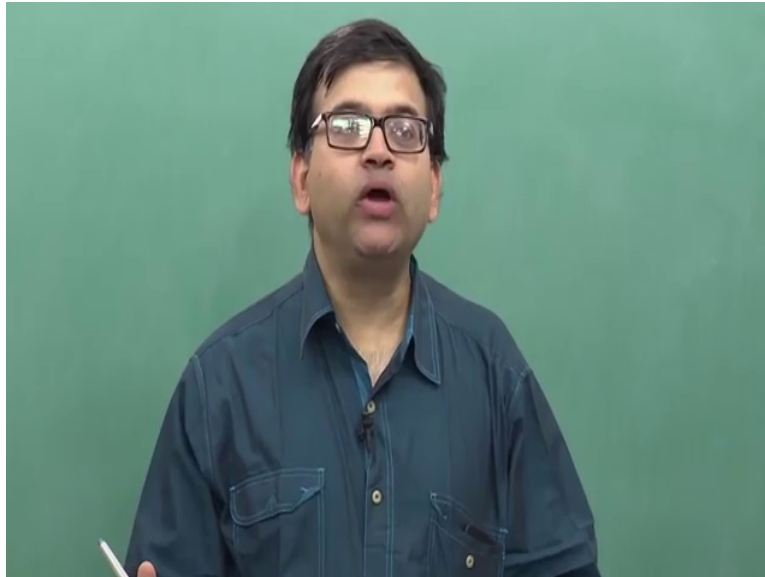


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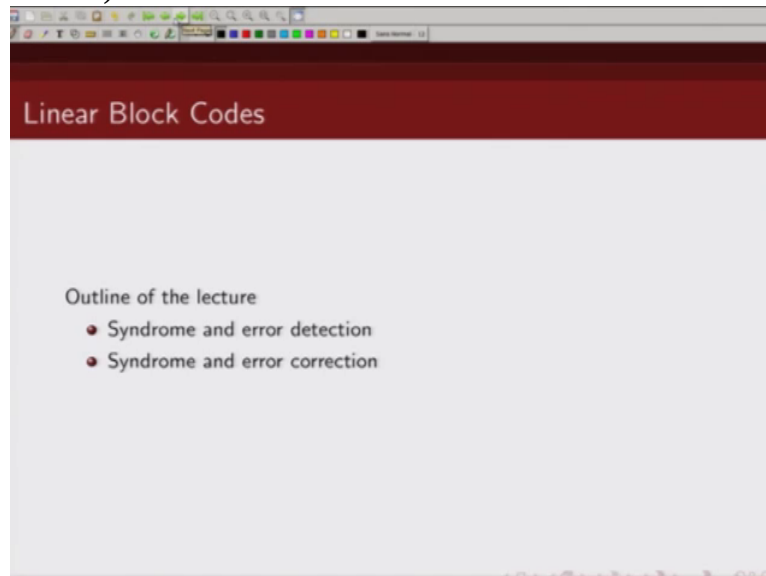
going to describe how we can use error correcting codes for error detection and error correction. So we will first describe what we mean by Syndrome and then we will show how we will use this Syndrome to do error correction and error detection. So this lecture

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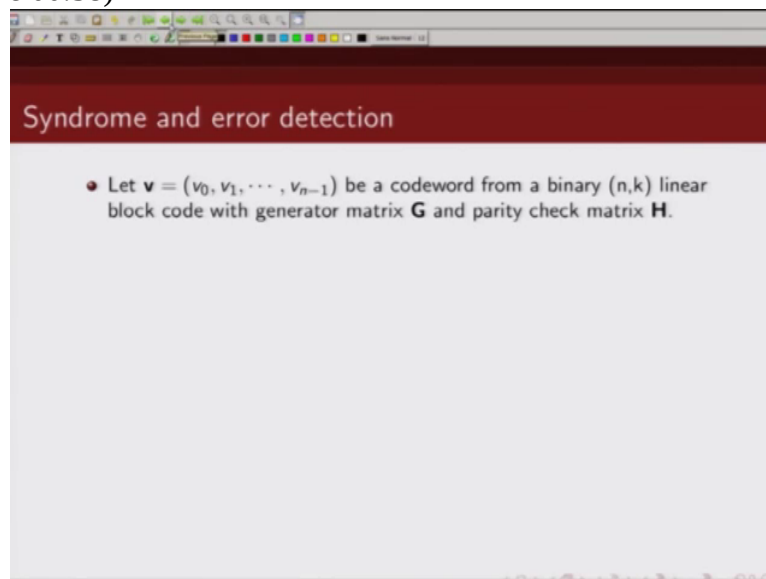
is about Syndrome and error correction and error detection We will first talk about what is a Syndrome and how we can use it for error detection. And then we will talk

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about how the Syndrome can be used for error correction

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So we know so far that if we have an n k linear block codes, so that means we have k information

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bits and n coded bits and this $n \times k$ linear block code is completely described by a generator matrix and a parity check matrix.

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n, k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,

$$\begin{aligned} \mathbf{r} = (r_0, r_1, \dots, r_{n-1}) &= \mathbf{v} + \mathbf{e} \quad (\text{modulo-2}) \\ &= (v_0, v_1, \dots, v_{n-1}) + (e_0, e_1, \dots, e_{n-1}) \\ &= (v_0 + e_0, v_1 + e_1, \dots, v_{n-1} + e_{n-1}), \end{aligned}$$

where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

So let us assume that we have codeword which is encoded using a linear $n \times k$ block encoder. So we have an output of an encoder which is of coded bits of length n . Now we want to transmit this

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codeword over a communication channel For simplicity let's consider a

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n,k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
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where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

binary symmetric channel Again let's recall what is a binary symmetric channel. So in a binary symmetric channel we have 2 inputs and 2 outputs, so binary inputs 0s and 1, binary output

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n,k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,

$$\mathbf{r} = (r_0, r_1, \dots, r_{n-1}) = \mathbf{v} + \mathbf{e} \quad (\text{modulo-2})$$

$$= (v_0, v_1, \dots, v_{n-1}) + (e_0, e_1, \dots, e_{n-1})$$

$$= (v_0 + e_0, v_1 + e_1, \dots, v_{n-1} + e_{n-1}),$$
 where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

0s and 1 and with probability let's say 1 minus, probability

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n,k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,

$$\mathbf{r} = (r_0, r_1, \dots, r_{n-1}) = \mathbf{v} + \mathbf{e} \quad (\text{modulo-2})$$

$$= (v_0, v_1, \dots, v_{n-1}) + (e_0, e_1, \dots, e_{n-1})$$

$$= (v_0 + e_0, v_1 + e_1, \dots, v_{n-1} + e_{n-1}),$$
 where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

1 minus p, we receive the bits correctly and there is

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Syndrome and error detection

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- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,

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$$= (v_0 + e_0, v_1 + e_1, \dots, v_{n-1} + e_{n-1}),$$
 where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

a crossover probability of p that the bits get flipped So this is a binary symmetric

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n,k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,

$$\mathbf{r} = (r_0, r_1, \dots, r_{n-1}) = \mathbf{v} + \mathbf{e} \quad (\text{modulo-2})$$

$$= (v_0, v_1, \dots, v_{n-1}) + (e_0, e_1, \dots, e_{n-1})$$

$$= (v_0 + e_0, v_1 + e_1, \dots, v_{n-1} + e_{n-1}),$$
 where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

channel Let's denote by \mathbf{r} our received sequence, received codeword which we sent over this binary symmetric channel. So the types of, as I said, the outputs of binary symmetric channel is also 0 and 1, so we can describe the output of a binary symmetric channel \mathbf{r} as our transmitted codeword plus some

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Syndrome and error detection

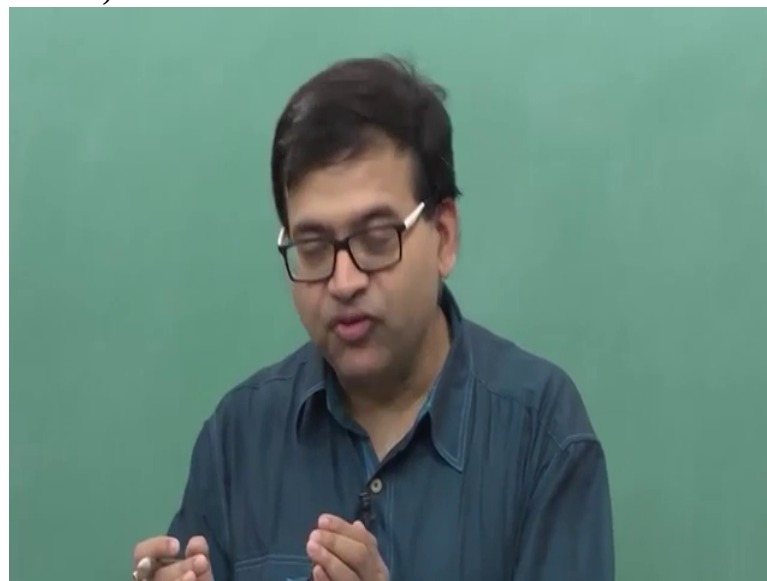
- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n, k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,

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where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

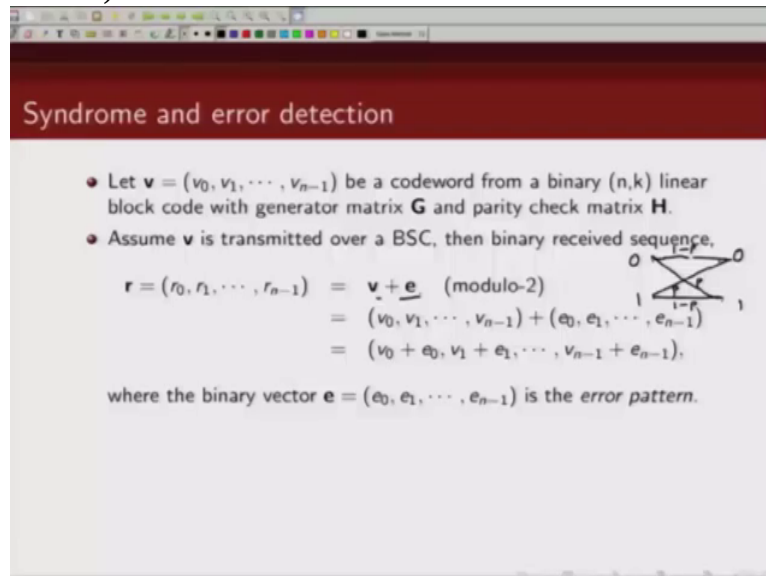
error vector So our transmitted codeword is an n-bit tuple. Similarly

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our error vector is also an n-bit.

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n, k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
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where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

The diagram shows a Binary Symmetric Channel (BSC) with input 0 and output 0, and input 1 and output 1. A central box labeled 'BSC' has a downward arrow from the input and an upward arrow to the output. A feedback loop labeled 'e' connects the output back to the input, indicating an error.

So we describe our error vector by $e_0, e_1, e_2, \dots, e_{n-1}$ and whenever e_i term is 1, that means that particular bit was not received correctly.

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So we can write an output of a binary symmetric channel.

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n,k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,

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where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

So the first bit that we would receive is basically

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n,k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,

$$\begin{aligned} \mathbf{r} = (r_0, r_1, \dots, r_{n-1}) &= \mathbf{v} + \mathbf{e} \quad (\text{modulo-2}) \\ &= (v_0, v_1, \dots, v_{n-1}) + (e_0, e_1, \dots, e_{n-1}) \\ &= (v_0 + e_0, v_1 + e_1, \dots, v_{n-1} + e_{n-1}), \end{aligned}$$

where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

v_0 plus e_0 , v_1 plus, second

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n, k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,

$$\mathbf{r} = (r_0, r_1, \dots, r_{n-1}) = \mathbf{v} + \mathbf{e} \pmod{2}$$

$$= (v_0, v_1, \dots, v_{n-1}) + (e_0, e_1, \dots, e_{n-1})$$

$$= (v_0 + e_0, v_1 + e_1, \dots, v_{n-1} + e_{n-1}),$$
 where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.

The diagram shows a 2x2 matrix with elements 0, 1, 1-r, 0. The top row is 0, 1-r and the bottom row is 1, 0. Arrows indicate a swap between the two columns.

bit will be v_1 plus e_1 , similarly v_2 plus e_2 and last finally we will get v_{n-1}

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n, k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,

$$\mathbf{r} = (r_0, r_1, \dots, r_{n-1}) = \mathbf{v} + \mathbf{e} \pmod{2}$$

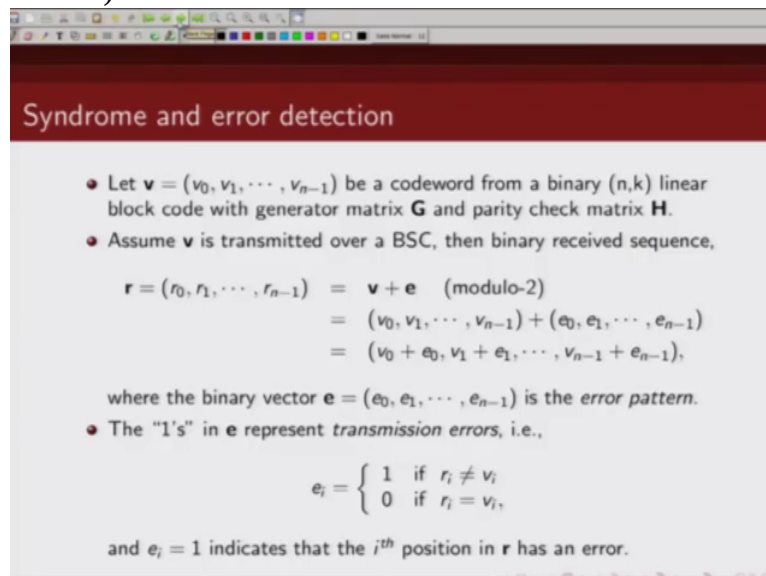
$$= (v_0, v_1, \dots, v_{n-1}) + (e_0, e_1, \dots, e_{n-1})$$

$$= (v_0 + e_0, v_1 + e_1, \dots, v_{n-1} + e_{n-1}),$$
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The diagram shows a 2x2 matrix with elements 0, 1, 1-r, 0. The top row is 0, 1-r and the bottom row is 1, 0. Arrows indicate a swap between the two columns.

and e_{n-1} where this e_0, e_1, e_2, e_{n-1} is my error pattern. Now when does an error

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n,k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
- Assume \mathbf{v} is transmitted over a BSC, then binary received sequence,
$$\mathbf{r} = (r_0, r_1, \dots, r_{n-1}) = \mathbf{v} + \mathbf{e} \quad (\text{modulo-2})$$
$$= (v_0, v_1, \dots, v_{n-1}) + (e_0, e_1, \dots, e_{n-1})$$
$$= (v_0 + e_0, v_1 + e_1, \dots, v_{n-1} + e_{n-1}),$$
where the binary vector $\mathbf{e} = (e_0, e_1, \dots, e_{n-1})$ is the *error pattern*.
- The "1's" in \mathbf{e} represent *transmission errors*, i.e.,
$$e_i = \begin{cases} 1 & \text{if } r_i \neq v_i \\ 0 & \text{if } r_i = v_i, \end{cases}$$
and $e_i = 1$ indicates that the i^{th} position in \mathbf{r} has an error.

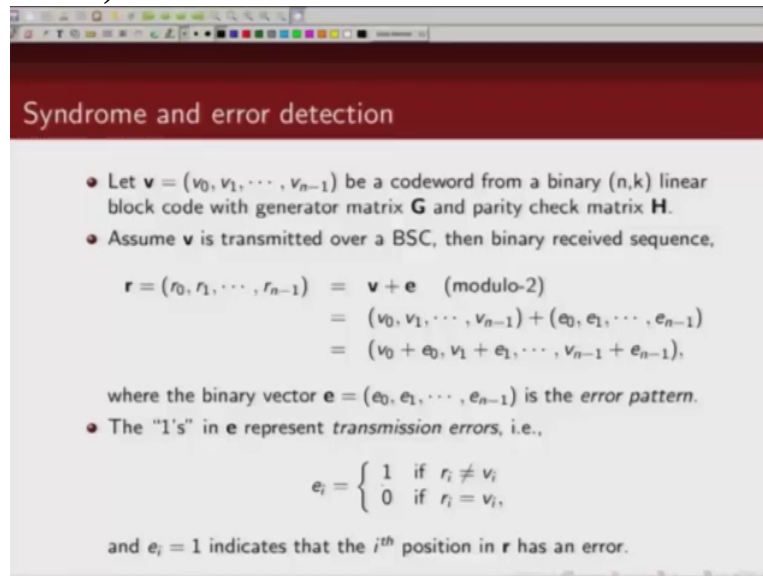
occur? If my received sequence is not same as my

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transmitted codeword, then there is an error. So the error is 1

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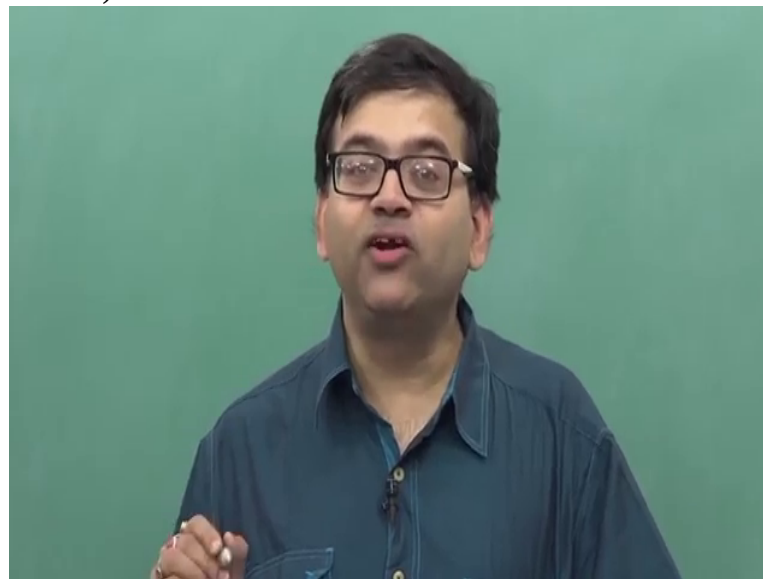


Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n, k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
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and $e_i = 1$ indicates that the i^{th} position in \mathbf{r} has an error.

only if my received bit is not same as my transmitted bit. If the received bit is same as my transmitted bit, there is no error. That I will keep that bit e_i as 0, Ok; now if a particular bit e_i is 1, what does it denote? It denotes that,

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that i^{th} bit is in error. Now when we are sending this codeword over a communication channel, what are we interested in? We are interested to find out whether any error has occurred. If any error has occurred, we are interested to find out the location where the error has occurred so that we can correct those errors, Ok.

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Syndrome and error detection

- Let $\mathbf{v} = (v_0, v_1, \dots, v_{n-1})$ be a codeword from a binary (n,k) linear block code with generator matrix \mathbf{G} and parity check matrix \mathbf{H} .
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and $e_i = 1$ indicates that the i^{th} position in \mathbf{r} has an error.

So first we are going to

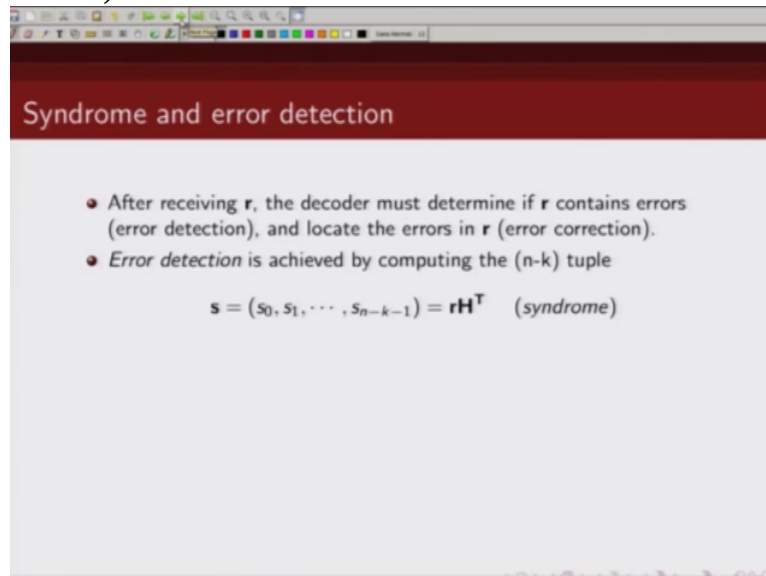
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Syndrome and error detection

- After receiving \mathbf{r} , the decoder must determine if \mathbf{r} contains errors (error detection), and locate the errors in \mathbf{r} (error correction).

show how we can detect error. So we

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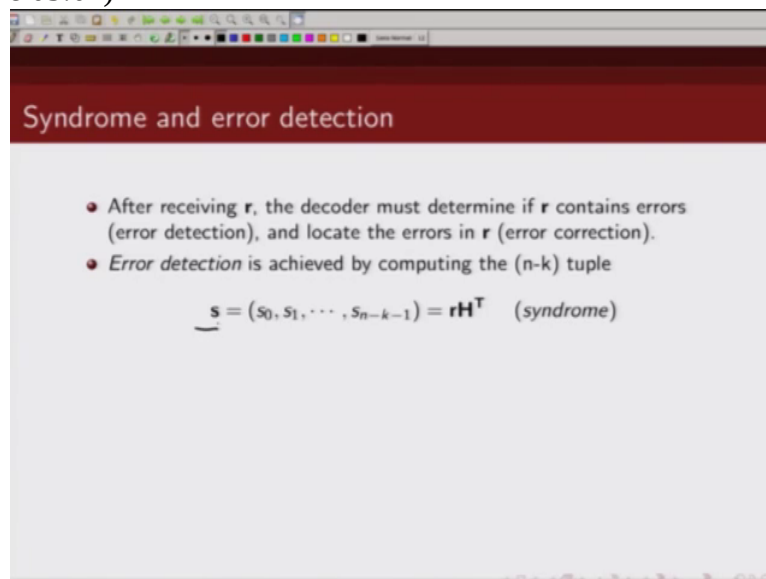
Syndrome and error detection

- After receiving \mathbf{r} , the decoder must determine if \mathbf{r} contains errors (error detection), and locate the errors in \mathbf{r} (error correction).
- *Error detection* is achieved by computing the $(n-k)$ tuple

$$\mathbf{s} = (s_0, s_1, \dots, s_{n-k-1}) = \mathbf{rH}^T \quad (\text{syndrome})$$

define a term which we call as Syndrome.

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Syndrome and error detection

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- *Error detection* is achieved by computing the $(n-k)$ tuple

$$\underline{\mathbf{s}} = (s_0, s_1, \dots, s_{n-k-1}) = \mathbf{rH}^T \quad (\text{syndrome})$$

What is a Syndrome? Syndrome is computed by computing this

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Syndrome and error detection

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- Error detection is achieved by computing the $(n-k)$ tuple

$$\underline{\mathbf{s}} = (s_0, s_1, \dots, s_{n-k-1}) = \underline{\mathbf{r}}\mathbf{H}^T \quad (\text{syndrome})$$

\mathbf{r} is basically $1 \times n$ vector. \mathbf{H} is $(n-k) \times n$

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Syndrome and error detection

- After receiving \mathbf{r} , the decoder must determine if \mathbf{r} contains errors (error detection), and locate the errors in \mathbf{r} (error correction).
- Error detection is achieved by computing the $(n-k)$ tuple

$$\underline{\mathbf{s}} = (s_0, s_1, \dots, s_{n-k-1}) = \underline{\mathbf{r}}\mathbf{H}^T \quad (\text{syndrome})$$

$1 \times n$

\mathbf{r} is $1 \times n$ vector, so \mathbf{H}^T is $n \times (n-k)$

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Syndrome and error detection

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- Error detection is achieved by computing the $(n-k)$ tuple

$$\underline{\mathbf{s}} = (s_0, s_1, \dots, s_{n-k-1}) = \underbrace{\mathbf{r}\mathbf{H}^T}_{\substack{1 \times n \quad n \times (n-k)}}$$

matrix So this term \mathbf{rH} transfer is known as Syndrome and this is, when this is non zero,

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Syndrome and error detection

- After receiving \mathbf{r} , the decoder must determine if \mathbf{r} contains errors (error detection), and locate the errors in \mathbf{r} (error correction).
- Error detection is achieved by computing the $(n-k)$ tuple

$$\mathbf{s} = (s_0, s_1, \dots, s_{n-k-1}) = \mathbf{r}\mathbf{H}^T \quad (\text{syndrome})$$

- \mathbf{r} is a codeword if and only if $\mathbf{s} = \mathbf{r}\mathbf{H}^T = \mathbf{0}$.

it indicates there is an error. So \mathbf{r} is a codeword if and only if the Syndrome is 0. So whenever the Syndrome is 0, basically if the Syndrome is 0, then \mathbf{r} is a codeword if and only if Syndrome is 0 and this is easy to check because we know

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that if v is a valid codeword, then vH^T will be 0. So if there is no error, my received sequence r would be just equal to v . And then the Syndrome would be

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A presentation slide with a red header and a white body. The header contains the text "Syndrome and error detection". The body contains two bullet points, a formula, and another bullet point. The formula is $s = (s_0, s_1, \dots, s_{n-k-1}) = rH^T$ (syndrome).

Syndrome and error detection

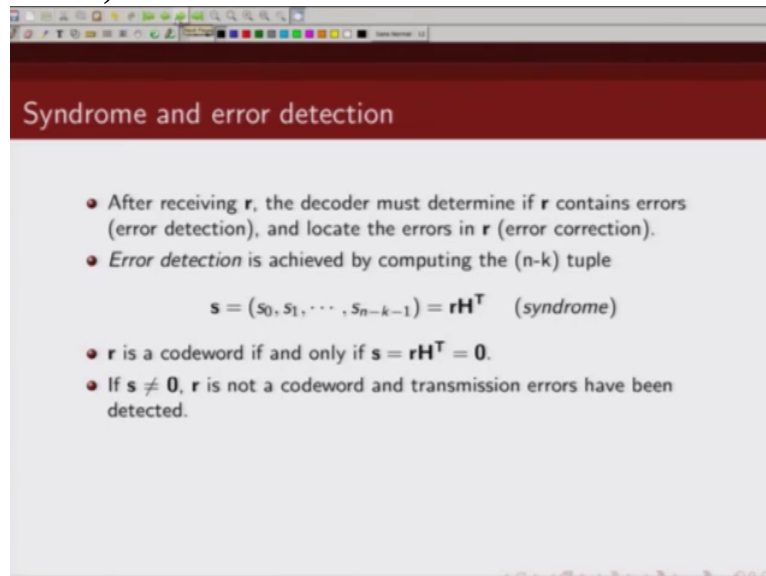
- After receiving r , the decoder must determine if r contains errors (error detection), and locate the errors in r (error correction).
- Error detection is achieved by computing the $(n-k)$ tuple

$$s = (s_0, s_1, \dots, s_{n-k-1}) = rH^T \quad (\text{syndrome})$$

- r is a codeword if and only if $s = rH^T = 0$.

vH^T which is 0. And if Syndrome

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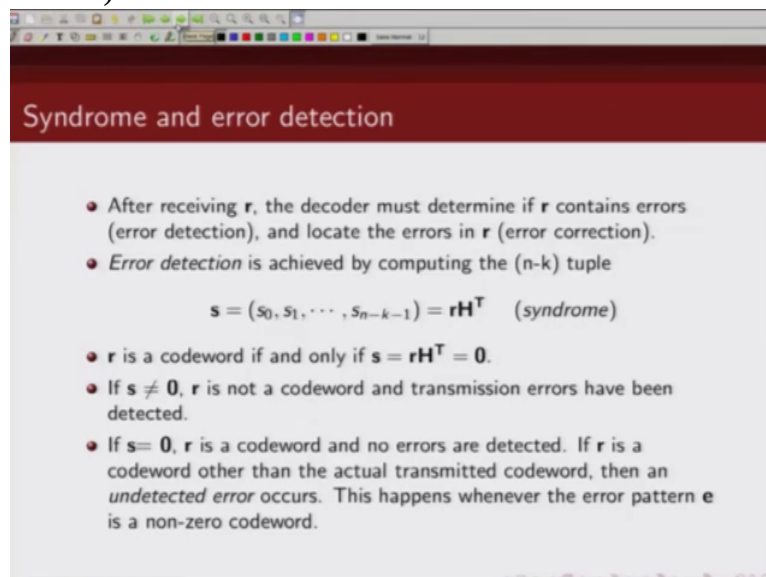


Syndrome and error detection

- After receiving \mathbf{r} , the decoder must determine if \mathbf{r} contains errors (error detection), and locate the errors in \mathbf{r} (error correction).
- Error detection is achieved by computing the $(n-k)$ tuple
$$\mathbf{s} = (s_0, s_1, \dots, s_{n-k-1}) = \mathbf{rH}^T \quad (\text{syndrome})$$
- \mathbf{r} is a codeword if and only if $\mathbf{s} = \mathbf{rH}^T = \mathbf{0}$.
- If $\mathbf{s} \neq \mathbf{0}$, \mathbf{r} is not a codeword and transmission errors have been detected.

is not equal to 0, then it means there is an error. Now if the Syndrome is 0, does it always mean that there is no error? No. There is a category

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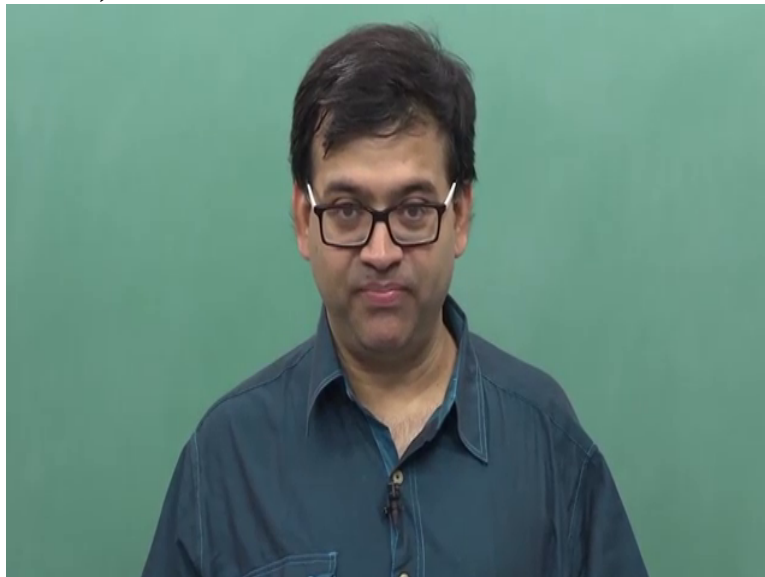


Syndrome and error detection

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- Error detection is achieved by computing the $(n-k)$ tuple
$$\mathbf{s} = (s_0, s_1, \dots, s_{n-k-1}) = \mathbf{rH}^T \quad (\text{syndrome})$$
- \mathbf{r} is a codeword if and only if $\mathbf{s} = \mathbf{rH}^T = \mathbf{0}$.
- If $\mathbf{s} \neq \mathbf{0}$, \mathbf{r} is not a codeword and transmission errors have been detected.
- If $\mathbf{s} = \mathbf{0}$, \mathbf{r} is a codeword and no errors are detected. If \mathbf{r} is a codeword other than the actual transmitted codeword, then an *undetected error* occurs. This happens whenever the error pattern \mathbf{e} is a non-zero codeword.

of error which we call as undetected error

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Now when does an undetected error happen? If your s

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Syndrome and error detection

- After receiving \mathbf{r} , the decoder must determine if \mathbf{r} contains errors (error detection), and locate the errors in \mathbf{r} (error correction).
- *Error detection* is achieved by computing the $(n-k)$ tuple
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is 0 and your received sequence \mathbf{r} is not the codeword that you transmitted but some other codeword, let's say I transmitted a codeword, \mathbf{v}_1 and my error \mathbf{e} was such that that it transformed it into another codeword \mathbf{v}_2 . So received sequence is \mathbf{v}_2 .

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Syndrome and error detection

- After receiving \mathbf{r} , the decoder must determine if \mathbf{r} contains errors (error detection), and locate the errors in \mathbf{r} (error correction).
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- \mathbf{r} is a codeword if and only if $\mathbf{s} = \mathbf{r}\mathbf{H}^T = \mathbf{0}$. $v_1 + e = v_2$
- If $\mathbf{s} \neq \mathbf{0}$, \mathbf{r} is not a codeword and transmission errors have been detected.
- If $\mathbf{s} = \mathbf{0}$, \mathbf{r} is a codeword and no errors are detected. If \mathbf{r} is a codeword other than the actual transmitted codeword, then an *undetected error* occurs. This happens whenever the error pattern \mathbf{e} is a non-zero codeword.

if I compute Syndrome because v_2 is a valid codeword then $v_2 H^T$ will be 0. So an undetected error happens when your error pattern is such that it transforms your one codeword to

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another codeword And that's what we have written here.

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Syndrome and error detection

- After receiving \mathbf{r} , the decoder must determine if \mathbf{r} contains errors (error detection), and locate the errors in \mathbf{r} (error correction).
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So if \mathbf{s} is 0 and \mathbf{r} is a codeword that means no errors are detected. However if \mathbf{r} is a codeword other than the actual codeword transmitted then an undetected error has happened. And this happens when an error pattern is a non-zero codeword; because we have said a property of linear block code that sum of 2 codewords is also a codeword. So for this scenario to happen, this \mathbf{e} has to be a

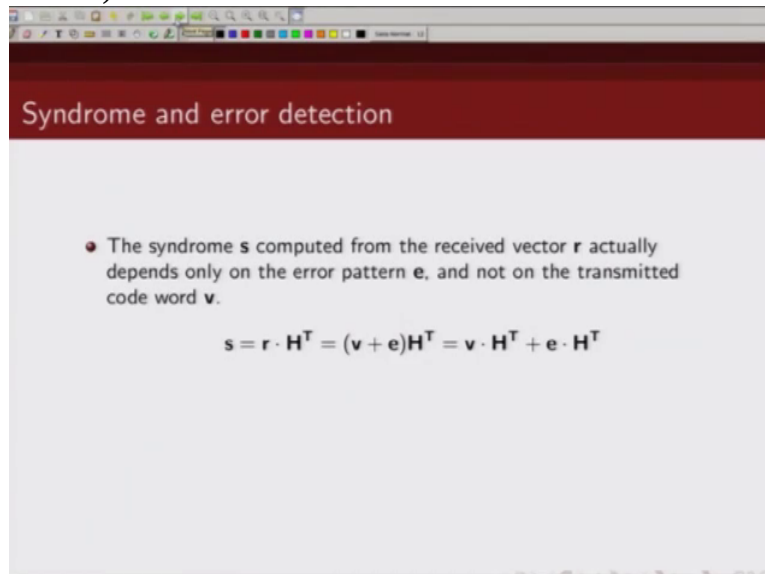
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Syndrome and error detection

- After receiving \mathbf{r} , the decoder must determine if \mathbf{r} contains errors (error detection), and locate the errors in \mathbf{r} (error correction).
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valid non-zero codeword So as I said

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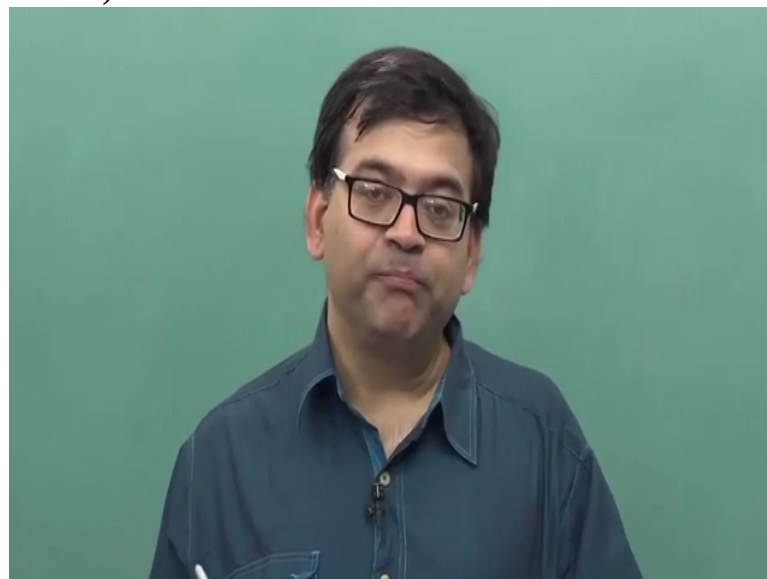
Syndrome and error detection

- The syndrome \mathbf{s} computed from the received vector \mathbf{r} actually depends only on the error pattern \mathbf{e} , and not on the transmitted code word \mathbf{v} .

$$\mathbf{s} = \mathbf{r} \cdot \mathbf{H}^T = (\mathbf{v} + \mathbf{e})\mathbf{H}^T = \mathbf{v} \cdot \mathbf{H}^T + \mathbf{e} \cdot \mathbf{H}^T$$

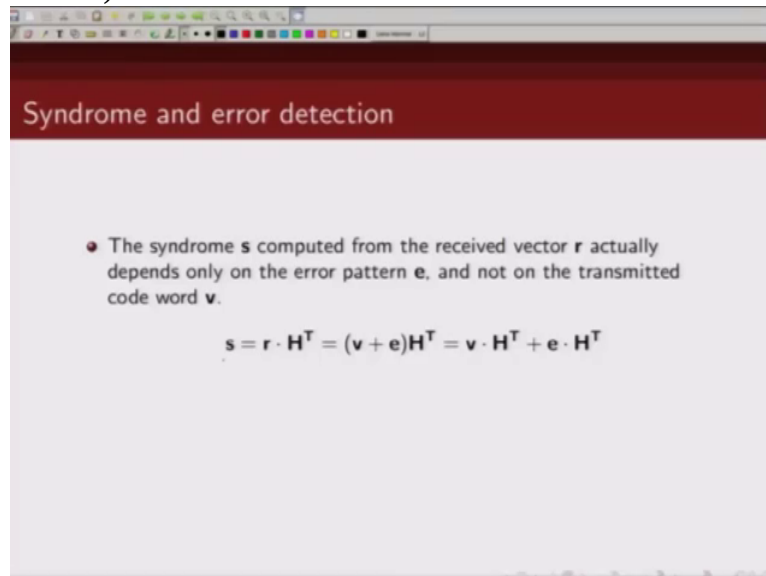
we compute the Syndrome from the received sequence \mathbf{r} , the interesting part is

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Syndrome depends only on the error pattern. It does not depend on what codeword was transmitted. And this is easy to see.

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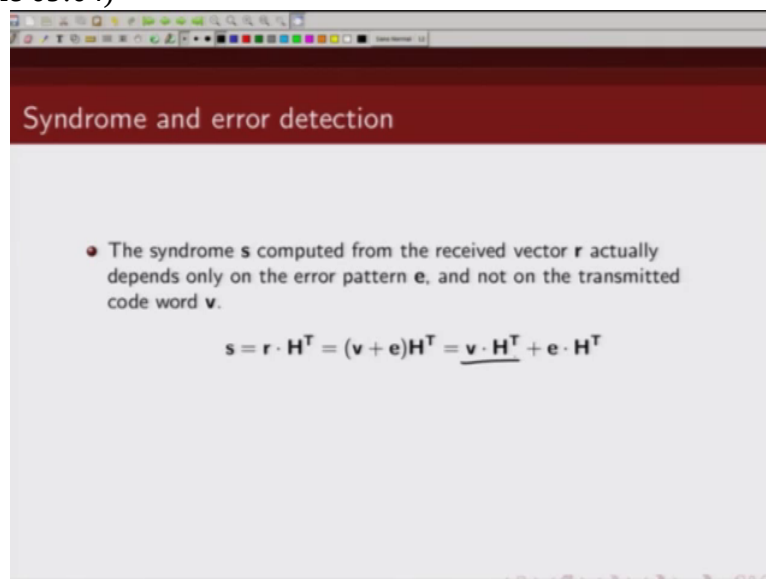
Syndrome and error detection

- The syndrome \mathbf{s} computed from the received vector \mathbf{r} actually depends only on the error pattern \mathbf{e} , and not on the transmitted code word \mathbf{v} .

$$\mathbf{s} = \mathbf{r} \cdot \mathbf{H}^T = (\mathbf{v} + \mathbf{e})\mathbf{H}^T = \mathbf{v} \cdot \mathbf{H}^T + \mathbf{e} \cdot \mathbf{H}^T$$

So Syndrome is $\mathbf{r} \cdot \mathbf{H}^T$ which we can write \mathbf{r} is my transmitted codeword plus error vector, this whole multiplied by \mathbf{H}^T . So this I can write as $\mathbf{v} \cdot \mathbf{H}^T$ and $\mathbf{e} \cdot \mathbf{H}^T$. Now what is $\mathbf{v} \cdot \mathbf{H}^T$?

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Syndrome and error detection

- The syndrome \mathbf{s} computed from the received vector \mathbf{r} actually depends only on the error pattern \mathbf{e} , and not on the transmitted code word \mathbf{v} .

$$\mathbf{s} = \mathbf{r} \cdot \mathbf{H}^T = (\mathbf{v} + \mathbf{e})\mathbf{H}^T = \underline{\mathbf{v} \cdot \mathbf{H}^T} + \mathbf{e} \cdot \mathbf{H}^T$$

$\mathbf{v} \cdot \mathbf{H}^T$ is 0, because \mathbf{v} is a valid codeword.

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Then Syndrome is nothing but $e \cdot H^T$.

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Syndrome and error detection

- The syndrome \mathbf{s} computed from the received vector \mathbf{r} actually depends only on the error pattern \mathbf{e} , and not on the transmitted code word \mathbf{v} .

$$\mathbf{s} = \mathbf{r} \cdot \mathbf{H}^T = (\mathbf{v} + \mathbf{e})\mathbf{H}^T = \mathbf{v} \cdot \mathbf{H}^T + \mathbf{e} \cdot \mathbf{H}^T$$

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Syndrome and error detection

- The syndrome \mathbf{s} computed from the received vector \mathbf{r} actually depends only on the error pattern \mathbf{e} , and not on the transmitted code word \mathbf{v} .

$$\mathbf{s} = \mathbf{r} \cdot \mathbf{H}^T = (\mathbf{v} + \mathbf{e})\mathbf{H}^T = \mathbf{v} \cdot \mathbf{H}^T + \mathbf{e} \cdot \mathbf{H}^T$$

- Since $\mathbf{v} \cdot \mathbf{H}^T = 0$,

$$\mathbf{s} = \mathbf{e} \cdot \mathbf{H}^T$$

So Syndrome does not depend on transmitted codeword. It only depends on the error pattern.

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Syndrome and error detection

Example 2.4: Consider a (7, 4) linear code with parity-check matrix

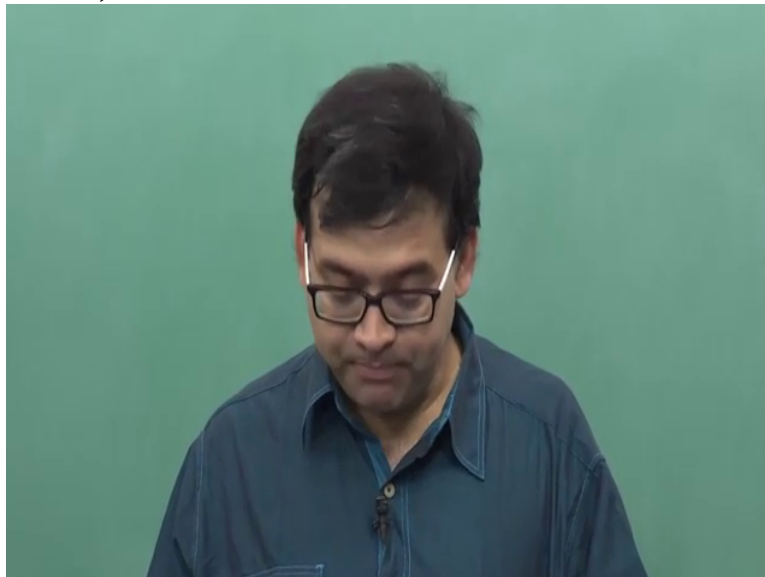
$$\mathbf{H} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array} \right]$$

Let $\mathbf{r} = (0\ 1\ 0\ 0\ 0\ 0\ 1)$. The syndrome of \mathbf{r} is

$$\begin{aligned} \mathbf{s} &= (s_0, s_1, s_2) = \mathbf{r} \cdot \mathbf{H}^T \\ &= (0\ 1\ 0\ 0\ 0\ 0\ 1) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = (1\ 1\ 1) \neq 0 \end{aligned}$$

So I have an example here. For 7 4 linear block code whose parity check matrix is given by this. Now let's say my received sequence is, received coded sequence is this. And I am interested to find whether there is any error in this received sequence. So how do I do that?

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So first I will compute the Syndrome. So what is Syndrome

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Syndrome and error detection

Example 2.4: Consider a (7, 4) linear code with parity-check matrix

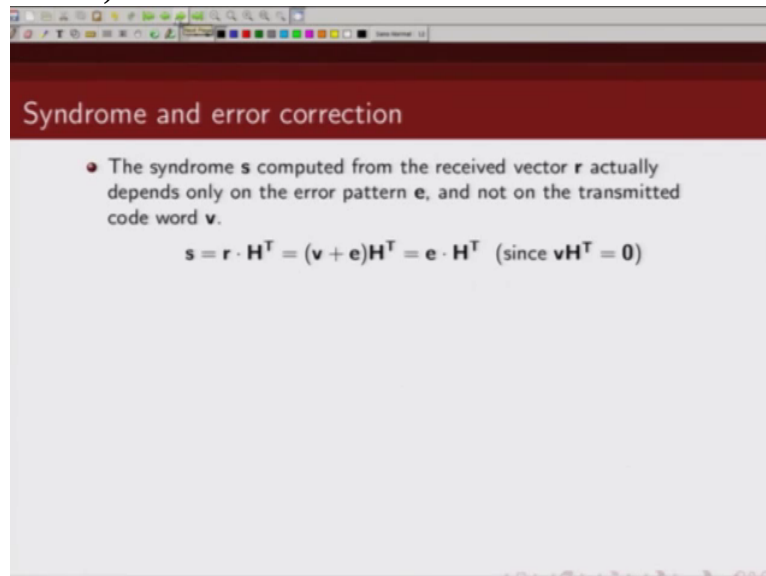
$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Let $\mathbf{r} = (0\ 1\ 0\ 0\ 0\ 0\ 1)$. The syndrome of \mathbf{r} is

$$\mathbf{s} = (s_0, s_1, s_2) = \mathbf{r} \cdot \mathbf{H}^T$$
$$= (0\ 1\ 0\ 0\ 0\ 0\ 1) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = (1\ 1\ 1) \neq 0$$

of \mathbf{r} ? This Syndrome is $\mathbf{r} \mathbf{H}^T$, so \mathbf{r} is this and \mathbf{H} is given, so \mathbf{H}^T is basically $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, so this is my \mathbf{H}^T and when I multiply this, I multiply this by this, multiply this by this, multiply this by this what I get is $(1\ 1\ 1)$ which is not 0 that means there is an error in my received sequence.

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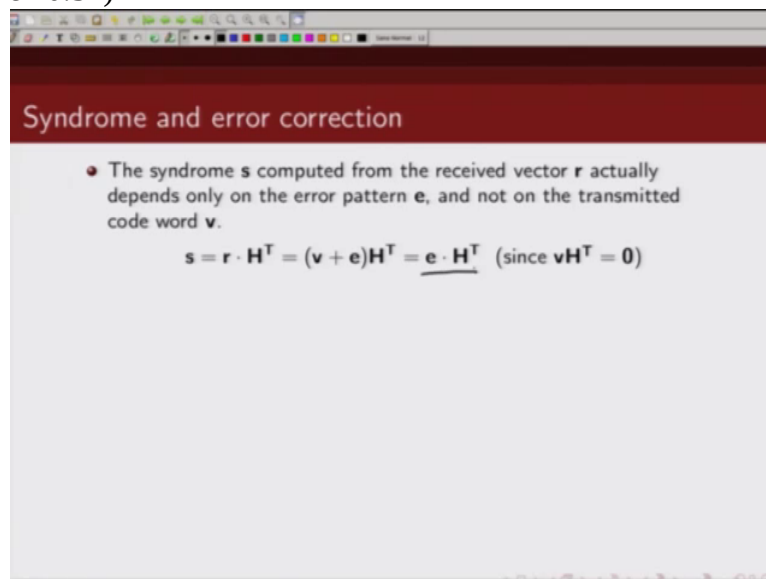
Syndrome and error correction

- The syndrome \mathbf{s} computed from the received vector \mathbf{r} actually depends only on the error pattern \mathbf{e} , and not on the transmitted code word \mathbf{v} .

$$\mathbf{s} = \mathbf{r} \cdot \mathbf{H}^T = (\mathbf{v} + \mathbf{e})\mathbf{H}^T = \mathbf{e} \cdot \mathbf{H}^T \quad (\text{since } \mathbf{v}\mathbf{H}^T = \mathbf{0})$$

As I said the Syndrome only depends on the error pattern. It does not depend

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Syndrome and error correction

- The syndrome \mathbf{s} computed from the received vector \mathbf{r} actually depends only on the error pattern \mathbf{e} , and not on the transmitted code word \mathbf{v} .

$$\mathbf{s} = \mathbf{r} \cdot \mathbf{H}^T = (\mathbf{v} + \mathbf{e})\mathbf{H}^T = \underline{\mathbf{e} \cdot \mathbf{H}^T} \quad (\text{since } \mathbf{v}\mathbf{H}^T = \mathbf{0})$$

on transmitted codeword.

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Syndrome and error correction

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- For error pattern $\mathbf{e} = \{e_0, e_1, \dots, e_{n-1}\}$, and \mathbf{H} given by

$$\mathbf{H} = [\mathbf{I}_{n-k} : \mathbf{P}^T]$$

$$= \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & p_{0,0} & p_{1,0} & \dots & p_{k-1,0} \\ 0 & 1 & 0 & \dots & 0 & p_{0,1} & p_{1,1} & \dots & p_{k-1,1} \\ 0 & 0 & 1 & \dots & 0 & p_{0,2} & p_{1,2} & \dots & p_{k-1,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & p_{0,n-k-1} & p_{1,n-k-1} & \dots & p_{k-1,n-k-1} \end{bmatrix}$$

the syndrome equations can be rewritten as

$$s_j = e_j + e_{n-k}p_{0j} + e_{n-k+1}p_{1j} + \dots + e_{n-1}p_{k-1,j}, \quad 0 \leq j \leq n-k$$

And if you have your parity check matrix in a systematic form then you can write your Syndrome equations in terms of error patterns like this. So these are your basically n minus k Syndrome equations.

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Syndrome and error correction

- This is a set of $n - k$ equations in n unknowns, e_0, e_1, \dots, e_{n-1} .

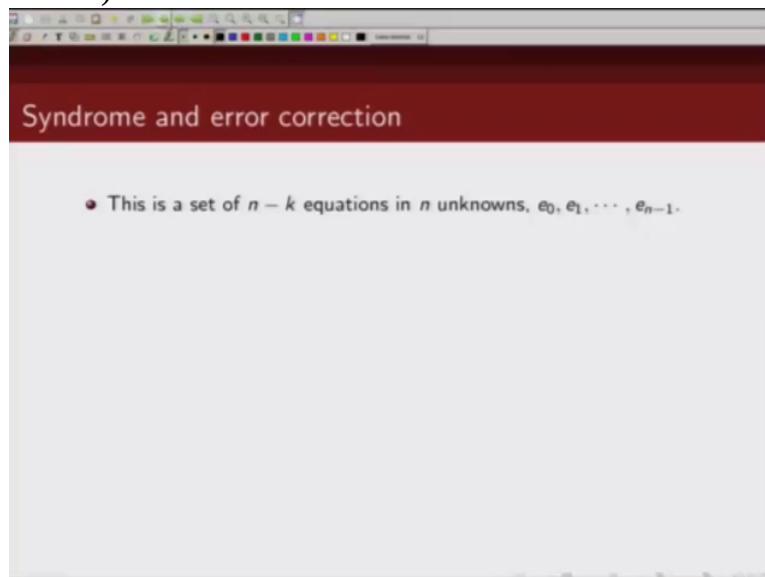
So to recap what we have said so far, we have said that for error detection we need to compute the Syndrome.

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And if the Syndrome is non-zero that means an error has occurred. Now we will move into error correction and we will talk about how we can use the Syndrome for error correction. So as we saw basically from the Syndrome

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equations

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Syndrome and error correction

- The syndrome \mathbf{s} computed from the received vector \mathbf{r} actually depends only on the error pattern \mathbf{e} , and not on the transmitted code word \mathbf{v} .

$$\mathbf{s} = \mathbf{r} \cdot \mathbf{H}^T = (\mathbf{v} + \mathbf{e})\mathbf{H}^T = \mathbf{e} \cdot \mathbf{H}^T \quad (\text{since } \mathbf{v}\mathbf{H}^T = \mathbf{0})$$

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the syndrome equations can be rewritten as

$$s_j = e_j + e_{n-k}p_{0j} + e_{n-k+1}p_{1j} + \dots + e_{n-1}p_{k-1,j}, \quad 0 \leq j \leq n-k$$

we had basically one cross n and h transpose was n cross n minus k. So we essentially had

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Syndrome and error correction

- This is a set of $n - k$ equations in n unknowns, e_0, e_1, \dots, e_{n-1} .

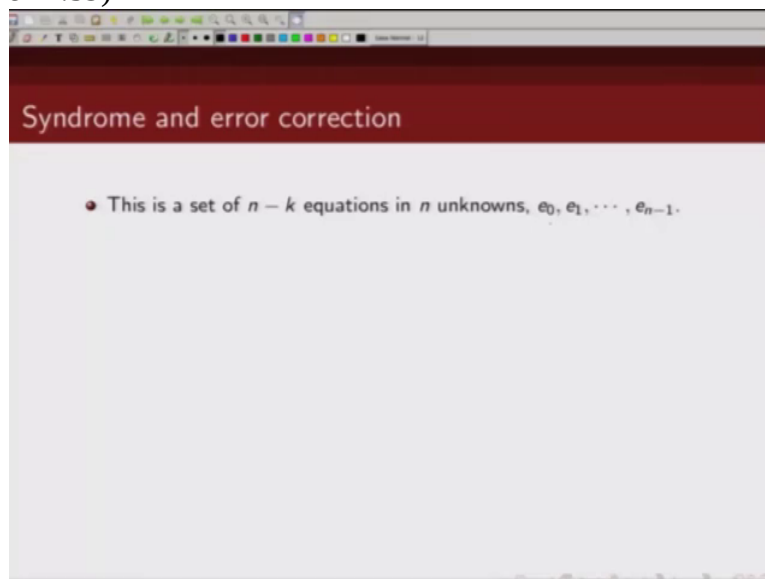
n minus k Syndrome equations So we had total n minus k equations and how many unknowns we have, we have n unknowns from location error at location 1 to error at location

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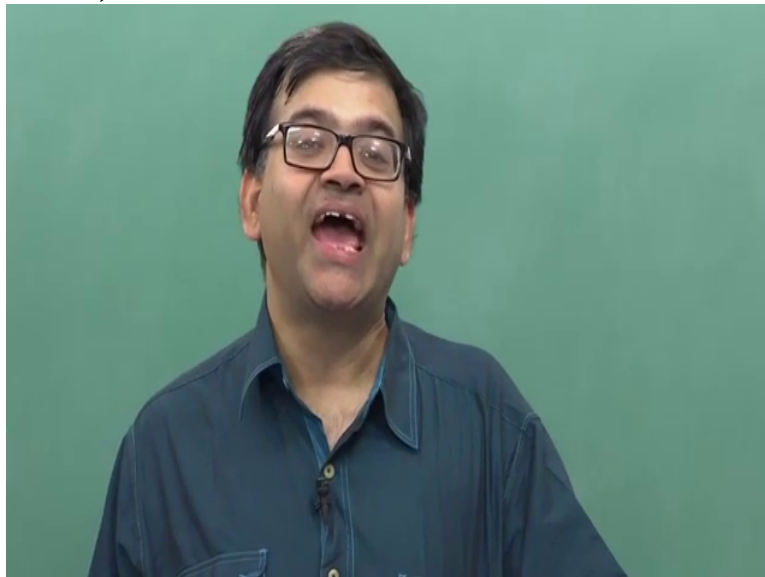
n. So we have n unknowns, $e_0, e_1,$

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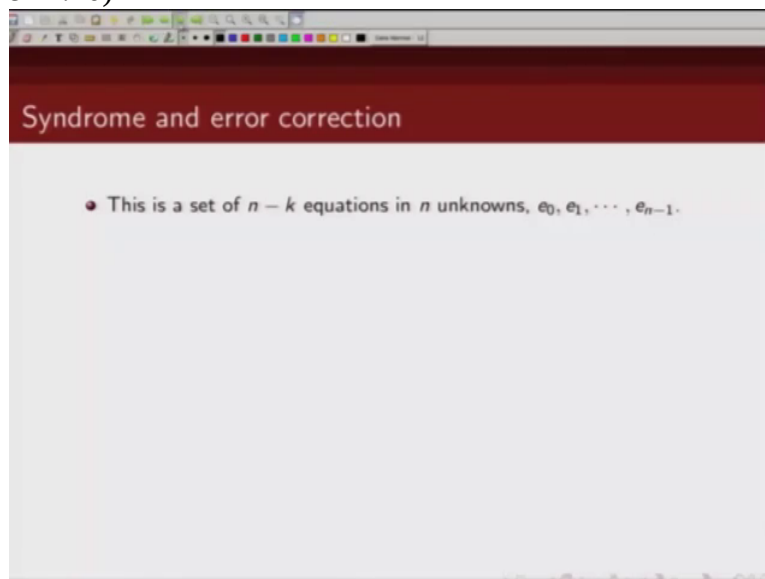
e_2, e_{n-1} where as we have only

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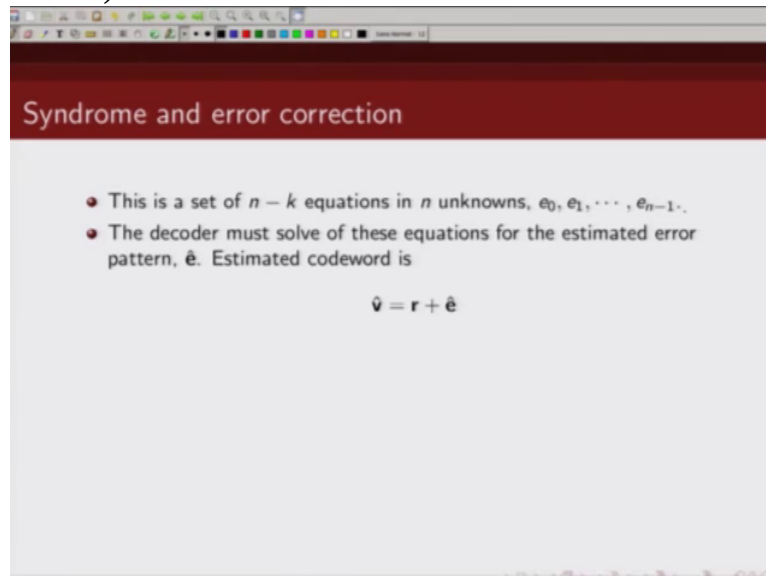
n minus k equations So we have less equations, more unknowns.

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And what we need to do is we need to solve this set of

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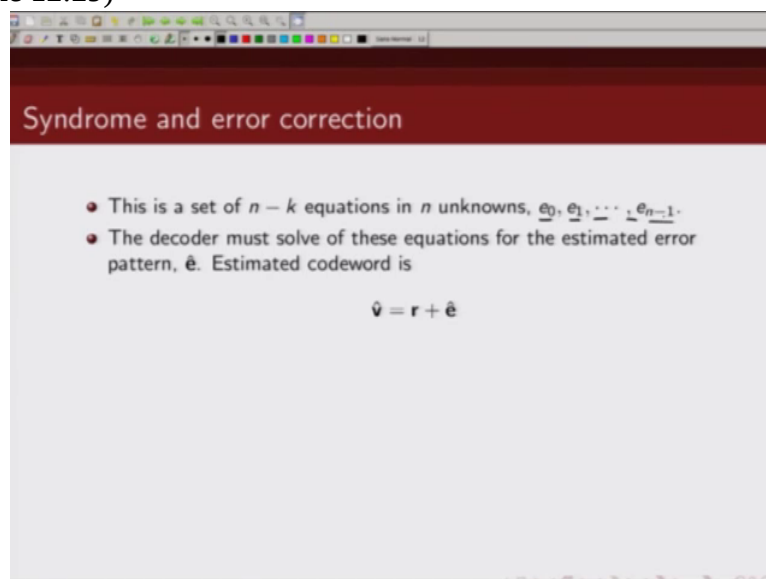
The slide has a dark red header with the title "Syndrome and error correction" in white. Below the header, there are two bullet points in black text:

- This is a set of $n - k$ equations in n unknowns, e_0, e_1, \dots, e_{n-1} .
- The decoder must solve of these equations for the estimated error pattern, \hat{e} . Estimated codeword is

Below the bullet points, the equation $\hat{v} = r + \hat{e}$ is displayed in black text.

equations to find out what these unknown quantities are Because to find out the error pattern, we need to know what these e i's are,

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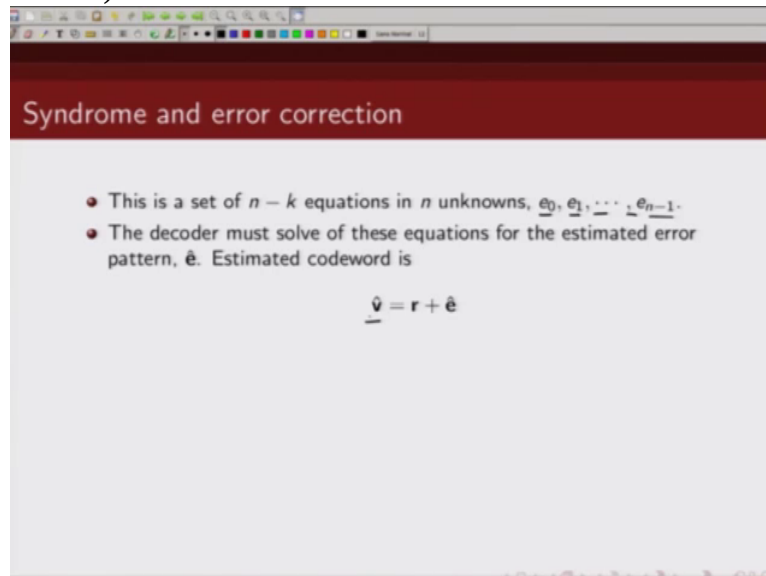
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Below the bullet points, the equation $\hat{v} = r + \hat{e}$ is displayed in black text.

Ok. So we need to solve these set of n minus k equations to get back our corrected sequence. And what would be our corrected sequence?

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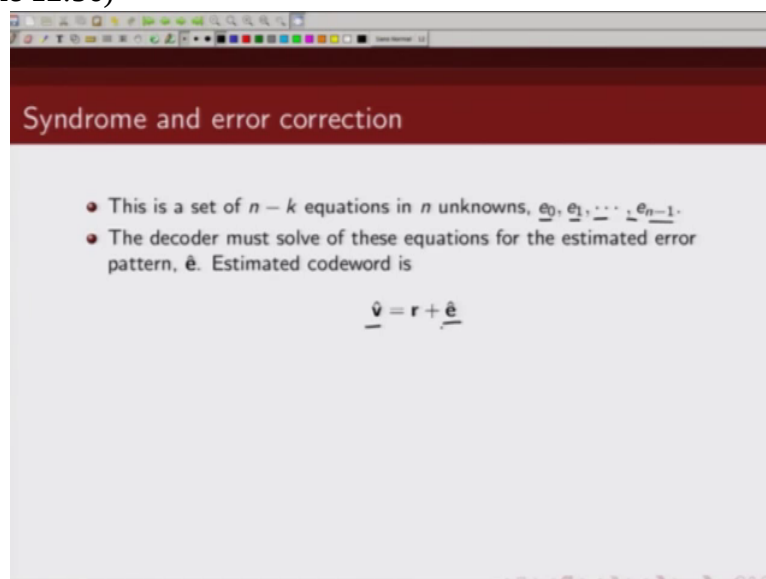
Syndrome and error correction

- This is a set of $n - k$ equations in n unknowns, e_0, e_1, \dots, e_{n-1} .
- The decoder must solve of these equations for the estimated error pattern, \hat{e} . Estimated codeword is

$$\hat{v} = r + \hat{e}$$

Estimated codeword would be nothing but r received sequence plus estimated error pattern. So we need to, when we do error

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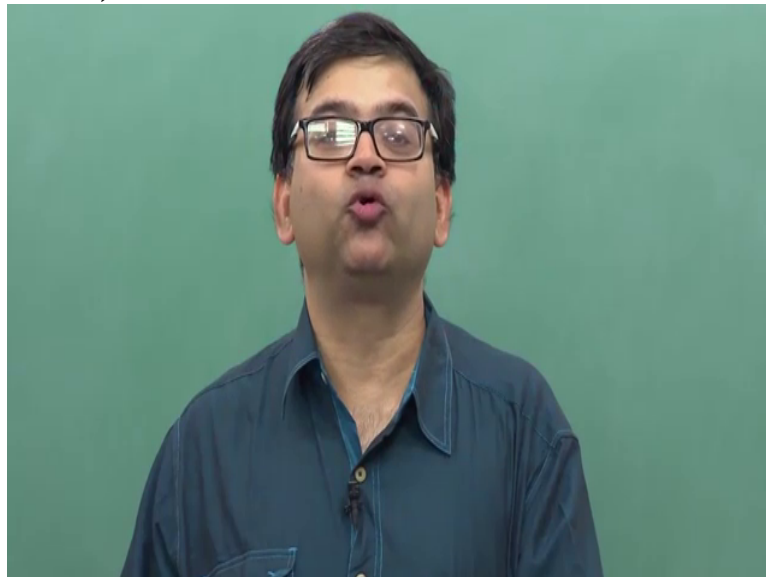
Syndrome and error correction

- This is a set of $n - k$ equations in n unknowns, e_0, e_1, \dots, e_{n-1} .
- The decoder must solve of these equations for the estimated error pattern, \hat{e} . Estimated codeword is

$$\hat{v} = r + \hat{e}$$

correction, essentially what we are trying to do is

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we are trying to estimate what the error pattern is. So as we can see

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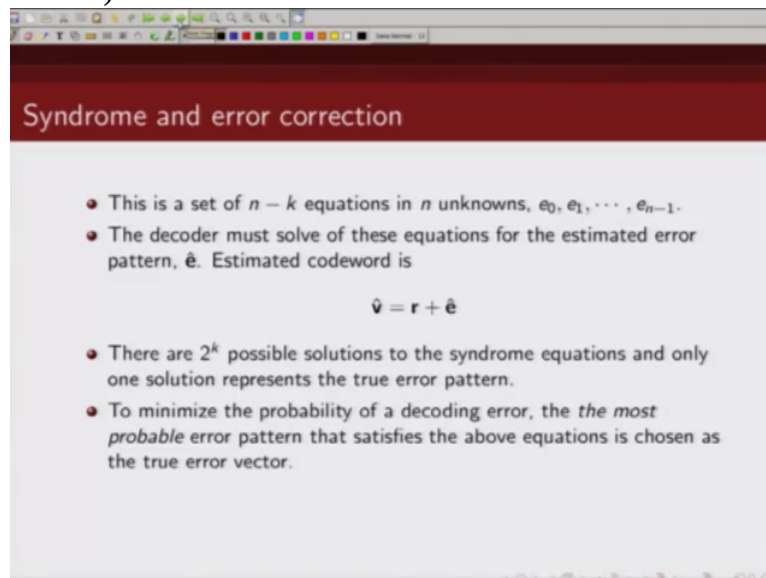
A screenshot of a presentation slide. The title is "Syndrome and error correction" in white text on a dark red background. Below the title, there are three bullet points in red. The second bullet point is followed by the equation $\hat{v} = r + \hat{e}$. The third bullet point discusses the number of possible solutions.

Syndrome and error correction

- This is a set of $n - k$ equations in n unknowns, e_0, e_1, \dots, e_{n-1} .
- The decoder must solve of these equations for the estimated error pattern, \hat{e} . Estimated codeword is
$$\hat{v} = r + \hat{e}$$
- There are 2^k possible solutions to the syndrome equations and only one solution represents the true error pattern.

from here, because we have $n - k$ equations and n unknowns, we have total 2^k solutions of these $n - k$ equations. So there are total 2^k solutions to these equations and out of them there is only one which is the correct error pattern. Out of those 2^k solutions there is only 1 error pattern

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Syndrome and error correction

- This is a set of $n - k$ equations in n unknowns, e_0, e_1, \dots, e_{n-1} .
- The decoder must solve these equations for the estimated error pattern, \hat{e} . Estimated codeword is

$$\hat{v} = r + \hat{e}$$

- There are 2^k possible solutions to the syndrome equations and only one solution represents the true error pattern.
- To minimize the probability of a decoding error, the *most probable* error pattern that satisfies the above equations is chosen as the true error vector.

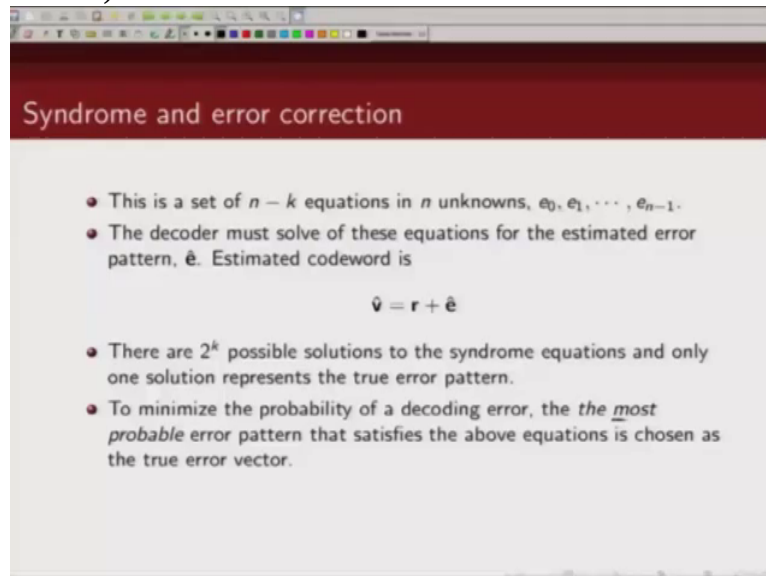
which is correct Now how do we choose

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the most likely error pattern from these 2^k solutions? How do we choose the most likely pattern? That's basically what our objective is. So when we try to minimize probability of error we want to choose

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Syndrome and error correction

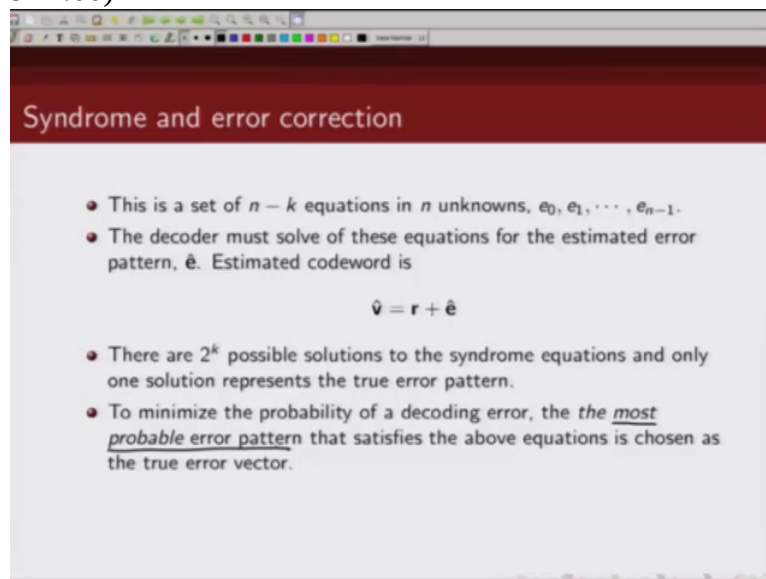
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the most probable error pattern

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Syndrome and error correction

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- There are 2^k possible solutions to the syndrome equations and only one solution represents the true error pattern.
- To minimize the probability of a decoding error, the the most probable error pattern that satisfies the above equations is chosen as the true error vector.

We want to choose the most probable error pattern from those 2^k solutions of this set of equations.

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And as we said, we did an exercise, you know,

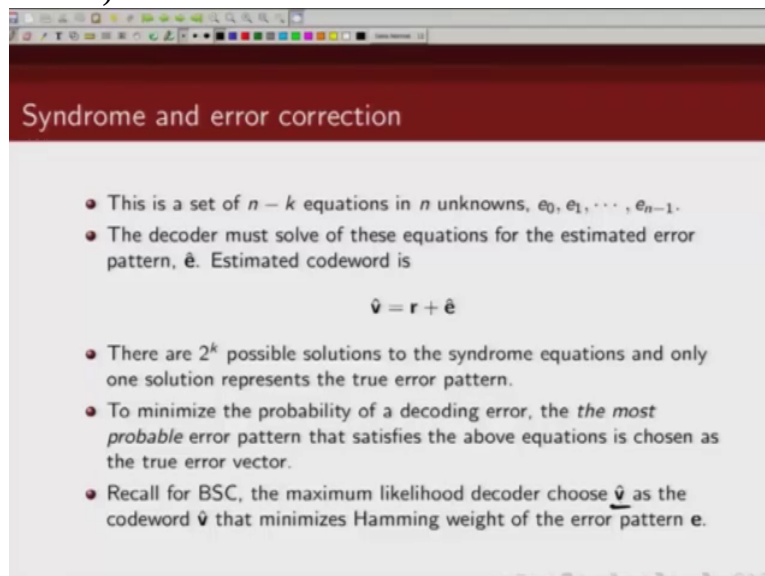
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Syndrome and error correction

- This is a set of $n - k$ equations in n unknowns, e_0, e_1, \dots, e_{n-1} .
- The decoder must solve of these equations for the estimated error pattern, \hat{e} . Estimated codeword is
$$\hat{v} = r + \hat{e}$$
- There are 2^k possible solutions to the syndrome equations and only one solution represents the true error pattern.
- To minimize the probability of a decoding error, the *the most probable error* pattern that satisfies the above equations is chosen as the true error vector.
- Recall for BSC, the maximum likelihood decoder choose \hat{v} as the codeword \hat{v} that minimizes Hamming weight of the error pattern e .

in previous lectures we have shown the decoding rule for maximum likelihood decoding rule for a binary symmetric channel and we have shown that for maximum likelihood decoder we will choose

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Syndrome and error correction

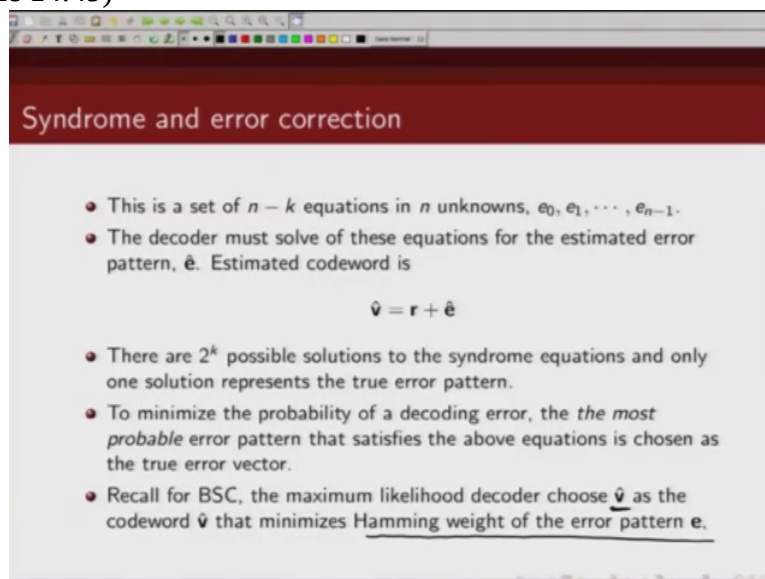
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- Recall for BSC, the maximum likelihood decoder choose \hat{v} as the codeword \hat{v} that minimizes Hamming weight of the error pattern e .

our estimated codeword as one that will minimize the Hamming distance between the received codeword and the transmitted codeword. In other words it would minimize the Hamming weight

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Syndrome and error correction

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- Recall for BSC, the maximum likelihood decoder choose \hat{v} as the codeword \hat{v} that minimizes Hamming weight of the error pattern e .

of the error pattern e So from the 2^k different solutions basically, the one which has the least Hamming weight, that's the best solution for the maximum likelihood decoding rule for a binary symmetric channel.

So let's take

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Syndrome and error correction

Example 3.1: Let

$$\mathbf{H} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array} \right]$$

Suppose $\mathbf{v} = (1\ 0\ 0\ 1\ 0\ 1\ 1)$ is transmitted and $\mathbf{r} = (1\ 0\ 0\ 1\ 0\ 0\ 1)$ is received. Then the syndrome of \mathbf{r} is

$$\mathbf{s} = (s_0, s_1, s_2) = \mathbf{r} \cdot \mathbf{H}^T = (1\ 1\ 1)$$

Let $\mathbf{e} = (e_0, e_1, e_2, e_3, e_4, e_5, e_6)$ be the error pattern.

an example now So we have a 7 4 code whose parity check matrix is given by this and our transmitted codeword is this and received codeword is this. We can see that there is an error in this location.

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Syndrome and error correction

Example 3.1: Let

$$\mathbf{H} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array} \right]$$

Suppose $\mathbf{v} = (1\ 0\ 0\ 1\ 0\ 1\ 1)$ is transmitted and $\mathbf{r} = (1\ 0\ 0\ 1\ 0\ 0\ 1)$ is received. Then the syndrome of \mathbf{r} is

$$\mathbf{s} = (s_0, s_1, s_2) = \mathbf{r} \cdot \mathbf{H}^T = (1\ 1\ 1)$$

Let $\mathbf{e} = (e_0, e_1, e_2, e_3, e_4, e_5, e_6)$ be the error pattern.

This was transmitted as 1 and this was received as 0, so there is an error in this location. Now how do we find out

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Syndrome and error correction

Example 3.1: Let

$$\mathbf{H} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array} \right]$$

Suppose $\mathbf{v} = (1\ 0\ 0\ 1\ 0\ 1\ 1)$ is transmitted and $\mathbf{r} = (1\ 0\ 0\ 1\ 0\ 0\ 1)$ is received. Then the syndrome of \mathbf{r} is

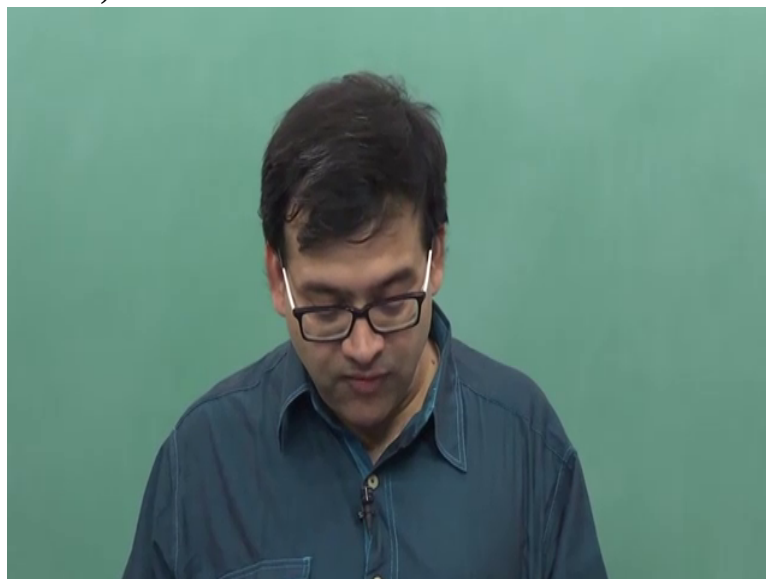
$$\mathbf{s} = (s_0, s_1, s_2) = \mathbf{r} \cdot \mathbf{H}^T = (1\ 1\ 1)$$

Let $\mathbf{e} = (e_0, e_1, e_2, e_3, e_4, e_5, e_6)$ be the error pattern.

Handwritten note: Error ↓ pointing to the 6th bit of r.

that there is an error and there is an error in this location? So first thing that we will do is we will compute the Syndrome. When we will compute

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the Syndrome which is $\mathbf{r} \cdot \mathbf{H}^T$ what we get is a

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Syndrome and error correction

Example 3.1: Let

$$\mathbf{H} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array} \right]$$

Suppose $\mathbf{v} = (1\ 0\ 0\ 1\ 0\ 1\ 1)$ is transmitted and $\mathbf{r} = (1\ 0\ 0\ 1\ 0\ 0\ 1)$ is received. Then the syndrome of \mathbf{r} is

$$\mathbf{s} = (s_0, s_1, s_2) = \mathbf{r} \cdot \mathbf{H}^T = (1\ 1\ 1)$$

Let $\mathbf{e} = (e_0, e_1, e_2, e_3, e_4, e_5, e_6)$ be the error pattern.

Handwritten notes: "Error" with an arrow pointing to the 6th bit of r.

non-zero and since this is non-zero this means

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Syndrome and error correction

Example 3.1: Let

$$\mathbf{H} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array} \right]$$

Suppose $\mathbf{v} = (1\ 0\ 0\ 1\ 0\ 1\ 1)$ is transmitted and $\mathbf{r} = (1\ 0\ 0\ 1\ 0\ 0\ 1)$ is received. Then the syndrome of \mathbf{r} is

$$\mathbf{s} = (s_0, s_1, s_2) = \mathbf{r} \cdot \mathbf{H}^T = (1\ 1\ 1) \neq 0$$

Let $\mathbf{e} = (e_0, e_1, e_2, e_3, e_4, e_5, e_6)$ be the error pattern.

Handwritten notes: "Error" with an arrow pointing to the 6th bit of r, and a handwritten "≠ 0" next to the syndrome calculation.

there is an error. Now next step is to find out where the error has occurred. So in this case

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Syndrome and error correction

Since

$$\mathbf{s} = \mathbf{e} \cdot \mathbf{H}^T$$

we have the following 3 equations:

$$\begin{aligned} 1 &= e_0 + e_3 + e_5 + e_6 \\ 1 &= e_1 + e_3 + e_4 + e_5 \\ 1 &= e_2 + e_4 + e_5 + e_6 \end{aligned}$$

we can write the Syndrome in terms of error bits. So we have 3 equations and we have total 7 unknowns, right? And these equations are basically given by this. We have our

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Syndrome and error correction

Example 3.1: Let

$$\mathbf{H} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array} \right]$$

Suppose $\mathbf{v} = (1\ 0\ 0\ 1\ 0\ 1\ 1)$ is transmitted and $\mathbf{r} = (1\ 0\ 0\ 1\ 0\ \textcircled{0}\ 1)$ is received. Then the syndrome of \mathbf{r} is

Error
↓

$$\mathbf{s} = (s_0, s_1, s_2) = \mathbf{r} \cdot \mathbf{H}^T = (1\ 1\ 1) \neq \mathbf{0}$$

Let $\mathbf{e} = (e_0, e_1, e_2, e_3, e_4, e_5, e_6)$ be the error pattern.

H matrix given by this This is our error pattern, so when we compute

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Syndrome and error correction

Since

$$s = e.H^T$$

we have the following 3 equations:

$$\begin{aligned} 1 &= e_0 + e_3 + e_5 + e_6 \\ 1 &= e_1 + e_3 + e_4 + e_5 \\ 1 &= e_2 + e_4 + e_5 + e_6 \end{aligned}$$

e H transpose we get set of these 3 equations. And

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Syndrome and error correction

• The solutions are:

(0 0 0 0 0 1 0)	(1 0 1 0 0 1 1)
(1 1 0 1 0 1 0)	(0 1 1 1 0 1 1)
(0 1 1 0 1 1 0)	(1 1 0 0 1 1 1)
(1 0 1 1 1 1 0)	(0 0 0 1 1 1 1)
(1 1 1 0 0 0 0)	(0 1 0 0 0 0 1)
(0 0 1 1 0 0 0)	(1 0 0 1 0 0 1)
(1 0 0 0 1 0 0)	(0 0 1 0 1 0 1)
(0 1 0 1 1 0 0)	(1 1 1 1 1 0 1)

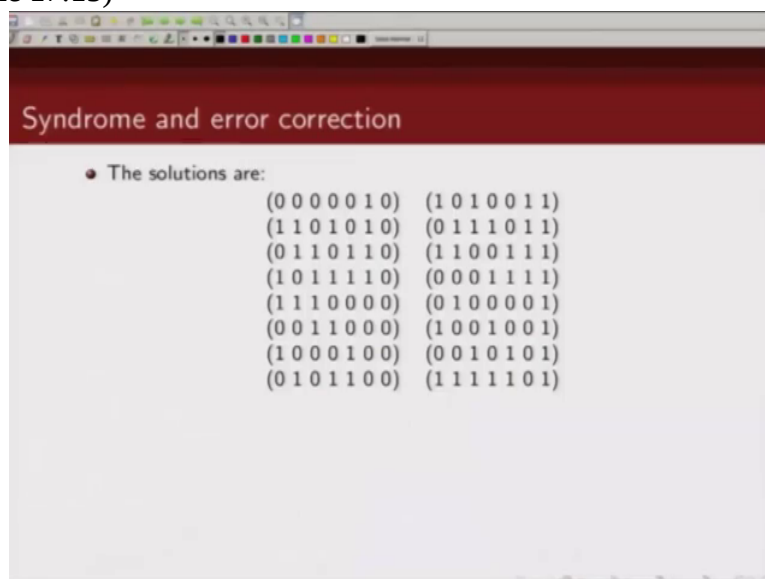
there are 16 solutions to this set of equations. This set of 3 equations where there are 7 unknowns; there are total 2 to power 4 different solutions. These are the 16 different solutions and as we said, the maximum likelihood decoding rule

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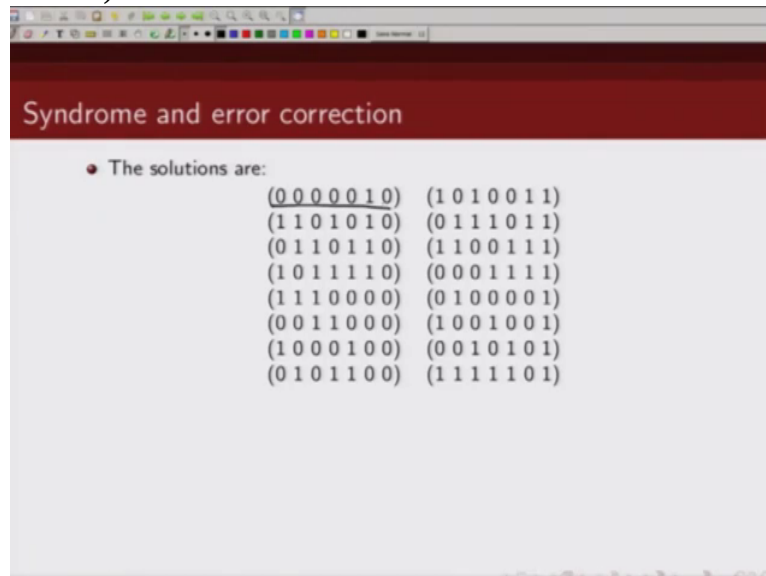
for a binary symmetric channel will chose an error pattern that has minimum Hamming weight so, which has minimum

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number of 1s So you can see among these 16

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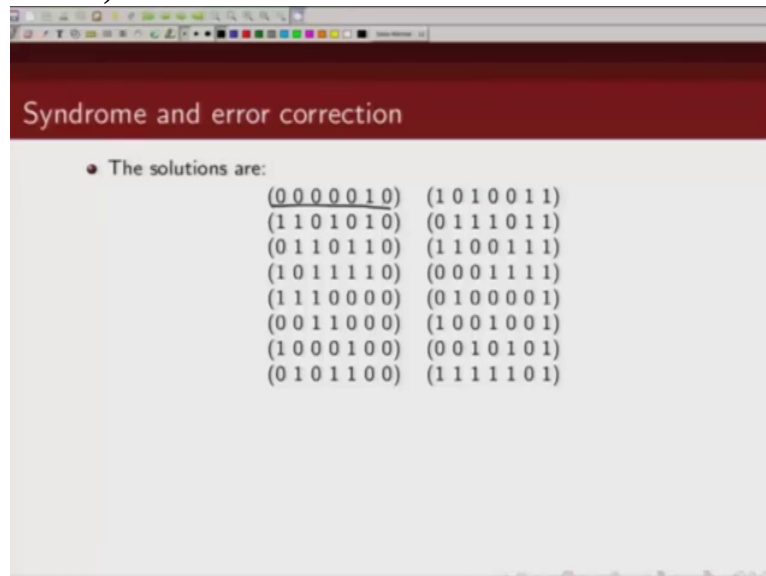
solutions the one that has minimum Hamming weight, minimum number of 1s is this. All others have, this has four 1's, this has four 1's, this has five 1's, three 1's, two 1's, you can check basically this one has the least number of 1's. So this

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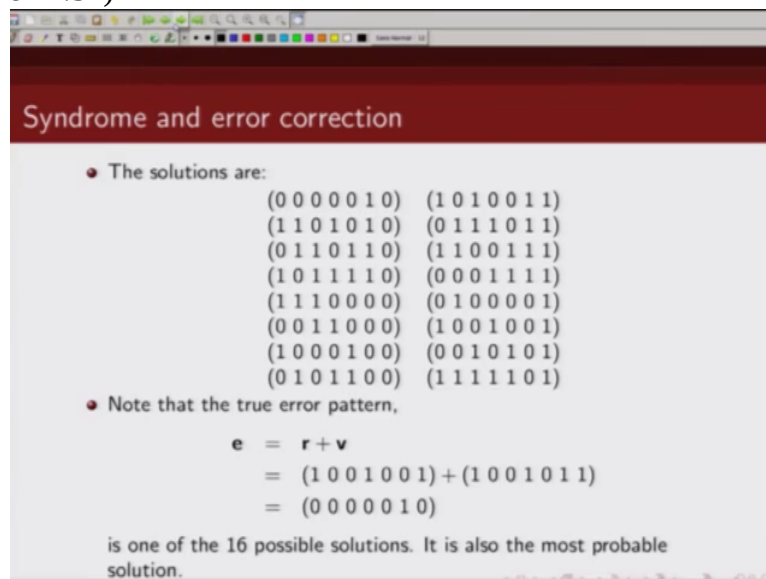
error pattern has the least of Hamming weight. So this is out of

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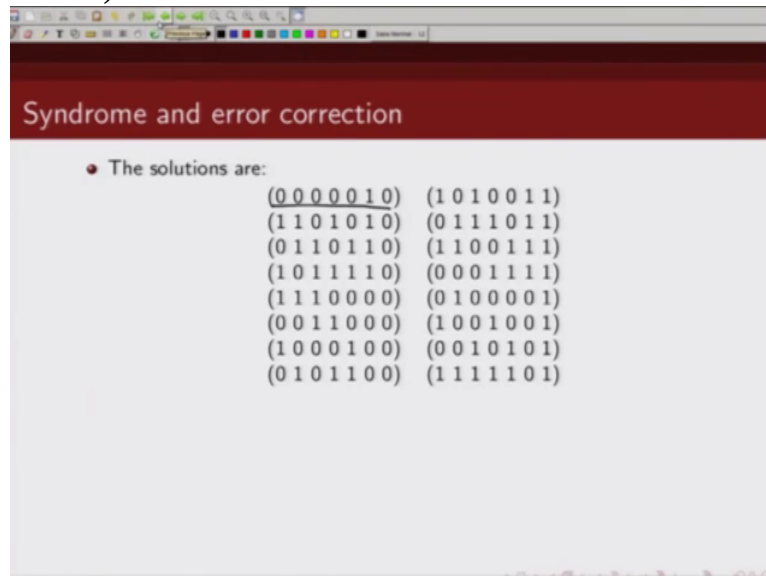
those 2 to the power 4 solutions, this is the most likely solution. And this we can verify also

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because we were given the received sequence and we knew what was the transmitted sequence was error pattern indeed was this, which we found out from, by solving these set of equations. So to recap then, now when we want to do error correction, what do we need to do, we need to solve

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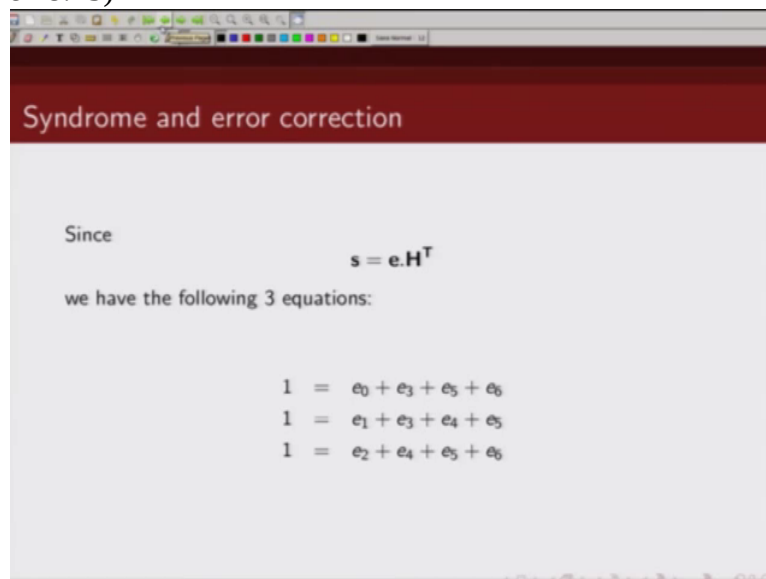
Syndrome and error correction

• The solutions are:

(0 0 0 0 1 0)	(1 0 1 0 0 1 1)
(1 1 0 1 0 1 0)	(0 1 1 1 0 1 1)
(0 1 1 0 1 1 0)	(1 1 0 0 1 1 1)
(1 0 1 1 1 1 0)	(0 0 0 1 1 1 1)
(1 1 1 0 0 0 0)	(0 1 0 0 0 0 1)
(0 0 1 1 0 0 0)	(1 0 0 1 0 0 1)
(1 0 0 0 1 0 0)	(0 0 1 0 1 0 1)
(0 1 0 1 1 0 0)	(1 1 1 1 1 0 1)

for

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Syndrome and error correction

Since

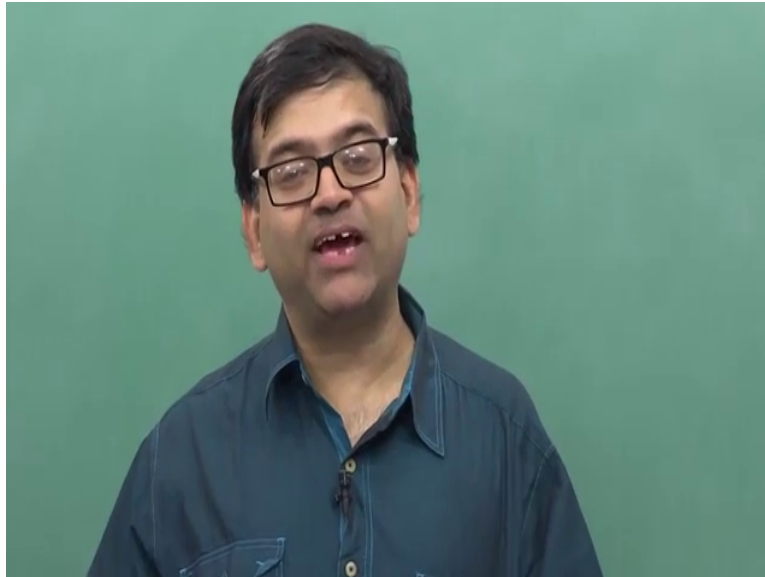
$$s = e.H^T$$

we have the following 3 equations:

$$1 = e_0 + e_3 + e_5 + e_6$$
$$1 = e_1 + e_3 + e_4 + e_5$$
$$1 = e_2 + e_4 + e_5 + e_6$$

this Syndrome equations And there are n minus k equations

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but there will be n unknowns so this will have 2^k solutions and we will have to pick the most probable solution from these set of 2^k solutions. The next lecture we are going to talk about a general decoding algorithm for a linear block code, thank you.