Principles of Communication Systems - Part II
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Lecture - 12
Introduction to Amplitude Shift Keying (ASK) Modulation

Hello, welcome to another module in this massive open online course. So, in this module let us start looking at another digital modulation scheme that is amplitude shift key.

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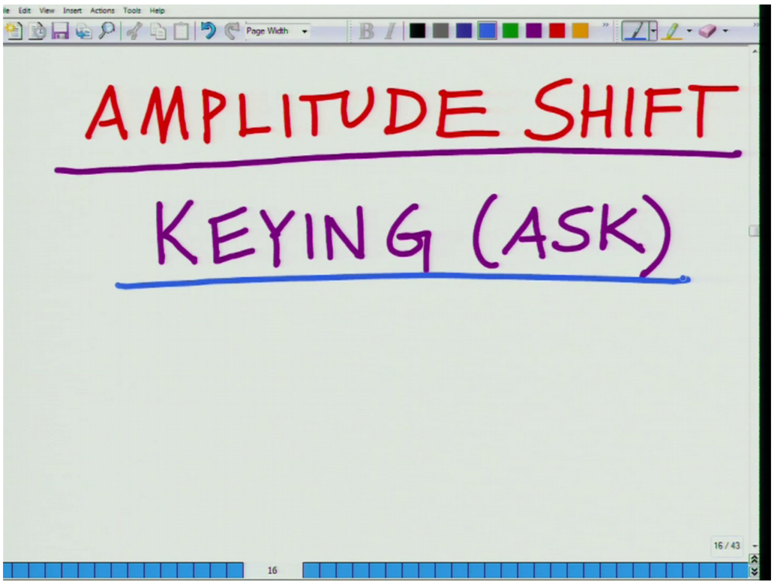

So, previously we have seen phase binary phase shift keying we will look at a different modulation scheme which as amplitude shift key. So, we want to start looking at different modulation scheme that is amplitude shift keying or ASK, this is a different digital modulation different digital modulation scheme.

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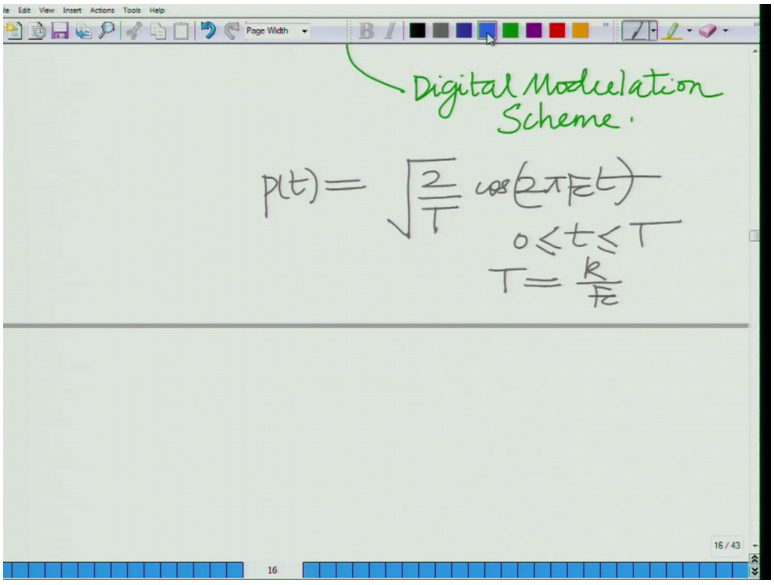

And in amplitude shift keying. So, this is another as we have already said this is another yet another digital modulation scheme, this is yet another digital modulation scheme in which we employ the pulse shape p t equals square root, again a similar pulse shape square root 2 pi f c cosine t for 0 less than equal to t a less than equal to capital T where capital T is the symbol duration and T equals k over F c.

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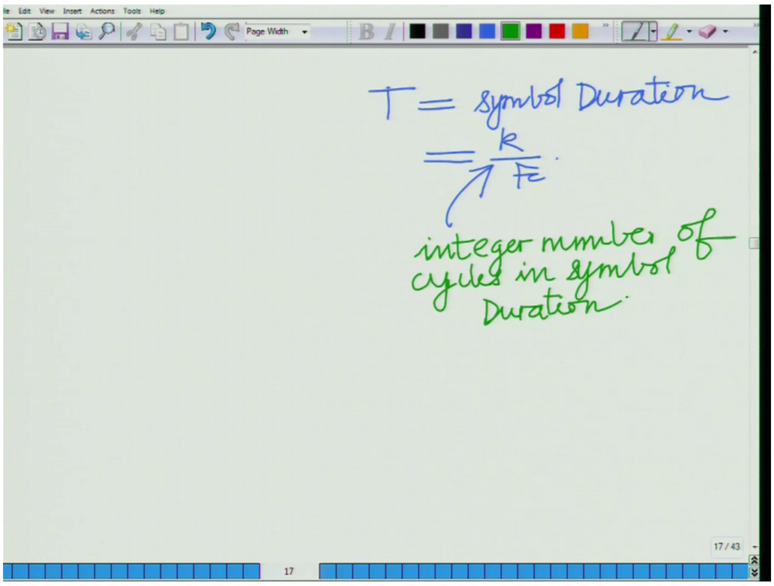

So, we have capital T equals the equals the, capital T equals the symbol duration and it was also k over F c which means basically it is an integer multiple similar to what we have seen for binary phase shift keying this is an integer multiple of the integer number of cycles. So, there are integer numbers of cycles of cosine 2 pi F c t in the symbol duration.

There are in integer number of cycles at the symbol duration.

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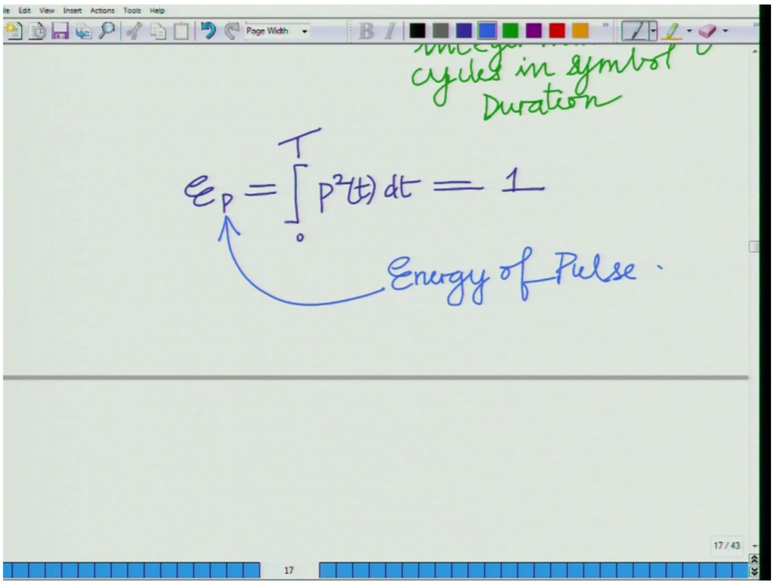

And we have already seen that energy of the pulse if you look at integral 0 to T, P square t that is energy of the pulse previously in binary phase shift keying, we have used the similar pulse we have seen that energy of the pulse, energy of the pulse is unity. We have already seen this.

Now, in binary phase shift keying remember the symbol a naught can belong to one of two amplitude levels minus A or plus A and we said that both of them differ in the both of the waveforms therefore, difference differ in the phase. One of the waveforms each of the waveforms is phase shifted by pi in comparison or with respect to the other in amplitude shift keying both the waveforms different amplitude, alright. So, I will one waveform has an amplitude A, the other waveform has an amplitude 0 that is whole point.

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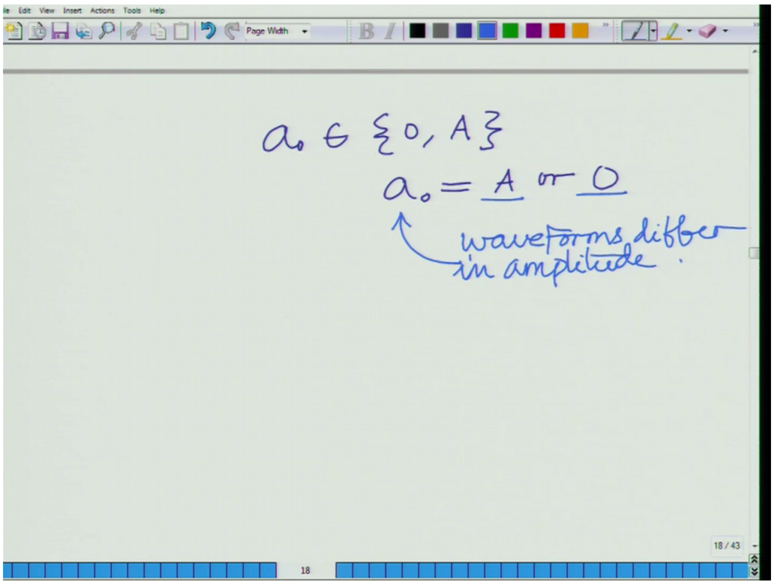

So, the symbol a naught belongs to the set 0 comma A. So, a naught equals either A or 0. So, there is difference, waveforms differ in waveforms differ in the amplitude. So, a naught can be either A or 0.

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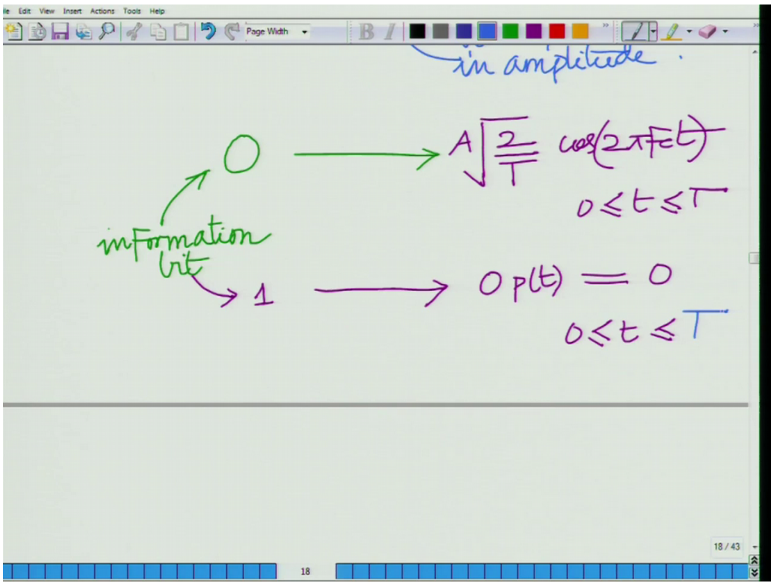

Therefore for instance let us take a look at this therefore, is the information bit 0 remember we have the information bit. So, this is an information bit.

The information bit 0 is coded as square root or A times square root 2 by T cosine 2 pi F c t 0 less than equal to t less than equal to T and information bit 1 this is also information bit that is 0 times p t which is simply equal to, which is simply equal to 0. For, of course, again goes without same saying 0 less than equal to t less than or equal to less than or equal to capital T.

So, corresponding to the information's bit 0 we are actually transmitting the waveform which amplitude A corresponding to the information bit 1, we are simply transmitting 0 amplitude - amplitude 0 means basically there is no signal.

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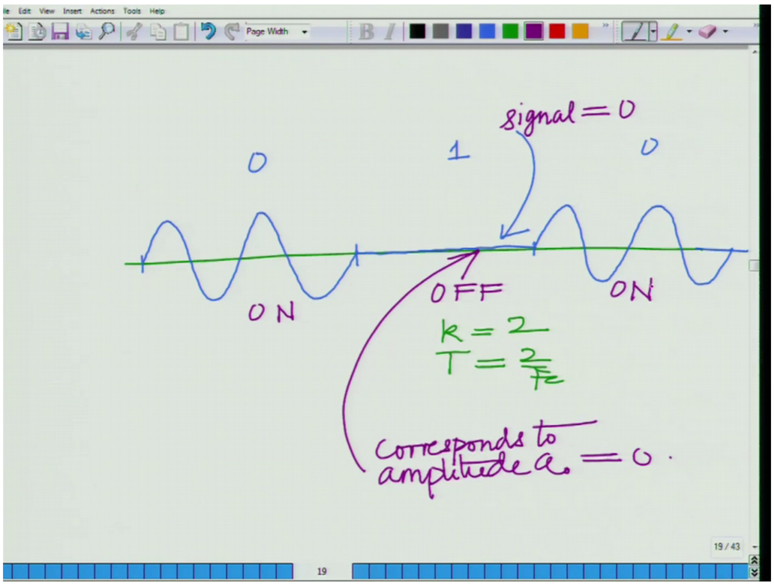

So, the transmission looks like for instance assuming k equal to 2 implies T equals 2 over F c. So, we have two cycles. So, basically the, this is let us say corresponding to the transmission of information bit 0 then corresponding to the transmission of information bit 1 we will have the transmission of 0 and again corresponding to the transmission of information bit 0. So, here the signal is basically, here the signal is basically signal is basically 0, corresponding to amplitude; corresponds to amplitude or a naught equal to 0, corresponds to amplitude a naught equal to 0.

Therefore if you can look at this, this is on, the signal is ON in this region or the transmission is ON transmission is OFF because amplitude is (Refer Time: 07:48) amplitude is 0 transmission is again ON.

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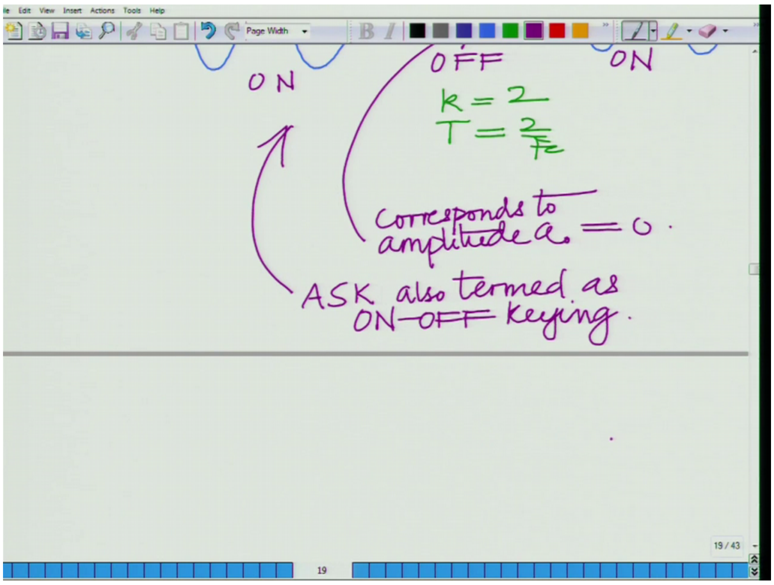

Therefore this is also known as that is ASK is also termed as ON OFF keying, this is also termed as ON OFF keying. It is as a if you are switching on the signal and wherever corresponded the information bit 1 the amplitude level is 0 that is signal is 0. So, you are switching off the signal. Again when the bit is 0 you are switching on it is as if you have turning on the signal. So, this is also known as ON OFF keying amplitude shift keying we (Refer Time: 08:29) shifting the amplitude between two levels 0 and capital A; can also be (Refer Time: 08:34) it is also termed as ON OFF keying.

Now, how to choose the energy A? We again come to the question remember - how to choose the energy or how to choose A which is the amplitude level.

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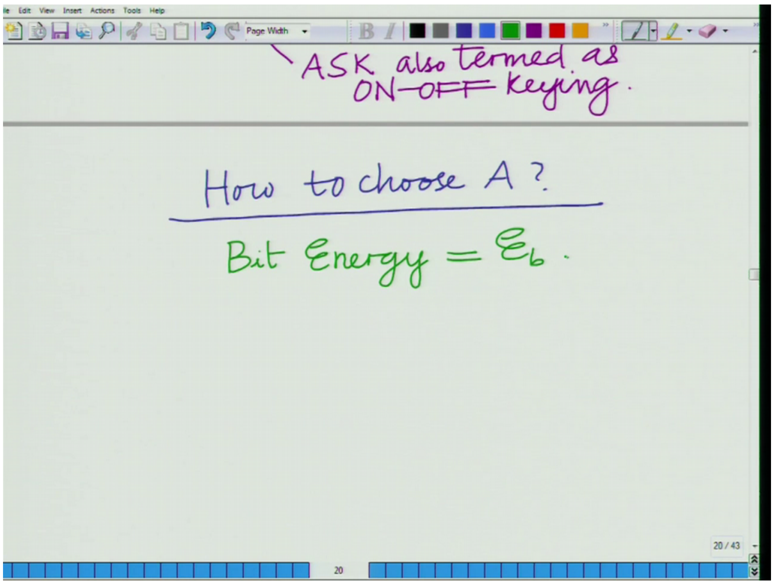

Now remember to choose A assume we have to keep the bit energy, bit energy is constant we are keeping the bit energy equal to E b.

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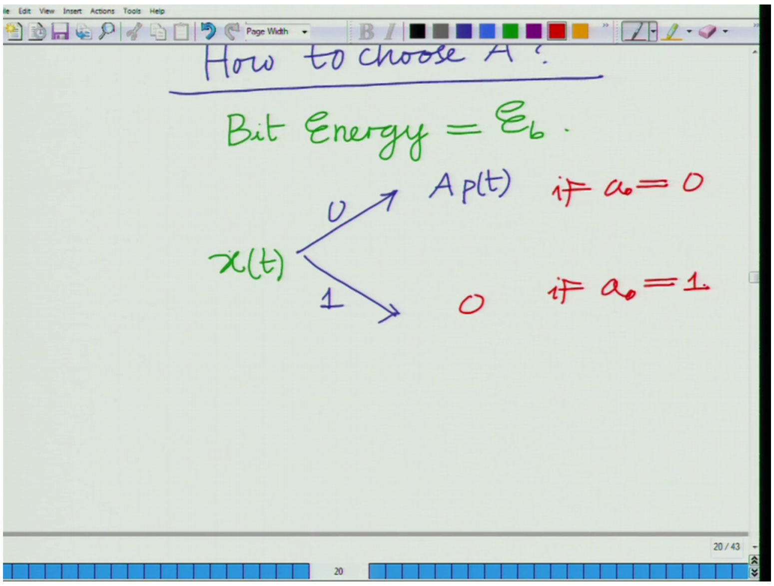

Now, in terms of for 0 x t remember for x t for 0 we are transmitting A times p t corresponding to (Refer Time: 09:28) symbol A or bit 1 we are transmitting 0. So, let us say this is if a naught equals 0, if a naught equals 1.

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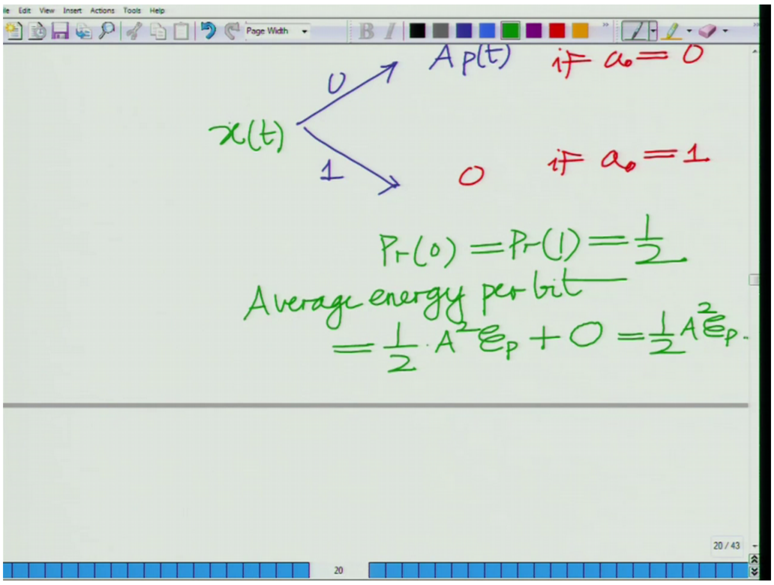

Now, let us say again let us assume a simple scenario let probability of 0 equals probability of 1 equals half, now average energy per bit is equal to half corresponding to 0 half a square energy in the pulse plus corresponding to bit 1 you are simply transmitting 0. So, the energy of that will be 0. So, this is simply half a square E p.

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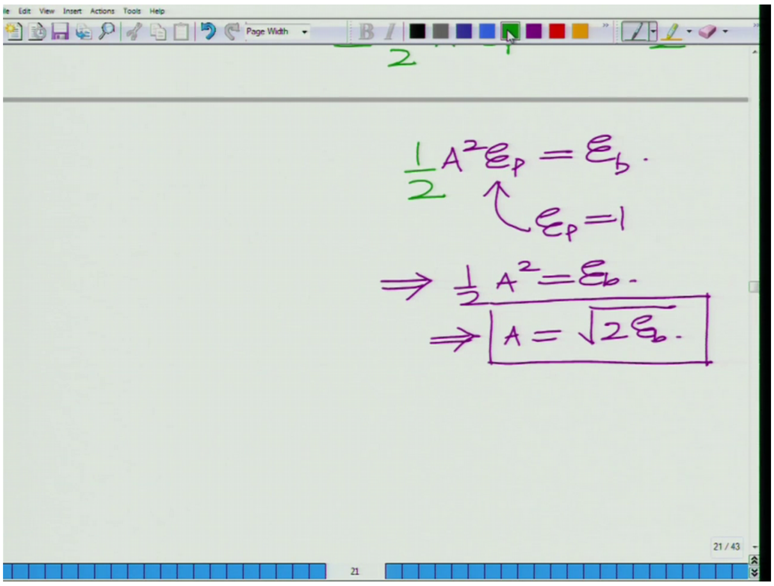

Now, given the average bit energy is E b, we need to have half a square E p is equal to E b or is equal to E b. Further we have E p equals one because of pulse is normalized to unity power which implies half A square is equal to E b which implies or which rather implies A is equal to square root of 2 E b. And remember this is important we have doing this again to maintain a constant energy E b per bit across keeps, for instance we had an energy per bit of E b in binary phase shift keying. We want to retain the same energy per bit E b for amplitude shift keying also. So, that will enable the fair comparison enable us to make a fair comparison between amplitude shift keying and binary phase shift keying. So, that is a reason we are keeping the energy per bit as constant across the different modulation schemes.

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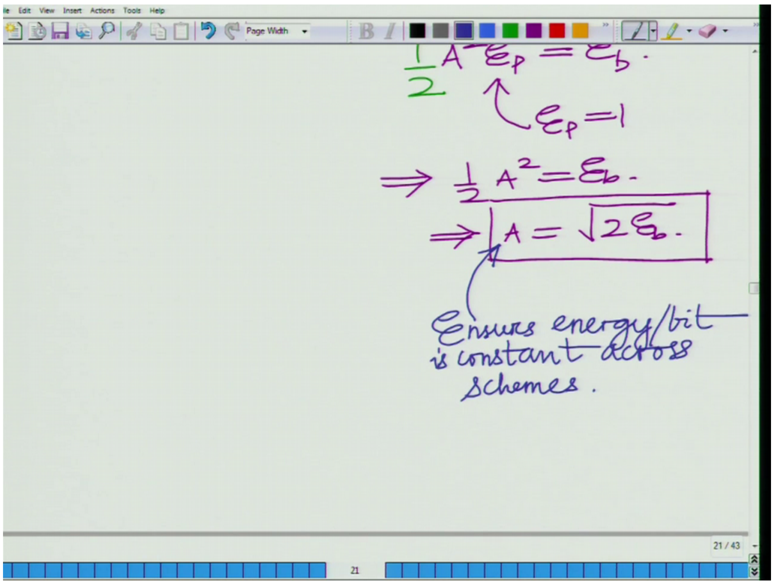

So, this ensures energy per bit is constant across the schemes.

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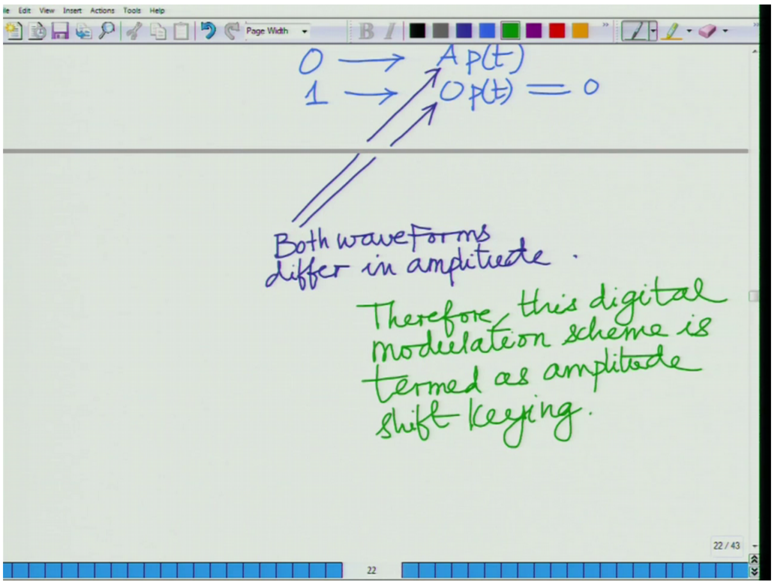

Now, as I have already told you here we have 0 being mapped to A times p t and formally stating this once again 1 is mapped to 0 times p t equals 0. Both the schemes differ in amplitude that is if you look at both the schemes both the waveforms differ in amplitude. Therefore, this digital modulation scheme is termed therefore; this digital modulation scheme is termed as amplitude shift keying. So, that is basically amplitude shift keying or ASK; amplitude shift keying or ASK.

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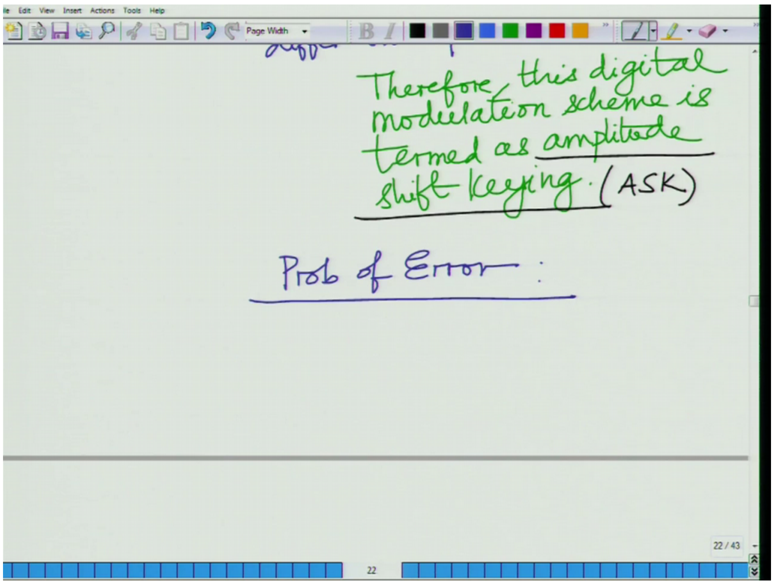

Now, we want to calculate what is the probability of the resulting probability of error for this scheme or the probability of bit error.

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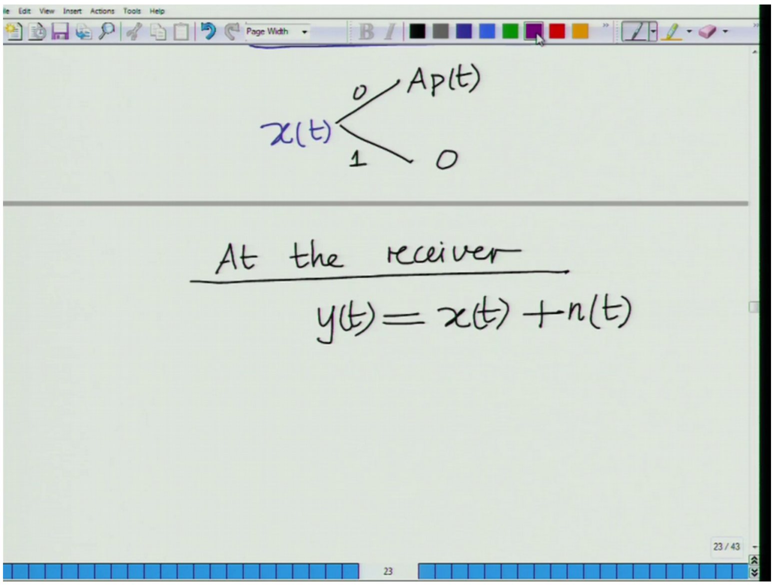

So, remember we are transmitting x t is either A times p t corresponding to information bit 0 or it is equal to 0 corresponding to information bit 1. At the receiver we have, at the receiver we have y t - y t equals x t plus n t where what is n t, we have already seen what is n t.

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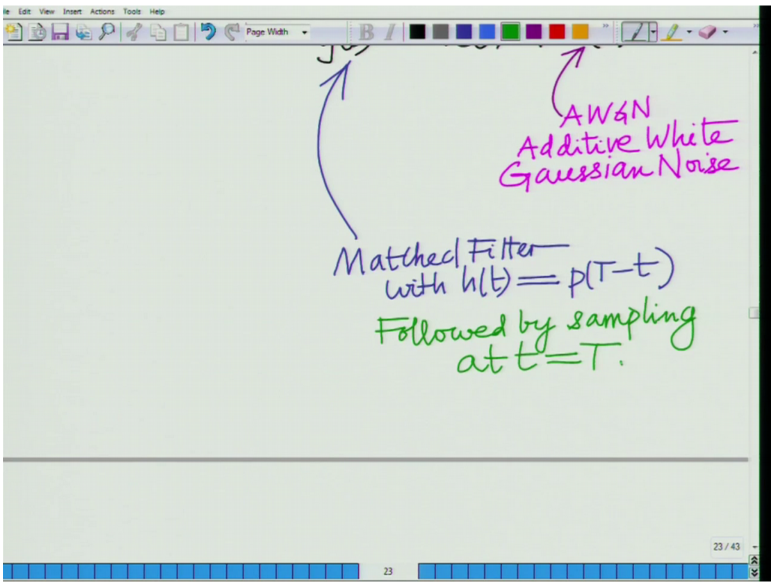

N t is basically well n t is additive, Additive White Gaussian Noise this is AWGN or this is Additive White Gaussian Noise. Now what is optimal processing at the receiver? Optimal processing at the receiver is we have to matched filter, we matched filter h t, matched filter with h t equals p T minus t that is matched filter followed by sampling at t equal to T, followed by sampling at t equal to T.

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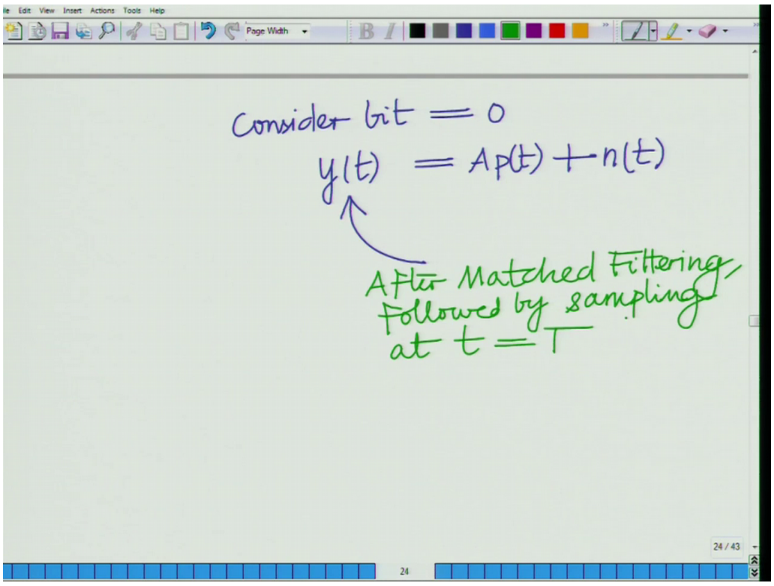

Now, consider y equal to, now consider the transmission of 0 which means x t equals A, A times p t plus n t. Now when you matched filter this, this is similar to again the transmission of bit 0 in binary shift keying when you are transmitting A times p t it is a same thing as binary shift keying when you are transmitting A times p t corresponding to the bit 0. Therefore, when you matched filter this with h t equals p of; h t equals p capital T minus t is followed by sampling at t equals to capital T where t is a symbol duration what we get is, well we get after matched filtering or this is not x t rather this is y t. So, after matched filtering followed by sampling at t equal to T or after matched filtering what we get is we get r T equals A times E p plus n tilde where n tilde is basically we have derived this before.

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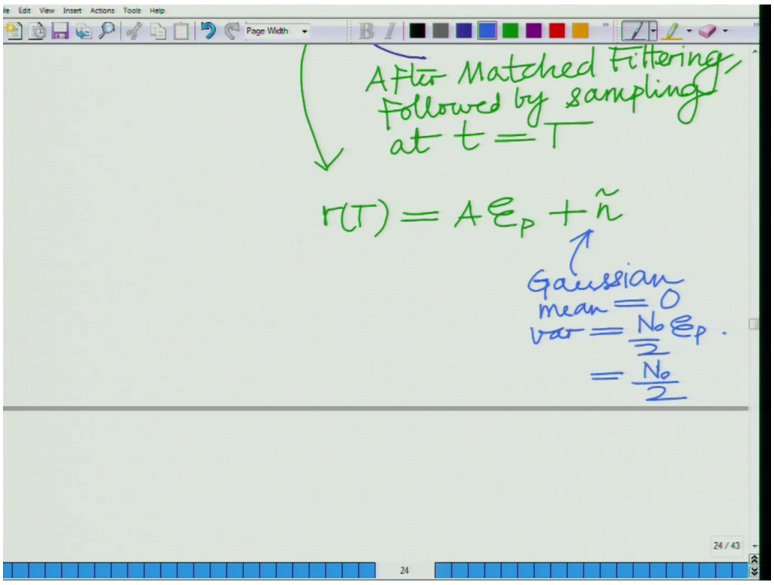

I am using a lot of results from what you have derive before for the general optimal received processing and binary phase shift keying scenario. So, n tilde is Gaussian with mean equal to 0 variance equals N naught by 2 times E p, but E p equals 1, so you can also say this is equal to N naught by 2.

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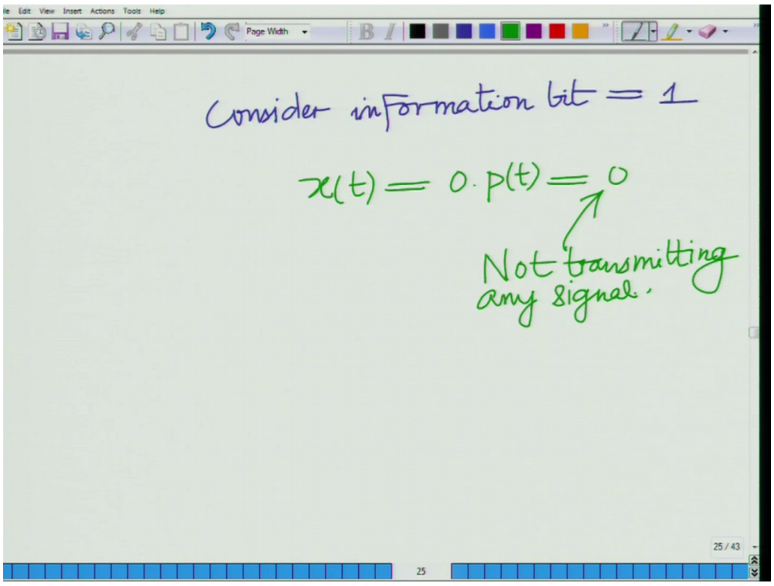

Now, consider the information bit 0 or consider the information bit 1; consider the information bit 1; when you look consider the information bit 1 what you have is basically well x t equals 0 times p t, x t equals 0 times p t that is basically equal to 0. So, you are not transmitting any signal, effectively not transmitting, effectively not transmitting any signal.

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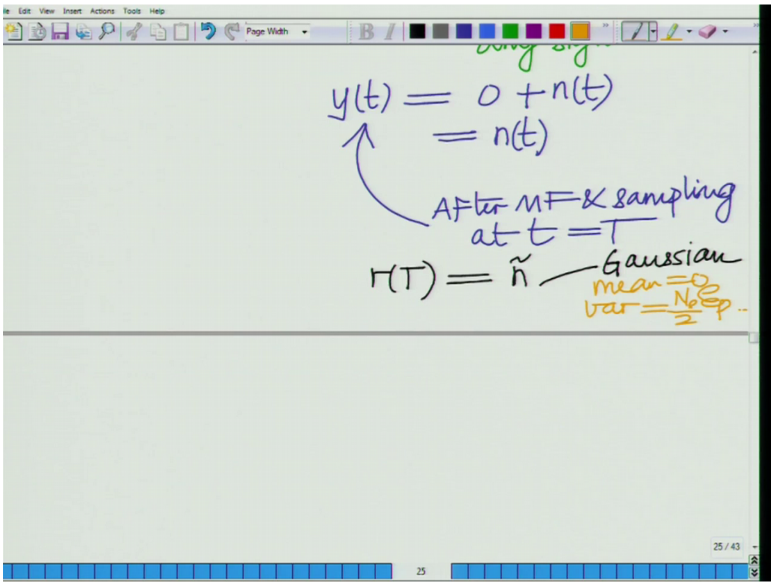

Now, therefore, what is r T therefore, y t is simply 0 plus n t equals n t. After matched filtering and sample with p sampling at t equal to T after M F and sampling at t equal to capital T what we get is r T is simply because there is no signal component there is only going to be the noise which is n tilde. Once again n tilde nothing changes n tilde is Gaussian with mean equal to 0, variance equals n naught by 2 times E p.

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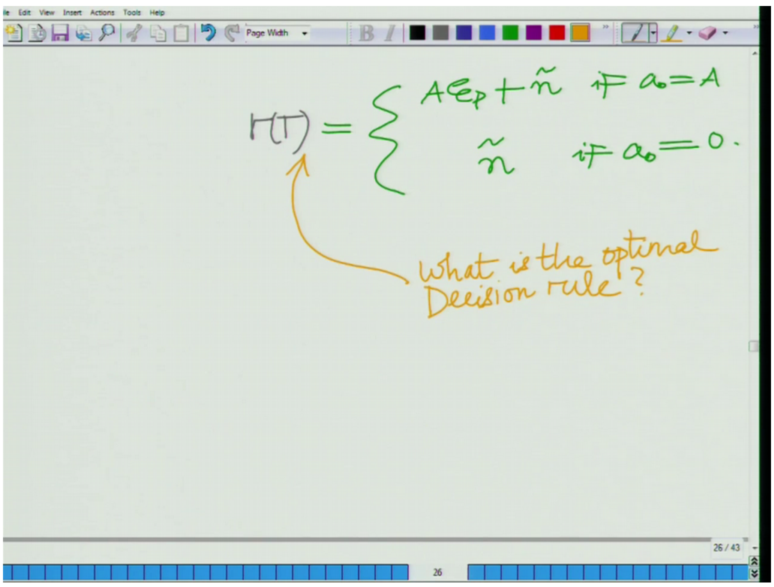

So, if you look at r T and this is important, if you look at r T you will realize that r T equals well A times E p plus n tilde if a naught equal to A this is simply 0 plus n tilde or rather simply just say n tilde if a naught is equal to 0.

Now, one can ask in this scenario; what is the optimal decision rule? One can ask: what is the optimal decision rule? Now previously the optimal decision rule, we said by symmetry we said by symmetry because we had mean shifted to minus A and mean shifted to plus or mean shifted to rather minus A tilde and mean shifted to plus A tilde. So, by symmetry the midpoint is the detection threshold, but here this is not symmetric. If you can see we have in this in the first scenario if you look at this, in this scenario corresponding to this r T is basically Gaussian with mean A p.

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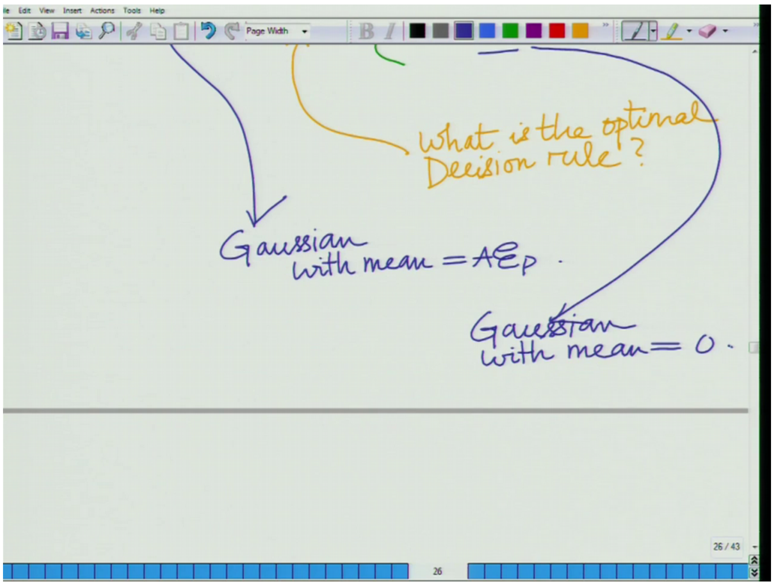

However in this r T simply n tilde which means it is a Gaussian with mean equal to 0. The variance is the same in both case.

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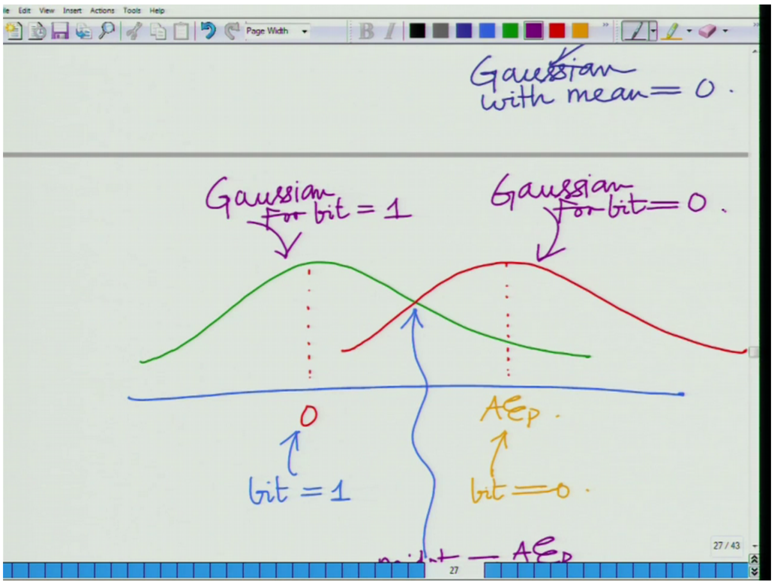

So, if you look at this, this is not shifted to minus n plus a but it is something very different. If you look at this, this is still two Gaussians, in first case the mean is 0 that is or in one case the first case rather the mean is A times E p, this corresponds to the bit 0 and this corresponds to the bit equals 1 and now you can see the midpoint is not 0, but by symmetry again midpoint is A E p divided by 2.

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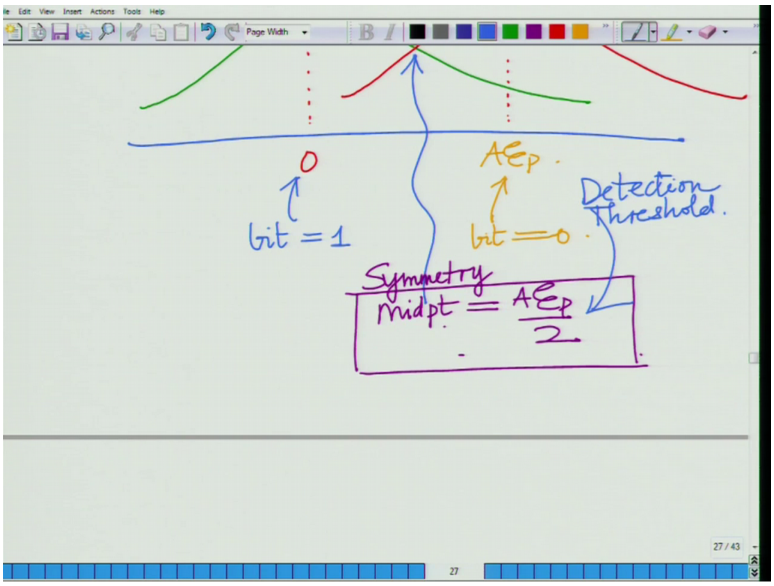

So, this is the Gaussian corresponding to bit 1, this is the Gaussian for bit equal to 0 by symmetry the midpoint is A E p divided by 2 which is the detection threshold which is also the detection which is also your detection threshold. So, by symmetry the midpoint is A E p divided by 2 which is also the detection threshold. Therefore, the optimal decision rule is not to compare r T with 0.

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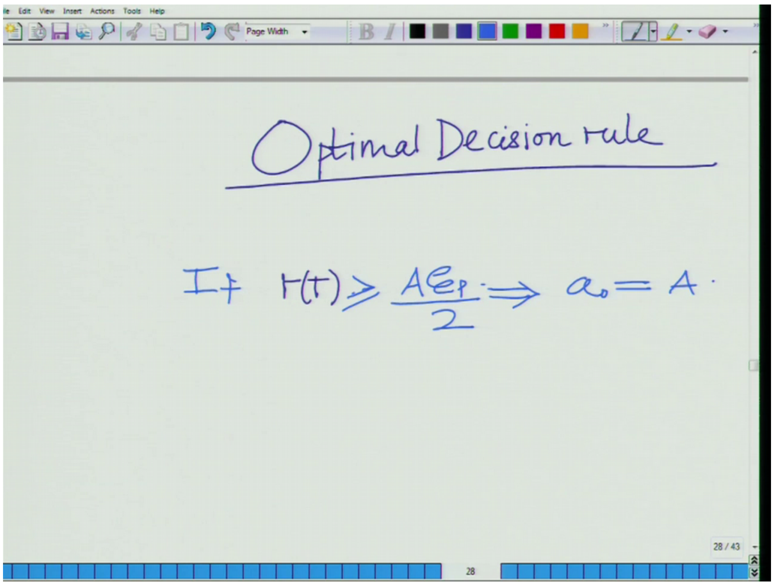

So, the optimal decision rule is not to compare r T with 0, but to compare r T with this threshold A E p divided by 2 which E p is 1. So, this is a by 2. If r T is greater than or equal to A E p divided by 2 implies, this implies the decision is a naught equal to A.

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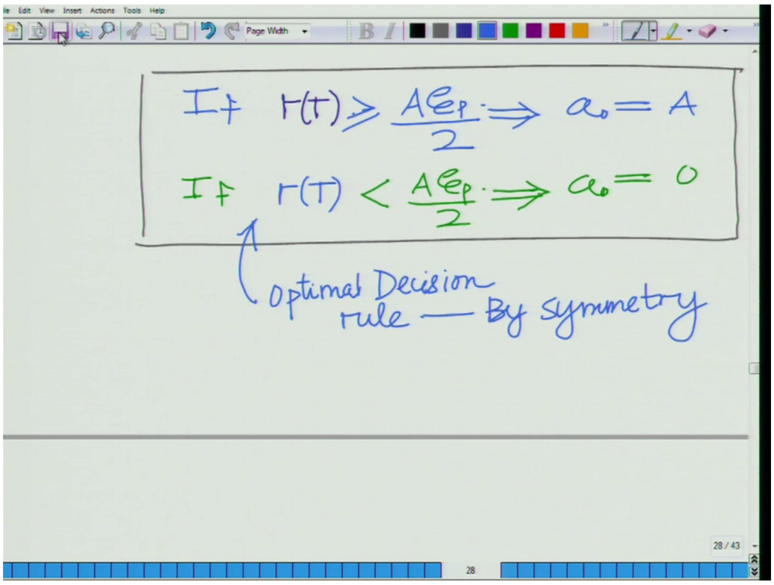

On the other hand if r T is less than A E p divided by 2 decision is a naught equals 0. So, this is basically our by symmetry this is our optimal decision rule where we are exploited the symmetry in the system, alright.

So, basically what we have seen so far is we have started looking at different digital modulation scheme that is amplitude shift keying in which the different waveforms shifted in terms of amplitude. One of them is amplitude A, other has an amplitude 0. We have seen; what is the amplitude level to keep the energy constant at E b or the energy per bit constant at E b? Again we have seen what are the different received what is the received symbol, what is the probability density function or what are the probability density functions of the sample r T corresponding the transmission of the waveform for bit 0 and the waveform corresponding to bit 1. We have seen that one is the Gaussian with mean A E p where E A is amplitude level E p is the pulse energy other is an a Gaussian with mean at 0. Therefore by symmetry the midpoint is at A E p divided by 2 which is the detection threshold.

If r t is greater than equal to A E p divided by 2 then the decision is a naught equal to A. If on the other hand r T is less than A E p divided by 2 the decision is a naught is equal to 0. So, that is the detection scheme. We will complete this in the next lecture and look at what is the resulting probability of bit error and also how it compares with the binary phase shift keying scheme that we have descried earlier.

Thank you very much.