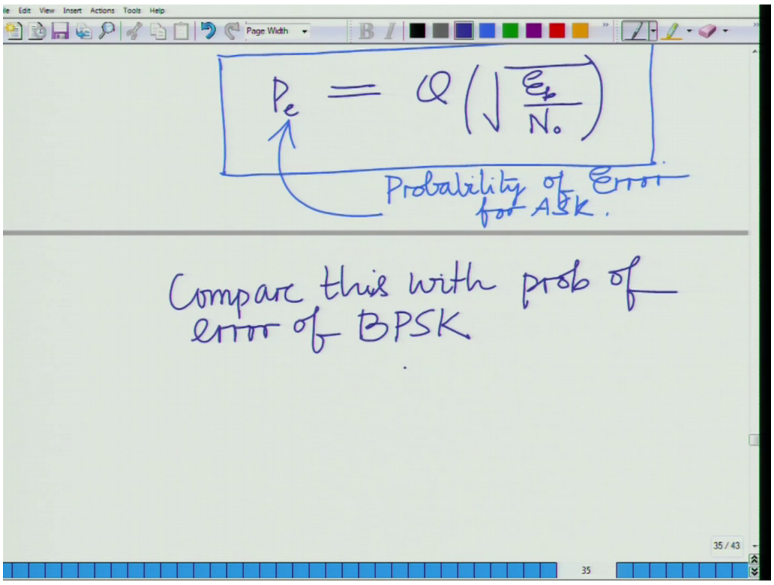

So, this is the probability of error for amplitude shift keying. This is the probability of error for amplitude shift keying. Now compare this with probability of error of binary phase shift keying compare it with probability of error of binary phase shift keying which is equal to q, remember this is Q square root of 2 E b over N naught this is the probability of error of binary phase shift keying.

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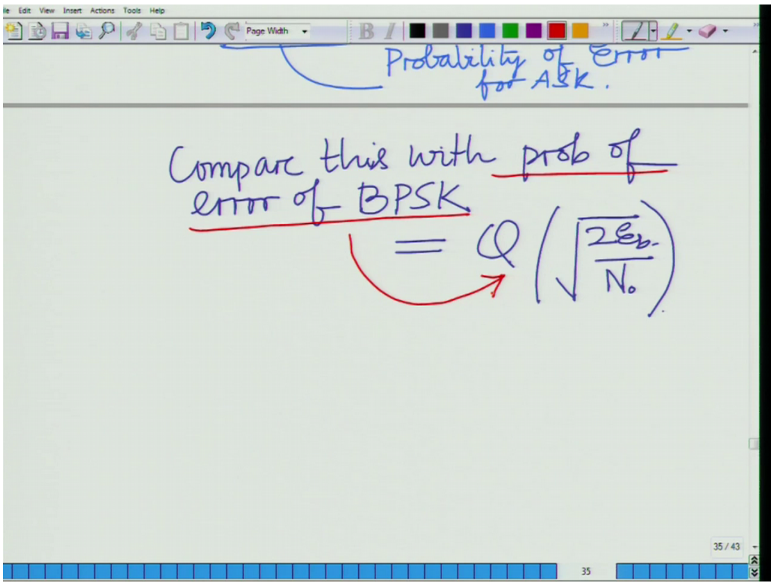

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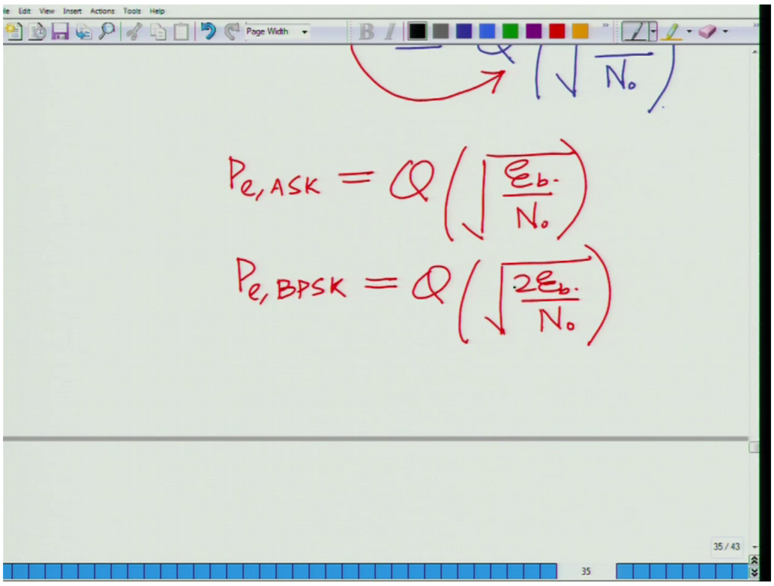

So, for amplitude phase amplitude shift keying it is equal to Q square root E b over N naught for binary phase shift keying probability of error, for binary phase shift keying is

Q square root 2 E b divided by. Look at this the argument of binary phase shift keying is higher that is this is square root of 2 E b over N naught, the argument that is argument of the Q function for amplitude shift keying is simply square root of E b over n naught. And the Q function is a remember it is important to realize that the Q function is a decreasing function of its argument because Q function represents the tail probability of the Gaussian probability density function correct, of the Gaussian random variable. Therefore, as the argument is increasing the tail probability is decreasing therefore the Q function is decreasing, alright.

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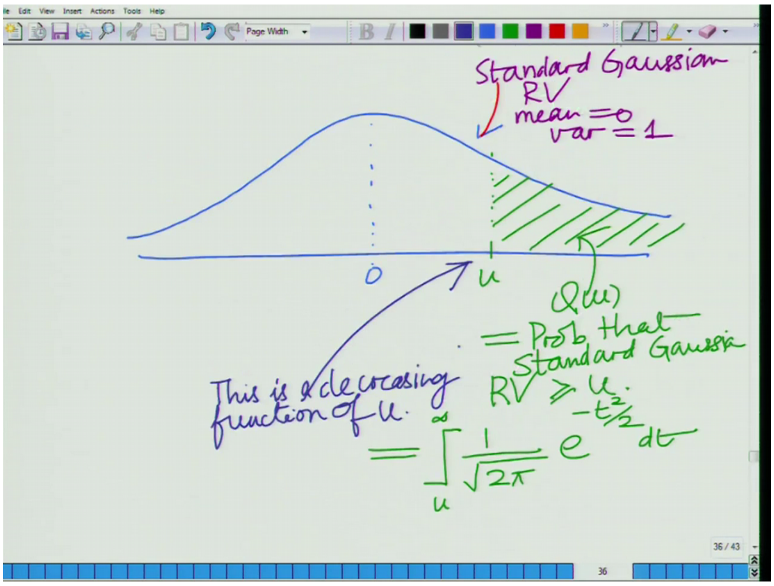

So, remember Q u if you look at Q u what is Q u this is the standard Gaussian random variable that is mean equal to 0 variance equal to 1, this is the standard Gaussian random variable Q of u is simply the probability that the standard Gaussian random variable, Q of u is simply this probability this is your Q of u equals probability that the standard Gaussian RV is greater than or equal to u which we have written as integral u to infinity one over square root of 2 pi correct, e to the power of minus t square by 2 d t and therefore, you can clearly see this is a decreasing function of u. As u is increasing this is a decrease this is decreasing, this is a decreasing function of u as u is increasing the Q function is decreasing.

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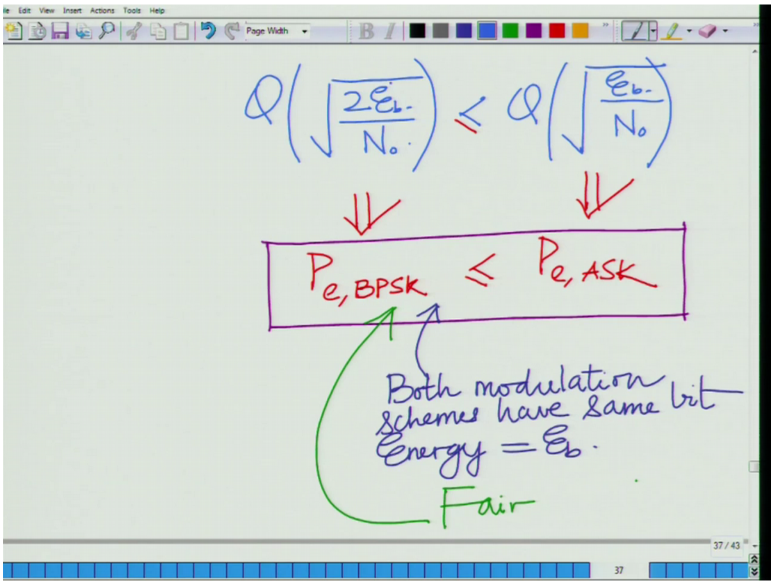

Therefore this means that Q the important point here is that Q square root of 2 E b over N naught is a less than Q of square root of well E b over N naught which means which implies that probability of errors for BPSK for the same bit energy is less than or equal to probability of bit error for amplitude shift keying this is an important result. And this comparison remember it is a fair comparison because we are comparing it for the same bit energy remember the important consideration here is that both modulation schemes have same bit energy equals E b both the modulation. Therefore, this is a fair comparison that is the important point. The comparison has to be same because if one of them has a higher energy right, if one of them has a higher energy. So, if one of them this is basically a fair comparison because if one of them has a higher, one of them is higher bit energy naturally the bit error rate is going to be low.

So, what we are doing is (Refer Time: 16:06) we are maintaining a constant average bit energy across the schemes and comparing the bit error rate performance and what we have seen is that the bit error rate for the same average bit energy E . The bit error rate or the probability of bit error of binary phase shift keying is lower in comparison to that of amplitude shift keying and this is an important observation.

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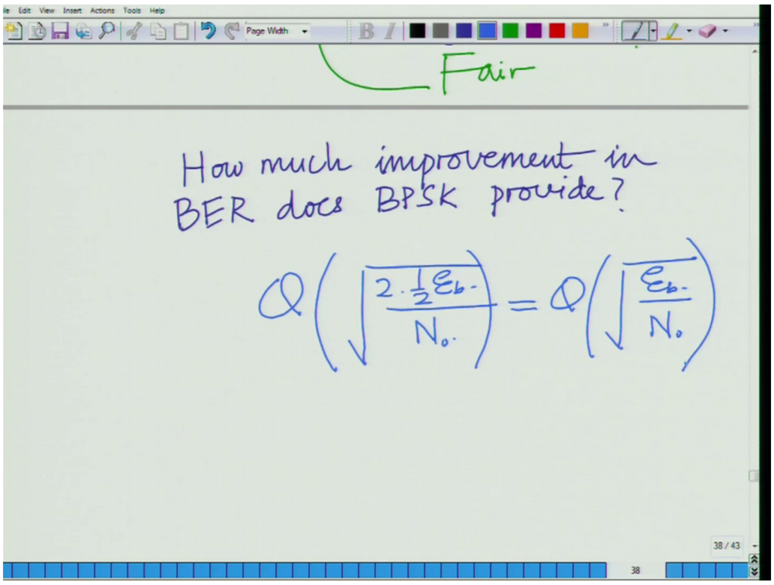

And how much improvement in bit error rate does binary phase shifts now. One can ask the question how much improvement or bit error rate or BER, how much improvement in bit error rate but does BPSK provide. Now if you look at this for the same bit error rate, if you look at this for the same bit error rate in BPSK we need half the average bit energy that is if you use half the average bit energy, bit error rate for BPSK that is bit error rate for BPSK is 2 half E b divided by N naught equals Q square root of well E b over n naught. So, this is bit error rate.

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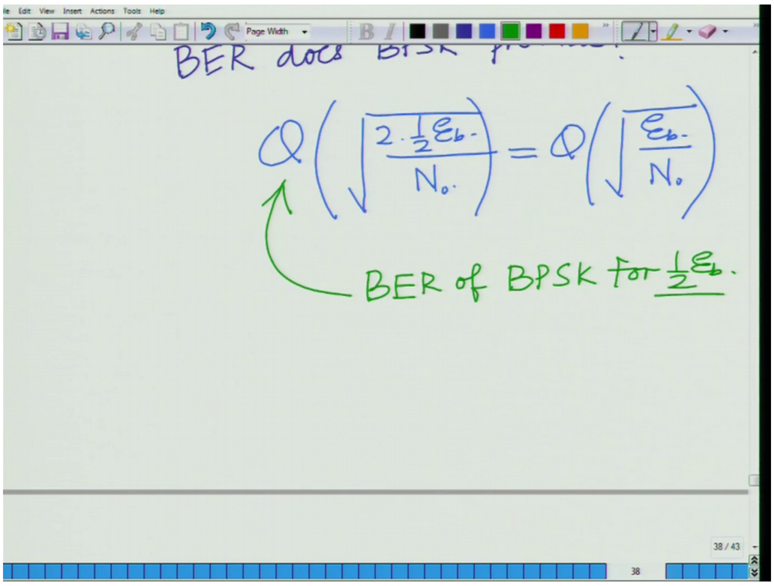

So, this is the bit error rate of BPSK for half bit energy. That is for that is the bit error rate of BPSK with half the average bit energy is same as that, is same as the bit error rate of amplitude shift keying with bit energy E b. So, to provide a similar bit error rate performance or provide a bit error rate performance similar to that of amplitude shift keying binary phase shift keying requires only half the half the bit energy as, half the average energy per bit as that of amplitude shift keying.

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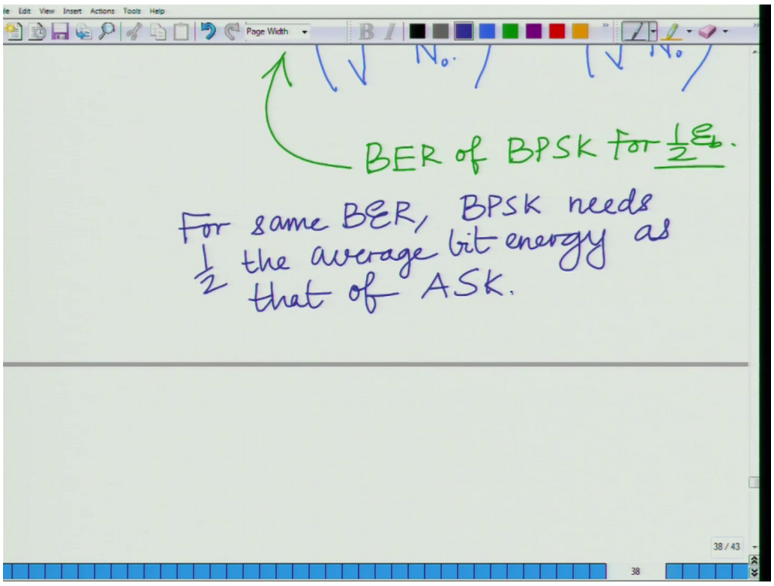

So, if you look at this for same bit error rate for an identical bit error rate, for the same bit error rate BPSK half the average bit energy as that of amplitude shift keying which implies that energy per bit of BPSK equals half energy per bit of amplitude shift keying.

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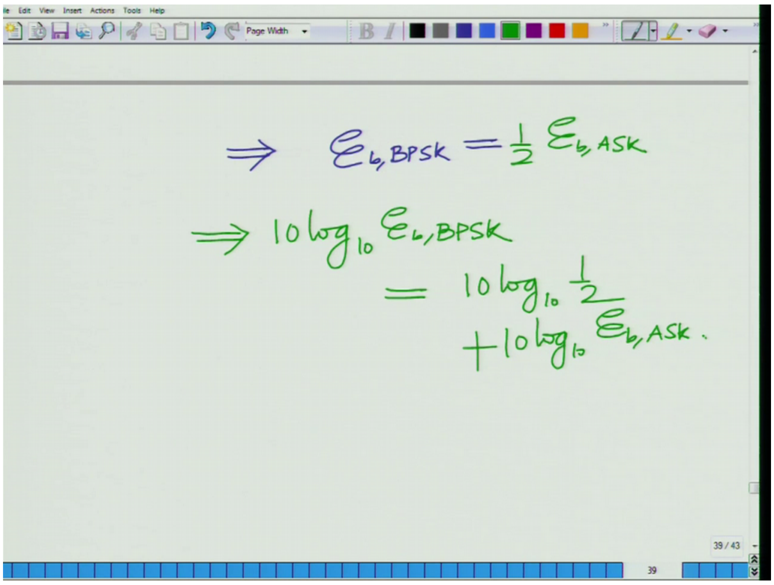

Now, if you convert this into dB terms this implies that 10 log 10 energy per bit of BPSK equals well minus equals 10 log 10 half plus 10 log to the base 10 energy per bit ASK.

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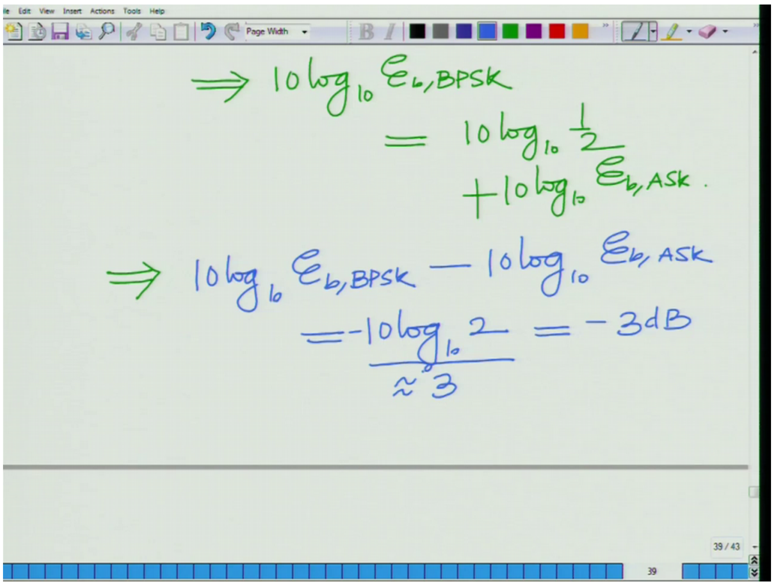

Which implies that the dB difference between the average bit energy of BPSK and ASK is 10 log to the base 10 that is the dB difference minus 10 log to the base 10 energy per bit ASK equals minus 10, equals minus 10 log to the base 10 2; 10 log to the base 10 of 2 is 3 this quantity is approximately 3 which means this is minus 3 dB.

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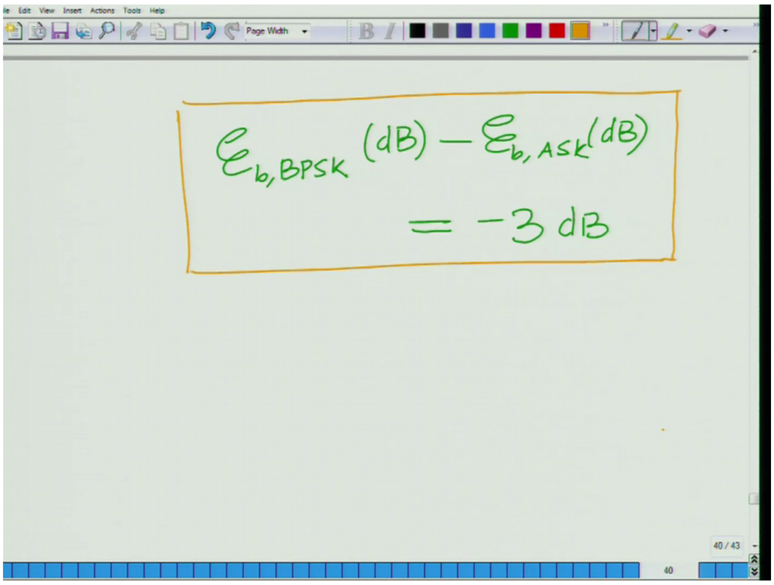

So, which means that if you look at the dB difference between the average bit energy correct, if you look at the dB difference if you look at energy per bit BPSK in dB terms let me make that clear; this is in terms of dB minus energy per bit of amplitude shift keying in dB this is equal to minus 3 dB. Or what this means is for the same bit error rate performance, for the same bit error rate performance binary shift phase shift keying requires three dB less average bit energy in comparison to amplitude shift keying. Or in other words amplitude shift keying requires 3 dB higher average bit energy in comparison to binary phase shift keying therefore, binary shift phase shift keying is more is much more efficient correct is more efficient than amplitude shift keying.

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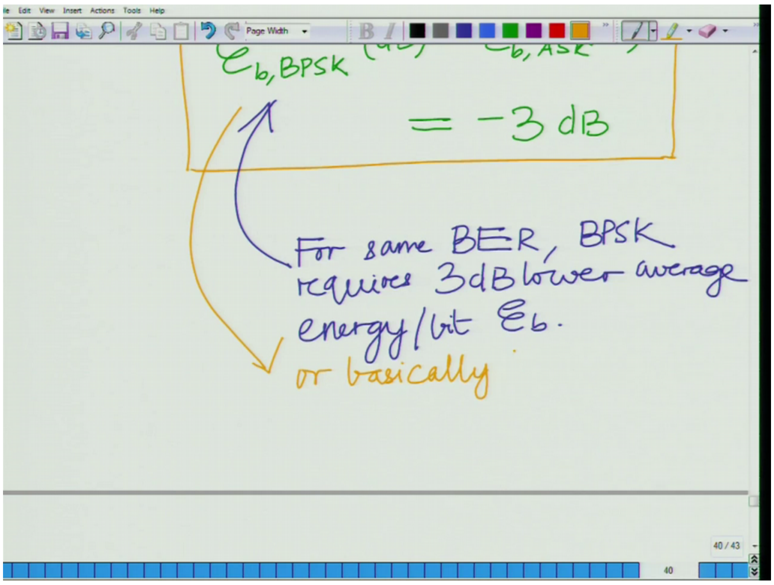

So, in dB terms for same BER BPSK lower average energy per bit or in other words or basically ASK amplitude shift keying 3 dB higher E b when compared to when compare to or basically amplitude shift keying requires 3 dB higher E b when compare to BPSK for the same bit error rate. So, that is the interesting observation that we would like to maintain.

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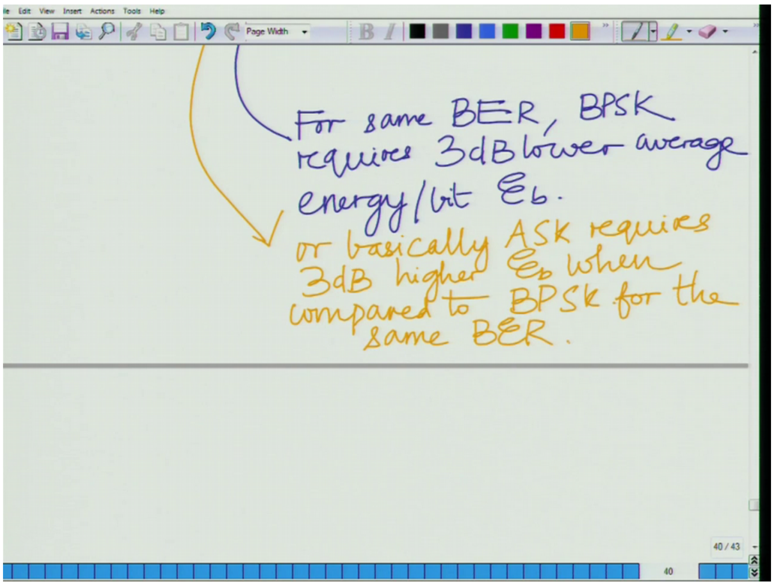

So, what we have seen in this module is we have completed our discussion of amplitude shift keying, we have described amplitude shift keying which is the different module for digital communication system, different modulation scheme for a digital communication system in which different waveforms are distinguish by their amplitude levels correct. We have seen; what is the optimal filter, optimal matched, filter receiver, what is optimal received signals or received signal test statistics after matched filtering sampling and also we have seen what is the optimal decision rule followed by the probability of error and what is the efficiency of amplitude shift keying in comparison to binary phase shift keying, alright. So, we will stop here and continue with other aspects in the subsequent modules.