Principles of Communication Systems - Part II
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Lecture – 11
Introduction to Binary Phase Shift Keying (BPSK) Modulation, Optimal Decision Rule and Probability of Bit-Error or Bit-Error Rate (BER)

Hello, welcome to another module in this massive open online course in this module. Let us start looking at binary phase shift keying. So, we want to look at this digital modulation scheme which is known as binary phase shift key.

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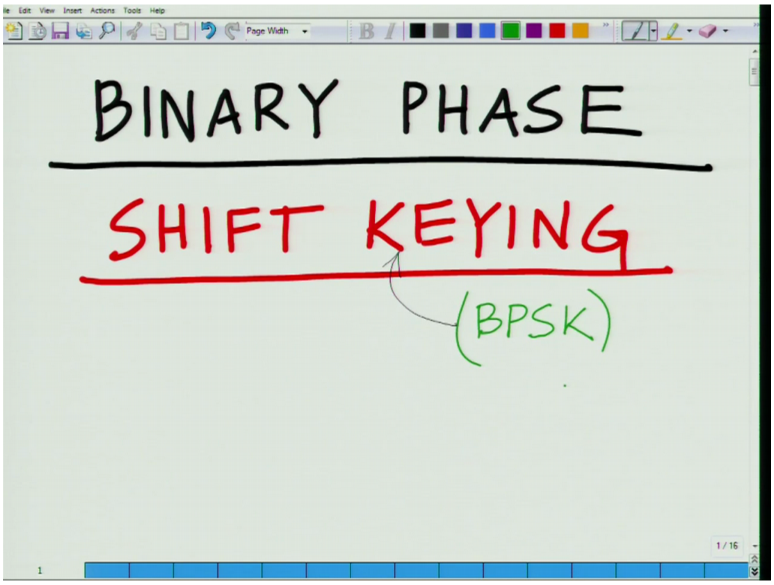

So, what I want to start looking at is a different modulation scheme which is termed as binary phase shift keying also termed as also we can call this a also abbreviated as rather BPSK; this is abbreviated as BPSK and this is basically it is a digital modulation scheme its one of the several schemes for digital modulation.

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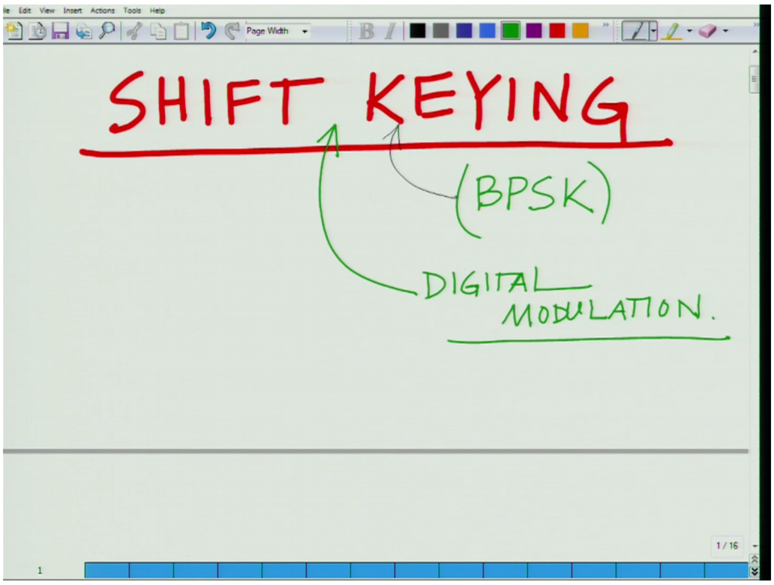

As we already described before; similar to analog modulation in a digital communication system we have digital modulation schemes.

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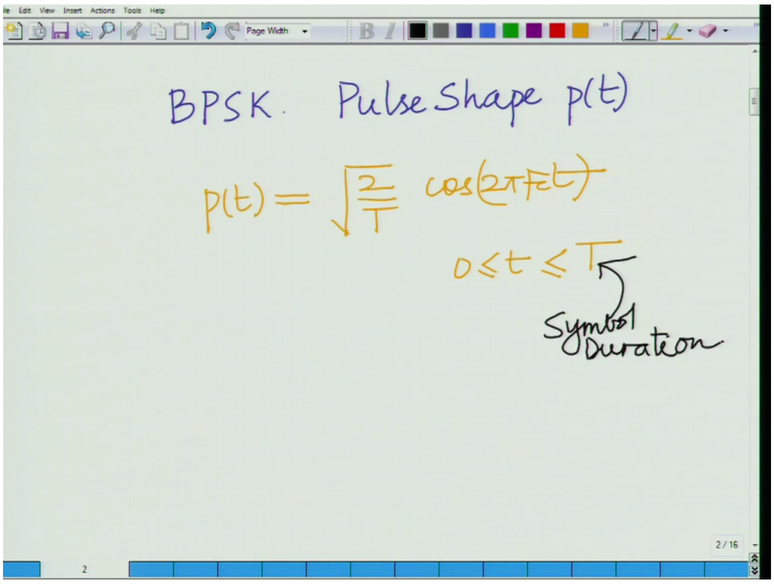

Now, what happens in by BPSK the pulse shape in BPSK the pulse shape or simply you can call this the pulse remember every digital communication system employs a pulse or a pulse shaping filter; the response of the pulse is given as well P T equals square root 2 over T; correct, square root 2 over T cosine 2 pi F c t such for 0 less than equal to t less than or equal to T where we know T is this is the symbol duration this is the symbol duration.

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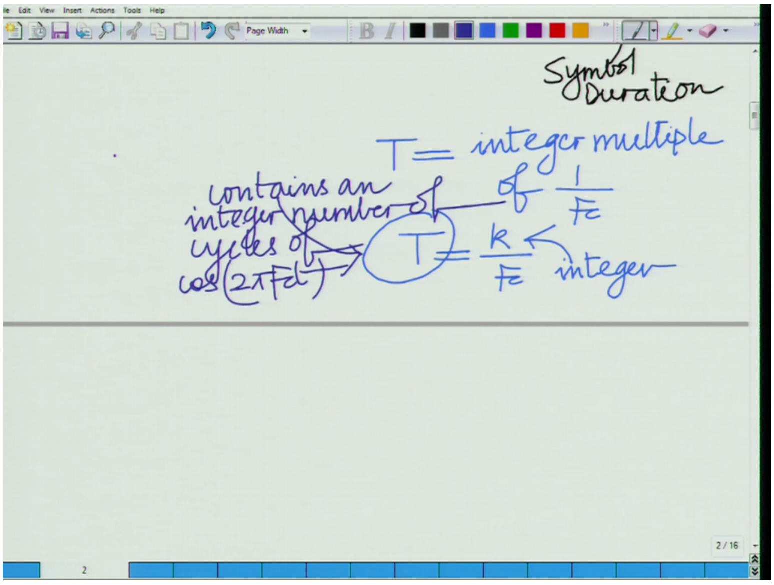

And further correct, we have this condition that T is an integer multiple of 1 over F c that is T equals k over F c where k is a integer; where k is a; this is a integer that is basically this duration T symbol duration contains an integer number of cycles of cosine 2 pi F c t and contains an integer number of cycles of cosine 2 pi F c t.

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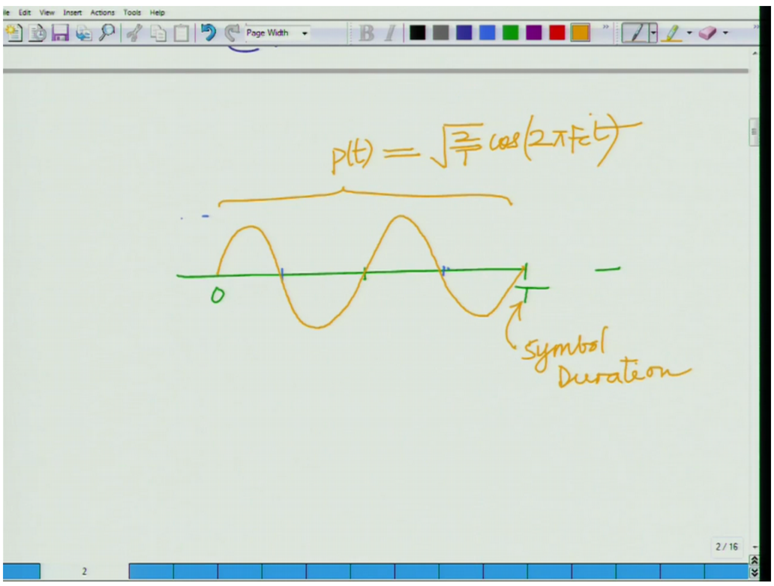

So, that is basically that is basically the idea and I can represent it pictorially it looks something like this is very simple this is 0 this is t let us say we have something. So, we have something that it looks something like this. So, if I were to draw this that will look this is basically your time duration. So, this is the duration symbol duration and this is basically your pulse and this is square root of 2 over T cosine 2 pi F c t for 0 less than equal to T where t is the symbol duration and you can see.

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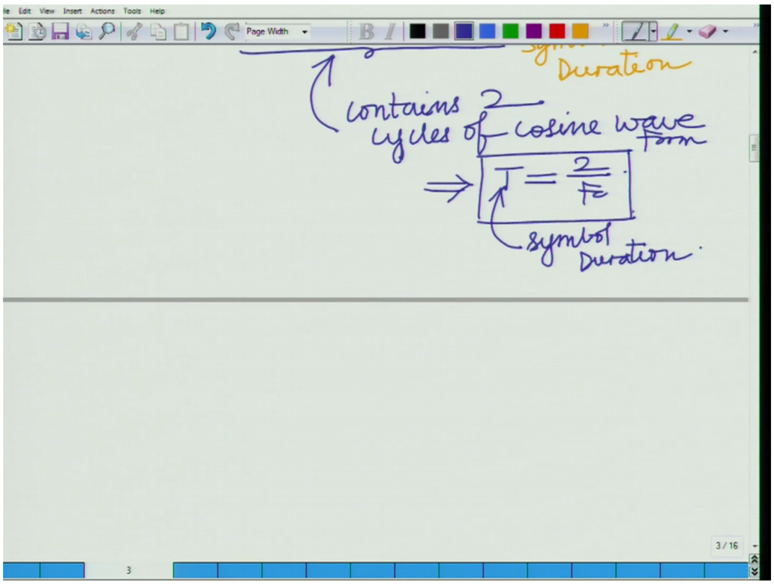

This contains basically contains 2 cycles contains 2 cycles of the cosine waveform implies T equals 2 over F c on over F c is the period. So, symbol duration is basically that is symbol duration equals 2 over F c. So, this is the symbol duration symbol duration equals 2 over F c.

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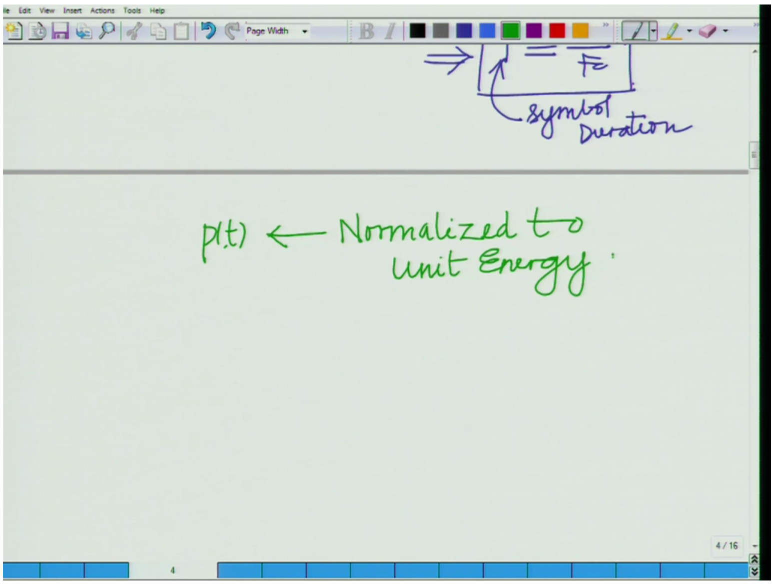

Now, further if you look at the energy of the pulse we can show that in this case pulse P T is normalized to unit energy what is the meaning of that.

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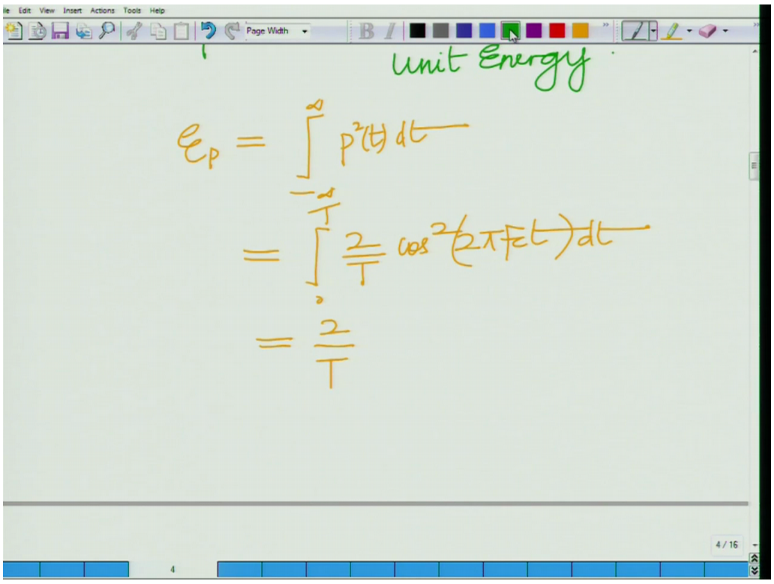

What is the meaning of that if we compute E P energy of the pulse that is minus infinity to infinity minus infinity to infinity P square t d t that is equal to well 0 to capital T 2 over T cosine square 2 pi F c t d t which is equal to 2 over T.

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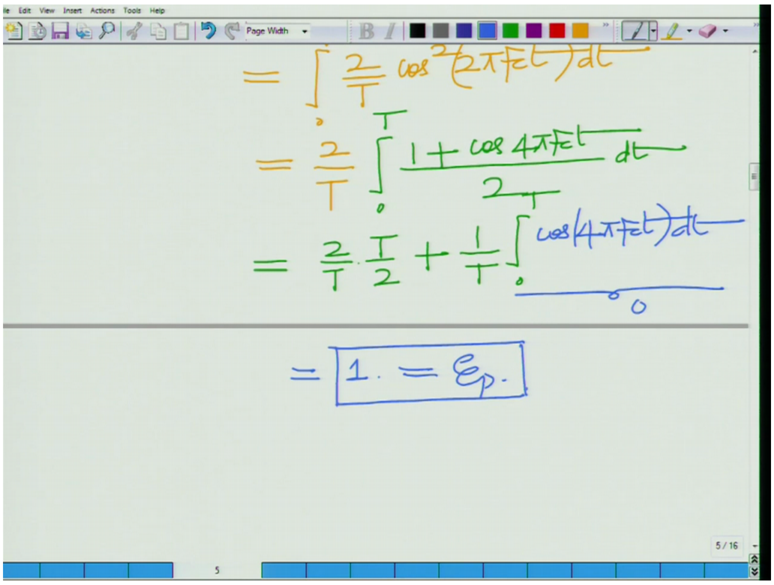

Well integral 0 to T 1 plus cosine 4 pi F c t divided by 2 d t which is equal to well 2 over T times t over 2 plus 1 over T times integral 0 to t cosine 4 pi F c t d t and you know this quantity or rather this integral this integral is 0 therefore, what is left is 2 over T times t over 2 equals 1 which is equal to your E P.

So, what is shown what will just shown although it is fairly obvious is that the energy of the pulse that is we look at square root of 2 over T 2 over capital T cosine 2 pi F c t the pulse has unit energy. So, the pulse energy has been normalized to unity.

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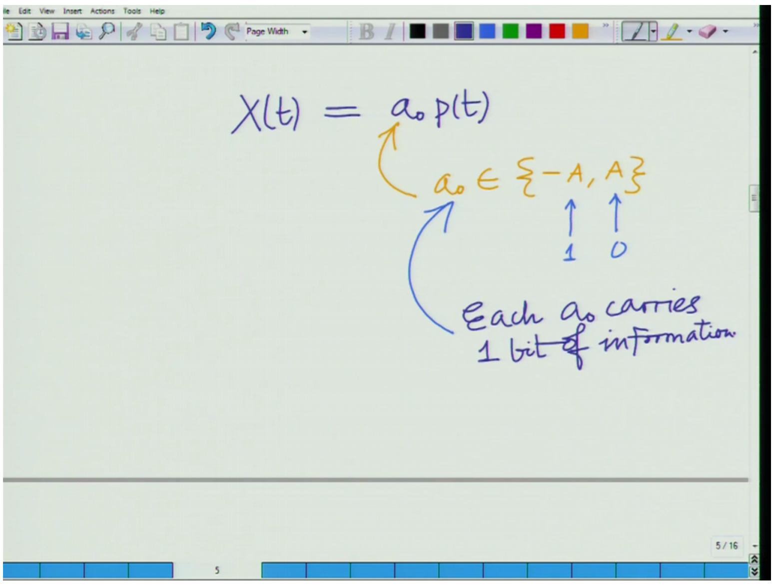

Now, a naught we have x t equals a naught times P T, now earlier we have assumed that a naught belongs to minus a or plus a now further what we are going to assume is that if you are transmitting 0 or 1 0 or 1 for instance let say for instance let say one is mapped to minus a 0 is mapped 2. So, each symbol can transmits. So, each a naught carries one bit of information each a naught carries one bit of one bit of information.

Now, therefore, if you want energy per bit now a standard way to normalize energy or compare this various schemes which is digital modulation schemes is to basically set a constant energy per bit which were going to denote by E b alright. So, this is the constant energy per bit that we are going to use throughout across all schemes.

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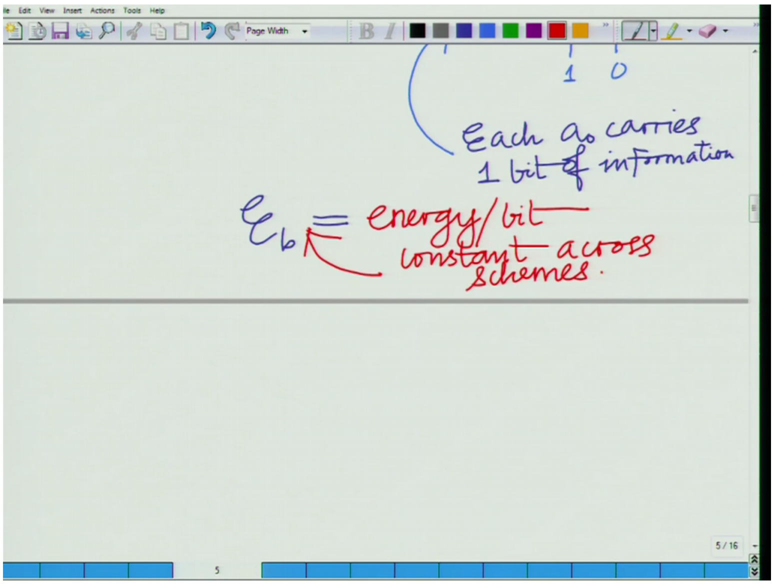

So, we are going to denote this quantity by E b; E b equals the energy per bit and this we are going to hold constant across this we are going to hold constant across the schemes.

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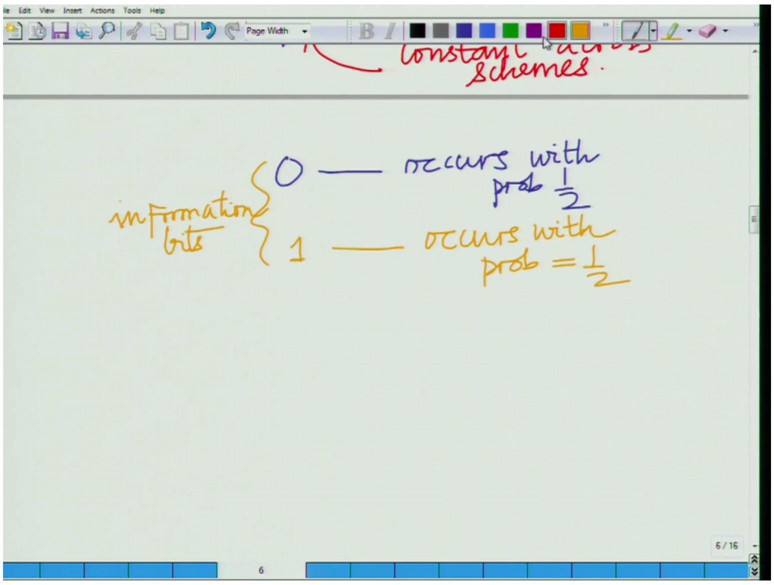

Now therefore if you look at this if 0 for instance occurs with probability half again and one that is information or these are information bits 0 occurs with probability half 1 occurs with probability half.

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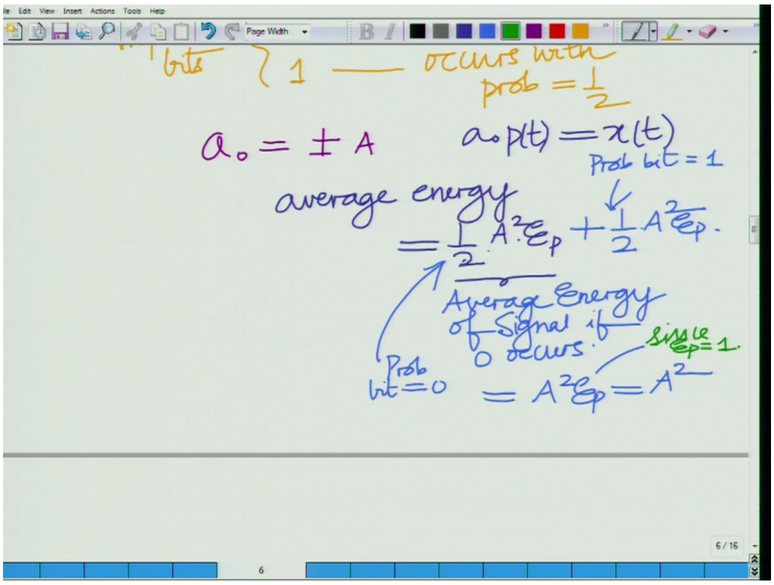

Now, average energy because a naught equals remember a naught equals plus or minus a implies average energy equals half well we are using the signal x t equals A; a naught times P T equals x t. So, half a square times E P that is the average energy corresponding to this average energy corresponding to if 0 occurs correct the average energy of the signal if 0 occurs plus again for one we are transmitting minus A times PT.

So, average energy is half a square half is the probability remember this half is probability of probability that of A the bit equals 0. So, and this half is probability of bit equals one when bit equals one with probability half multiplied by a square which is amplitude level times the energy of the pulse E P which is equal to half a square E P half A square E P that is A square E P and further we have set in this scheme E P equals 1. So, this is a square since E P equals 1 that is we are normalize the pulse to unit energy since E P is equal to 1.

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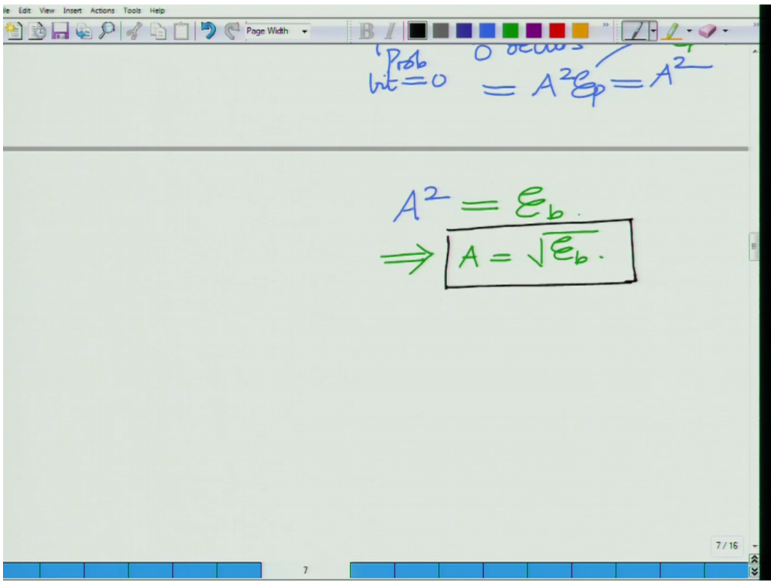

Now, if we want to set constant bit energy A square. So, if you want. So, this is our average energy. So, this is the average energy per bit. In fact, correct. So, this; what we have calculated is average energy per bit which is A square. So, we want to set average energy per bit equal to E b which means a square has to be equal to E b which means a equals square root of E b. So, if A equal to square root of E b if we set a equal to square root of E b that is the amplitude A; remember we are using the fact that the pulse has been normalized to have unit energy.

So, if you set the amplitude a equal square root of E b; correct then the transmitter then a equal to square root of E b then the average energy expended per bit in this digital modulation scheme or in this binary phase shift keying scheme is going to be E b that is the point. So, A equal to square root of E b.

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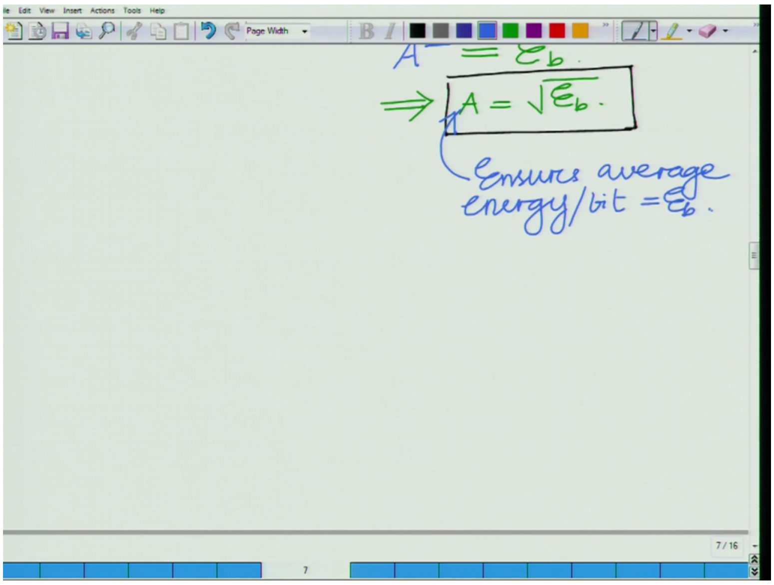

This ensures that average energy per bit equals E b therefore, the transmitter waveforms.

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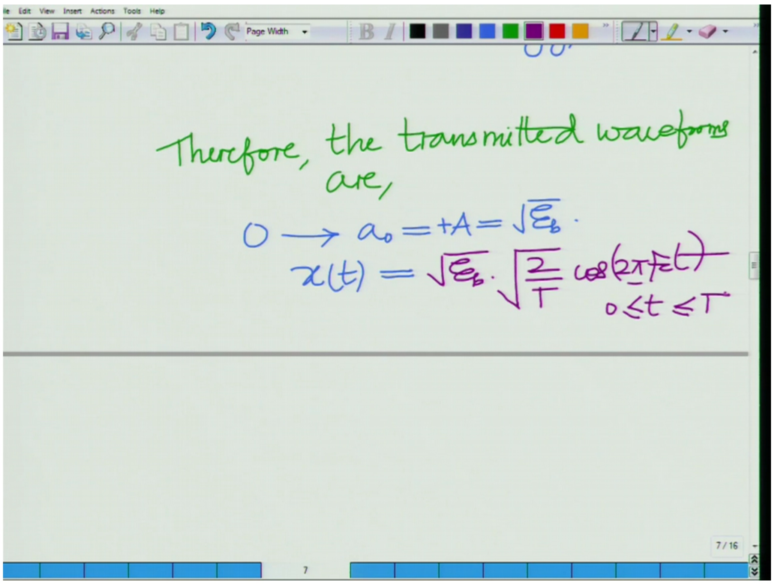

Now, if you look at the transmitted wave forms if you have 0 that is mapped to a naught equals plus A and correspondingly we transmit x t equals a naught times P T which is a naught equals plus A which is square root of E b by the way since you are setting the energy per bit equals E b as we have just shown. So, this is square root of E b times square root of 2 t cosine 2 pi F c t of course, it goes without saying 0 less than equal to t less than equal to T.

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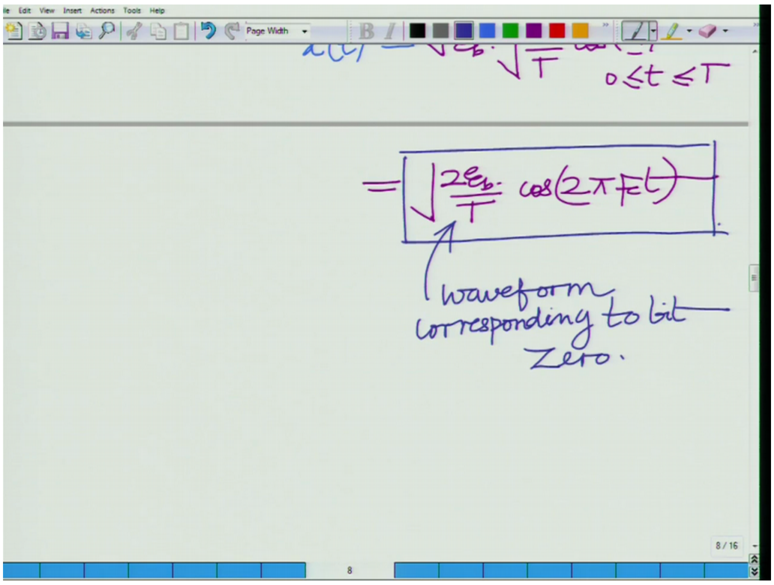

Which is equal to again square root of 2 E b over T cosine 2 pi F c t, this is the wave form corresponding to 0, this is the wave form just to summarize this once again although it should be very clear this is the wave form corresponding to this is the wave form corresponding to bits 0, this is the waveform corresponding to bit 0.

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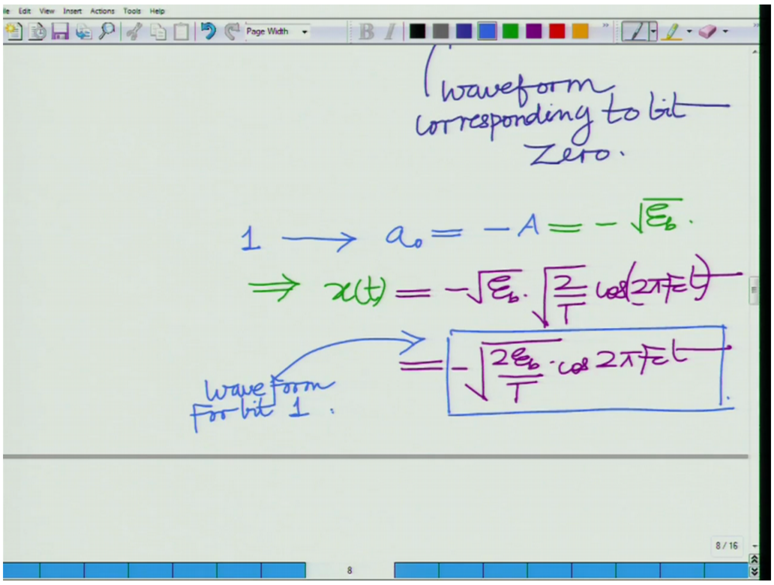

Now, on the other hand if you look at the bit one if you look at the information bit one then I have a naught equals minus A which implies x t which is basically your minus square root of E b which implies x t equals remember a naught times P T minus square root E b times root 2 over T cosine 2 pi F c t of course, again 0 less than equal to t less than equal to capital T which is minus square root 2 E b by T cosine 2 pi F c t, this is the wave form corresponding to your bit 1; this is the waveform corresponding to bit 1.

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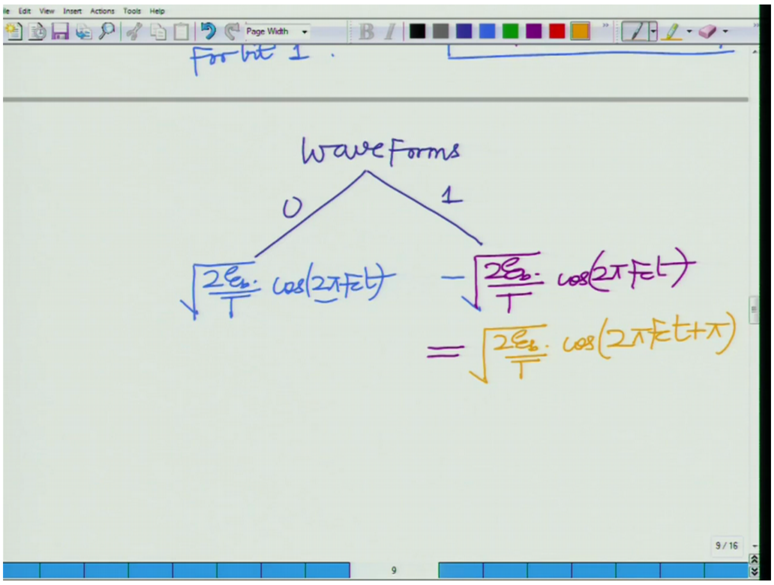

Now, if you observe both the waveforms if you observe the waveforms that are employed in this scheme corresponding to bit 0 corresponding to bit one for instance corresponding to bit 0, we have 2 square root E b by t cosine 2 pi F c t corresponding to one the only difference is we have exactly identical waveform except we have a minus sign in front of it minus 2 square root of E b over T cosine 2 pi F c t which if you realize is nothing, but square root 2 E b over T times cosine 2 pi F c t plus pi.

So, if you look at the waveform for bit one it is exactly identical to the waveform it is identical to the waveform identical under sense it is similar to the waveform of bit 0 except that it is shifted by a phase of pi. So, in this digital modulation scheme when you look at the different information bits the waveform of one of the bits in that the waveforms of each of the bits is shifted from the that of the other by a phase of pi of course, if you take this shift by pi like it the waveform corresponding to bit one add another pi to it that is this becomes cosine 2 pi F c t plus 2 pi that is again cosine 2 pi F c t.

So, if you shift the waveform corresponding to bit 1 by pi you get the waveform corresponding to bit 0, if you shift the waveform corresponding to bit 0 by pi, you get the waveform corresponding to bit 1. So, the waveforms are shifted by pi and therefore, this is a digital modulation scheme based on shifting the phases alright this is the digital modulation scheme based on shifting of the phases between the waveforms corresponding to the different information bits further you are transmitting 2 waveforms there are is a there are 2 waveforms, alright. So, its binary in nature its either the corresponding to information bits 0 or corresponding to information bit one therefore, this is binary phase shift keying phase shifting because you are shifting the phase binary because you are employed 2 waveforms therefore, this digital modulation scheme is known as binary phase shift key.

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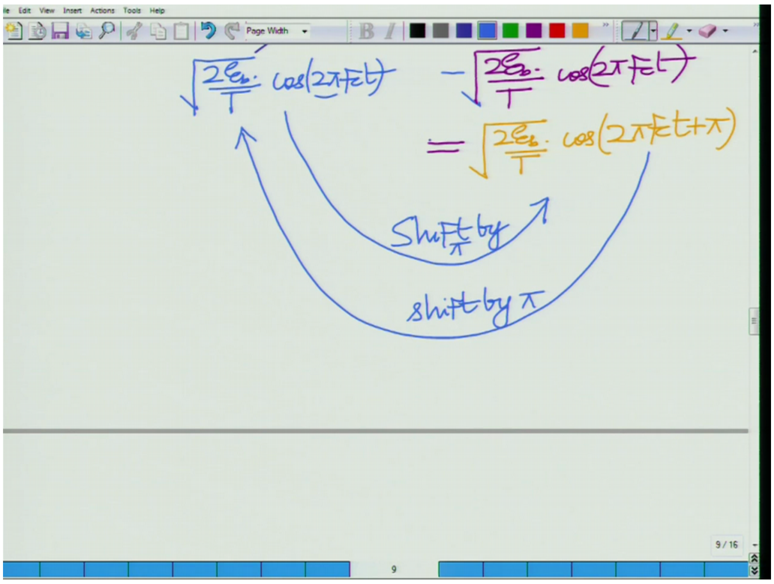

So, you will realize that if you shift the phase of this by pi you get this if you shift the phase of this a waveform corresponding to information bit by one if you shift by pi you will get this.

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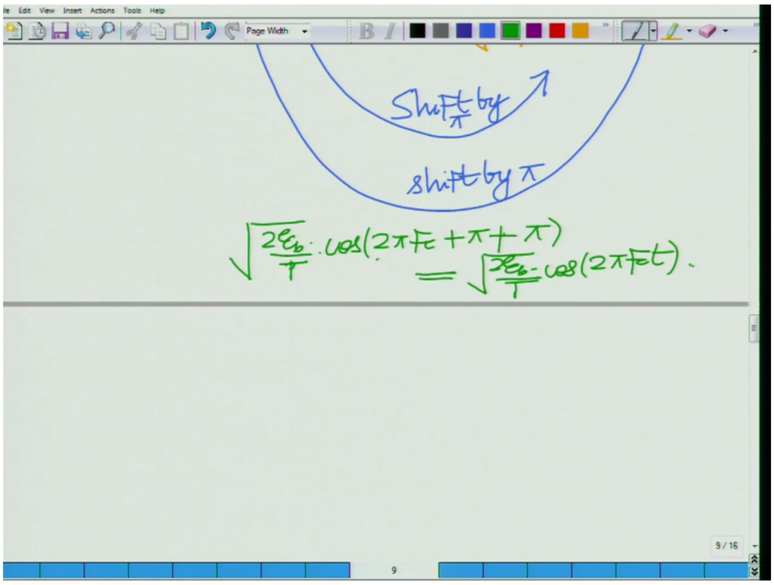

So, what we have for instance you can really see that I mean if you have square root of 2 E b by t cosine 2 pi F c t plus pi if I add other pi to it that will give you square root 2 E b by t nothing, but cosine 2 pi F c t.

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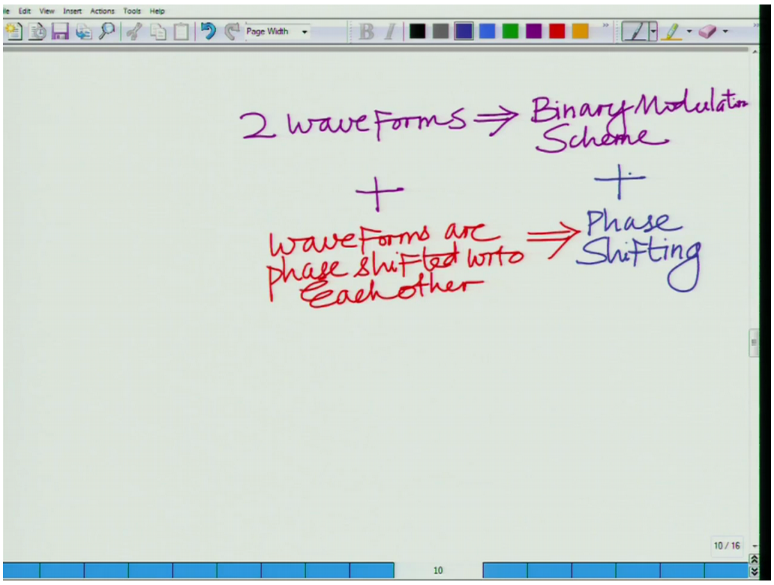

So, basic is, so basically again we have 2 waveforms implies this modulation scheme is a binary modulation scheme plus waveforms are phase shifted with respect to each other.

This implies phase shifting or the digital modulation scheme is based on phase shifting. So, you have binary plus phase shifting that gives rise to the name which is binary and this is important it is not just an arbitrary name there is a motivation behind every nomenclature.

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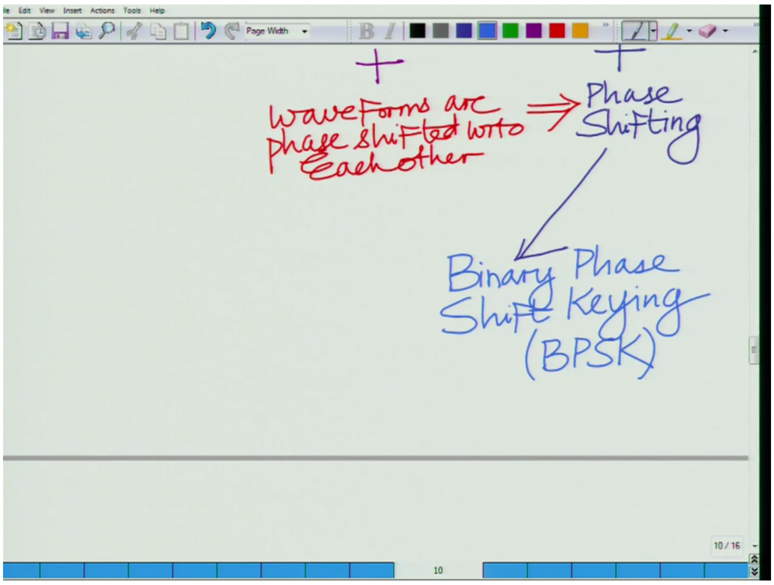

So, the motivation behind this calling this binary phase shift keying is basically there are binary waveforms 2 waveforms which are phase shifted with respect to each other therefore, this is known as binary phase shift keying and now we know what is optimal now we know this is what is. So, we are employing 8 A times P T minus A times P T we already see the similar scheme for instance.

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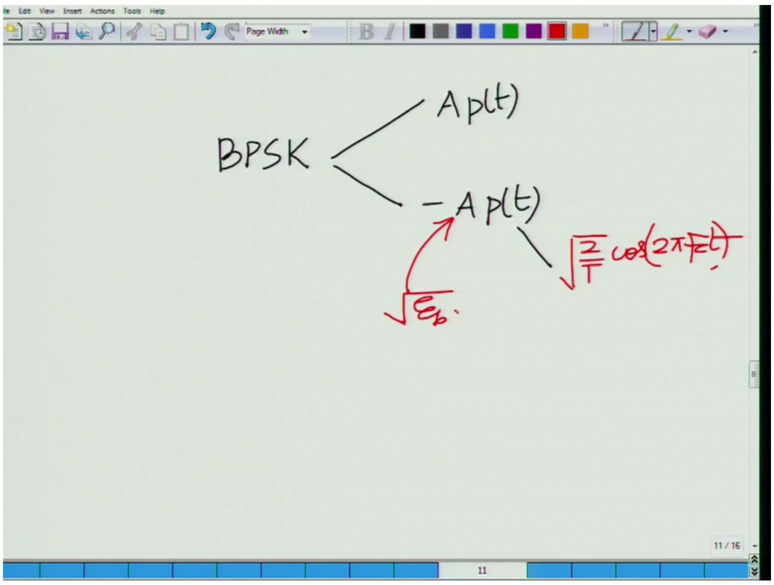

So, in this BPSK we are employing 2 waveforms a times or a naught times or let us put it this way a times P t A times P t minus; A times P t, the only thing is that P T equals square root of 2 over T cosine 2 pi F c t and a is square root of E b.

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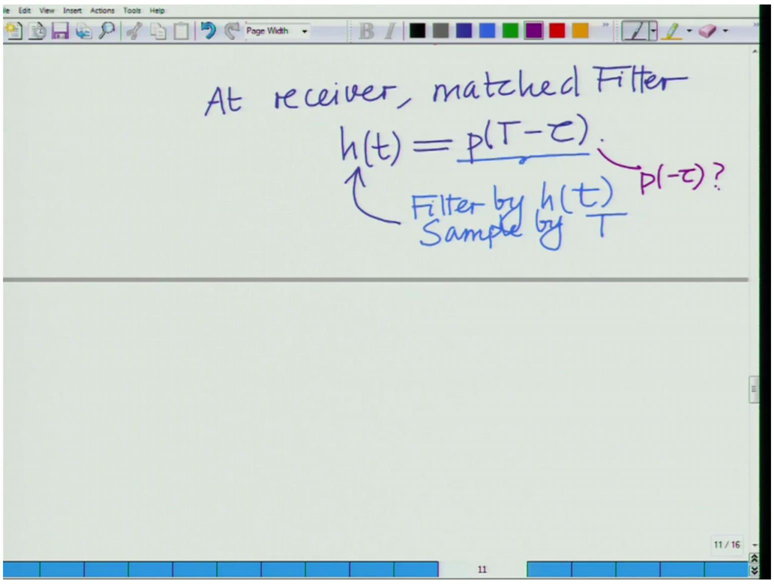

So, we know the match filter match filter is given for this the match filter at receiver match filter is h t equals P T minus tau that is filter with h t optimal processing is filter by h t followed by sample at capital T sample by t and now you are wondering why we are using this P capital T minus tau that also probably is worth clarifying because rather than using simply P minus T why are we shifting it by capital T.

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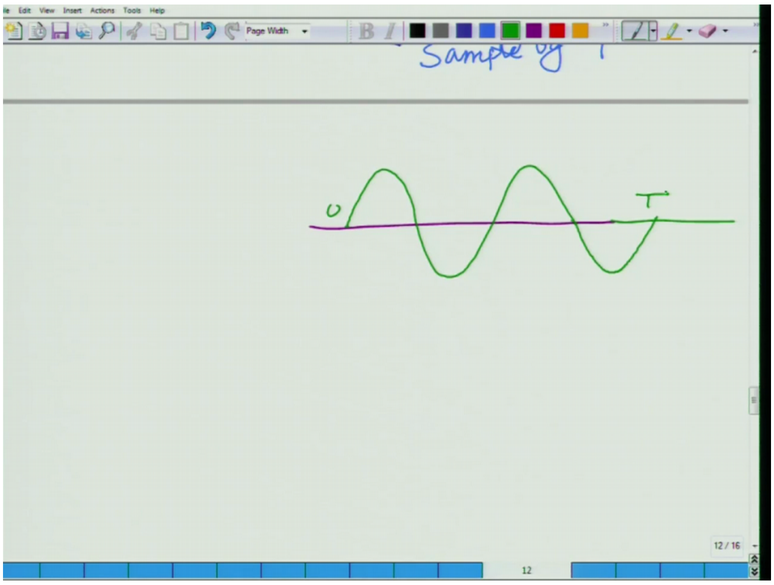

If you look at it this and this is a good spot to explain this if you look at this waveform this is my P of 2 cycles in 0 to T.

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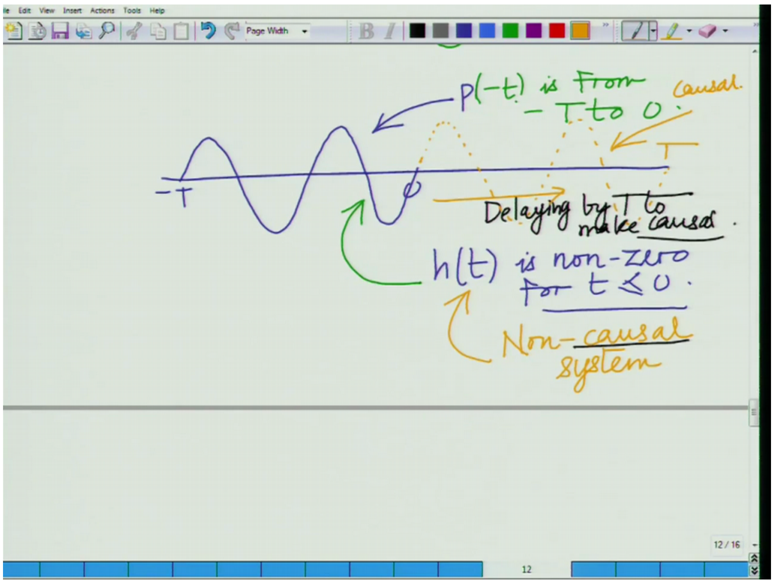

So, if you look at P of minus t if you simply look at P of minus t that will simply be shifted with respect to. So, it will be from 0 to. So, it will simply be from minus t to 0. So, P of minus t is from minus T to 0 and therefore, you want to filter by this the problem is this is a non causal system h of T is nonzero for t less than or equal to 0 this is a non causal system which causes problems this is a non causal system this is a non causal system and therefore, what we are doing is we are shifting this what we are doing is we are shifting this by capital T to make it causal.

So, shifting by capital T causal that is at important and if you understand the theory of linear time invariant systems you will realize that any impulse response which is non0 for t less than equal to 0 is a non causal system alright and one cannot easily implement one cannot implement non causal systems. So, therefore, we are shifting it advancing it by T or delaying it that is the correct word delaying it by t because shift can be either left shift or right shift we are delaying it by capital T to make it a causal system. So, let me just correct that it is not simply shifting it is rather it is rather delaying by t that is important that is we are performing t minus t naught. So, that what happens is basically now when you delay this by T you get this from 0 to T, this is causal.

When you delay it you get it from 0 to t which is causal and now, therefore, we know after match filtering.

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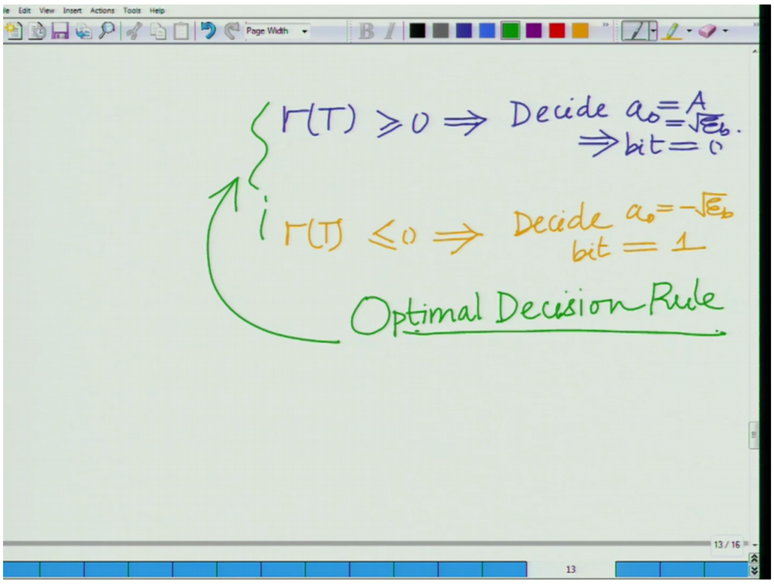

We also know what is optimal detector optimal detector is take the test static r T which is and compare it with the threshold r T, I am just repeating this again once again for the sake of clarity r T greater than 0 implies decide a naught equals A which or a naught equals square root of E b which implies bit the bit equals 0 or if r T, let me put it this way if r T; if r T is less than equal to 0 this basically means decide a naught equals minus square root of E b or bit is equal to 1.

So, this is the decision rule and we also call this as the optimal decision rule although it not justified its optimality you can see by symmetry you cannot have any other decision rule which is optimal this is the optimal decision rule purely by a symmetric arguments of symmetry.

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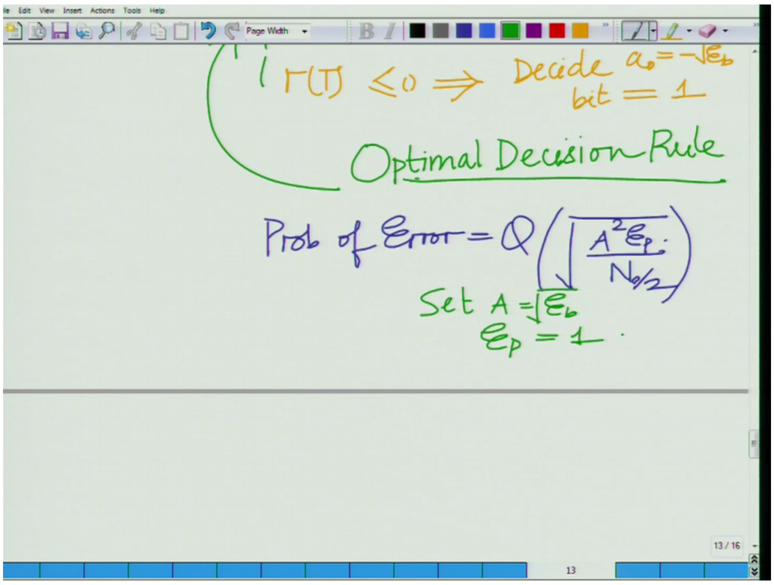

And the probability of error that is the probability 0 will be decided as one and one will be decided as 0 we have derived that that is Q of square root E square E P by n naught by 2 which is basically now you have to set in this set a equals square root of E b and E P equals 1.

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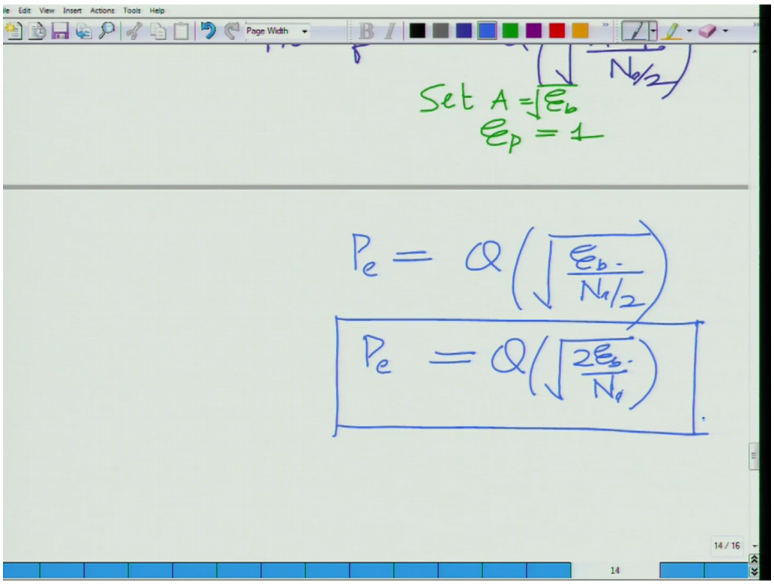

And therefore, probability of error if I denoted by P E is equal to Q square root of A square is E b over N naught by 2 which is equal to Q square root of 2 E b over N naught that is your P E which is the probability of error of this binary phase shift key system.

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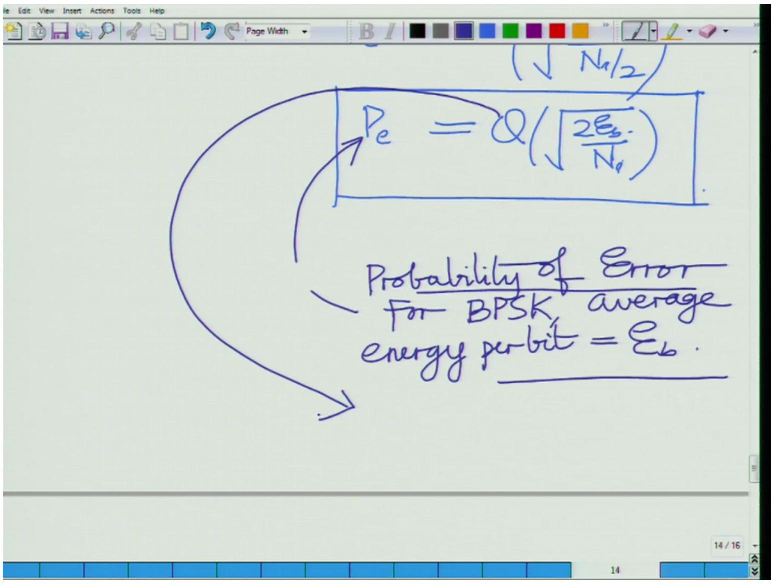

And this is an important result. In fact, this is one of the most also one of the most fundamental systems because BPSK as you have seen is a simple modulation scheme and is very efficient and very robust.

So, this is the probability of error of error for BPSK with average energy per bit equals E b that is the whole point and this is a very popular and very standard result that is the probability of error of BPSK is given as Q square root of 2 E b by N o and remember again just to be extremely clear the Q function is the complementary cumulative distribution function or the tail probability of the standard Gaussian density probability density function which is defined as integral x to infinity one over square root of 2 pi E power minus t square by 2 d t.

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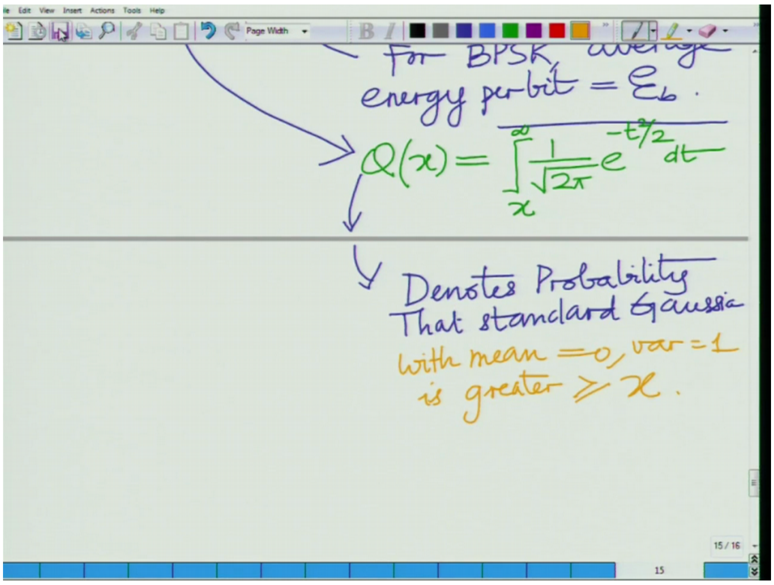

This is the probability it denotes the probability that denotes the probability that standard Gaussian with mean equal to 0 variance equal to one is greater than or equal to x that is basically what this is t naught t.

So, in this module we explored one of the most one of the simplest one of the most robust I would say popular and robust scheme. So, digital modulation that is BPSK binary phase shift keying which uses cosine waveforms shifted by a phase of pi 2 cosine waveforms that is binary shifted from each other by a phase of pi. Therefore, it is known as binary phase shift keying we have seen the transmitted the transmitted waveform we have seen the optimal processing at the receiver with match filtering and also seen what is the resulting probability of bit error, alright. So, let us stop here.

Thank you very much.