An Introduction to Coding Theory
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Module 08
Lecture Number 31
Convergence of turbo codes

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Today we are going to talk about how to analyze the performance of turbo code in low S N R.

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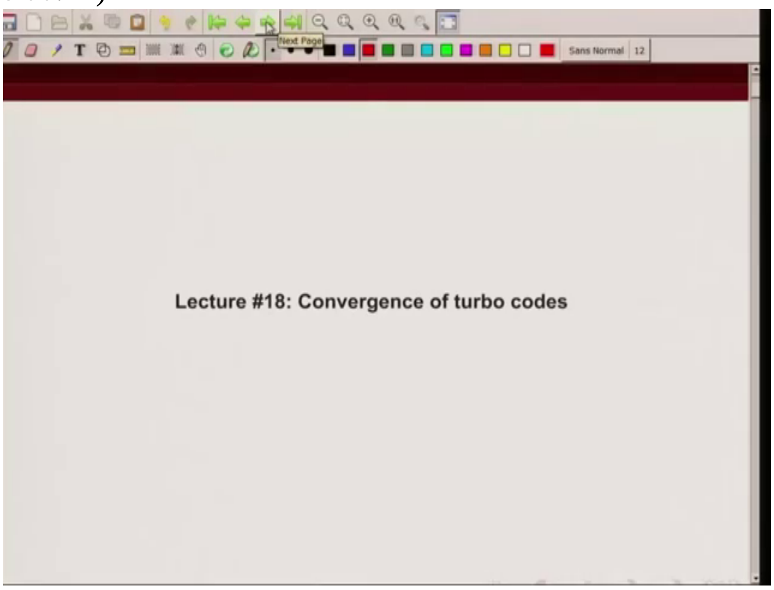

So we are going to talk about convergence, how to track the convergence

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of turbo iterative decoding algorithm and that's the topic of our discussion, convergence of turbo codes. So with brief introduction, we will talk about

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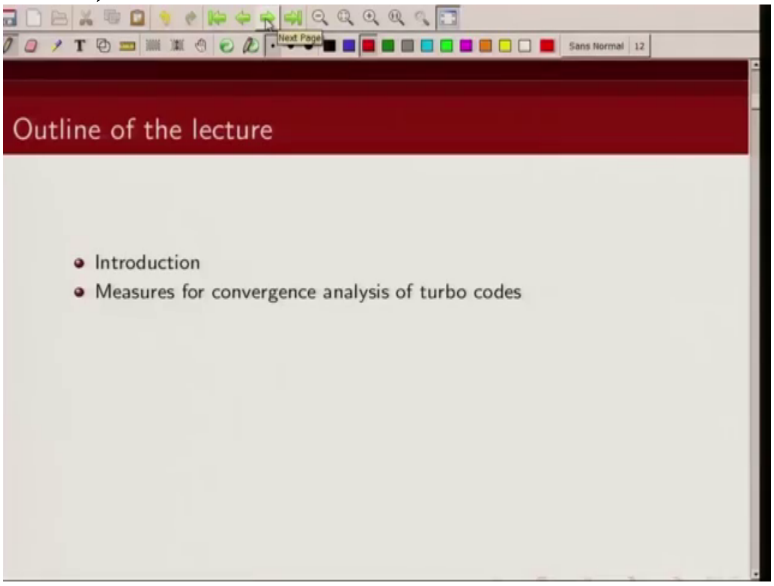

what are the various measures for convergence analysis

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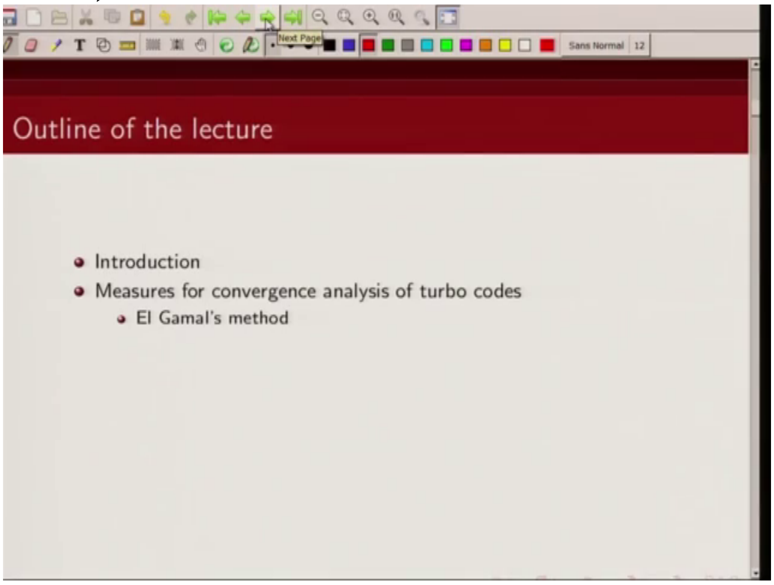

and in particular we will, are going to talk about these three methods, the first method

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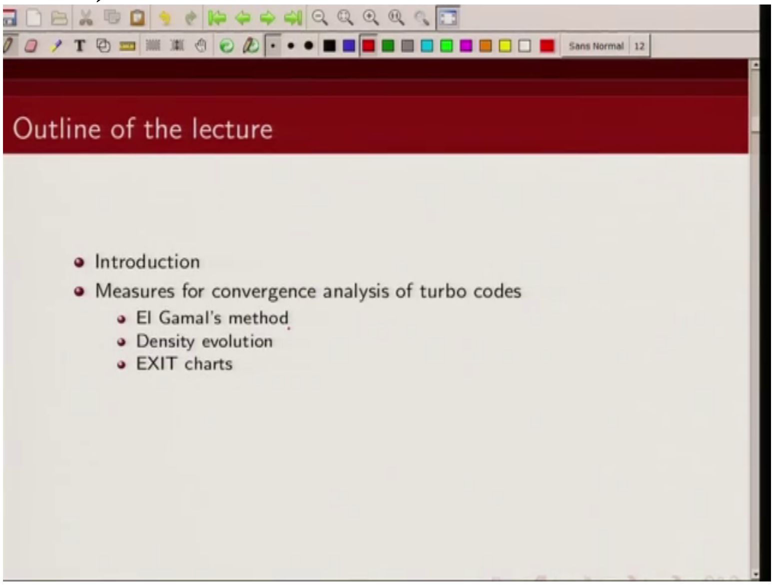

which is based on Gaussian approximation and which involves tracking the mean of the extrinsic values, a method proposed by El Gamal. Next we will talk about a method which is proposed by Divsalar and others using density evolution and then a method which is based on mutual information, tracking mutual information proposed by ten Brink.

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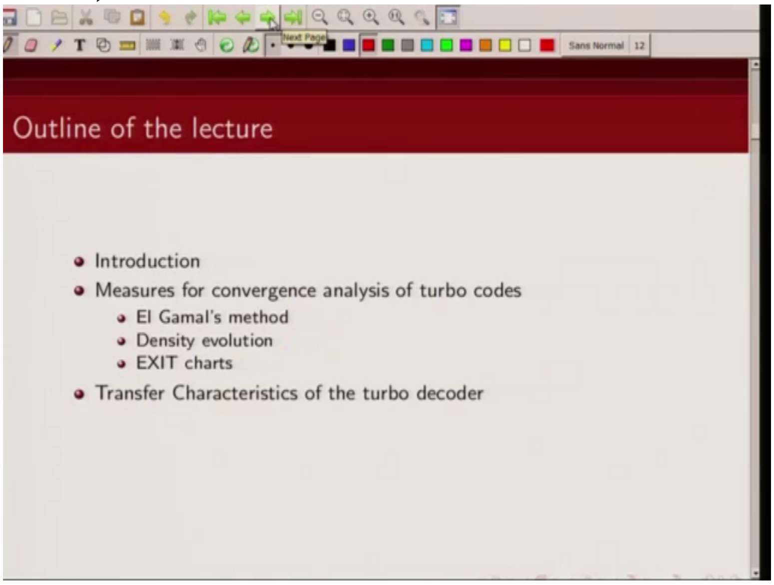

And then we will talk about what do we mean by a

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transfer characteristic of a turbo

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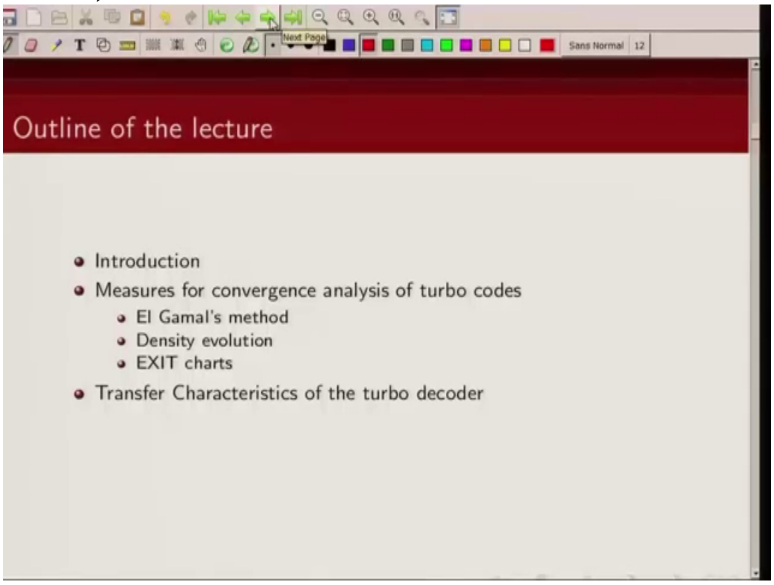

decoder and how we can use it to compute the convergence threshold

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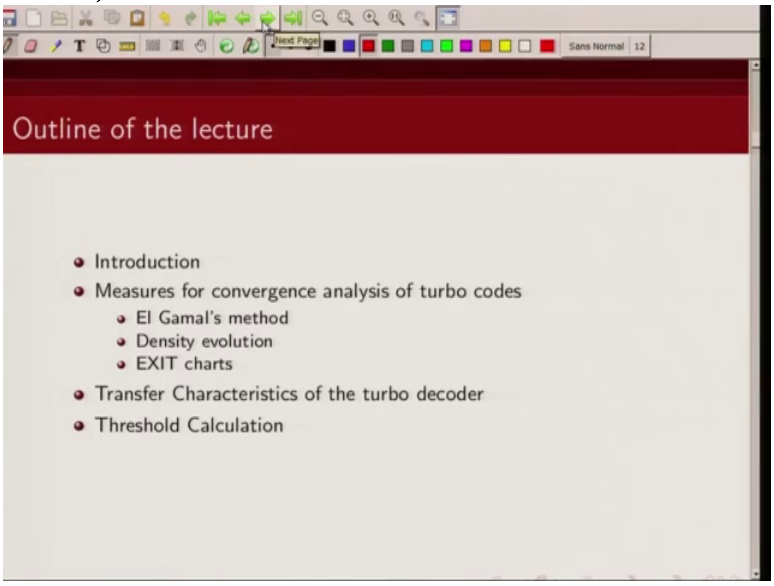

of a turbo code.

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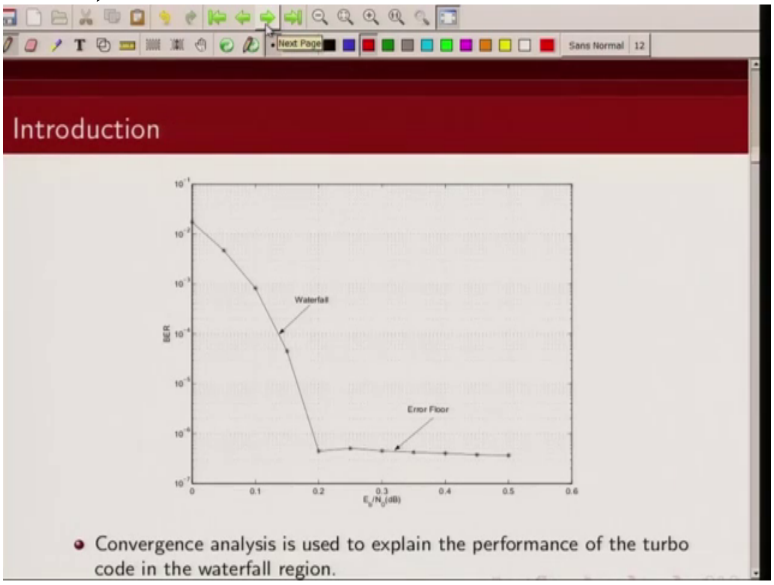

So this is a typical performance of a turbo code. If we take a larger block size, this is for a block size, I think 65000 plus, so if you take a large block size, this is typical performance of a turbo code. On x axis, I have signal to noise ratio and on the y axis, I have plotted

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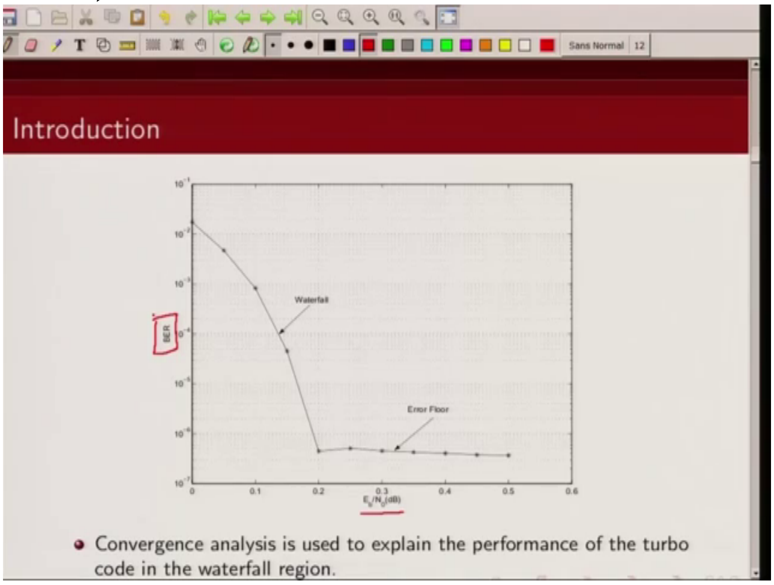

bit error rate. Now you will see there is a region, so this region which we are calling

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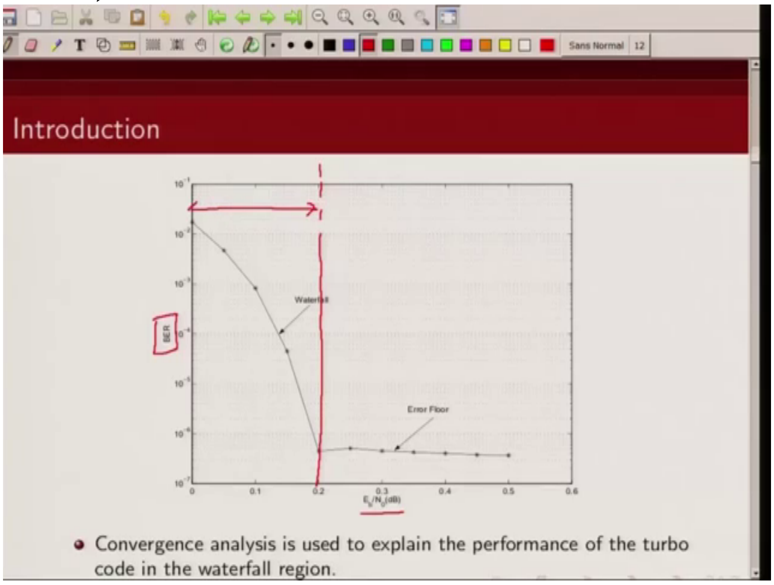

waterfall region where there is a steep fall

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in bit error rate performance and there

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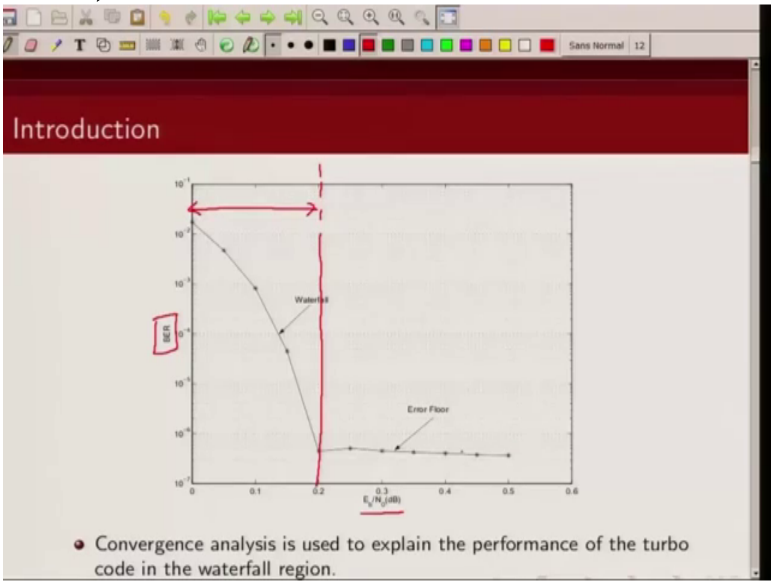

is a region, we call it error flow region

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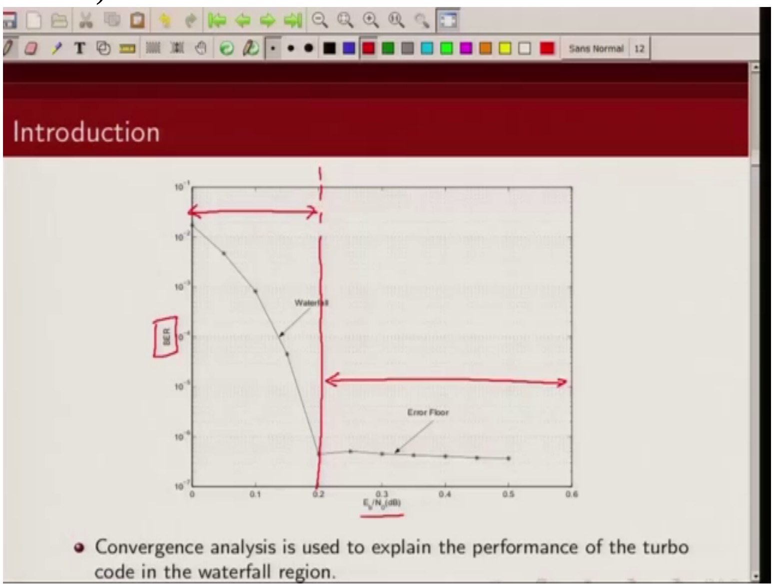

where

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the b e r does not improve much. So today's

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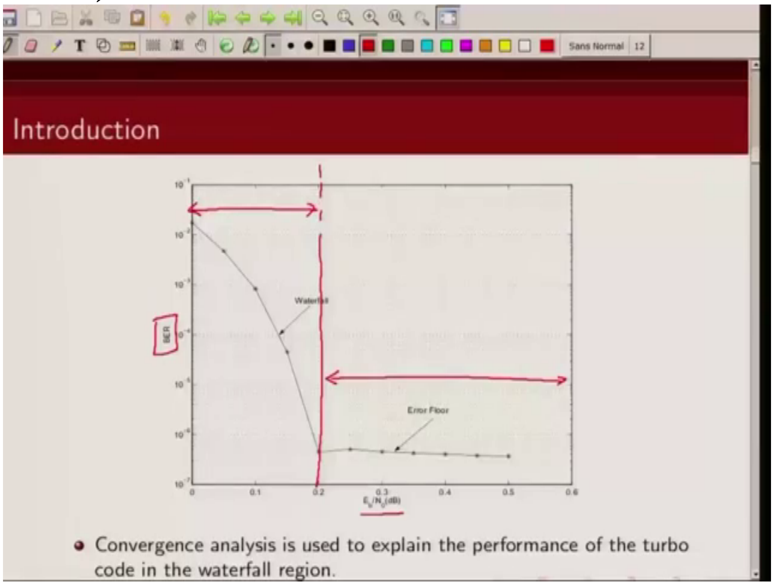

topic of discussion is this waterfall region. What determines

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the performance of turbo code in this region where it falls sharply and how can we get some guidelines on how to choose constituent encoders so that we get a steep fall like this.

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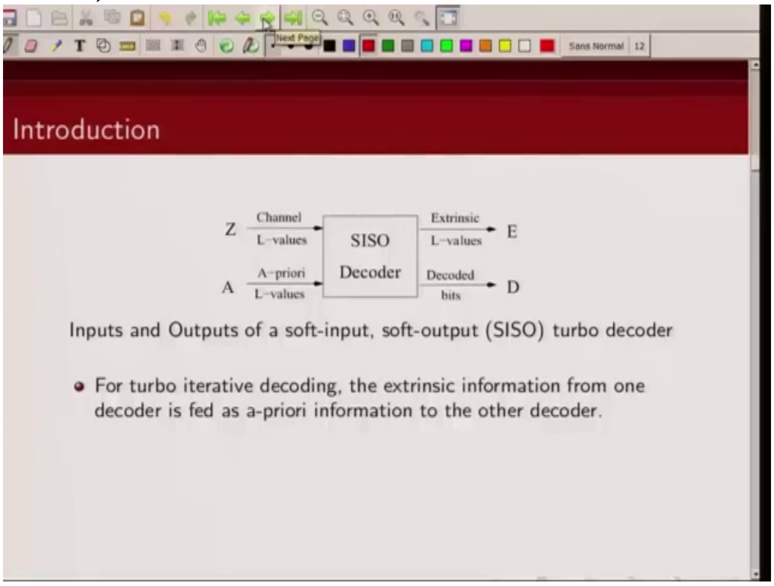

So before we study the convergence

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analysis, convergence of turbo code, let's pay close attention to the basic block diagram of our turbo decoder.

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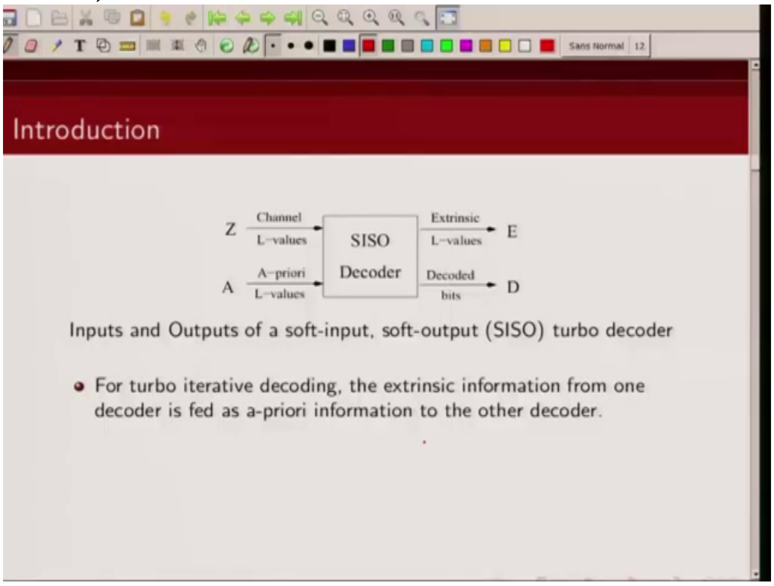

The heart of the turbo decoder is the soft input soft output decoder and if you recall this soft input soft output

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decoder takes in as input the channel

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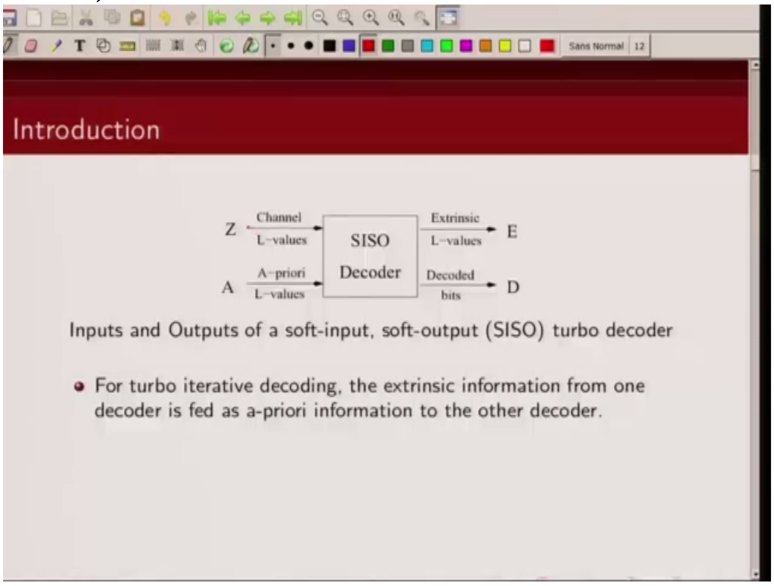

received values corresponding to the information

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and parity bits,

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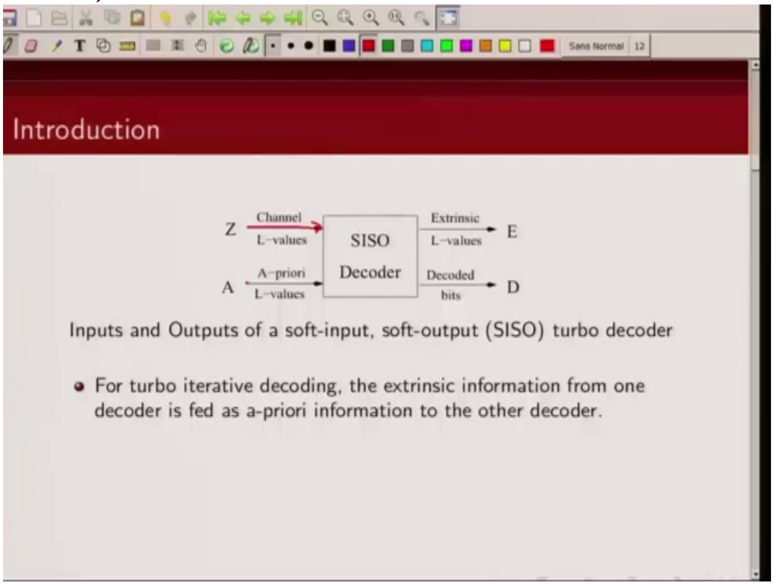

a priori value which

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it receives from the other decoder, which are the extrinsic values passed on to the other decoder and it

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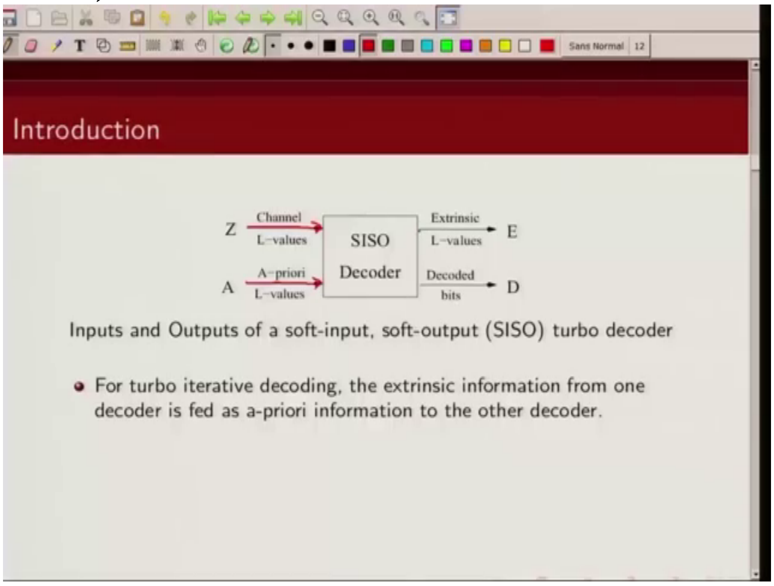

computes extrinsic values as well as A P P L values

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where you take a hard decision to get back your decoded bits. So if you look at a turbo decoder, this is the heart of the turbo decoder. There are two such soft input soft output decoder and if you look for a particular signal to noise ratio, if we look at turbo decoder as a function of iteration you will notice the only thing changing with iteration is this

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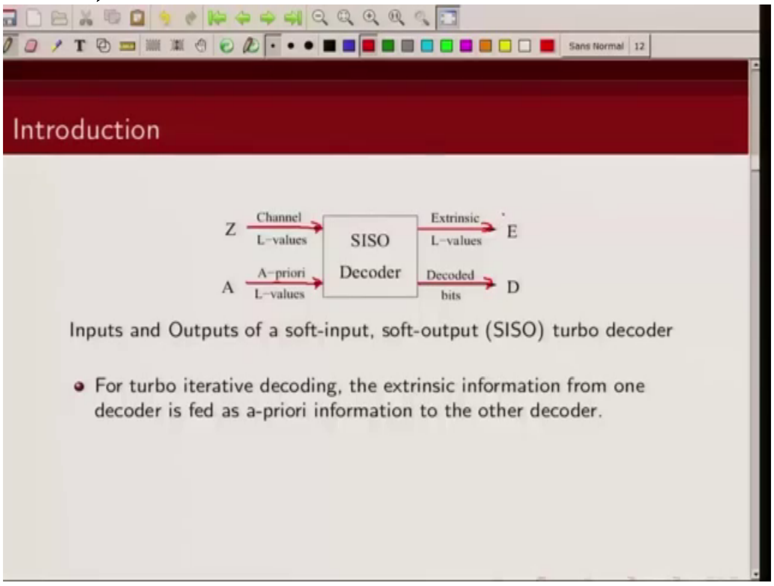

extrinsic value and

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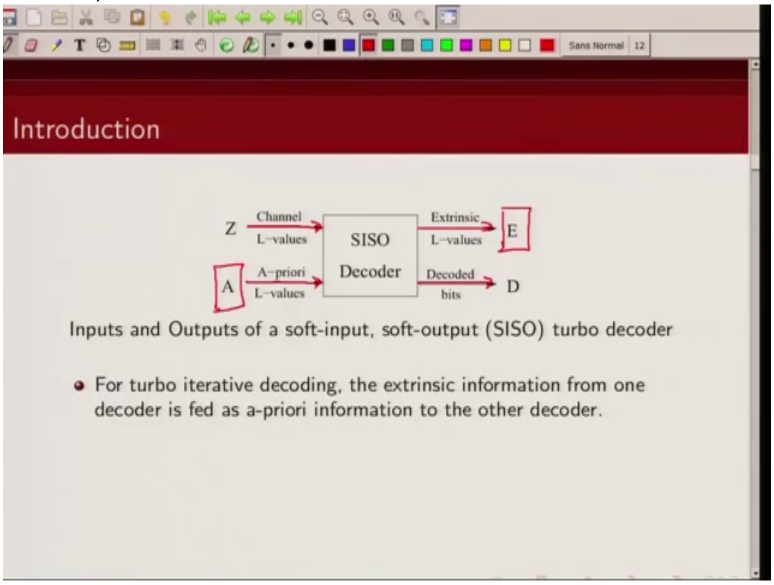

a priori value. So with iteration,

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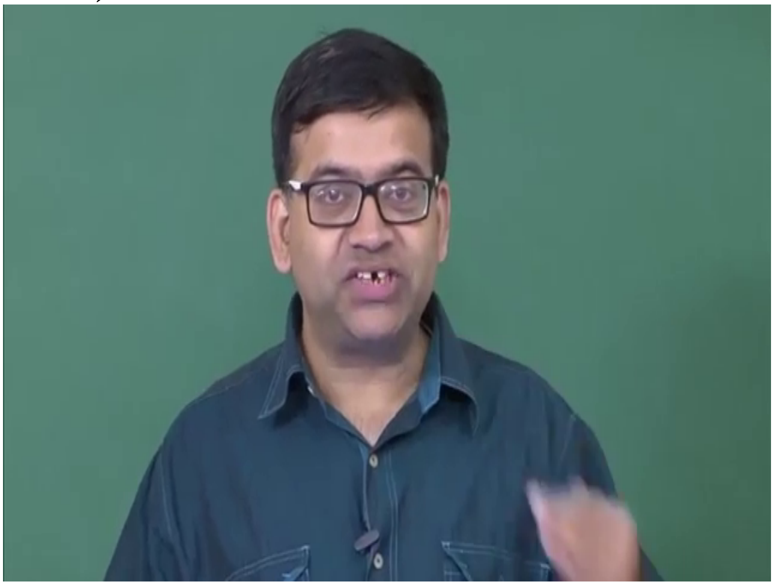

your, initially you do not have any estimate on a priori value, you assume that the bits are equally likely to be zero and 1 but subsequently with iteration when your extrinsic values are generated, those are passed on as a priori value. Now the channel L values remain same for a fixed signal to noise ratio; for a received bit, the channel L value remains same. Only thing changing with iteration are these two quantities,

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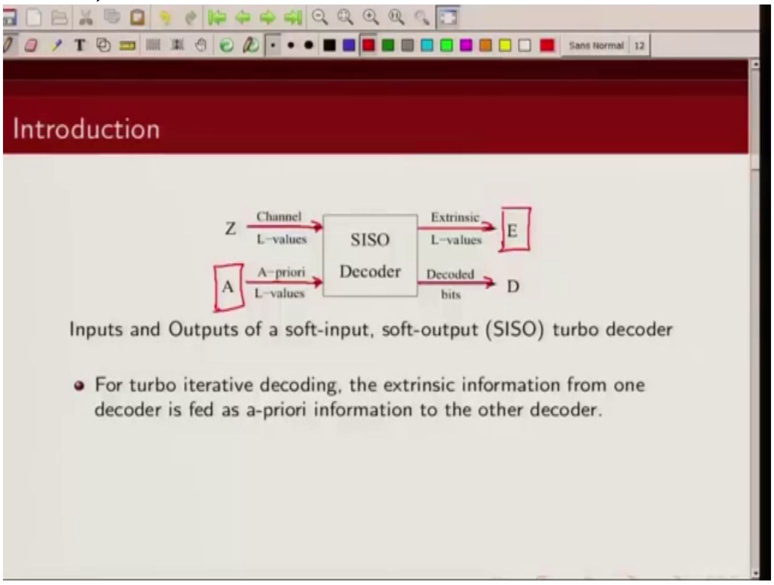

this a priori value and the

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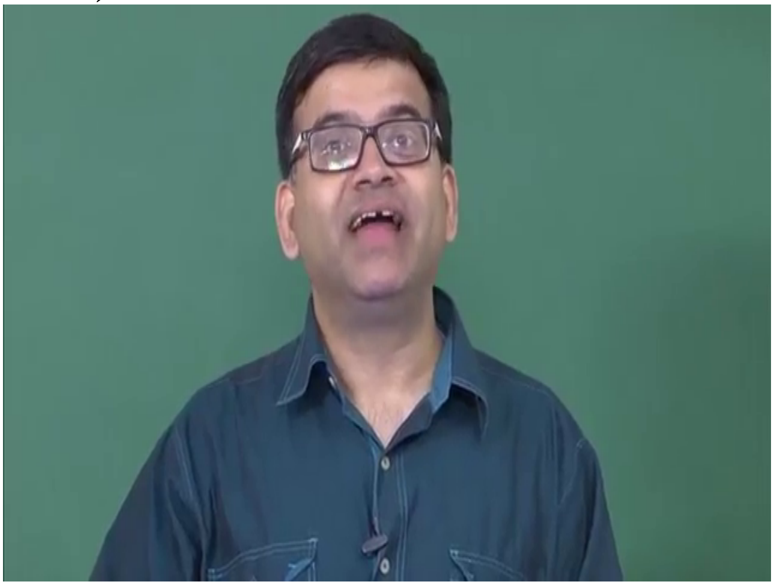

extrinsic value. So if we can track with iteration how our extrinsic information is growing with this a priori information, that will give us some clue about the performance of turbo code at waterfall region.

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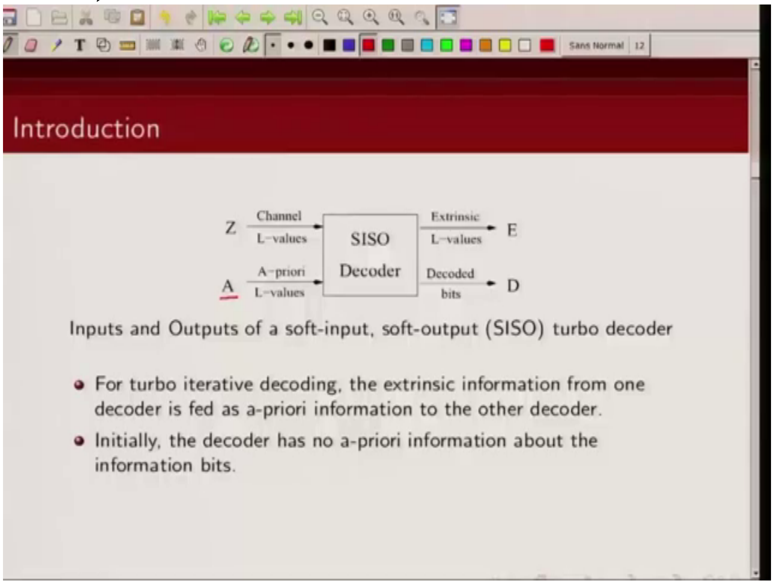

So as I said, initially we do not have any a priori value but subsequently after one half iteration

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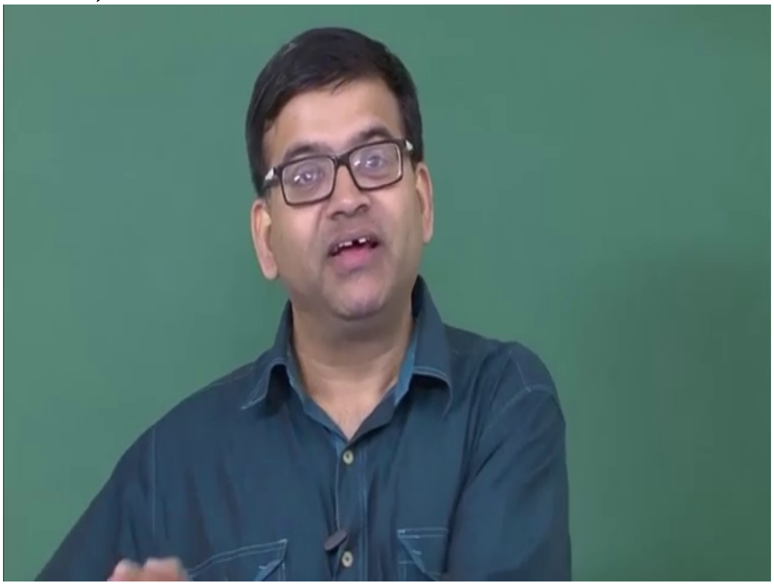

extrinsic information are generated and that's passed on as a priori value

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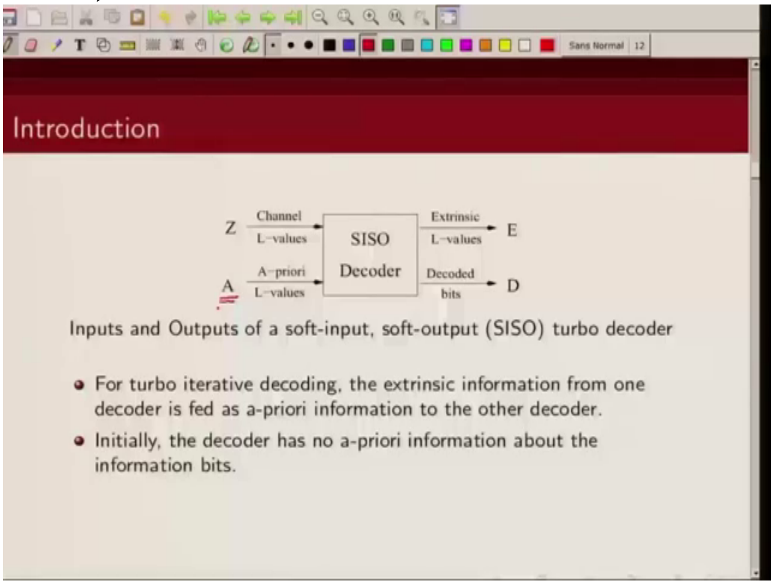

to this soft input soft output

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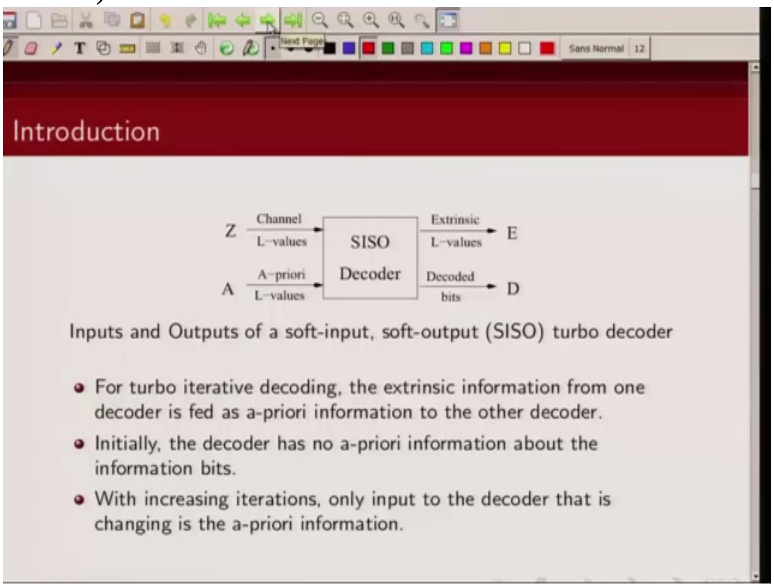

decoder. And again I emphasize, the only thing changing with iteration are

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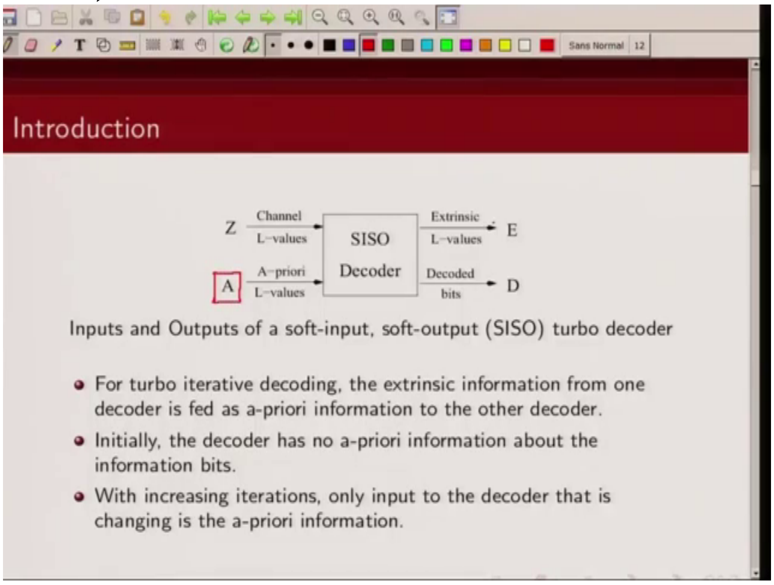

these extrinsic values and

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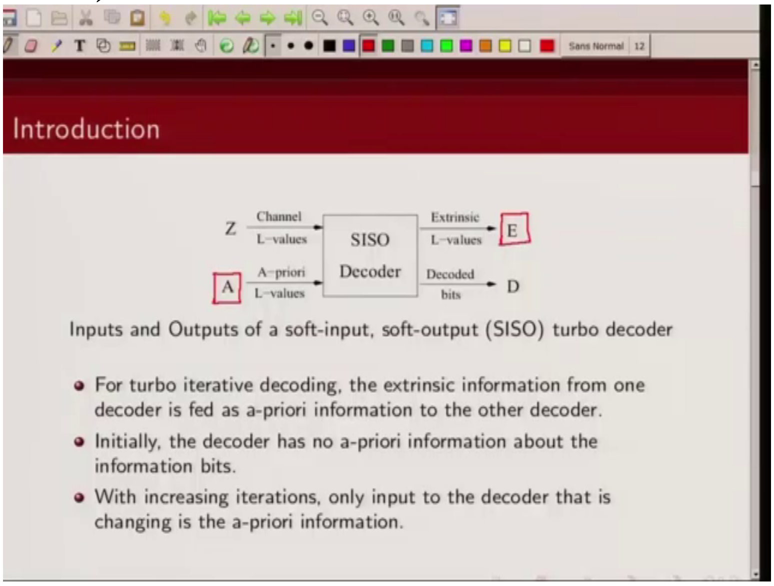

a priori values. So if

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you want to track how your turbo decoder is working with iteration, you need to track these two quantities

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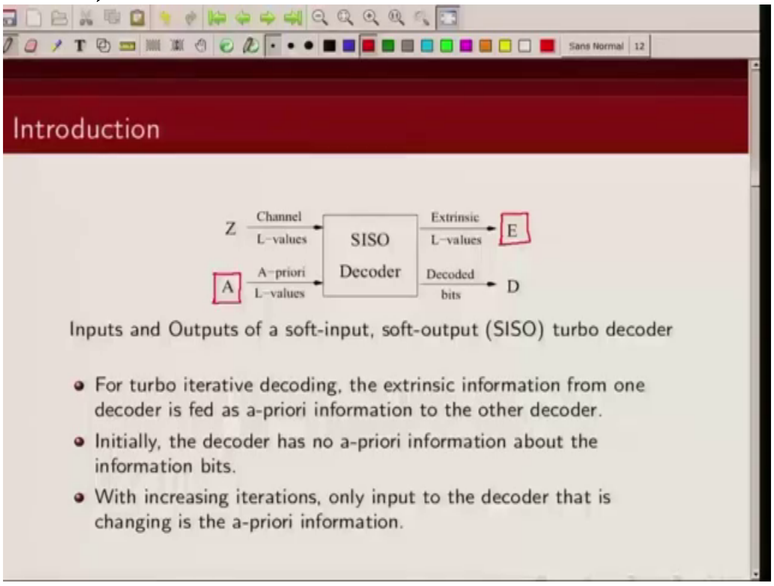

and we are going to talk about what are the various measures

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that we can use to track these two quantities.

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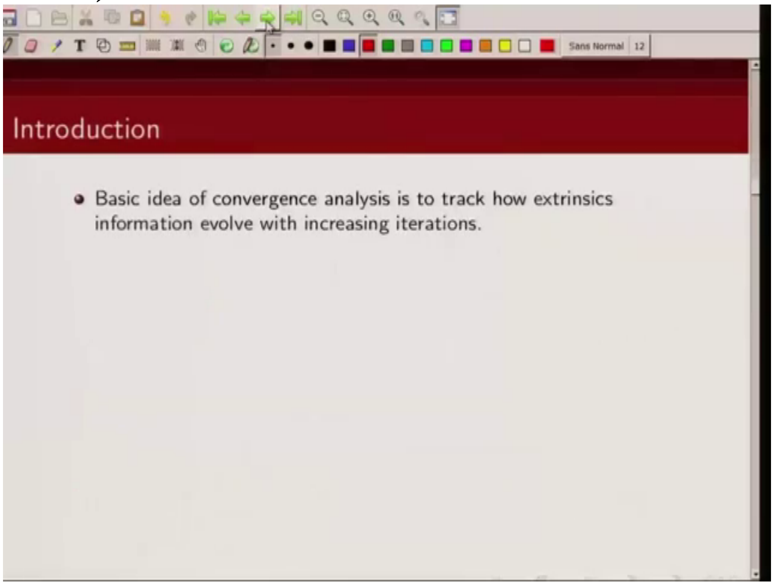

So basic idea of convergence of turbo code, convergence analysis of turbo code is to track how these extrinsic information are evolving with increased iteration. So if you feed in

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better a priori value, how is your extrinsic information

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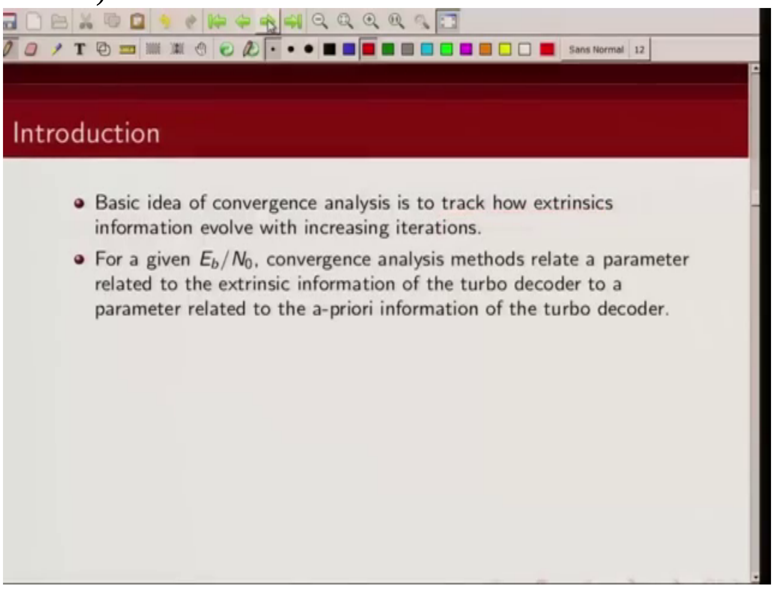

evolving? So what we do is, for a fixed signal to noise ratio we have a set of received values. So what we do is we try to relate a parameter which is related to the extrinsic information of the turbo decoder and

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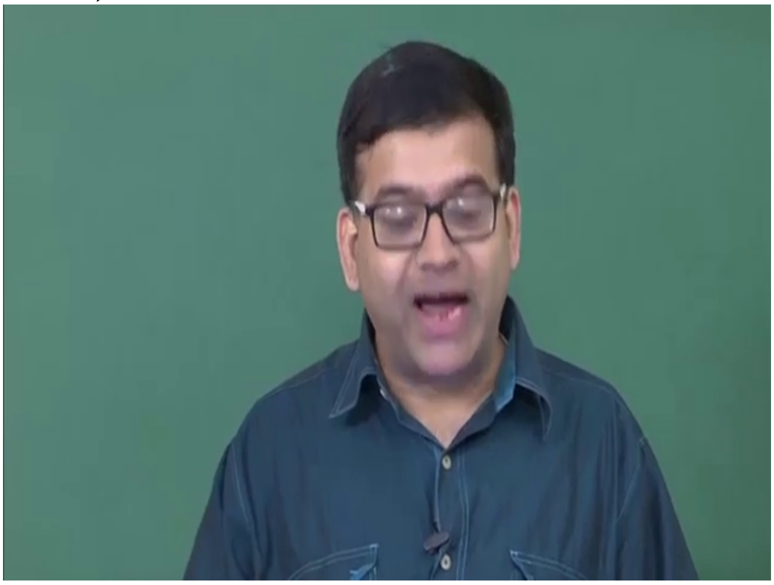

we try to relate it to the parameter

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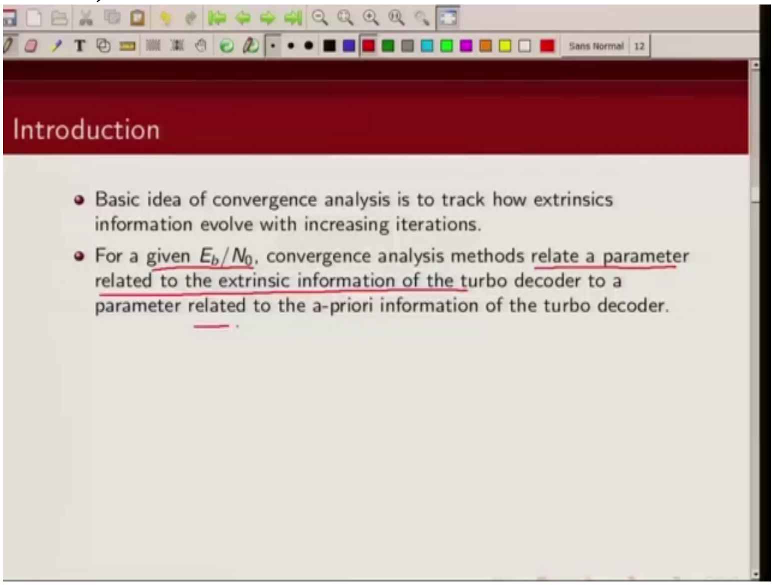

which is related to the a priori information. As

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I said in this soft input soft output decoder, only thing changing is this a priori information and this extrinsic information. So we want to track how these extrinsic information and a priori information are growing with iteration. So what we are going to do in this convergence analysis is we are going

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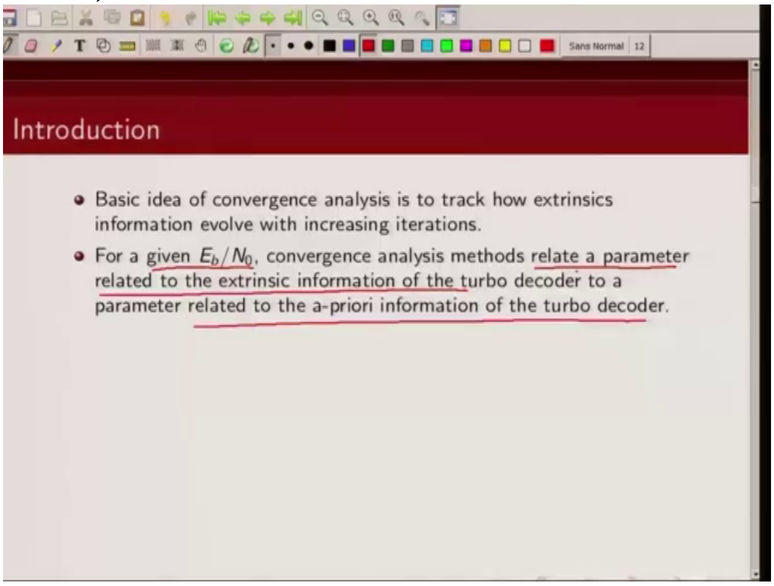

to track a parameter which is related to extrinsic

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information and we will see how that parameter will change when the parameter at the input side which is a priori value is also changed.

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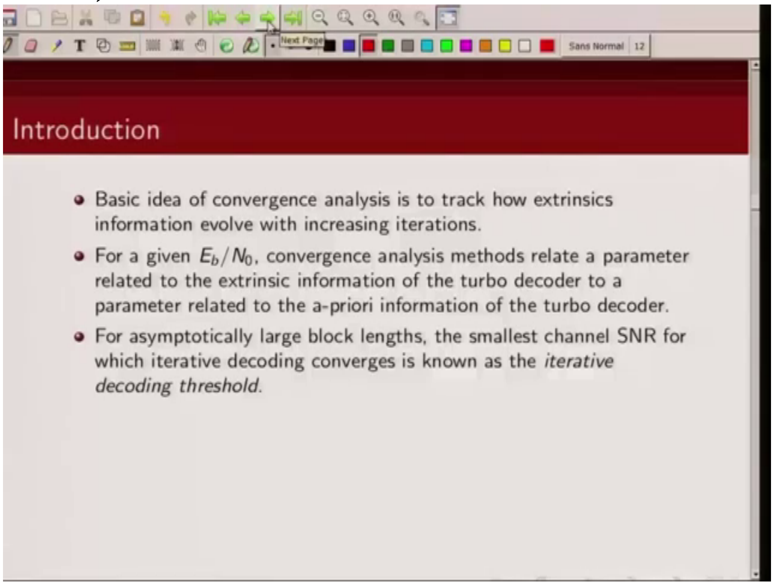

And for an asymptotically large block size the smallest channel S N R for which iterative decoding algorithm converges is known as decoding threshold. So this iterative

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decoding threshold will be away from your channel capacity, typically.

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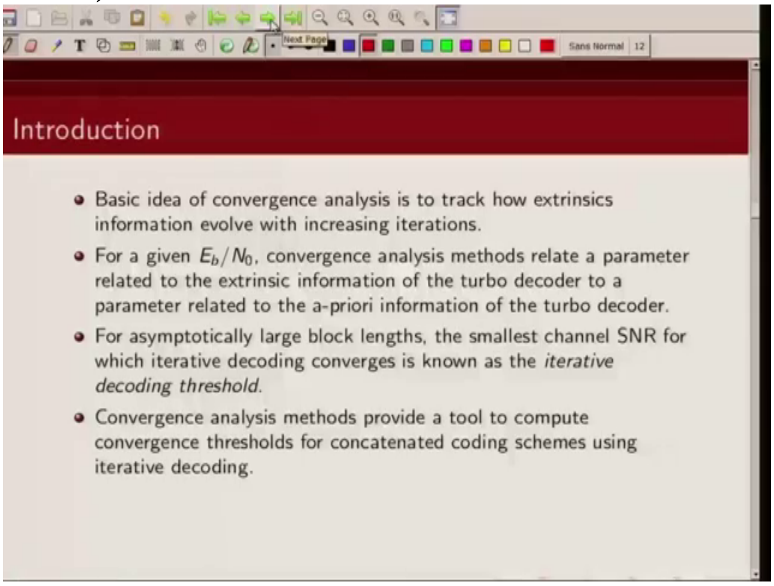

Now this convergence analysis tool is a very, very powerful tool to analyze these kinds of

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iterative decoding algorithms. It gives us tool to analyze the performance of concatenated schemes that use iterative decoding algorithm. It gives us tool to design our constituent encoders. It gives us tool to design our puncturing pattern, uh so it is a very, very interesting tool for analysis in the waterfall region.

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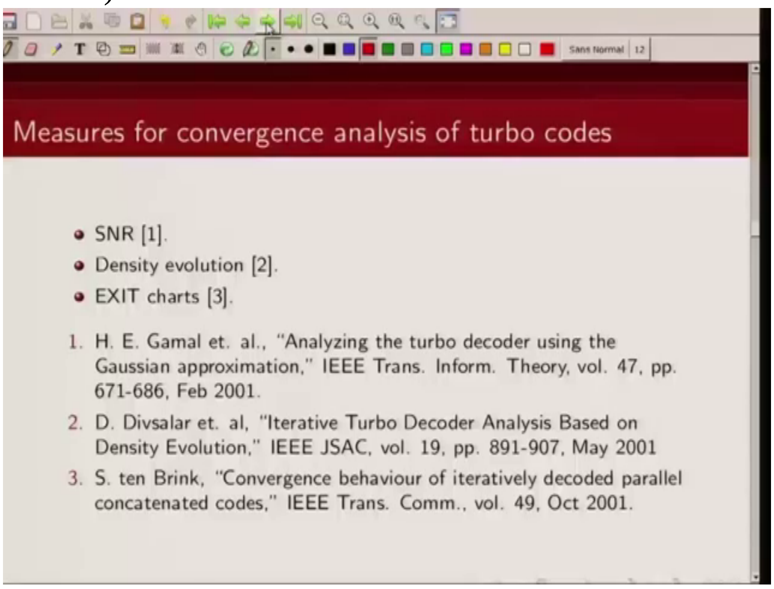

So as I said there are three

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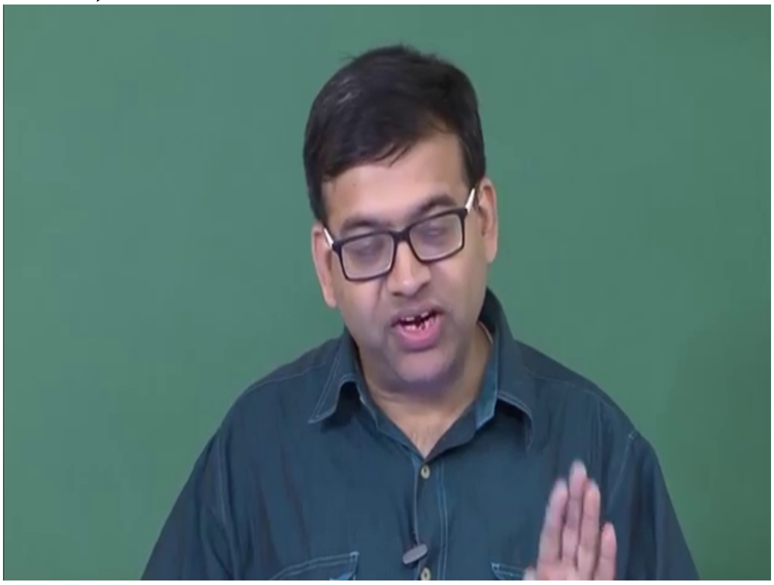

popularly known techniques for

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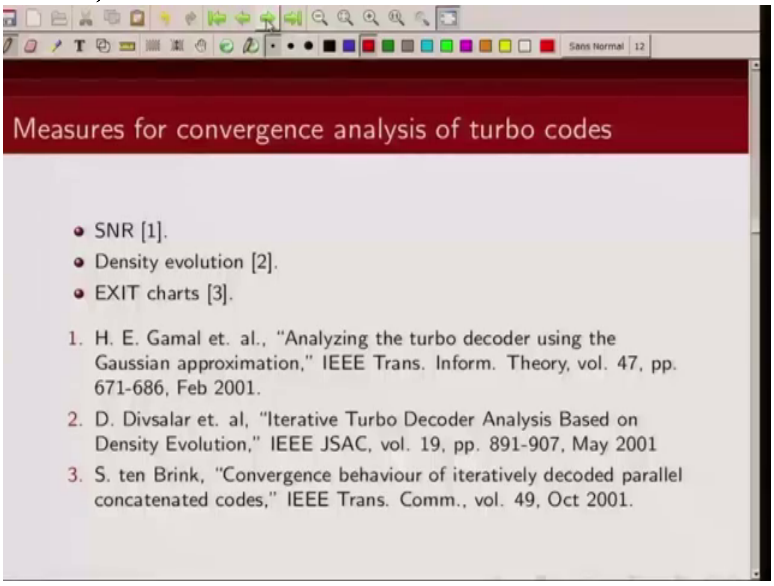

convergence analysis and as I said the idea

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of these techniques is track one parameter which is related to the extrinsic information and track the same parameter related to the a priori information. So this technique by El Gamal

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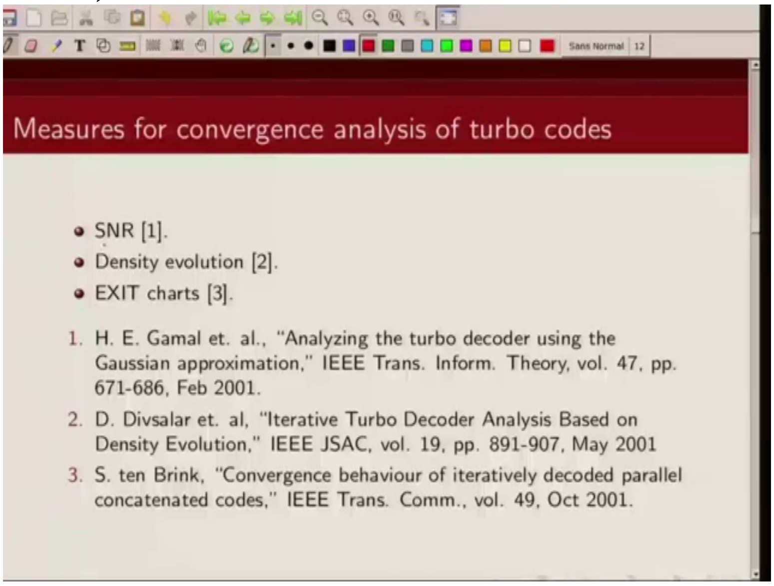

makes use of Gaussian approximation and it tracks the signal to noise ratio, so it tracks the signal to noise

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ratio of the extrinsic information and observes how this S N R extrinsic information grows when you change the S N R of the a priori information. In the density

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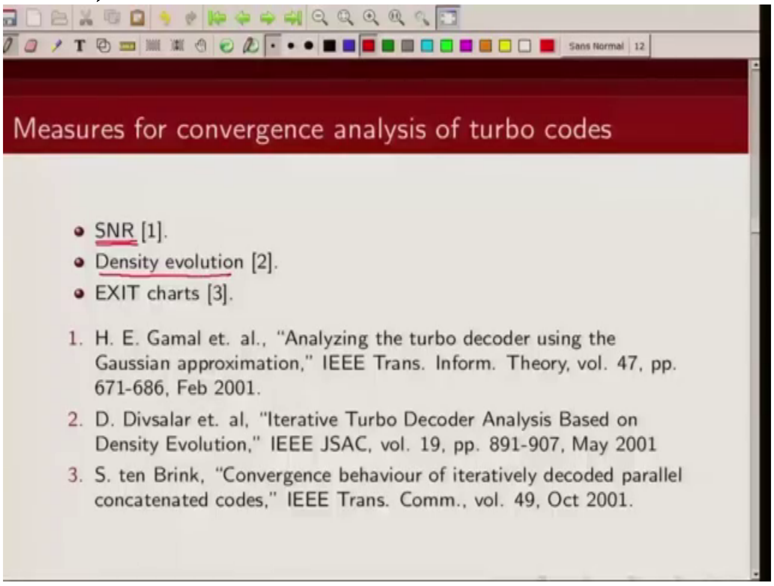

evolution method by Divsalar and others they actually see the

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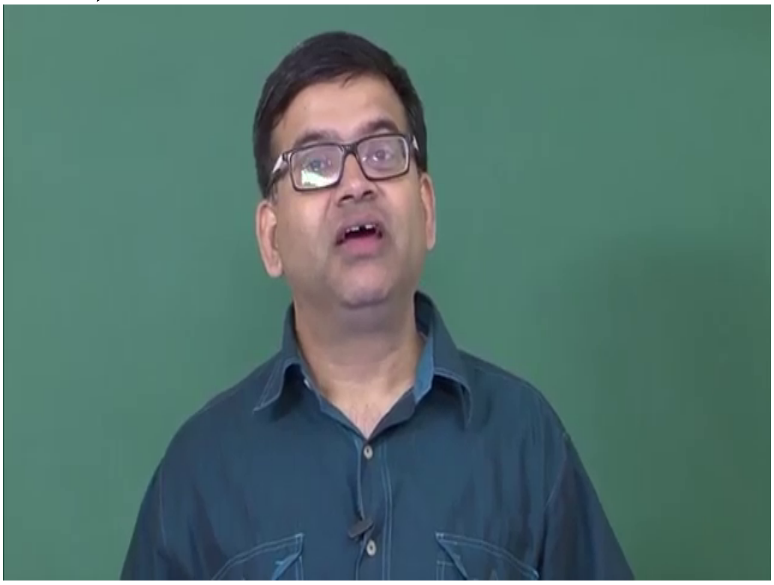

density of this extrinsic information, how does it grow with iteration and this

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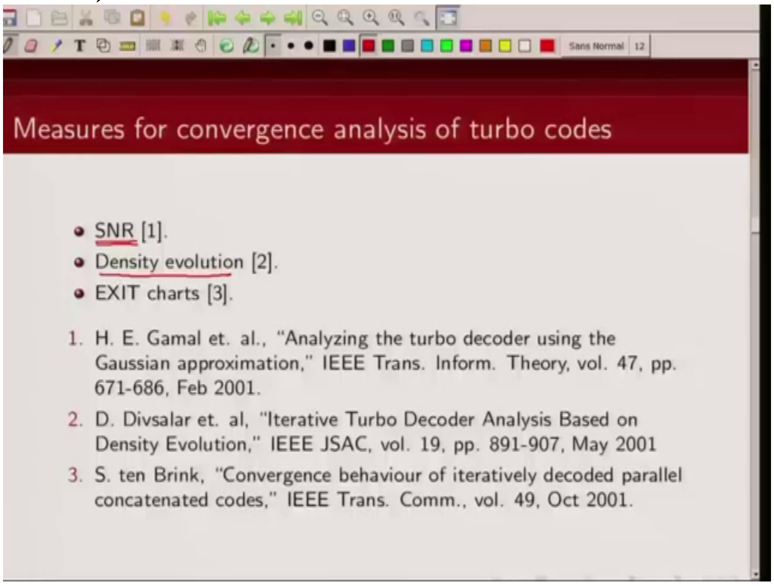

approach of ten Brink which is known as extrinsic information transfer chart, it

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uses mutual information as a parameter to

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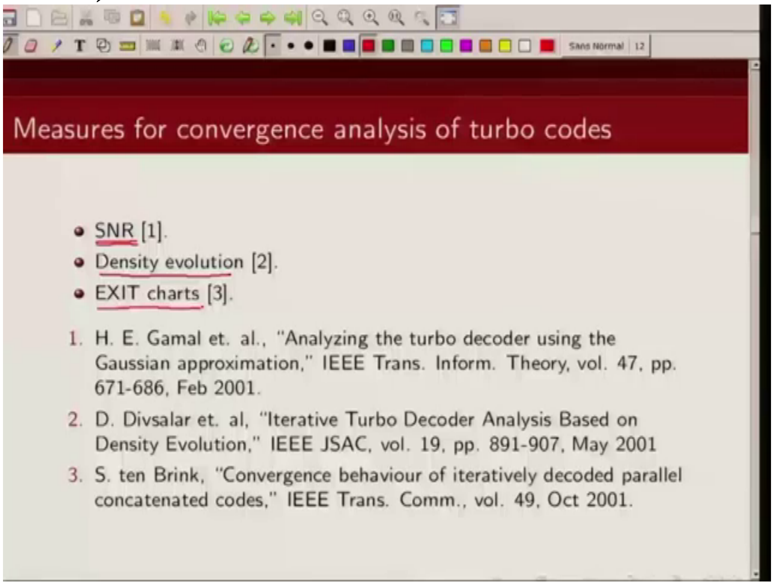

observe how, with iteration your extrinsic information is growing. And these are the three references, the first one corresponding to this S N R technique, the second one corresponding to this density evolution technique and third corresponds to this EXIT chart technique.

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So the El Gamal approach is based on Gaussian

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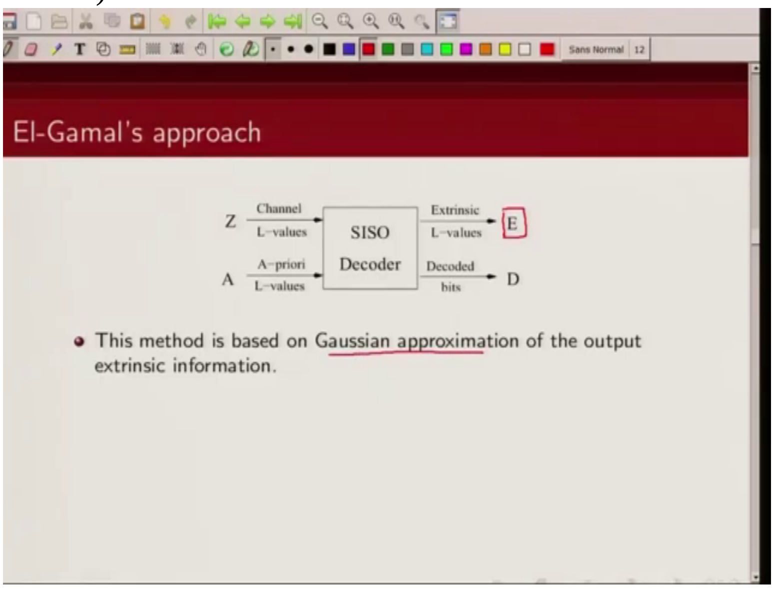

approximation of this output extrinsic information. So note,

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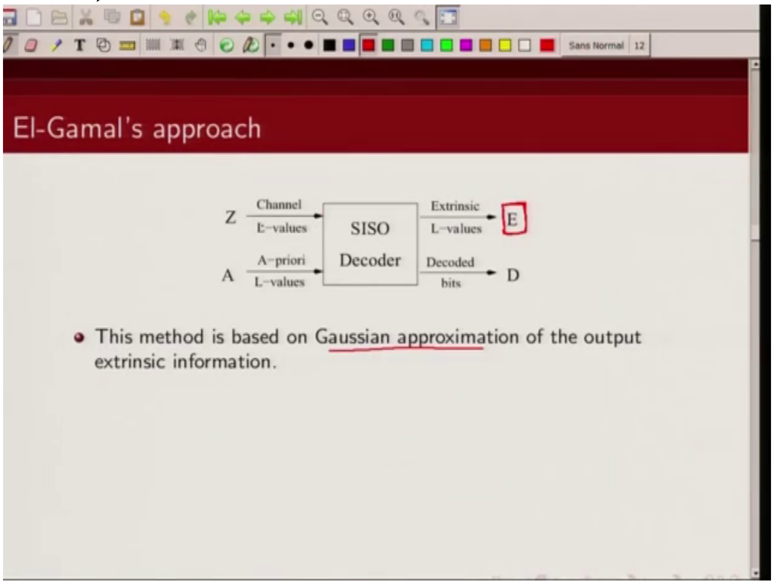

there are 2 inputs to my soft input soft output decoder; one which I am referring by Z

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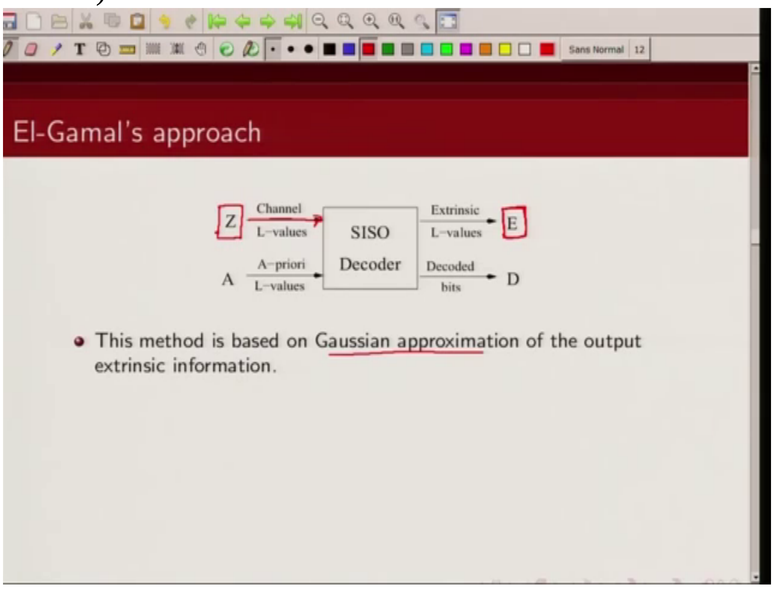

which is just channel received L

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values. The second

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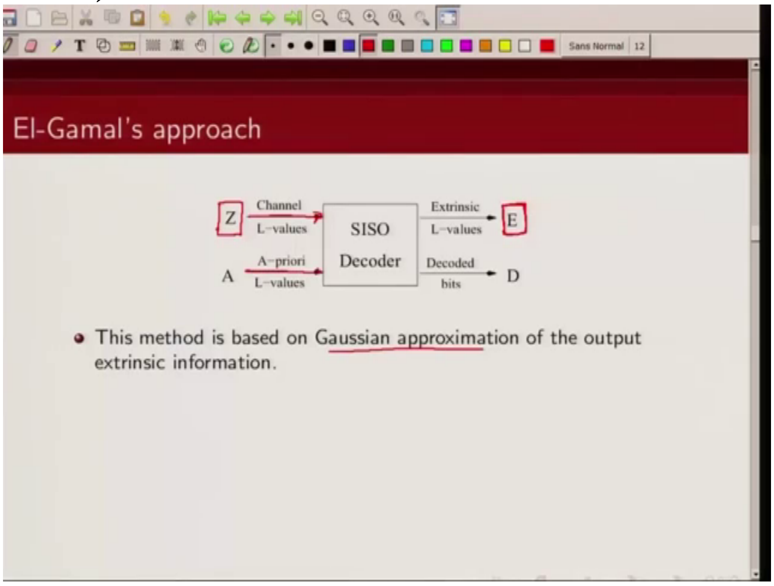

one is this a priori

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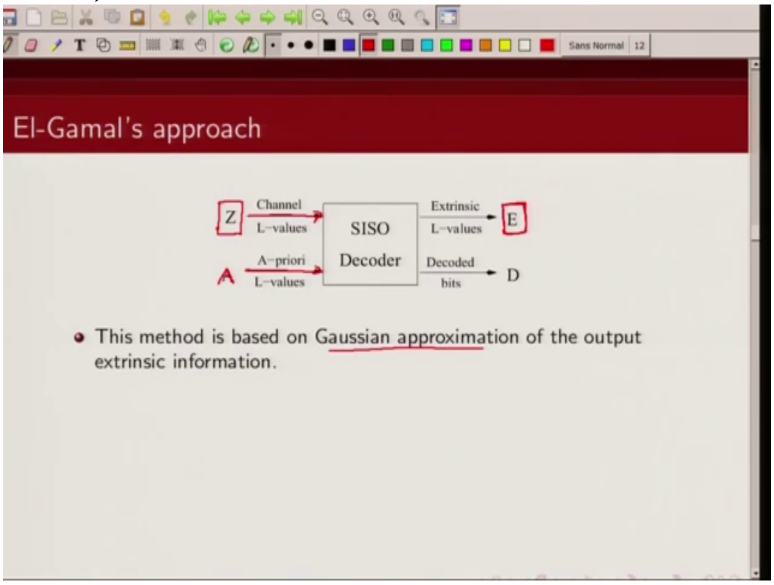

values and there are 2 outputs, one is this extrinsic information and other one is A P P L values, if I take a hard decision

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on that, what I get is my decoded bits.

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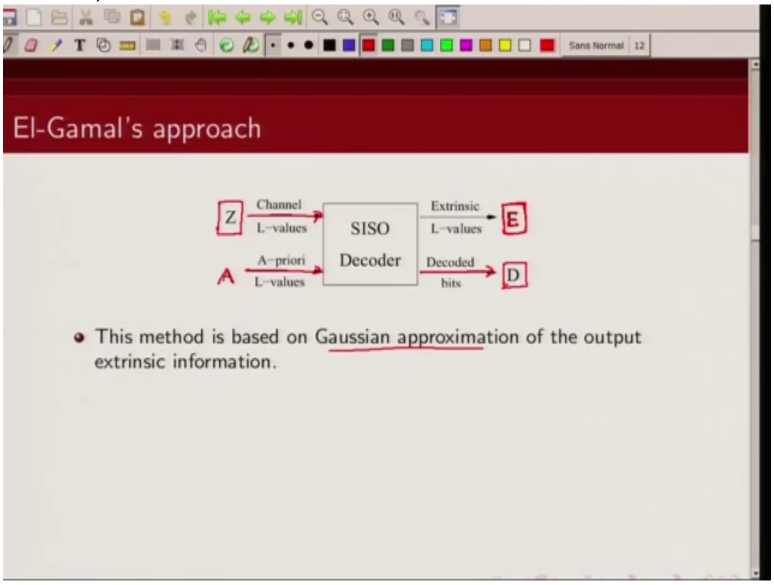

Now

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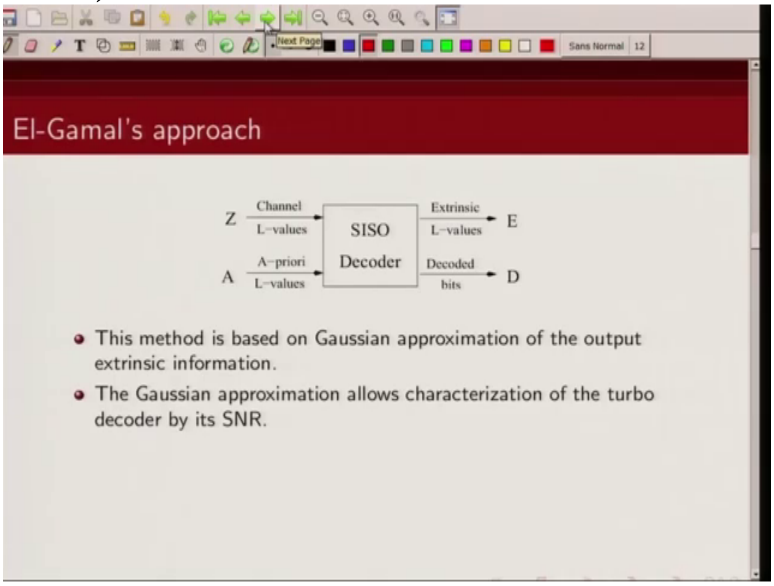

we are using this Gaussian

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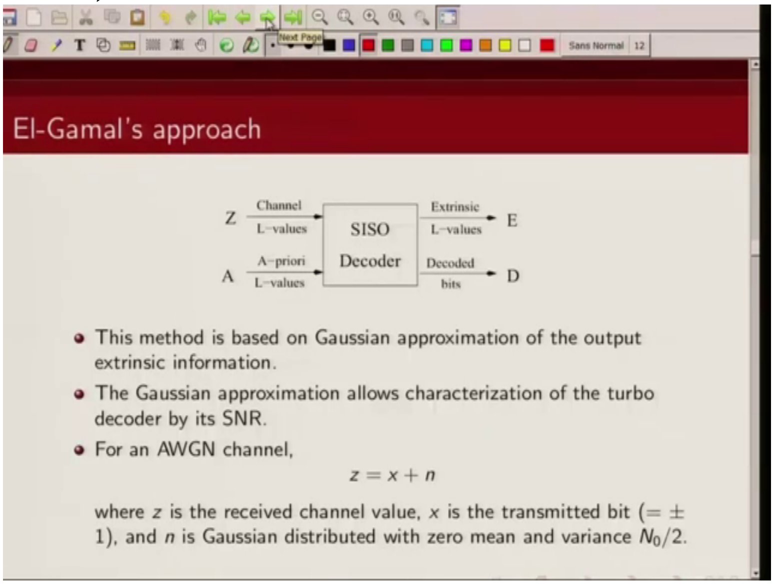

approximation so assume, so we have Gaussian channel. So if x was your modulated signal and n is my Gaussian noise, so what I receive is

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Z.

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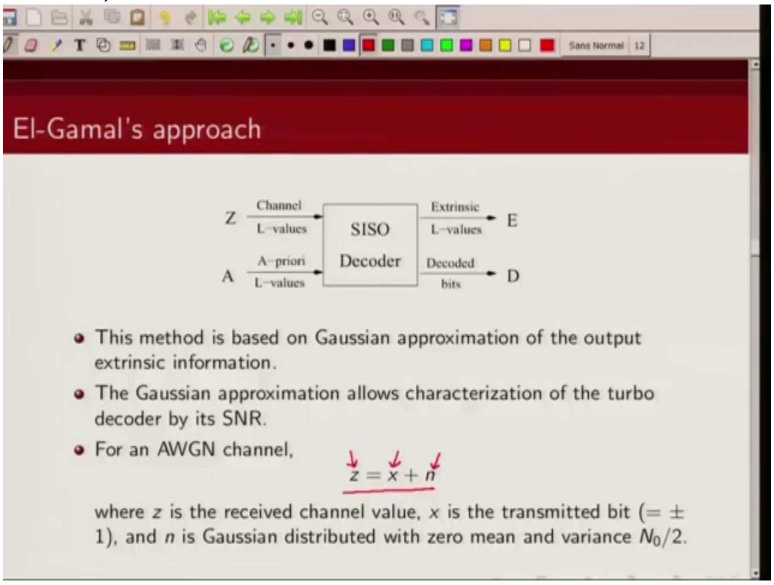

Now

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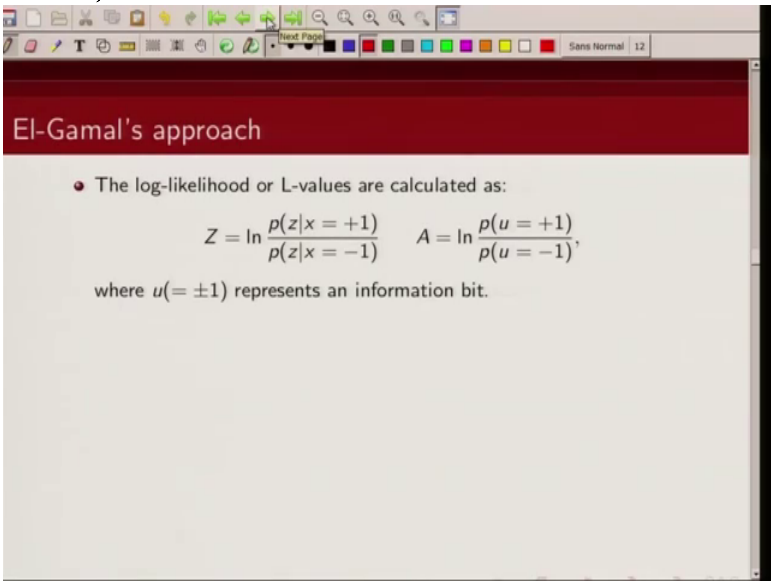

the likelihood ratio of Z we can write it like this, similarly this a priori information, the L value of that I can write

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it like this.

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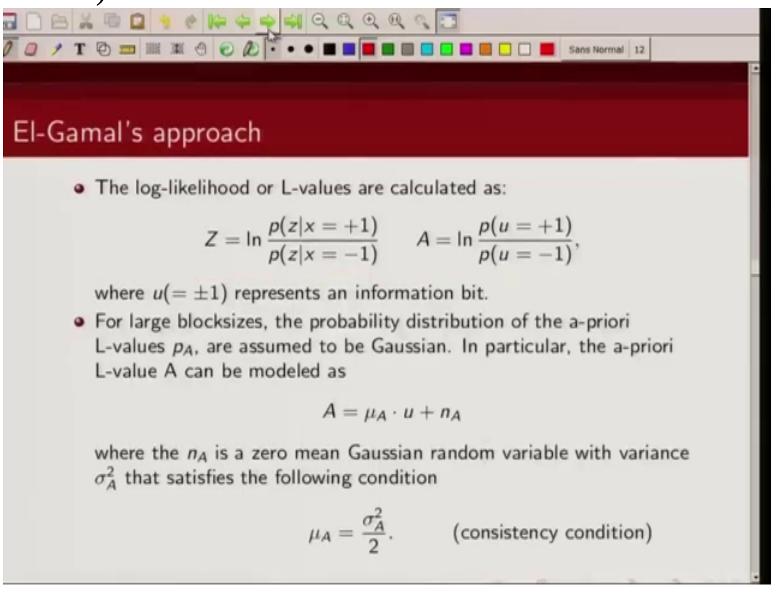

Now for large block sizes this a priori distribution is assumed to be Gaussian. So we model this a priori L value in this particular way in this

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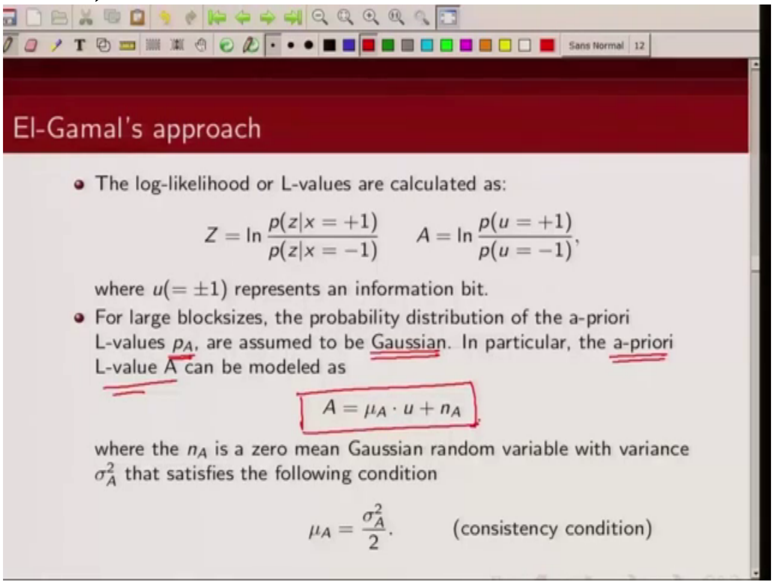

El Gamal's approach. So in El Gamal's approach we modeled our a priori information as Gaussian and we generated like this, A is mu A times input plus some Gaussian noise and they have also observed what they call consistency condition.

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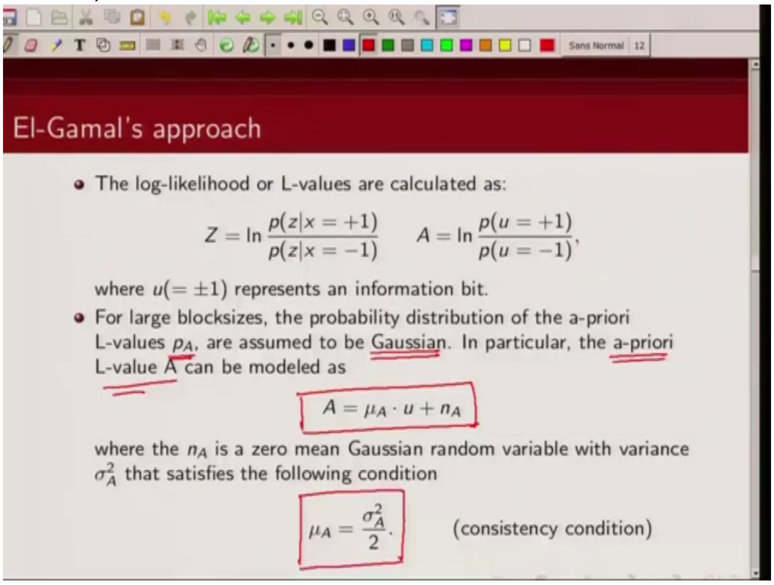

So they assume the mean and variance are related in this particular fashion. So what happens is if you make this Gaussian assumption and you make this assumption that mean and variance are related, then you essentially need to track only

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one parameter. So you, for example, with just the mean you can track your

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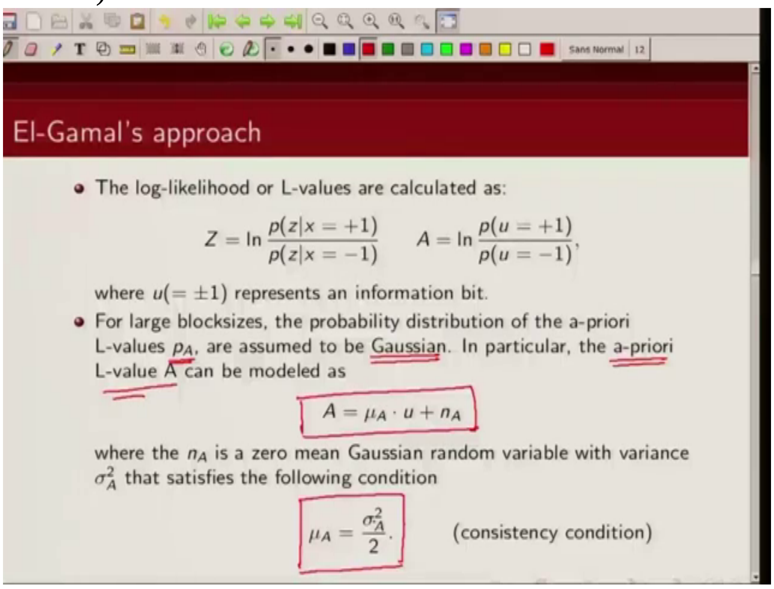

Gaussian distribution because mean and variance are related.

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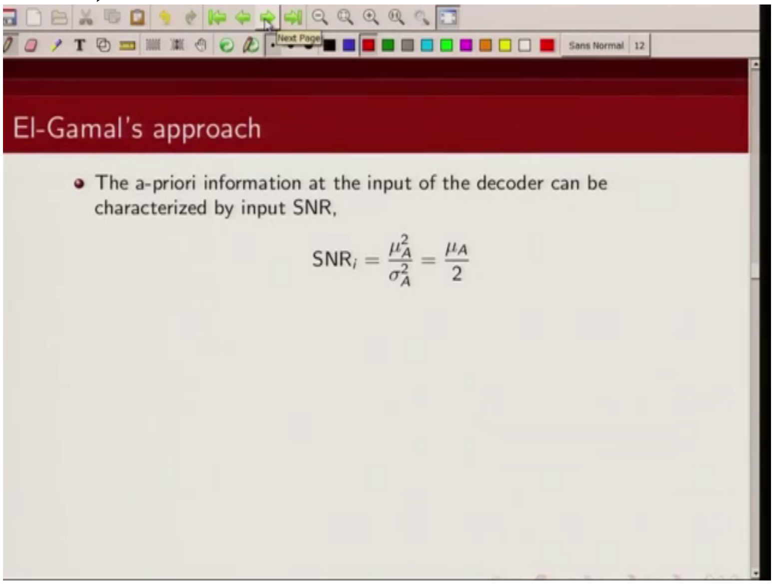

Now similarly we can define input S N R of the a priori information.

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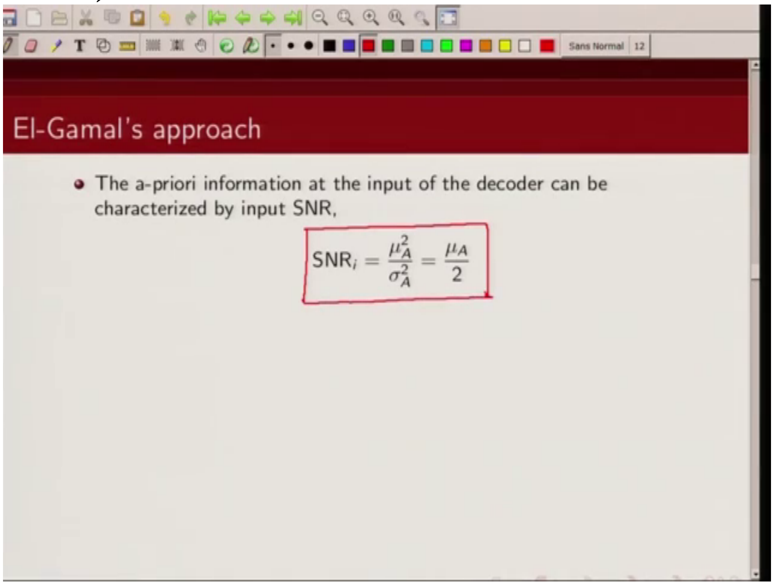

This is mu A square by sigma square. Now sigma square by 2

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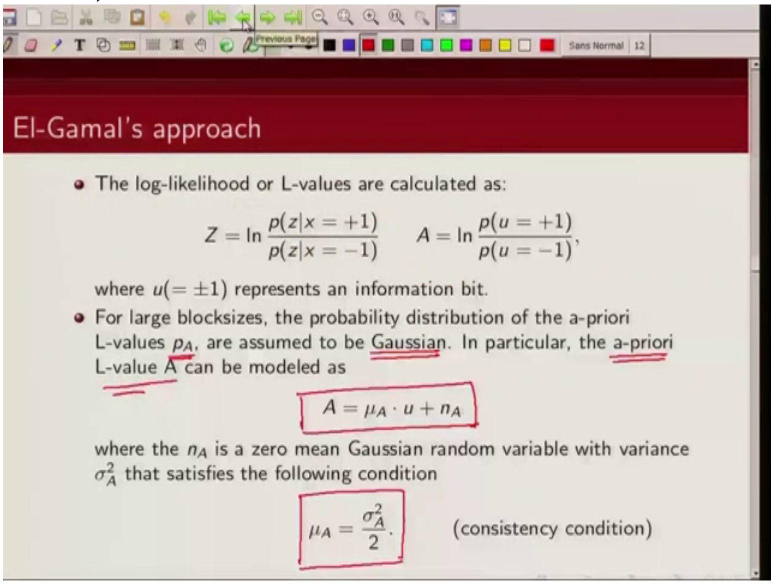

is mu A. So our

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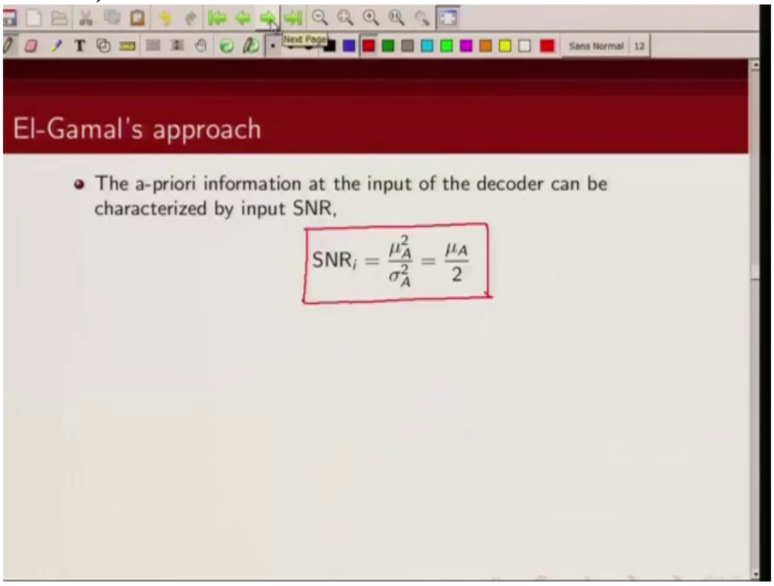

input S N R is given by the mean of the a priori information divided by 2.

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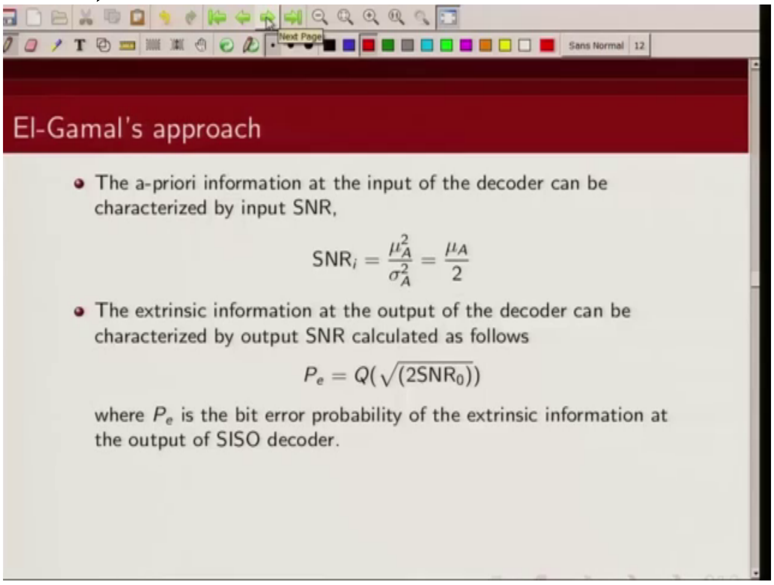

And since our output is approximated as Gaussian, so we can calculate the output probability of error as a function of

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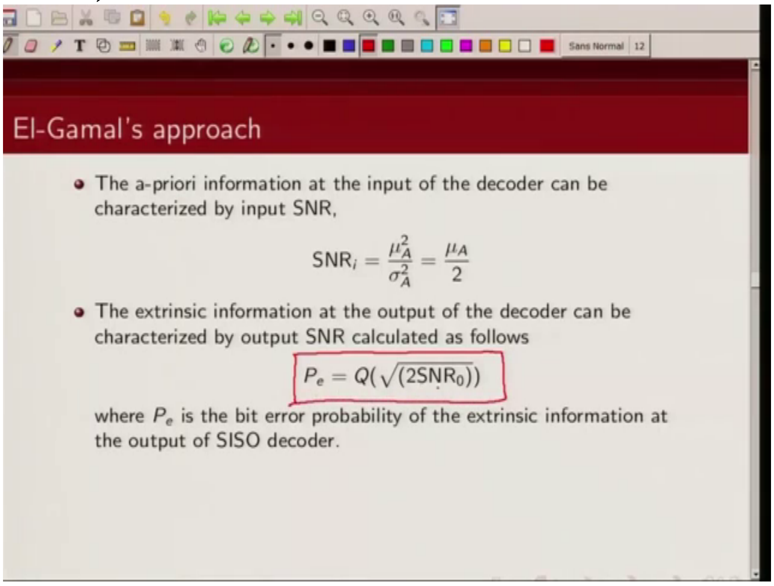

output S N R and they are related to the, using this Q function. Now,

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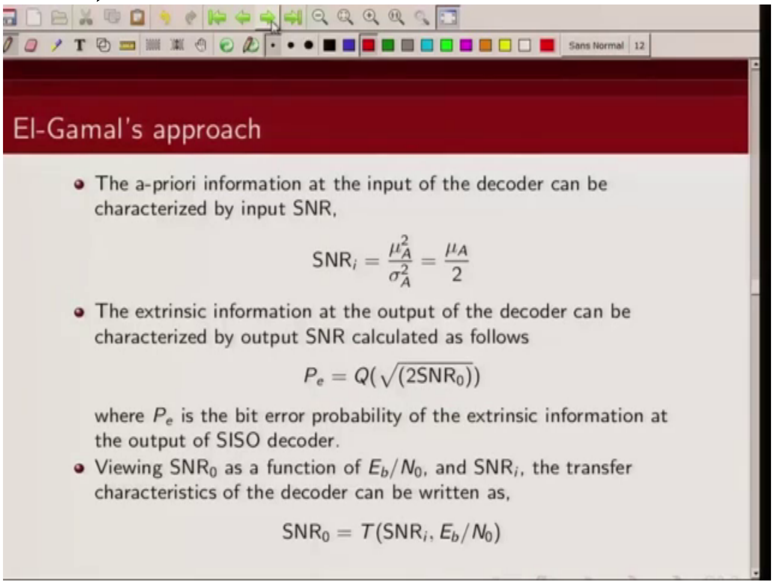

so what we can do is we can write this output S N R

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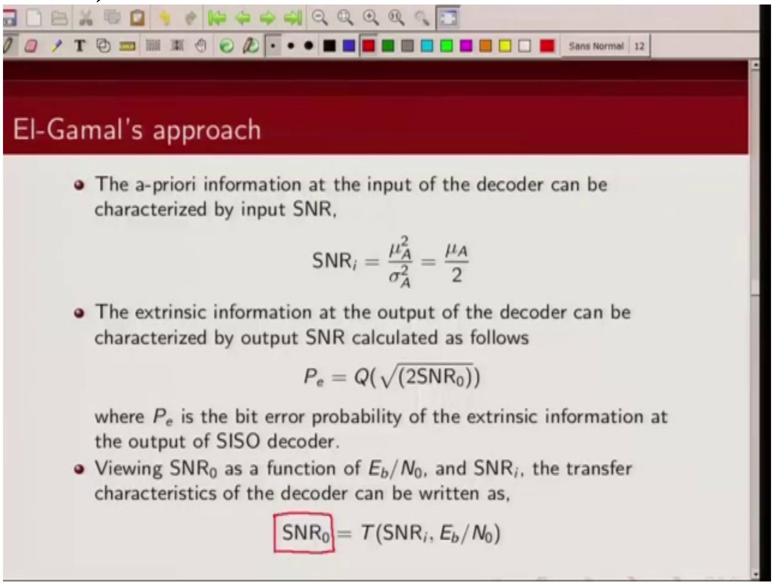

in terms of input S N R and our operating signal to noise ratio. So what we can do is we can view the output S N R of the extrinsic

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information as a function of input S N R of a priori information as well as the channel operating signal to noise ratio. So

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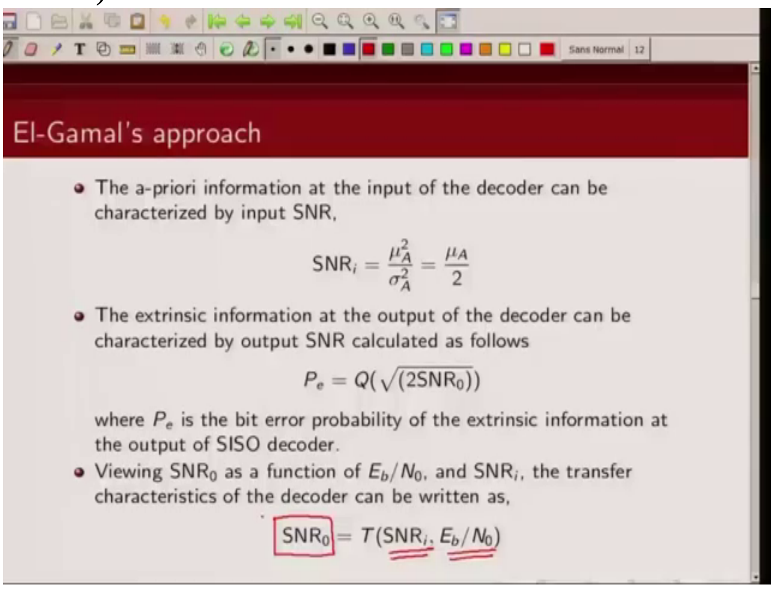

this is crucial, so this is basically what I call the transfer characteristics of the decoder. Because my decoder is a function of

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a priori inputs as well as channel received values. Now channel received value is the function of channel operating S N R and what I get, a priori information is the function of a priori input S N R. So I can view S N R of the extrinsic information, I can

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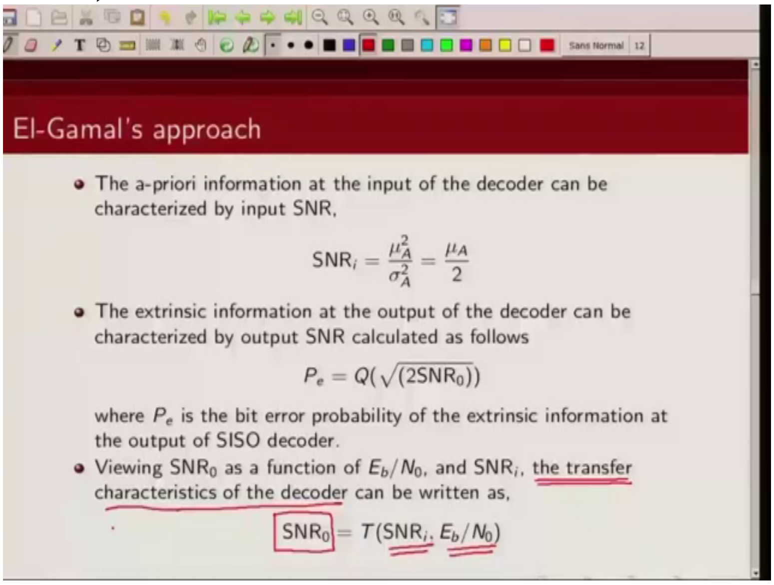

view it as a function of input S N R of a priori values as well as channel, operating channel signal to noise ratio. So this relation characterizes how my decoder will behave. Because remember with iteration your extrinsic information is changing as a function of

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a priori value and what is your operating channel S N R. So this transfer function will give

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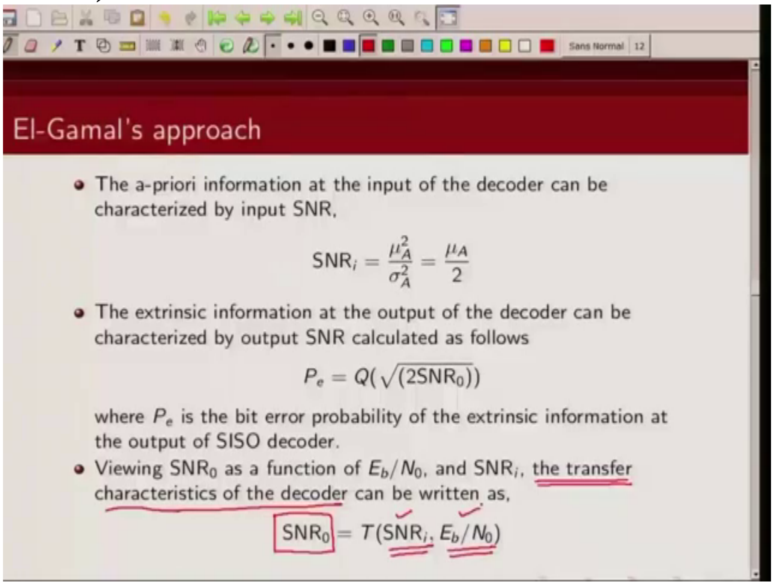

me how my decoder, this soft input soft output

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decoder, how it will perform as a function of a priori value and the channel operating S N R.

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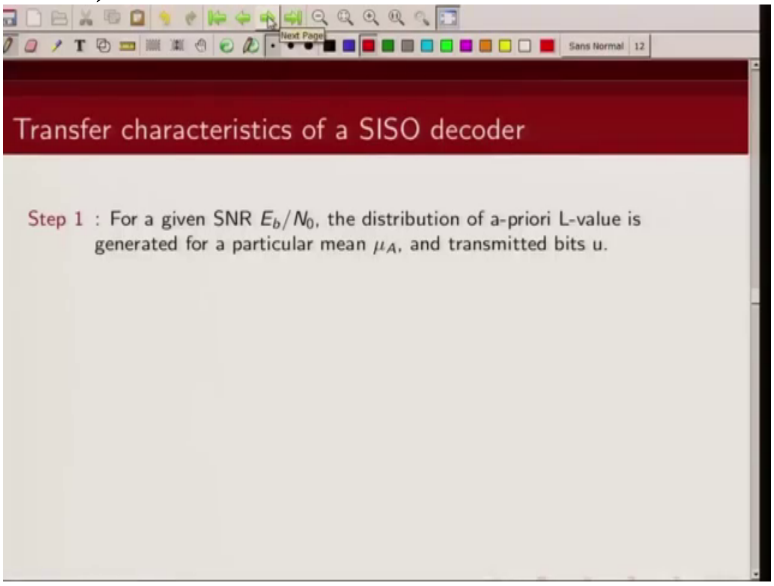

So then how do we draw the transfer characteristics? For a given signal to noise ratio, the distribution of a priori L values is generated for a particular mean mu a and transmitted bit u. How?

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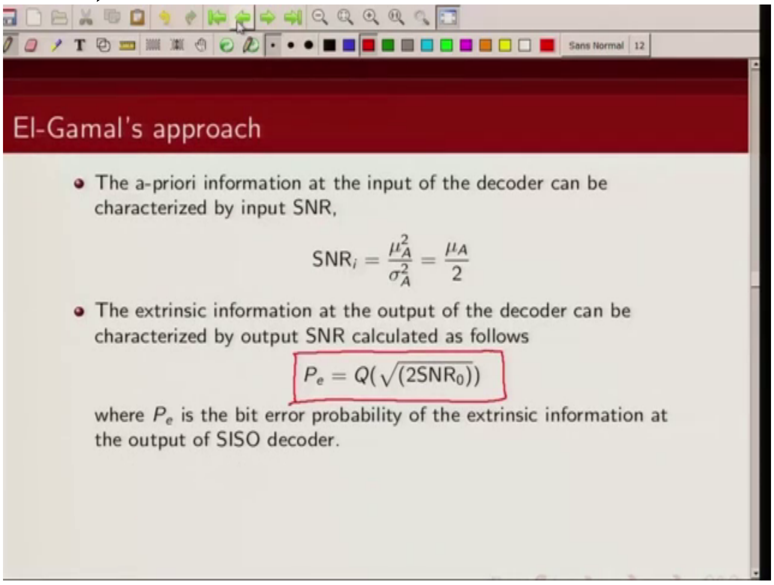

We know that we are modeling

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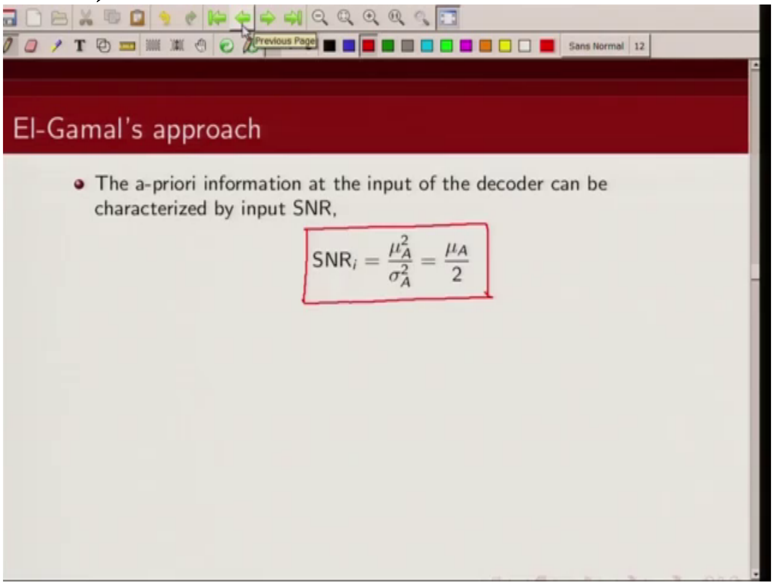

our a priori

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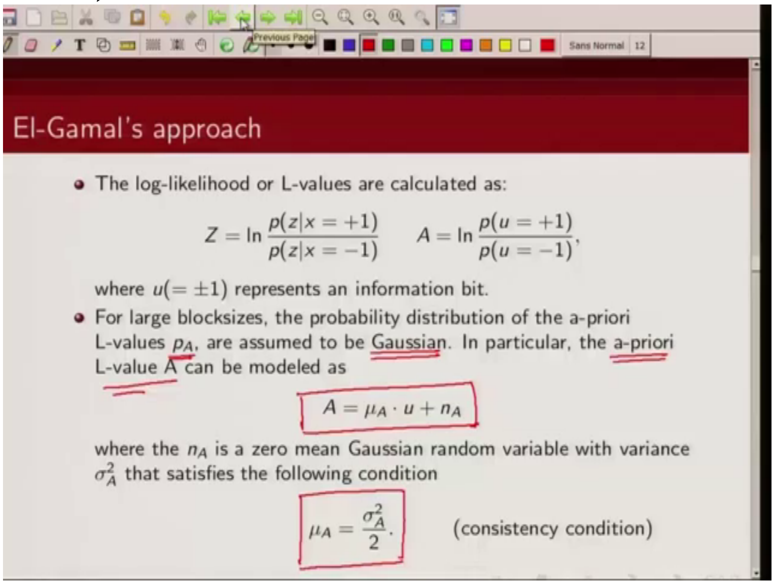

information like this. And of course we are assuming consistency condition so the mean and

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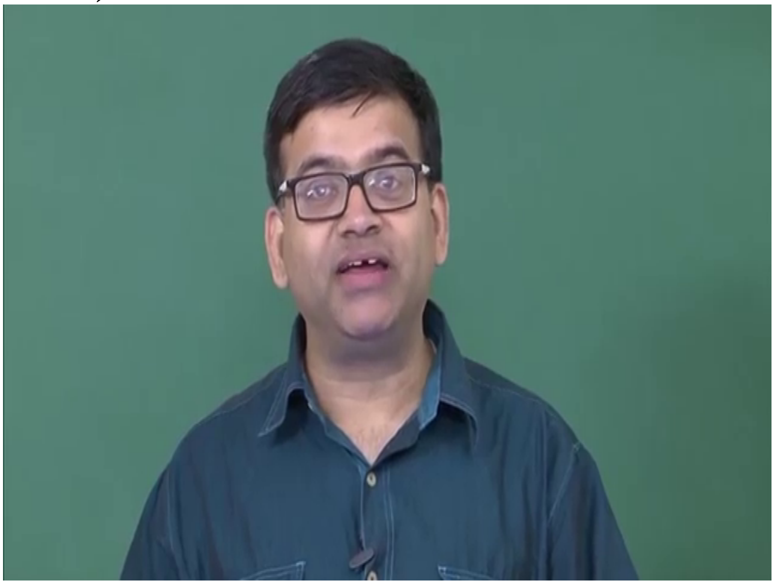

variance of the mutual, the a priori information is related like this.

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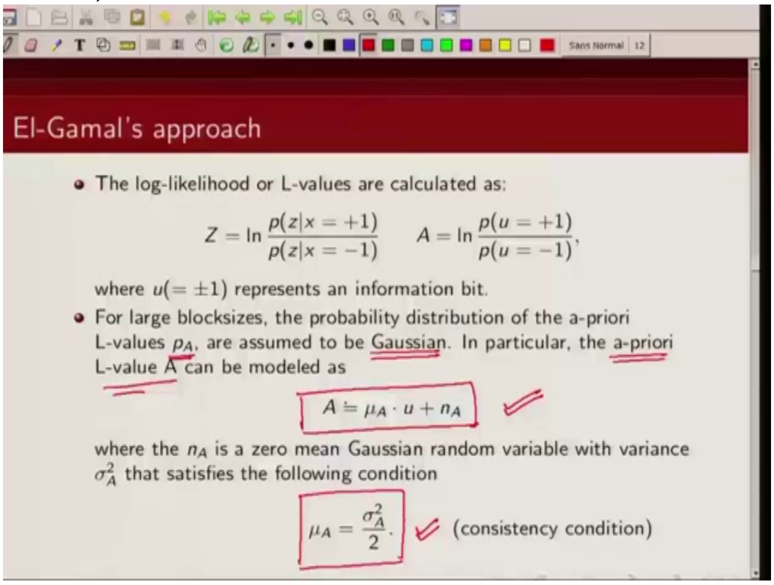

So next

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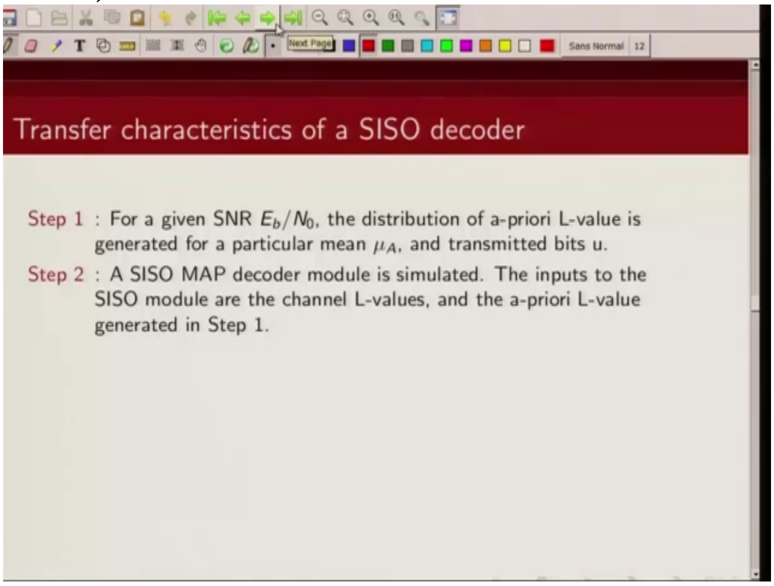

step is we simulate a soft input soft output decoder. So we feed in these two input. One is this channel received

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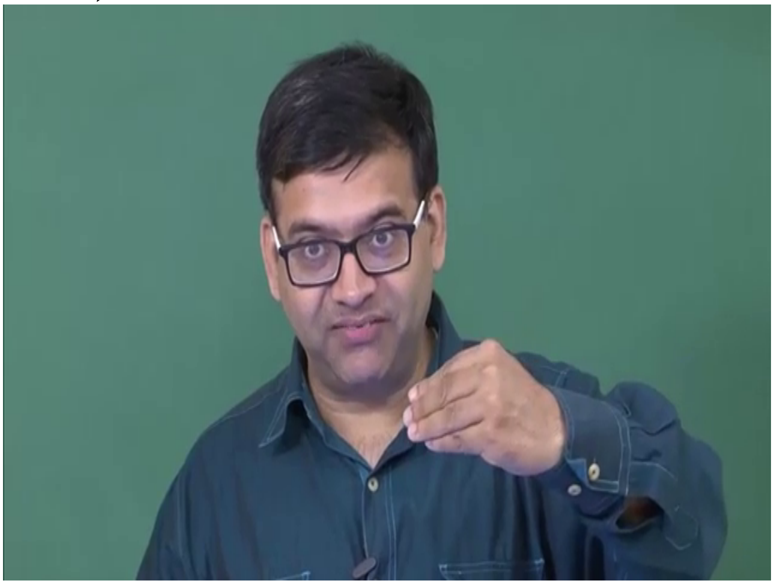

S N R and other is this a priori information which

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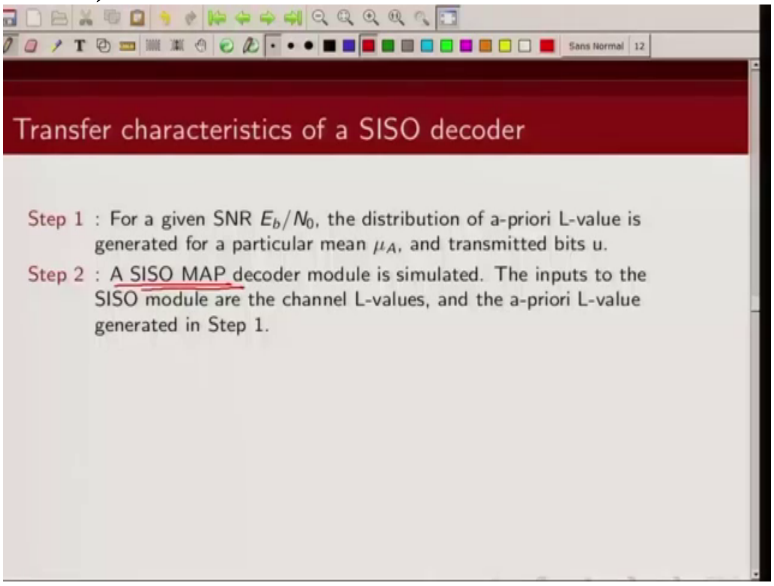

we modeled as Gaussian. We feed these two inputs to the decoder and what comes out as output

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are these extrinsic values.

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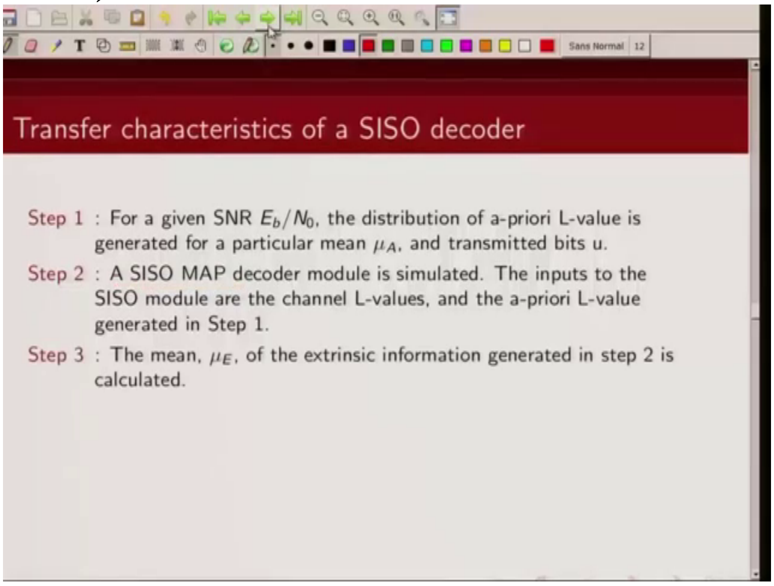

And we compute the mean of the extrinsic values.

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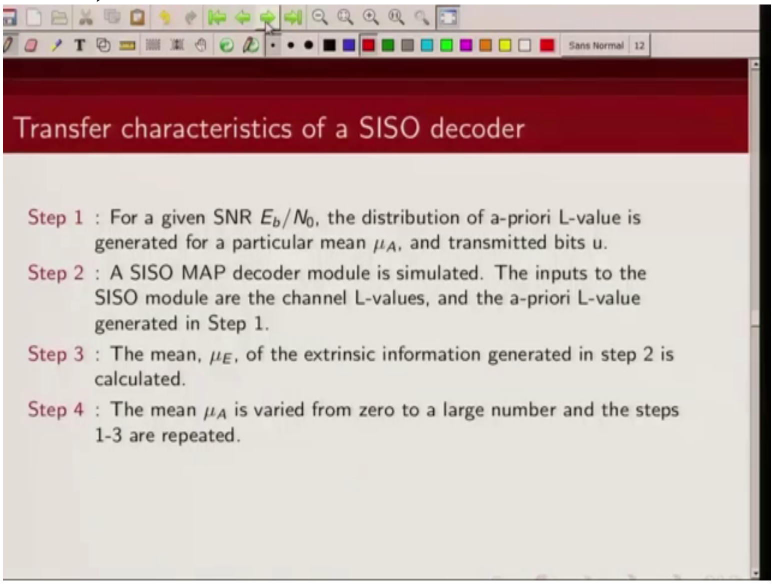

Now we know that our signal to noise ratio, because we are making

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Gaussian assumption, our signal to noise ratio is related to the mean. Now as I said with iteration, my a priori information is changing. So now we are going to

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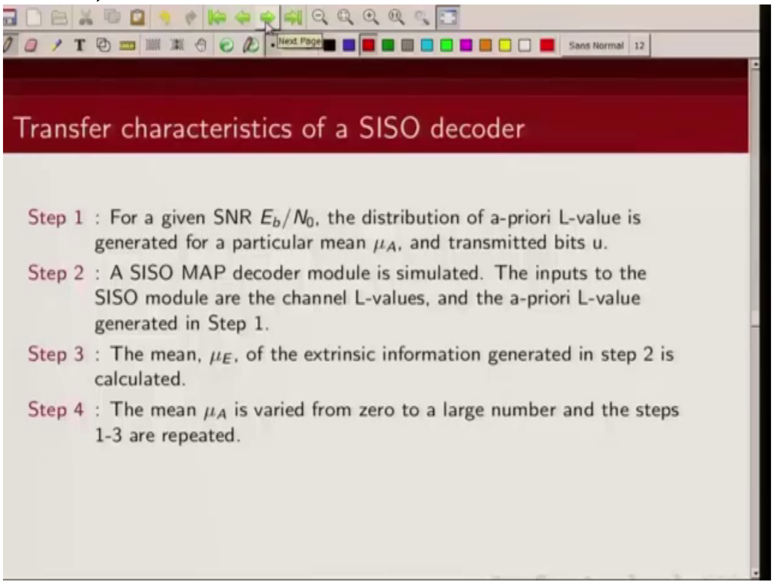

change the mean of the a priori information. And then we will again simulate

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this soft input soft output decoder and we will try to see what happens to the extrinsic information mean. How much it is growing with change in input a priori information mean?

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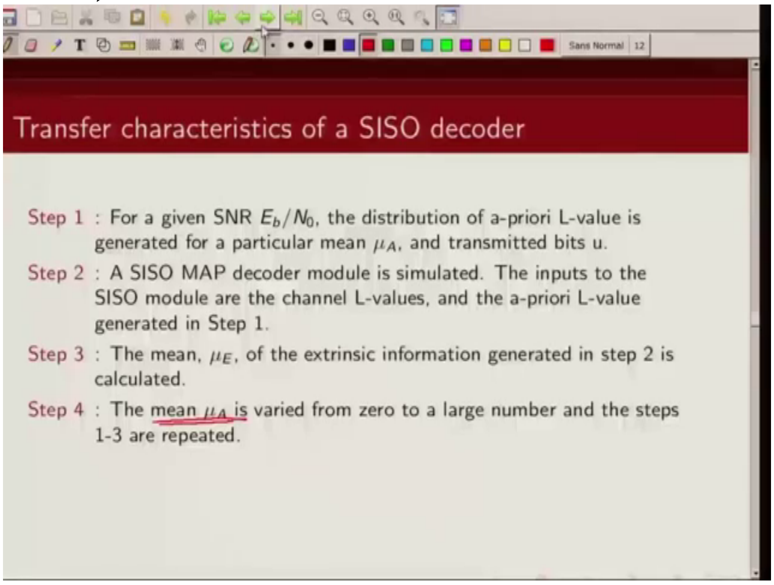

So this process is done. So we repeat this by varying our a priori information mean.

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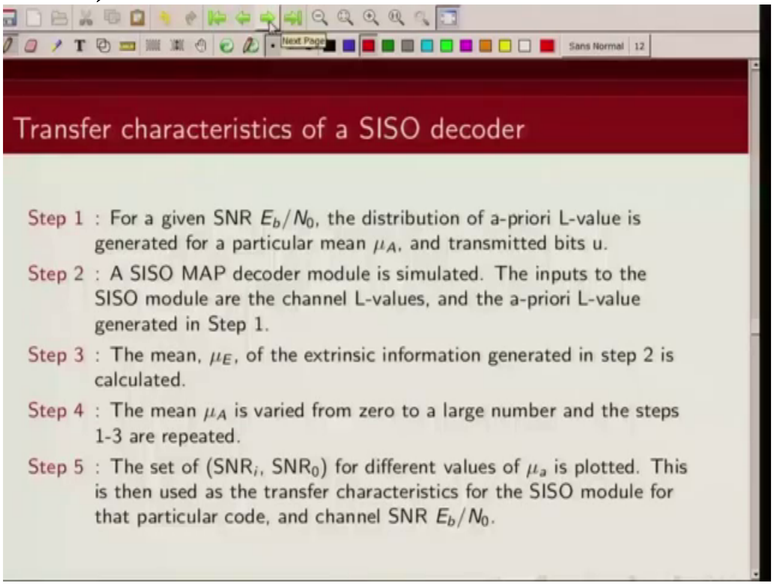

And finally what we do, we plot this input output relation for a particular channel S N R. So this is my input a priori S N R, this is the extrinsic information S N R. We plot it for a particular value of signal to noise ratio and this is my transfer characteristic for that particular decoder

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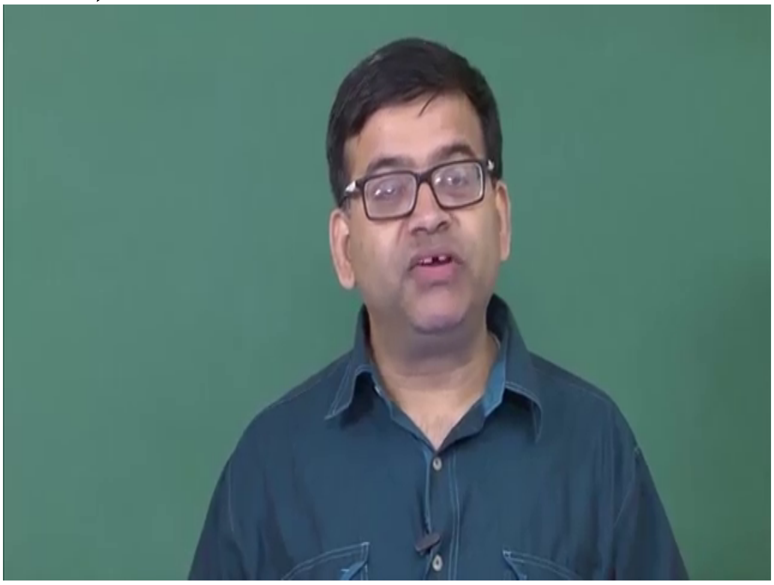

which is a function of channel operating S N R and of course it is the function of the constituent encoders that I have used.

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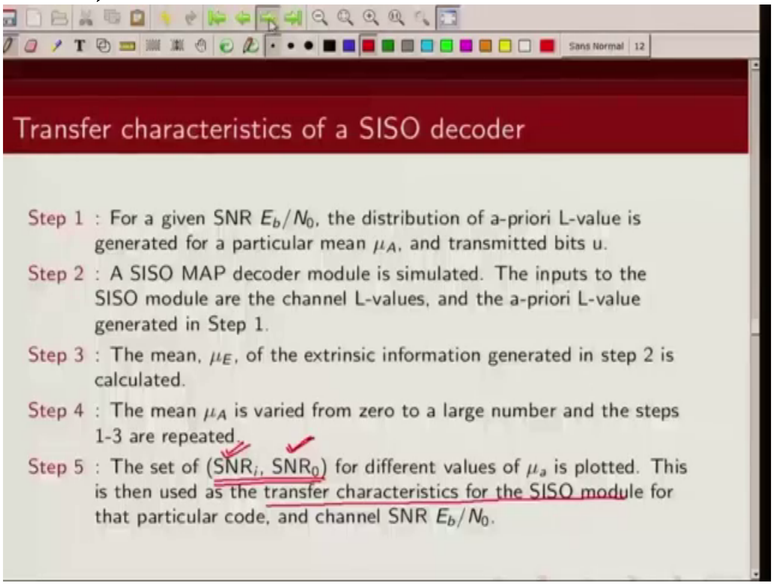

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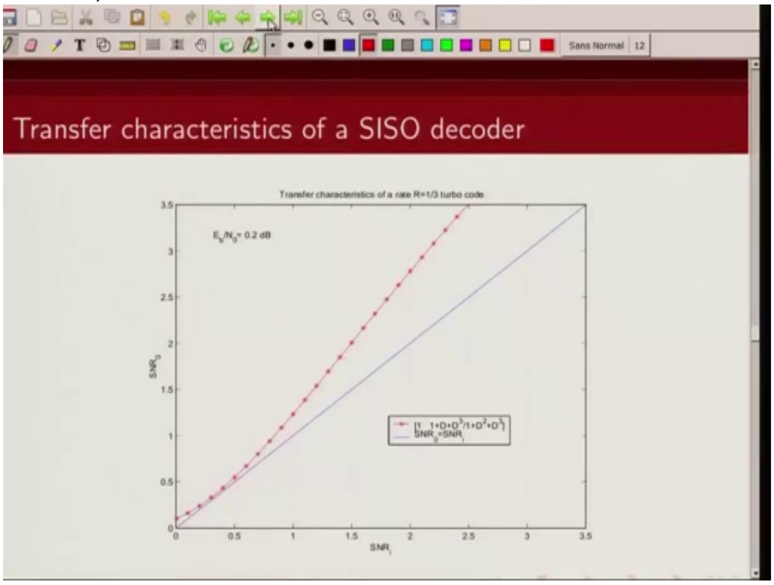

So here basically I have plotted, with red curve I have plotted transfer characteristics of one such code. It is a 8 state code. What I have here at the input side is

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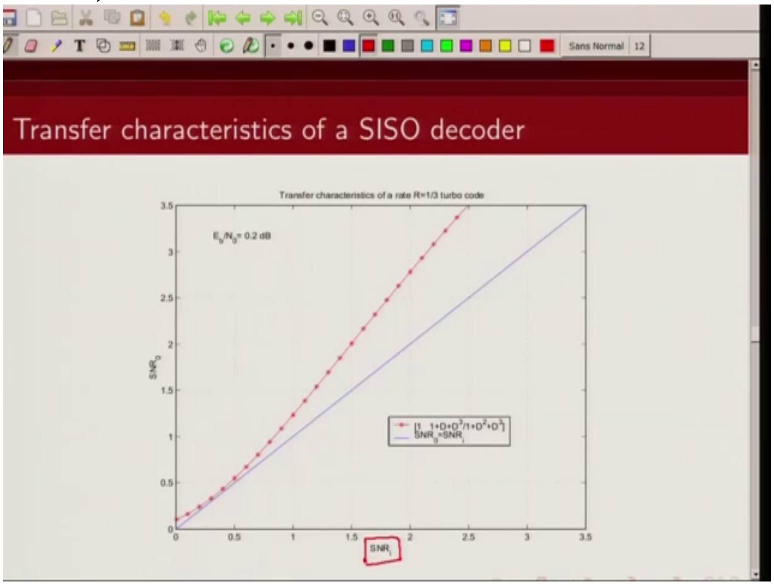

S N R of a priori information and what I have here on the output side is

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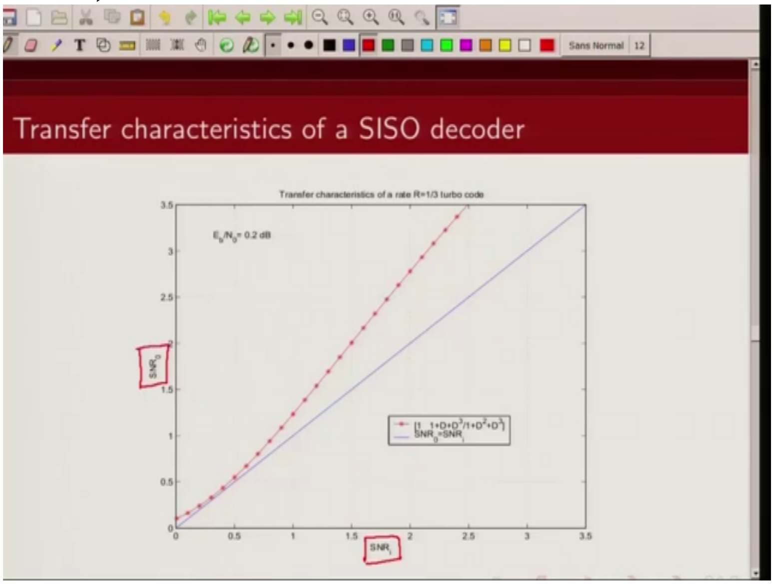

S N R of the extrinsic information. And this is how my; so initially

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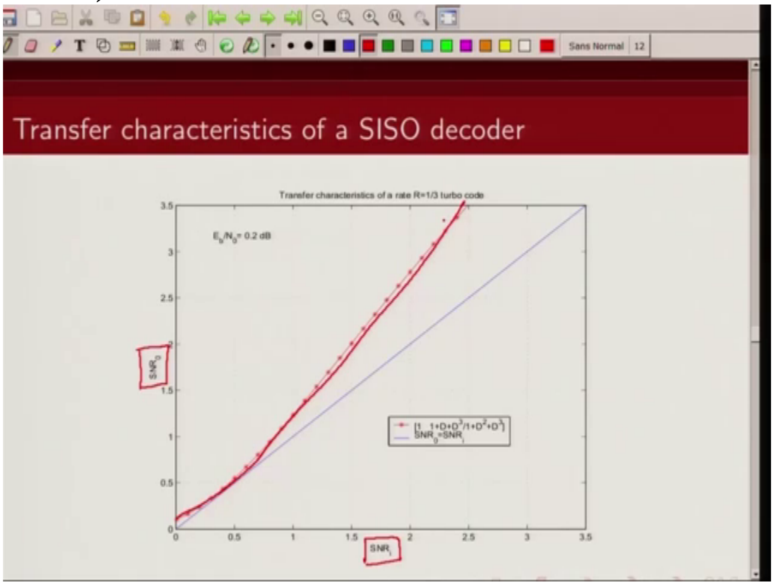

I don't have any a priori knowledge, the extrinsic information will, this is the amount of extrinsic information which is generated. So this transfer characteristics will tell me, if I have a particular input a priori information then what is the corresponding

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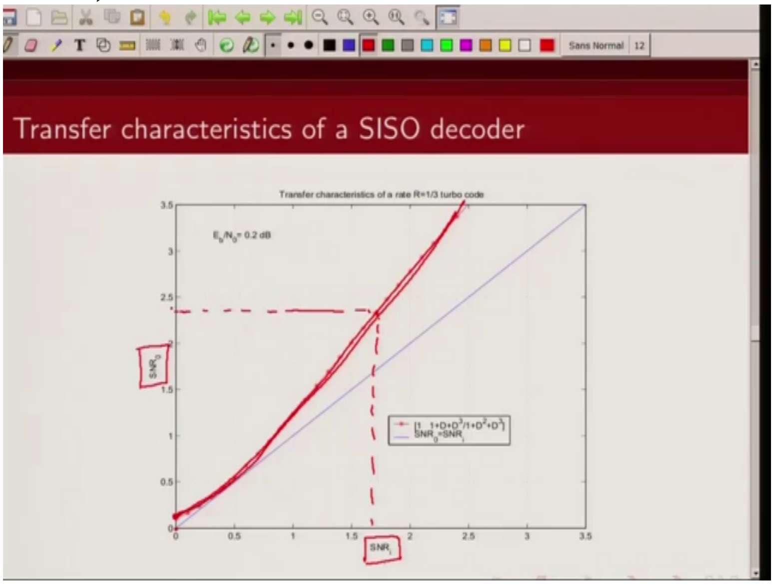

extrinsic information S N R. And for comparison sake I have drawn this line which is the S N R in

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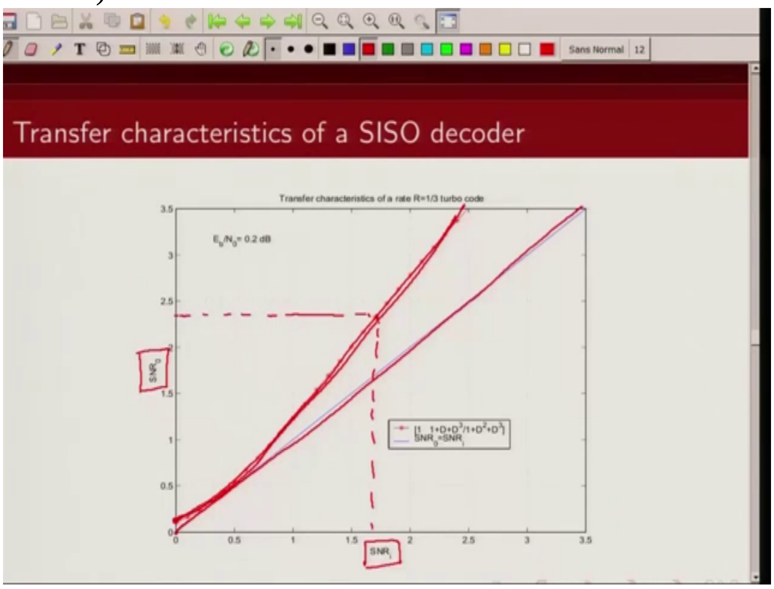

equal to S N R out. Now if you have a symmetric turbo code, you obviously would like your transfer characteristics to be above this line.

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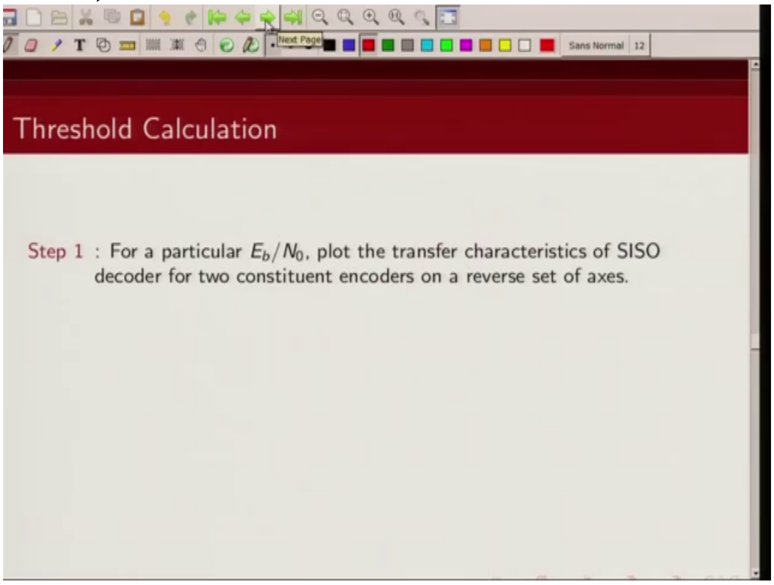

Now how do we compute, how do we use these

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transfer characteristics to compute the decoding threshold? So how do we find out the S N R, minimum S N R under which our iterative algorithm will converge? For that we need to do this threshold computation. So how do we do this threshold computation? So for a

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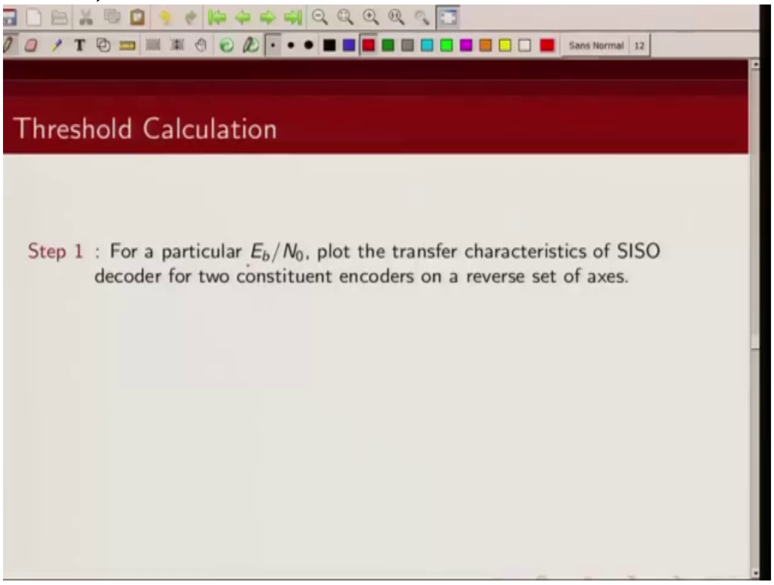

particular signal to noise ratio, we plot the transfer characteristics of this soft input soft output decoder. We plot them on reverse set of axes. Now what do I mean by reverse set of axes? So for the first, my S N R in is on x axis, and

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S N R out is on the y axis. Now for the second decoder, my S N R in is on the y axis and S N R out is on the x axis. Now why do I do this? Because the extrinsic information of first decoder is input to the second decoder. So S N R out of the first decoder becomes S N R in of the second decoder. And that's why I put the S N R in of the second decoder as y axis and the S N R out of the second decoder is S N R in for the first decoder because the extrinsic information from the second decoder is coming as input to the, as a priori input to the first decoder. And that is the reason I plot these transfer characteristics on reverse axes.

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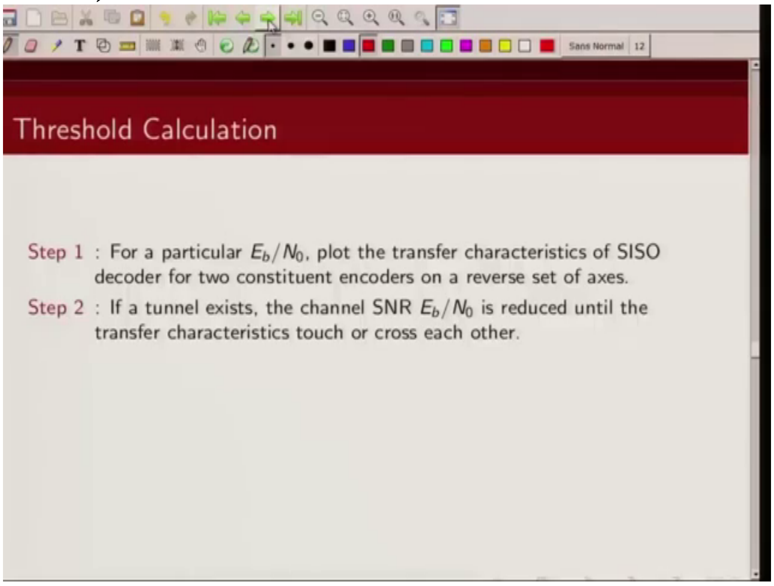

Now if these transfer characteristics do not cross, there is a tunnel in the sense they do not touch each other, then what we do is the channel, operating channel S N R is reduced until these transfer characteristics just about touch.

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So what is the effect of channel S N R? So as you reduce the channel S N Rs these transfer characteristics which have been plot on reverse axes, they come closer when you reduce the channel S N R. So the smallest S N R for which there is still a tunnel, that's your decoding threshold for that particular

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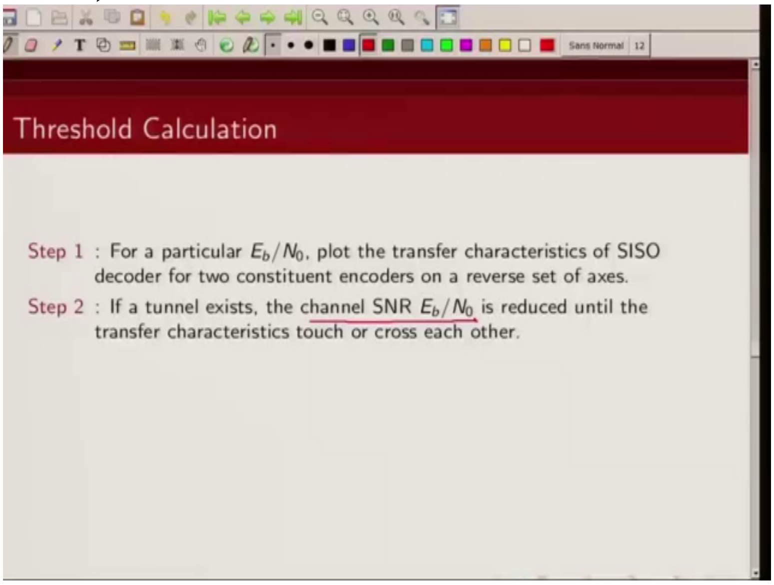

code.

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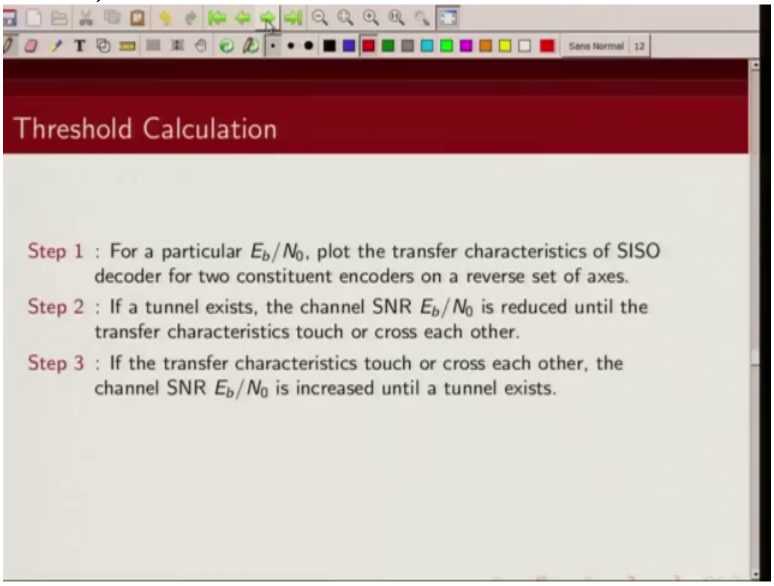

So if the transfer characteristics touch or cross each other, what we need to do is we need to increase the S N R until there is a tunnel, still a tunnel.

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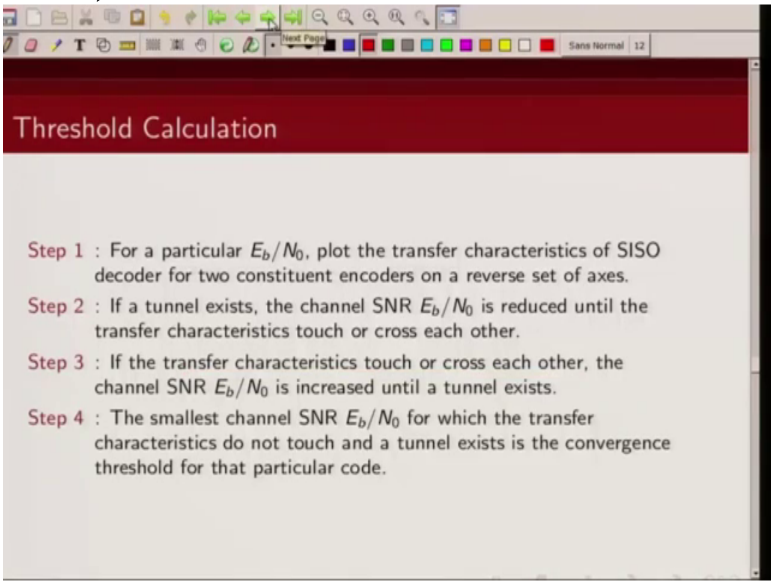

So the smallest channel S N R for which these two transfer characteristics which have been plotted on reverse axes, they do not touch and a tunnel exist is basically the convergence threshold for that particular code. So that would give the S N R, minimum S N R under which that particular code will converge and it will have a

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waterfall kind of behavior if you take large enough block size.

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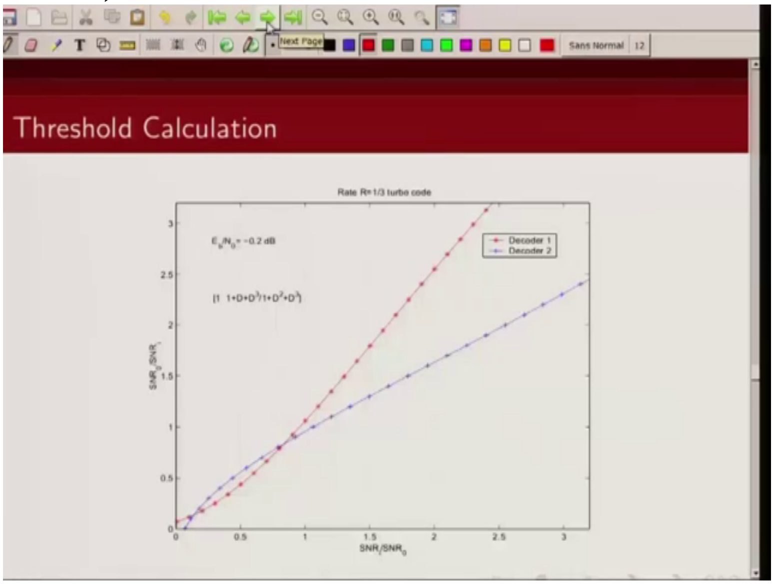

This is one example. Now note here, this is plotted for channel operating

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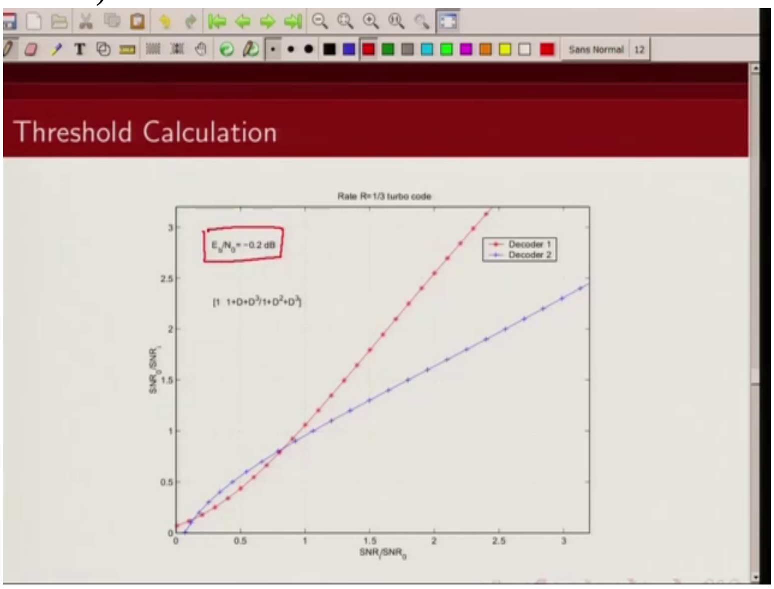

S N R of minus point 2 d B so this is, in red curve is my decoder 1 and

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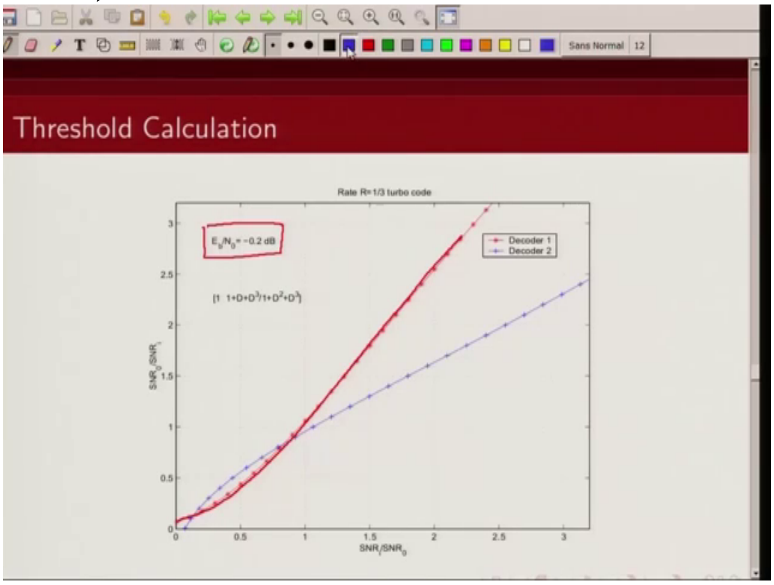

in blue curve I have decoder 2. Note that these 2 are crossing each other so there is no tunnel.

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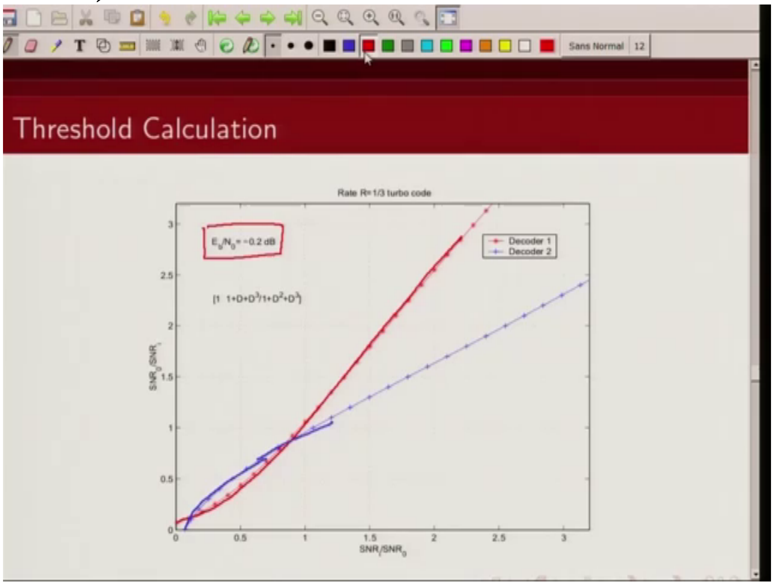

Now

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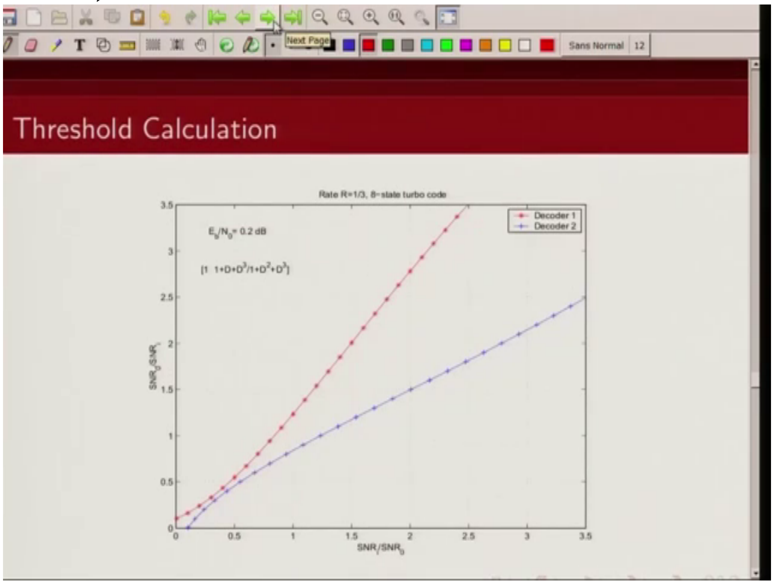

same code, now I increase my S N R and I have made it point 2 d B. Now you can see there is a tunnel between them. There is a tunnel, Ok.

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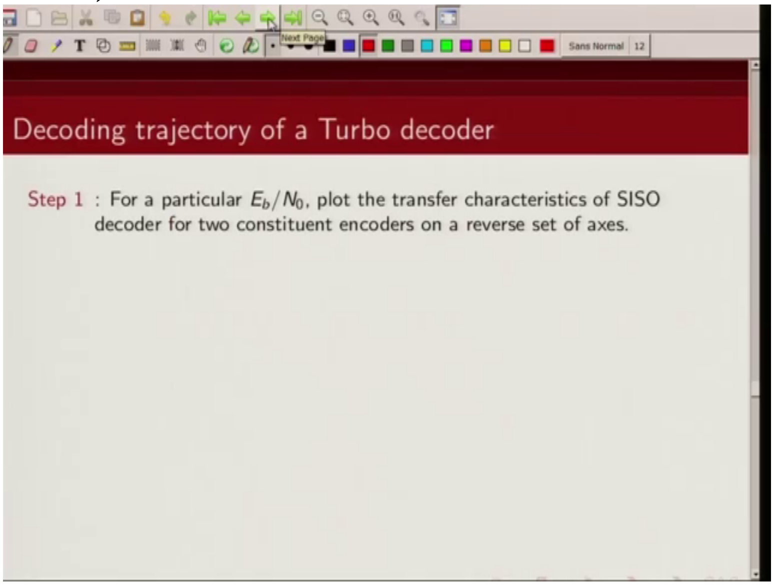

Now let us see how

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we can draw a decoding trajectory of a turbo decoder with the help of these transfer characteristics. So

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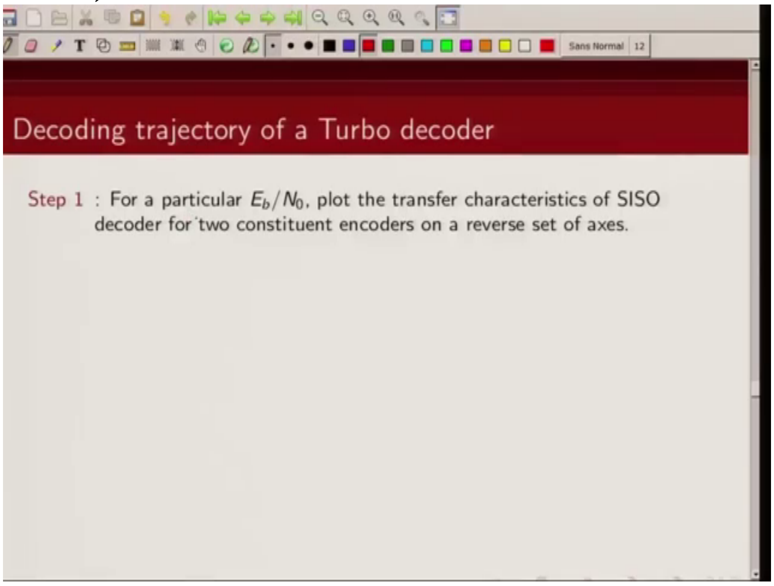

what we do is for a particular signal to noise ratio as I said, we plot these transfer characteristics of two constituent encoders on reverse set of axes. So for decoder 1, S N R in will be on

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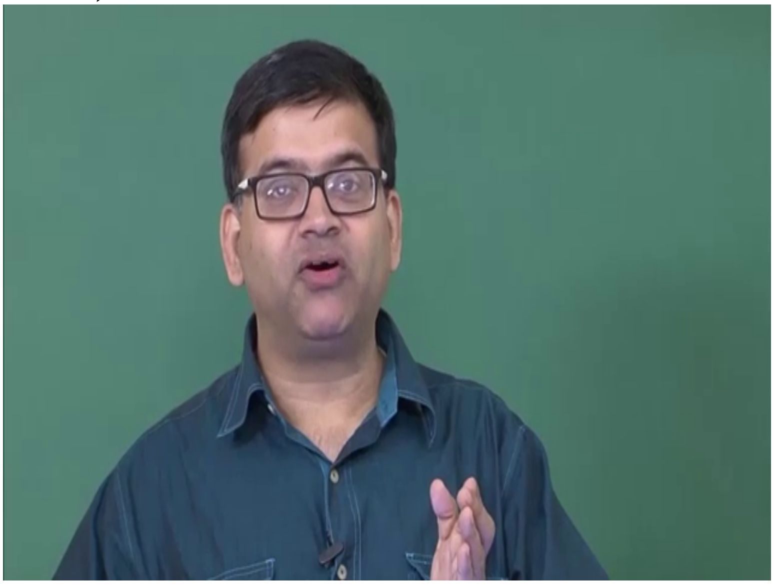

x axis, S N R out will be on y axis, where as for decoder 2, S N R in will be on y axis and S N R out will be on x axis.

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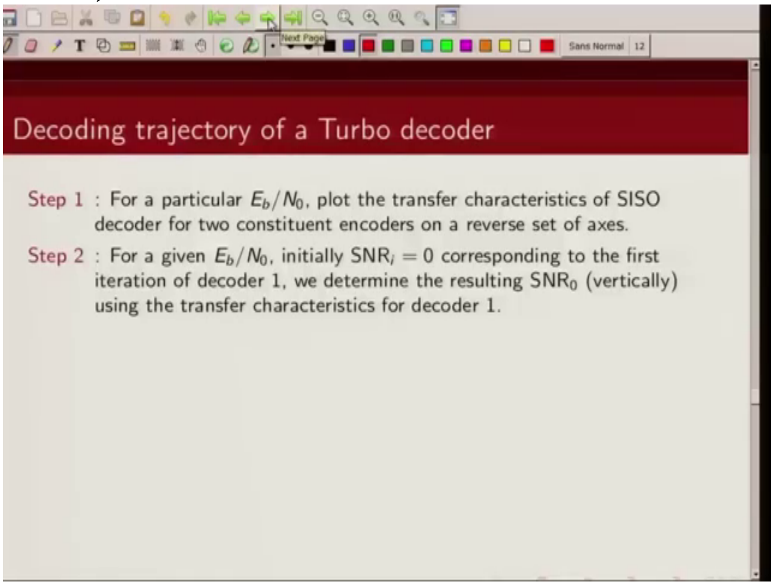

So initially, because you don't have any a priori knowledge about the information bits, so initially the

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a priori S N R is zero. And this

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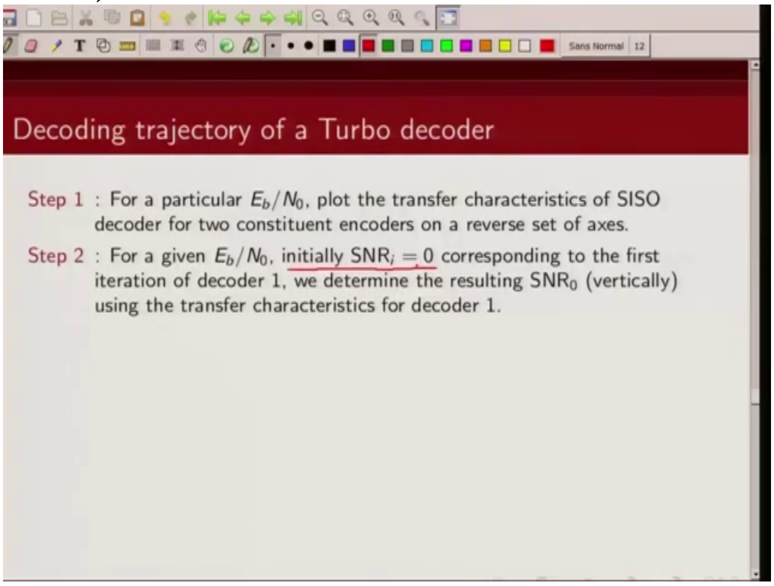

corresponds to, and so we are first going to look at the transfer characteristics of the first decoder. So input we will get zero, so we will try to see what is the

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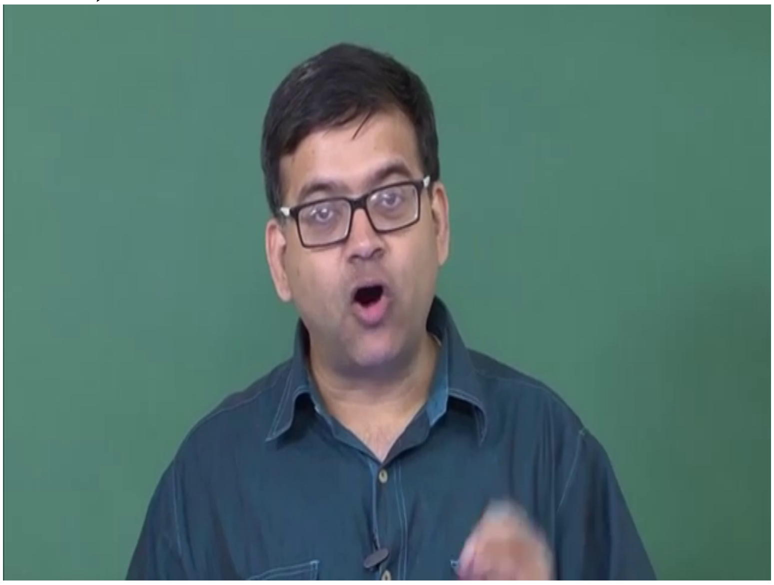

output S N R corresponding to this decoder 1. So we determine

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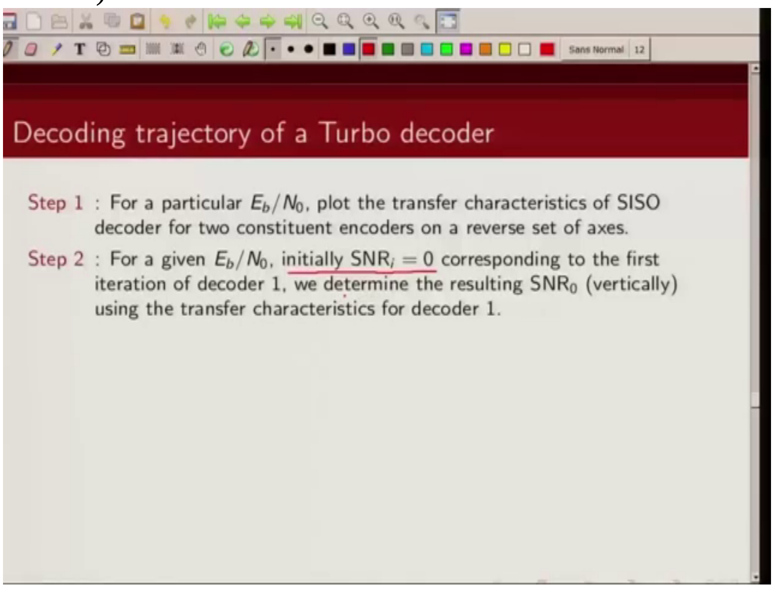

the resulting output S N R which we look vertically for using the transfer characteristics for decoder 1.

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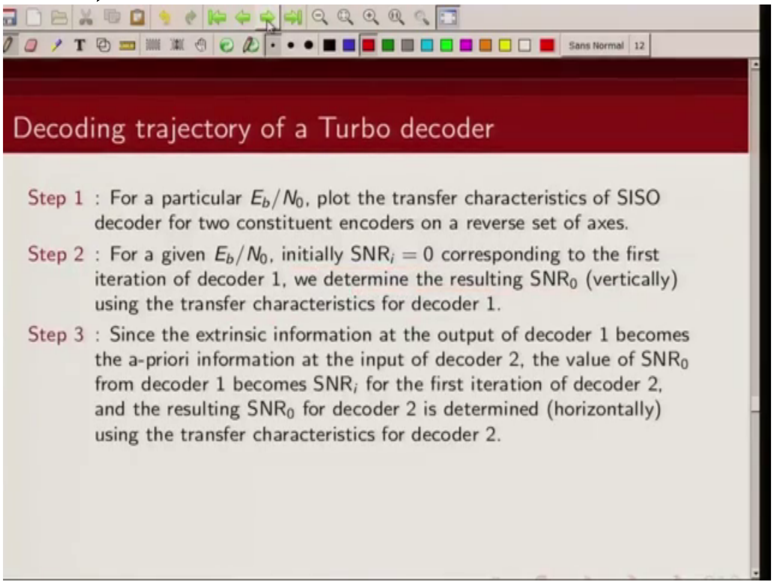

Now as I said, since the extrinsic information from the first decoder is actually a priori value for the second decoder, so what we are going to do is that particular extrinsic information will now become S N R in for the decoder 2. So the

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S N R out that we got from the transfer characteristics of decoder 1, that is our new a priori S N R in for decoder 2. Now we are going to look at the transfer characteristics of decoder 2 and we are going to go horizontal and find a point corresponding to that particular a priori S N R what is the output S N R.

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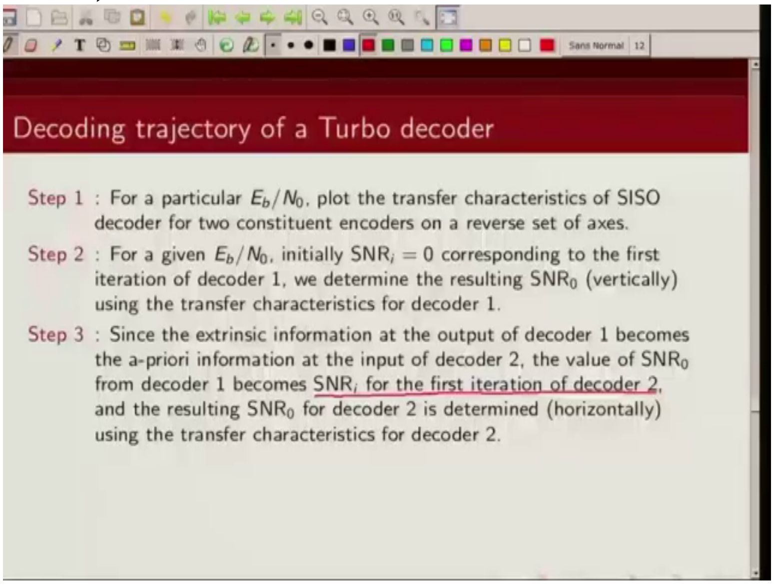

And this process we are going to repeat to draw the decoding trajectory of turbo decoder.

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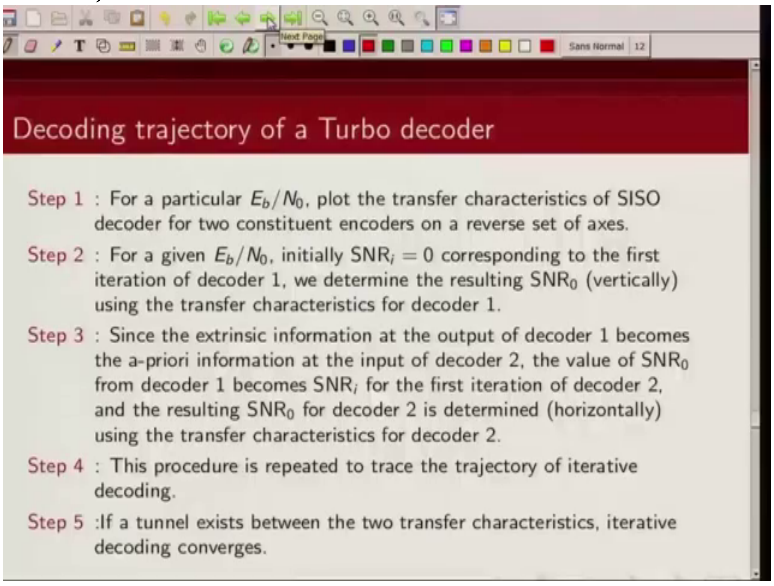

If while drawing this decoding trajectory, our

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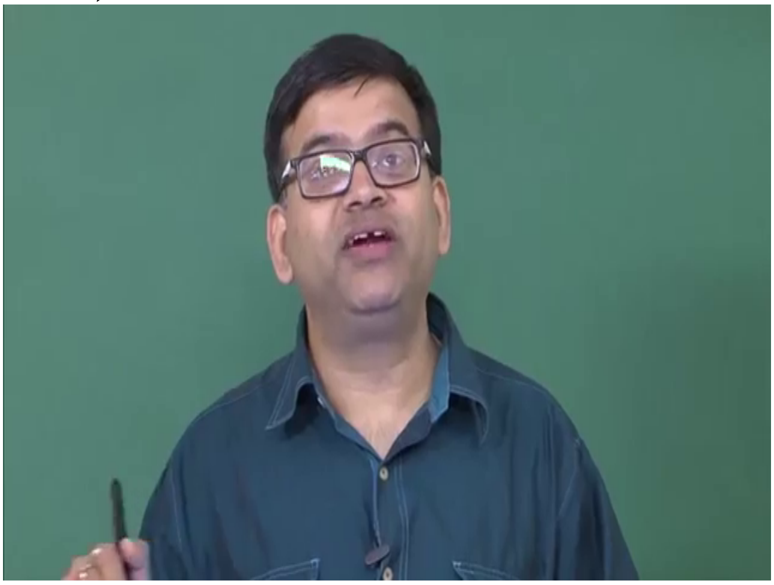

decoding trajectory does not get stuck, our decoding trajectory will not get stuck if there is a tunnel and if there is these transfer characteristics cross each other, then our decoding trajectory will get stuck.

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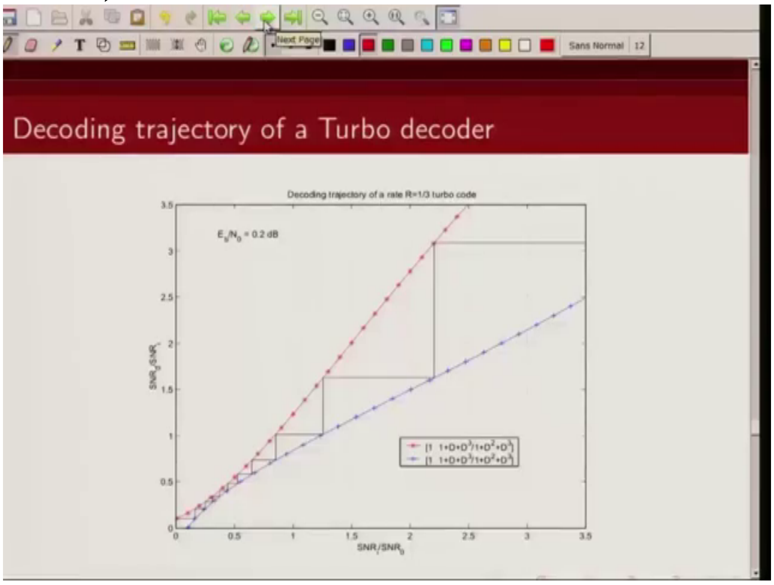

So this is an example. So I have this with red that you see, that is the transfer characteristics of the first decoder. This is decoder 1. This is transfer characteristics of decoder 1.

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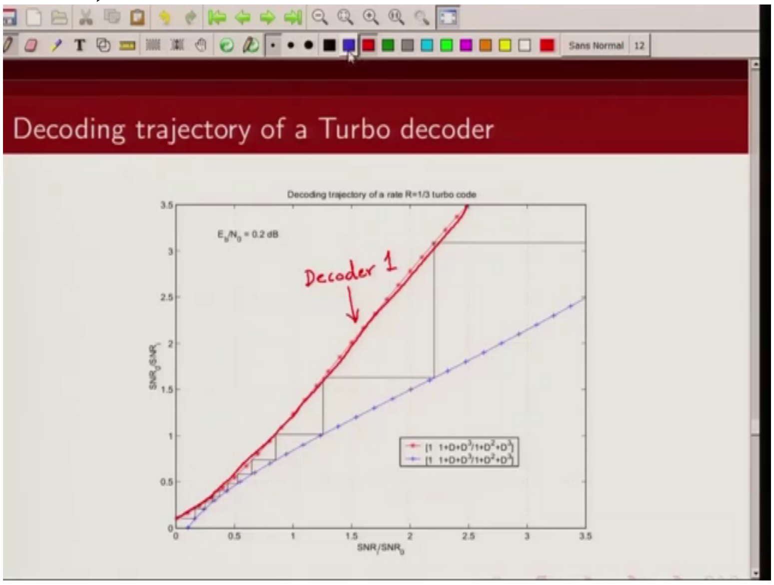

And what you see in blue is the transfer characteristics of decoder 2. They are the

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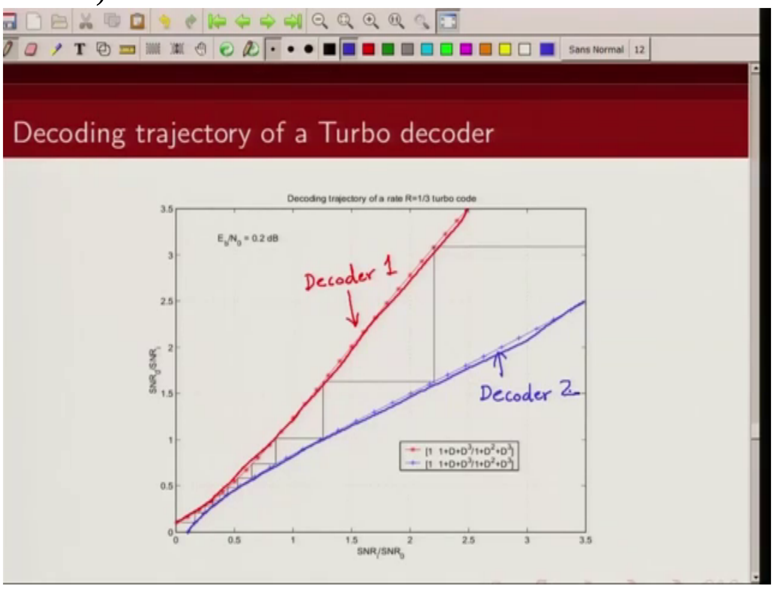

same encoder; this is the symmetric turbo code I am considering. So how do I start? So initially I will look at the transfer characteristics of the first decoder. This is where I will look. So initially I don't have any a priori knowledge. So I will start from this point and I am looking at this curve. So this is my extrinsic S N R corresponding to zero input. Now note that this extrinsic information that we are getting from decoder 1 is going to be the a priori information for decoder 2. So then what we will do? So we will now look at this curve which is transfer characteristics of decoder 2. For decoder 2, this side is input and this side is output, this is input and this is output. So we will look here and we will look horizontally. So this is the point. So this is the point corresponding to S N R out corresponding to decoder 2. Now note this extrinsic information is getting fed as a priori information to

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decoder 1. So we will look at

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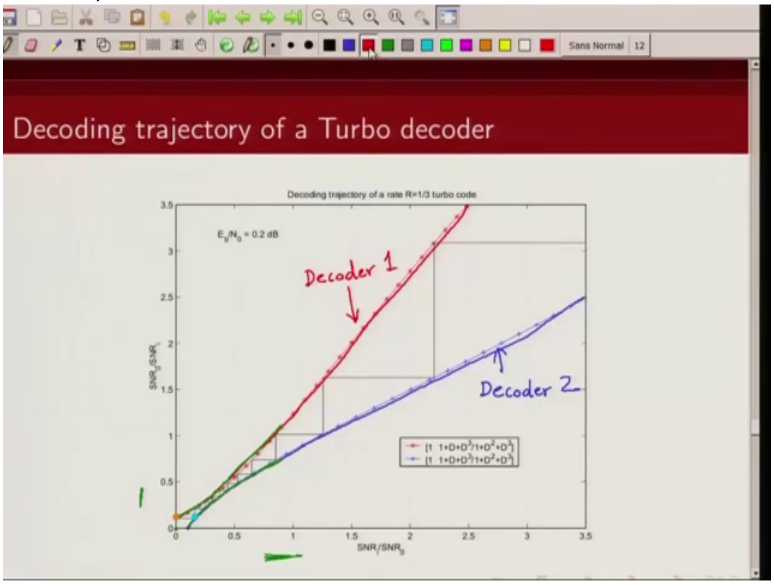

decoder 1 transfer characteristics and this is the point. So you can see I am going like this. You see

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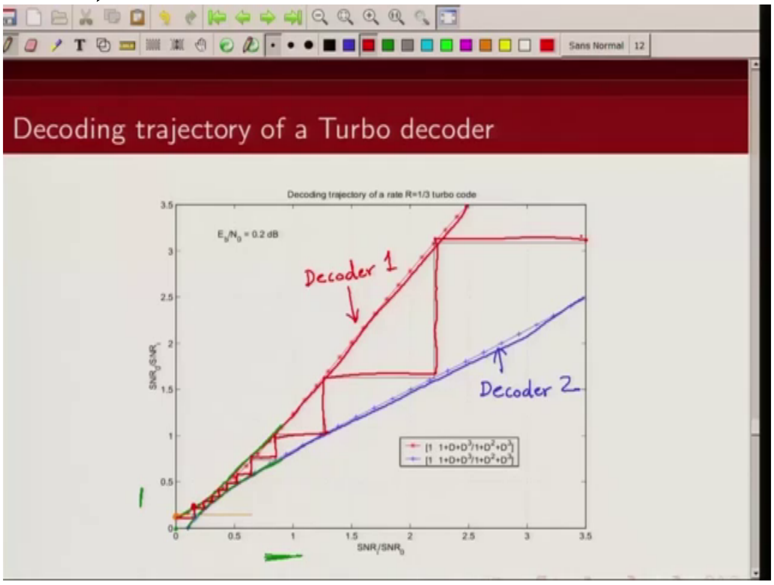

this is how basically my decoding trajectory of my turbo decoder is happening.

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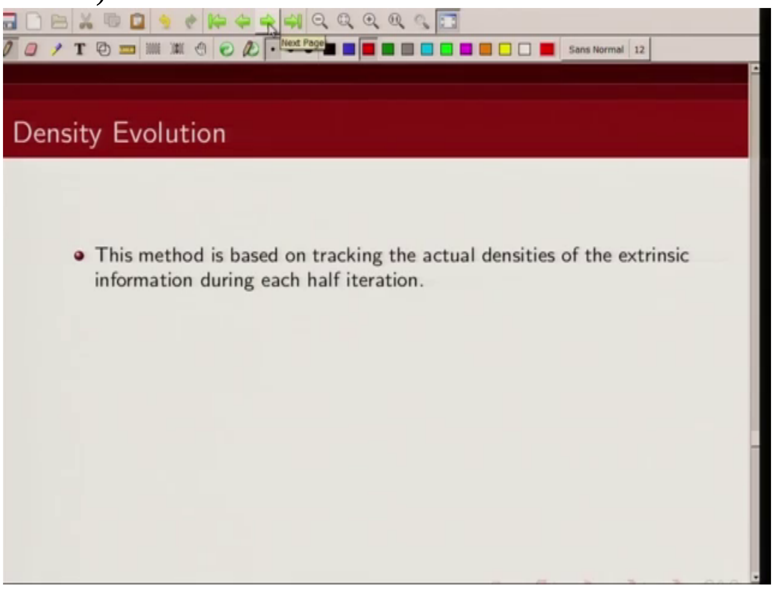

Now what would have happened

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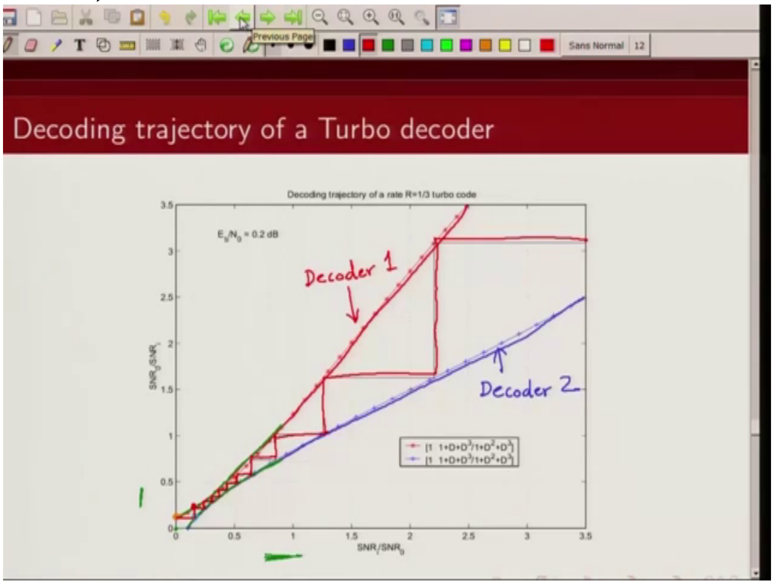

if these curves would have got crossed? So let's look at scenario. Let us say I had some curves which are like this. So let's say this is my decoder 1 and this is my decoder 2.

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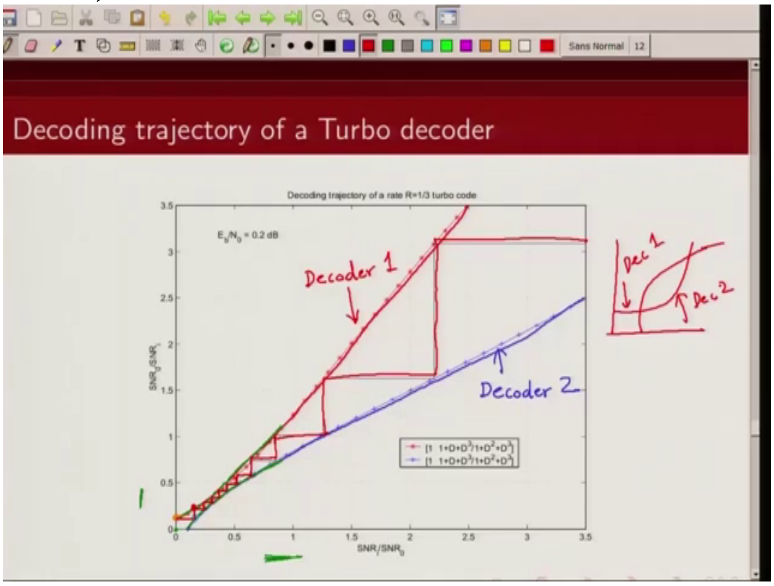

Then what would have happened is, so I would have initially started with zero, I have got this, then I got this. Let me draw slightly better transfer characteristics. So (()) second. So you draw it, basically you draw it like this, Ok. Now

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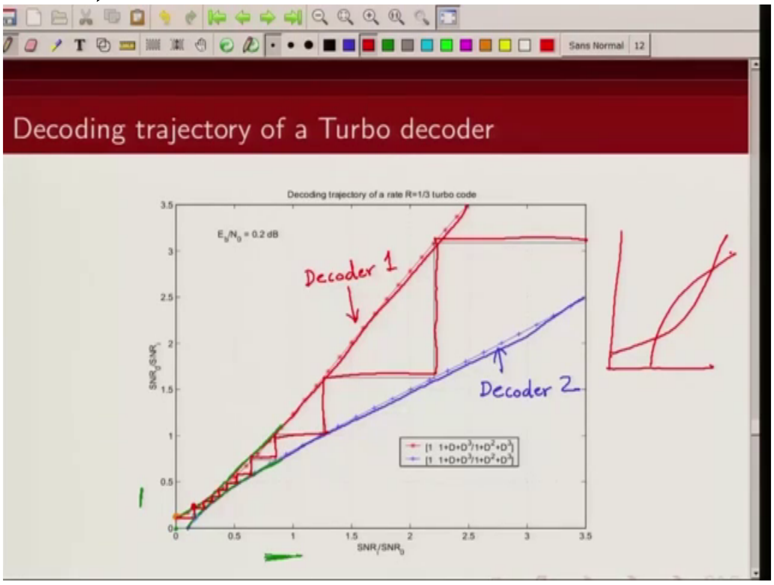

let's draw the decoding. So this is transfer characteristics of decoder 1

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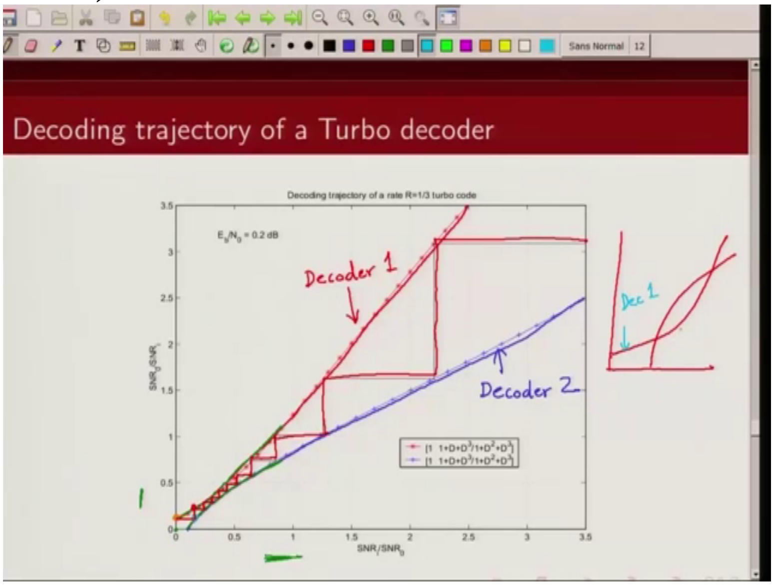

and this is transfer characteristics of decoder 2. So what

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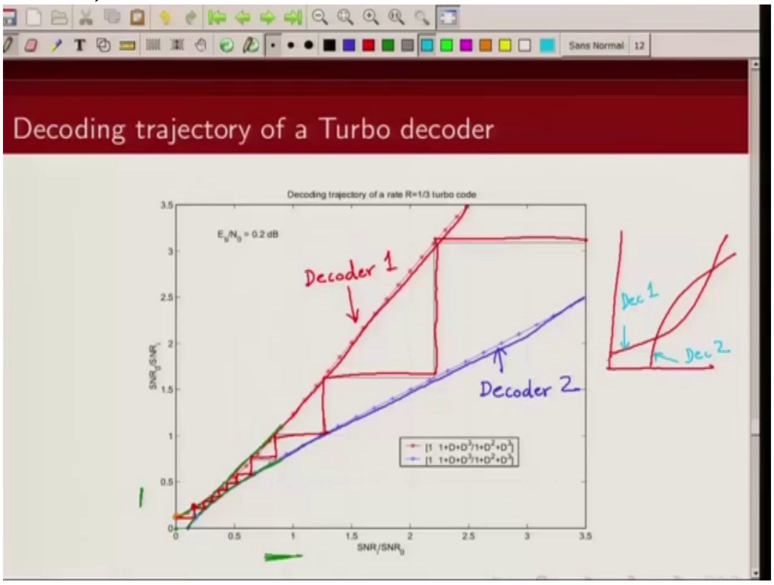

happens here? So you start off with S N R 0 point, you are getting this output S N R from the decoder 1. Now this is input to decoder 2. So you will get to this point. Then from here you will get to this point. Then you get to this point. And then here you are stuck because these 2 graphs cross each other. So what you will notice is if there is no tunnel then your decoding algorithm

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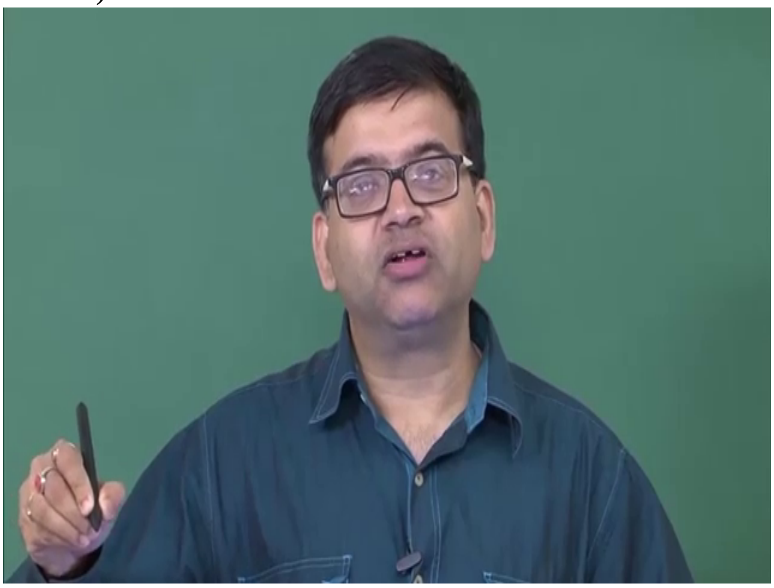

will get sruck and the extrinsic values will not improve whereas if there is a tunnel existing

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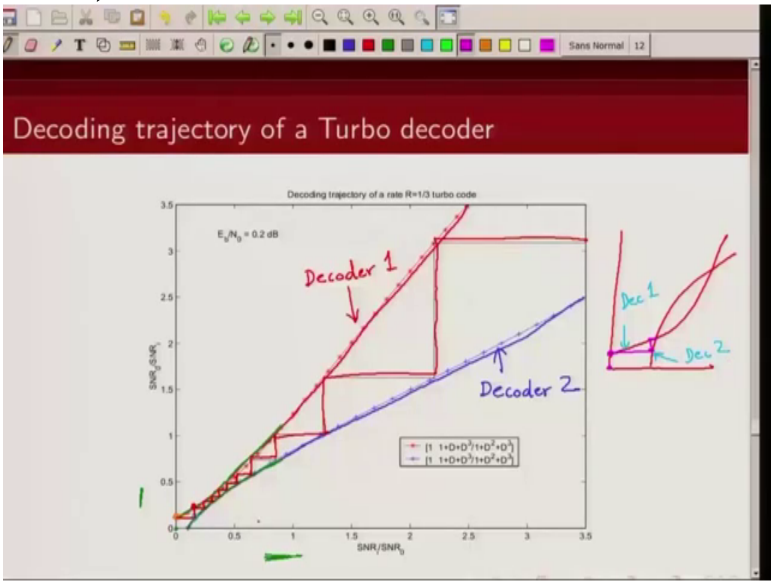

like in this particular case, you saw that, with iterations your extrinsic information is growing. And that's what we would like. So we would like to choose our encoders in such a way such that they match up in a way that there is a tunnel if we plot

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the decoding trajectories on reverse axes.

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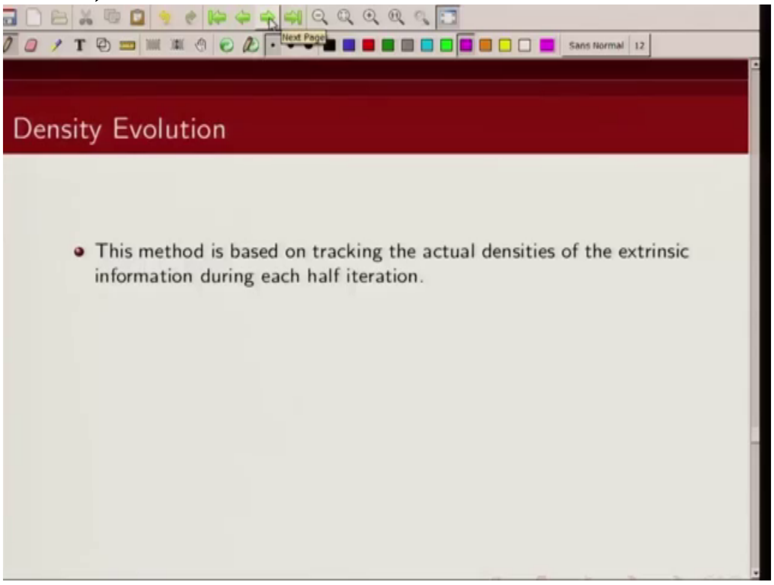

This was the method of El Gamal.

Now the method of Divsalar, they actually used the actual densities of the extrinsic information and they track it for finding

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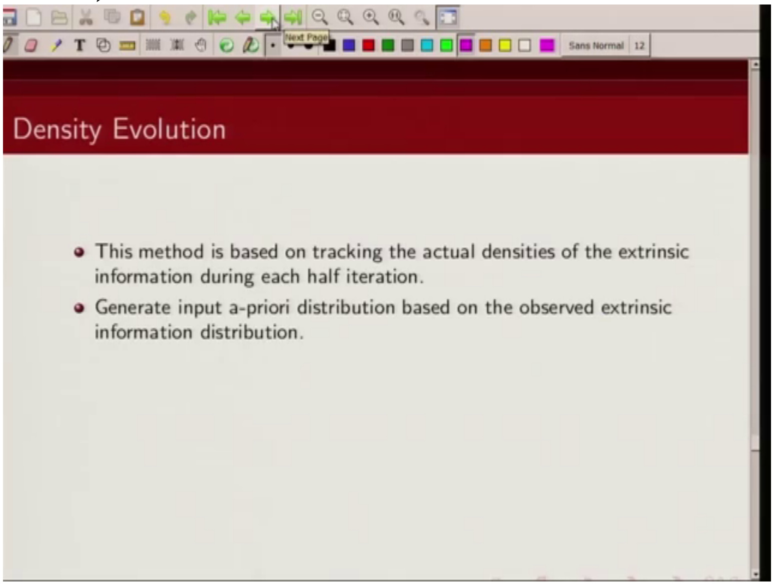

out how it is growing for iteration. So they generated some input a priori distribution based on observed extrinsic information and then they

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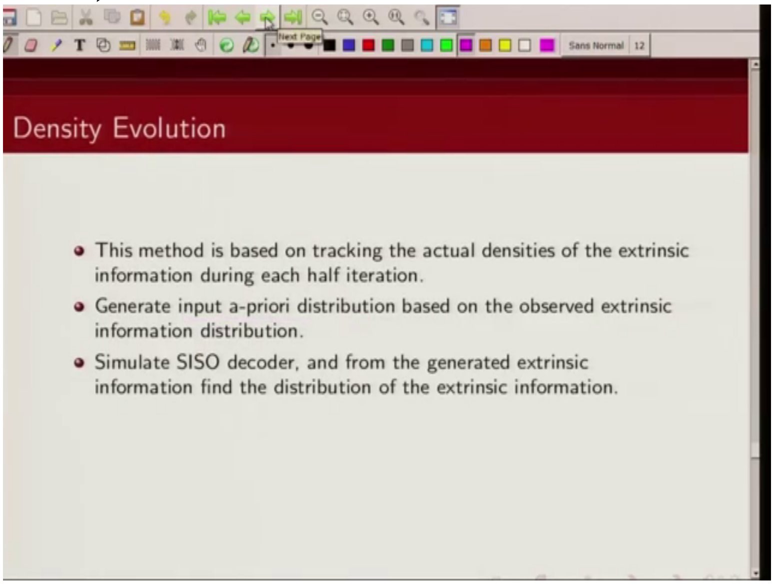

simulate this soft input soft output decoder using this generated distribution of a priori information and they find out the distribution of extrinsic

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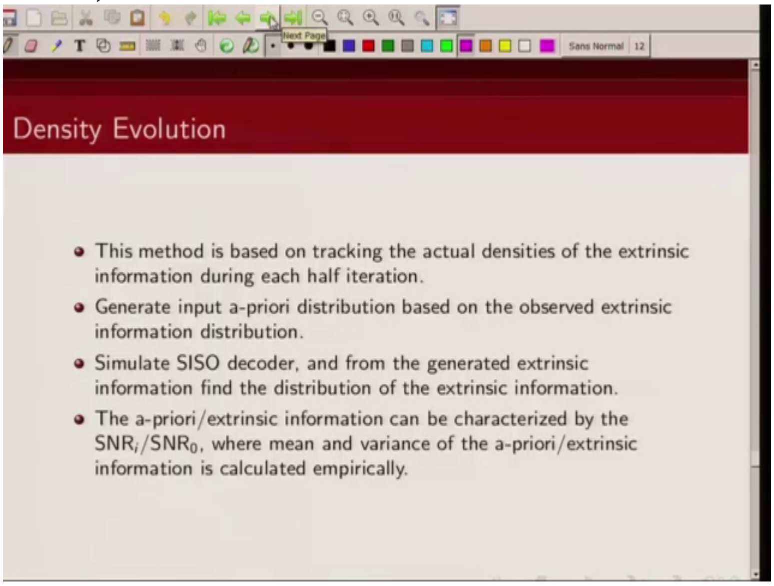

information. And similarly they characterized

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the S N R of the input distribution as well as the output distribution using mean and

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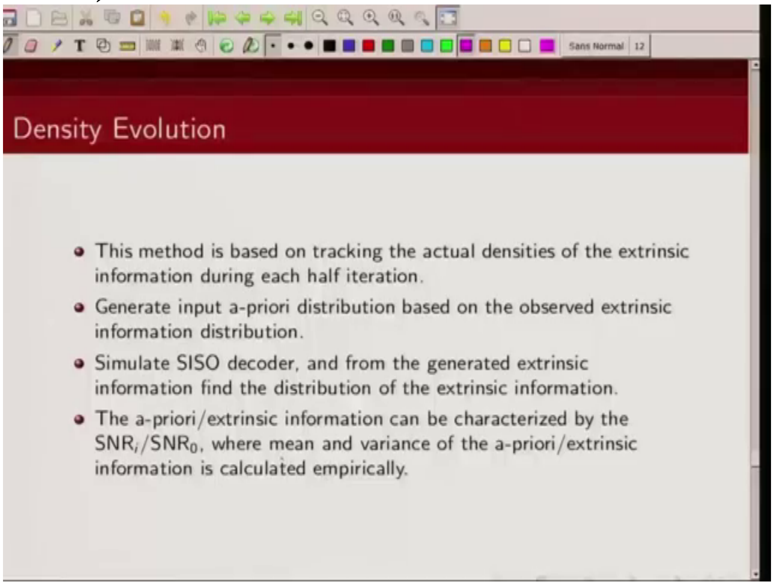

variance which was empirically computed. So they did not assume that consistency criteria which El Gamal and others did, they actually

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used the observed density. They generated a priori information based on the observed distribution of the extrinsic information.

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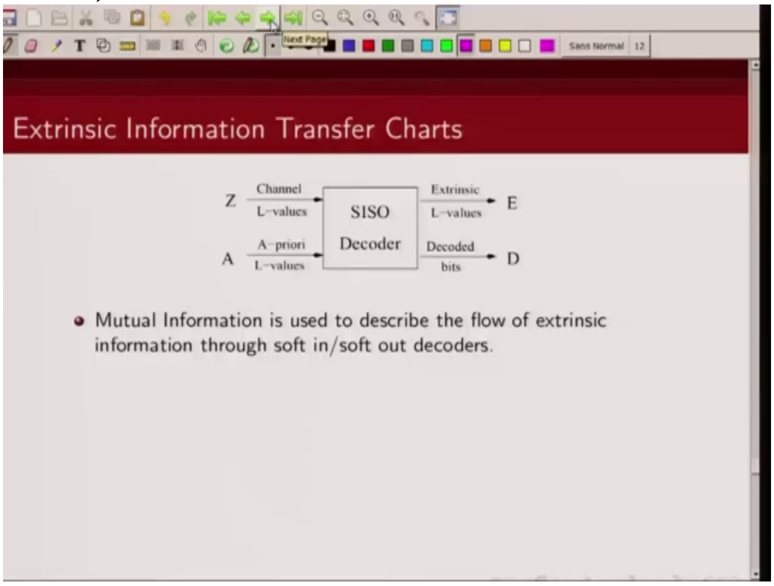

The third method which was proposed is based on mutual information. So mutual information was used to describe the flow

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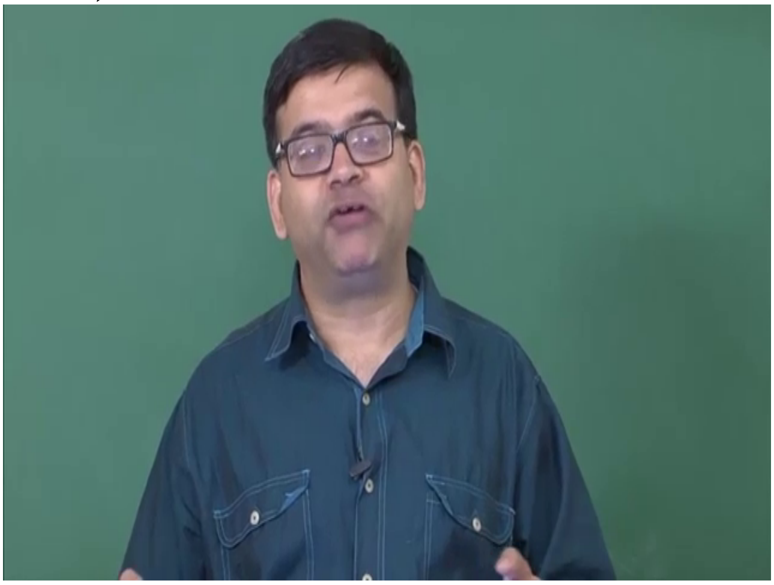

of information through this soft input soft output decoder. So there

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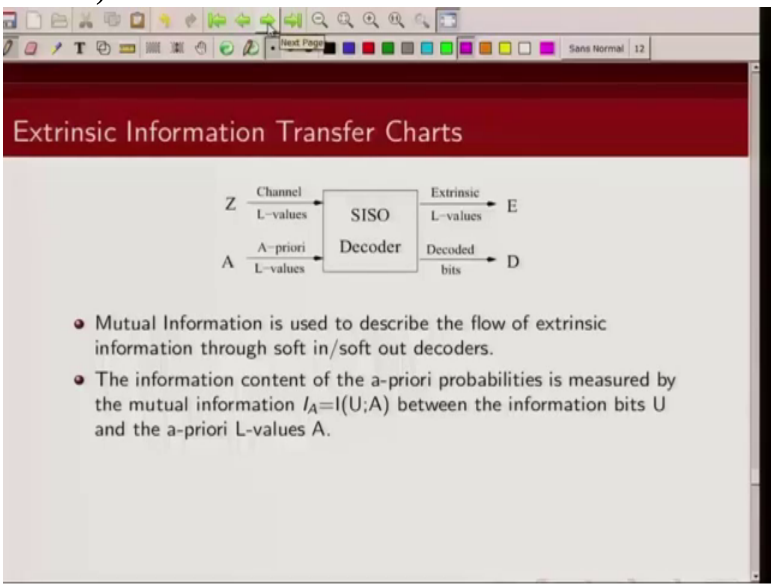

were 2 quantities which were described here. Basically one was this input mutual information which is the mutual information between the information bits and the a priori value and the second

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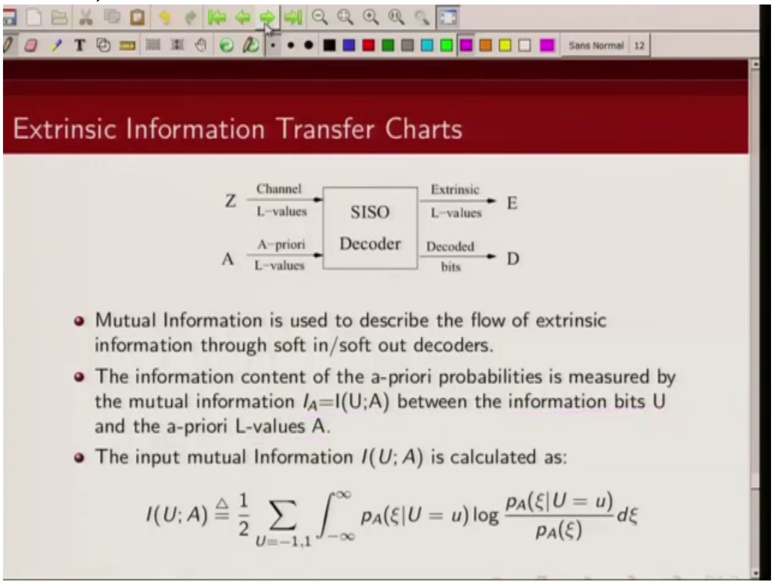

term which was defined here was

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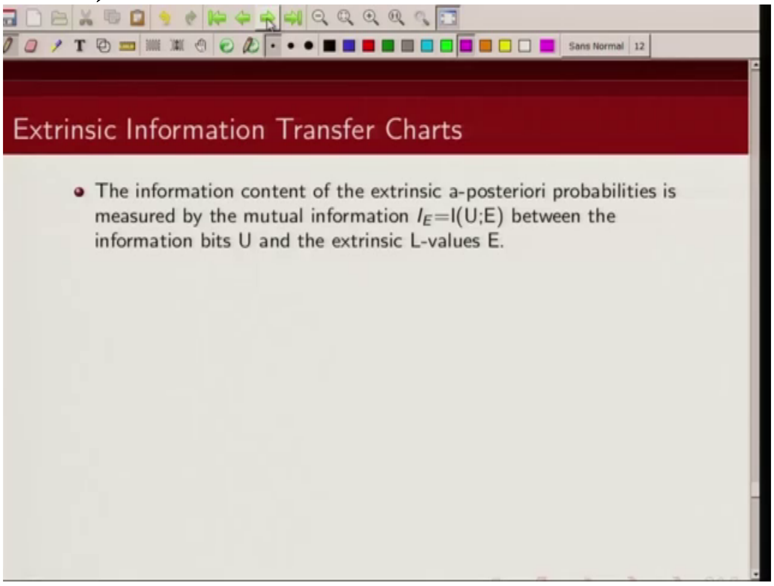

the extrinsic mutual information which is the mutual information between the input bits and the extrinsic values. So

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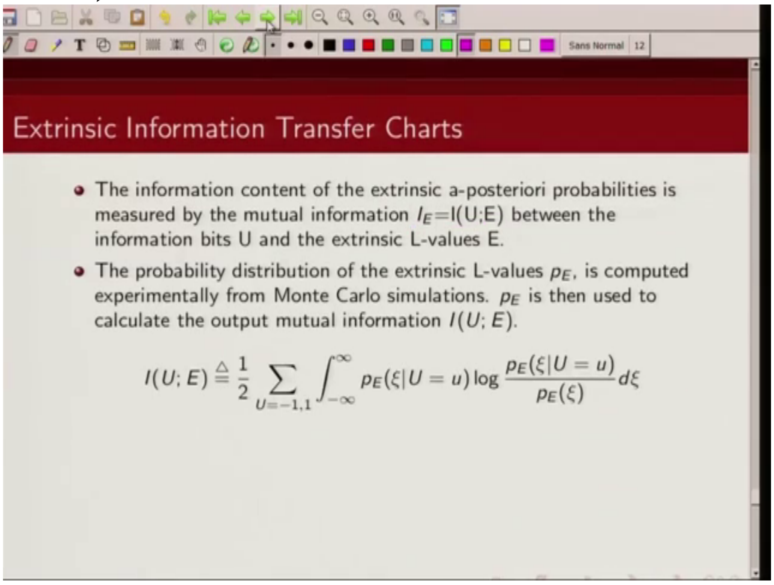

what was done in

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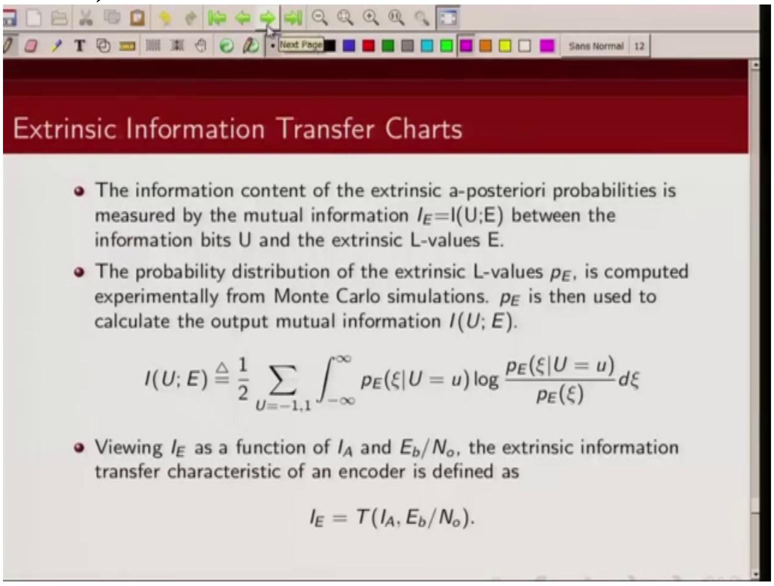

this technique was you can view the mutual information corresponding

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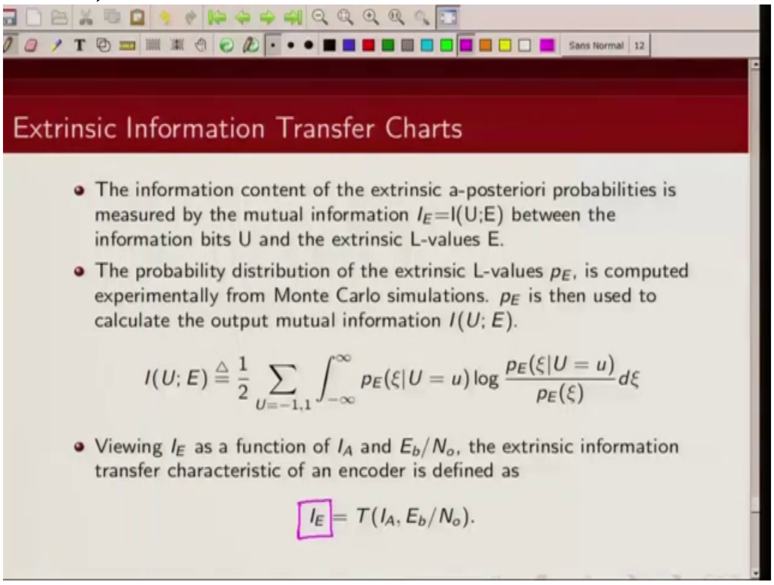

to the input and extrinsic value as a function of mutual information of a priori values and information bits and operating signal to noise ratio. So this was the transfer function which was considered in this extrinsic information chart. That viewing the output mutual information between the extrinsic information and the information bit as a function of mutual information between the a priori and the information bits and signal to noise ratio.

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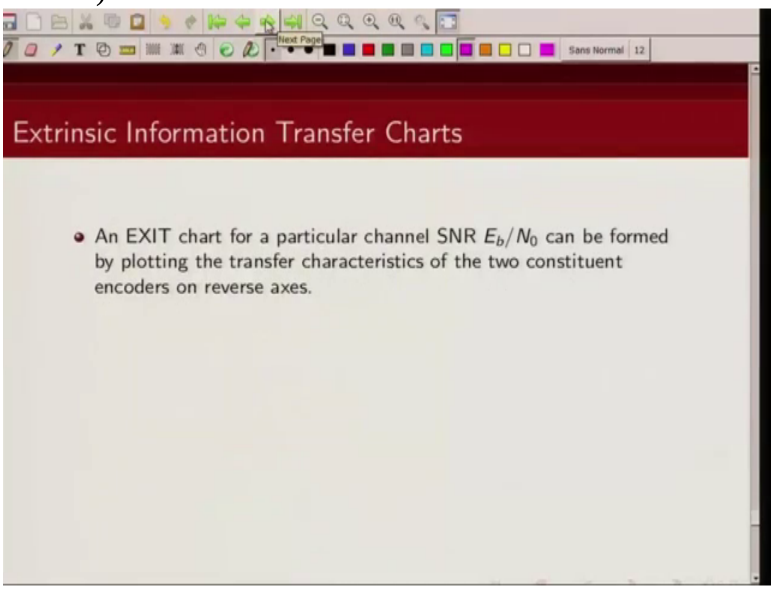

So how was EXIT chart created? So they plotted these transfer characteristics which was given by this.

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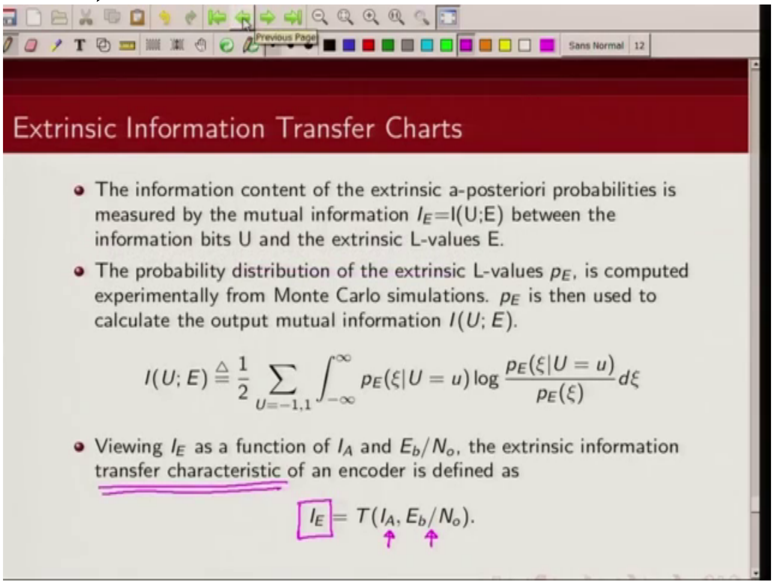

They plotted these transfer characteristics

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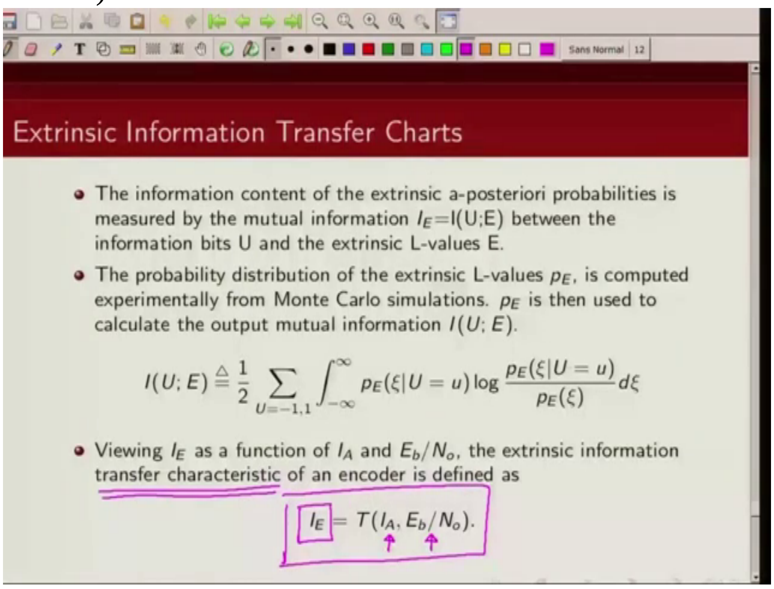

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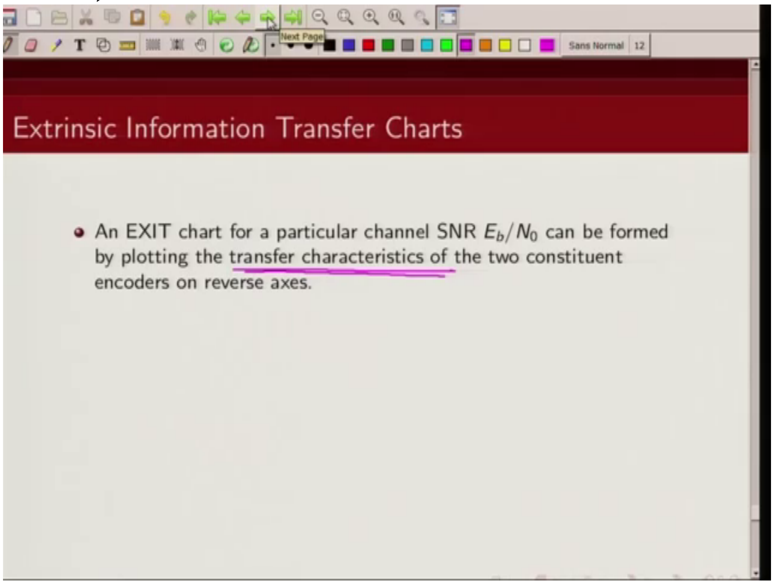

for two constituent

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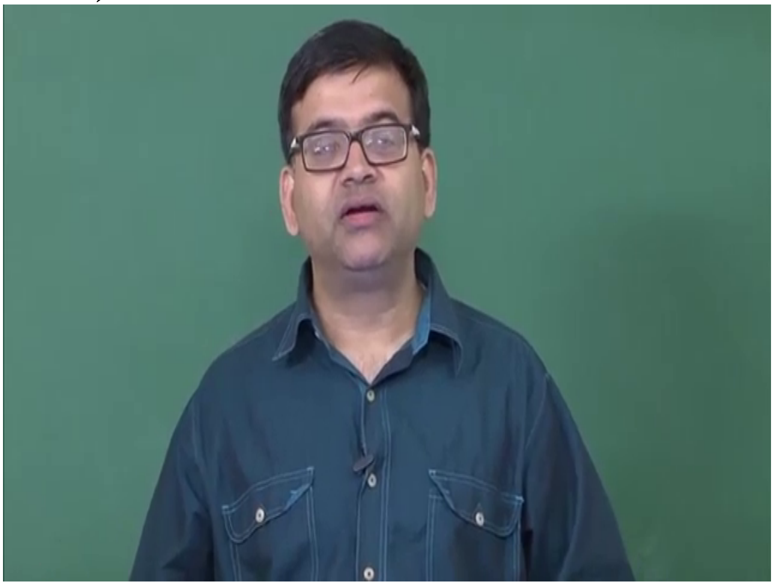

decoders on reverse axes

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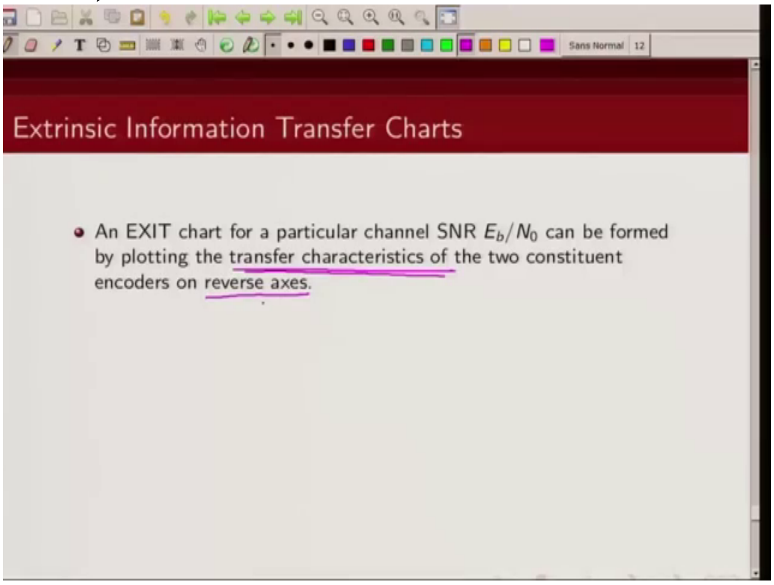

similar to El Gamal's technique, the difference is

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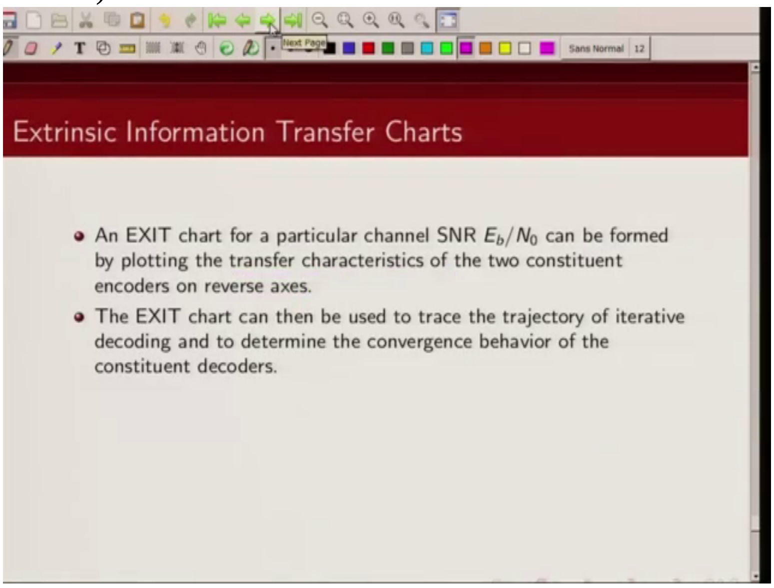

El Gamal used mean as

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S N R, here they used mutual information.

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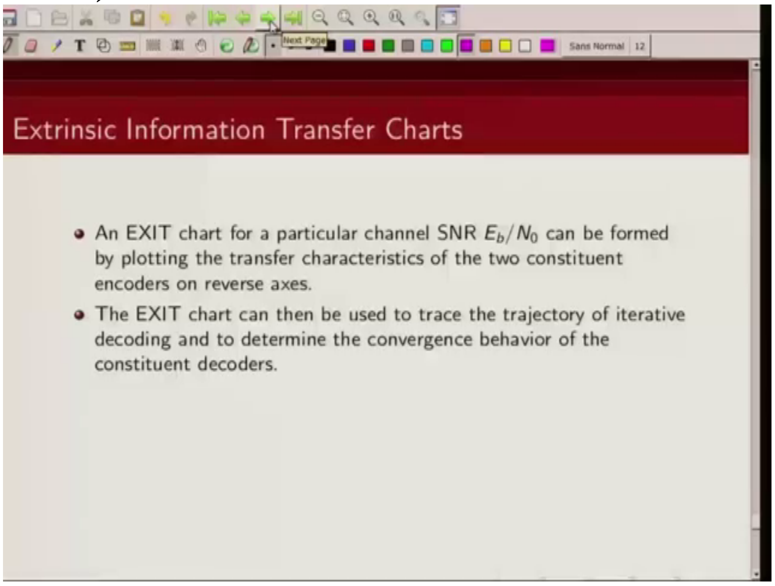

So very similar idea, so

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these transfer functions were plotted on reverse axes. Initially you don't have any a priori knowledge, so the input a priori mutual information is zero. And then after one half iteration, you get some extrinsic information. So you have some positive mutual information. And then you pass that as input to second decoder. And the decoding will progress if there is a tunnel otherwise it will get stuck.

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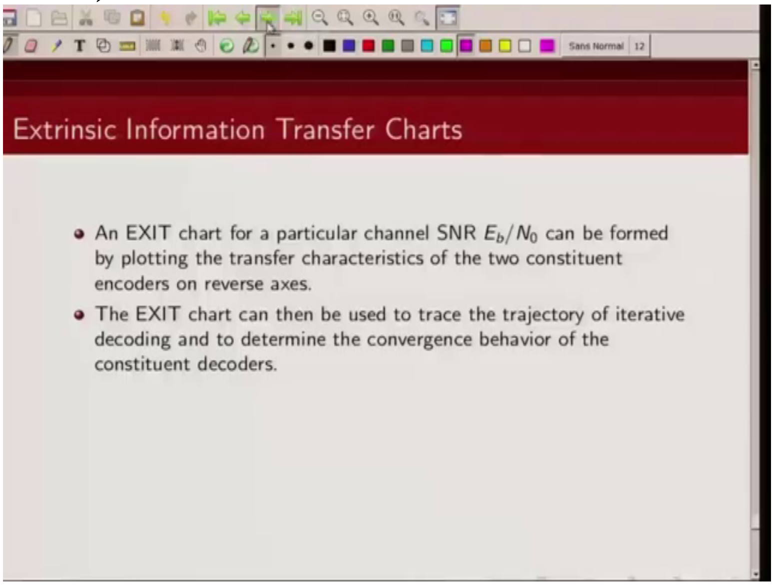

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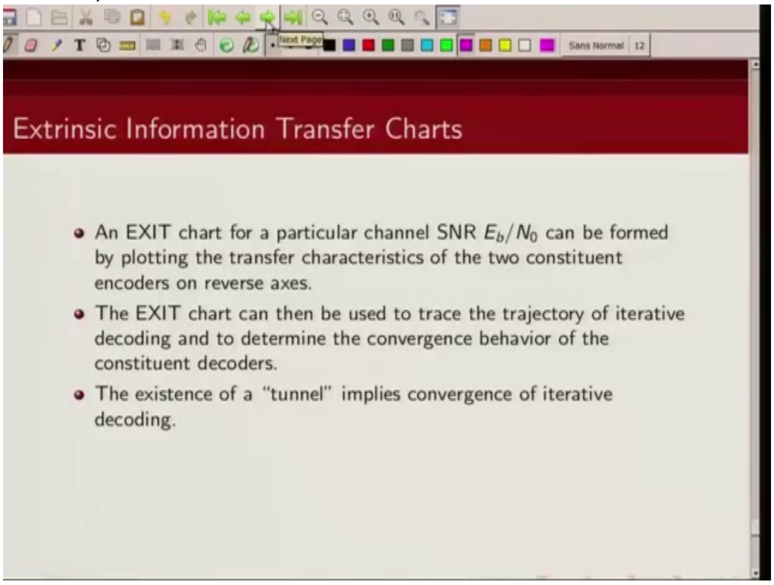

So as I have said, whether the decoding algorithm will converge or not, is, can be viewed by

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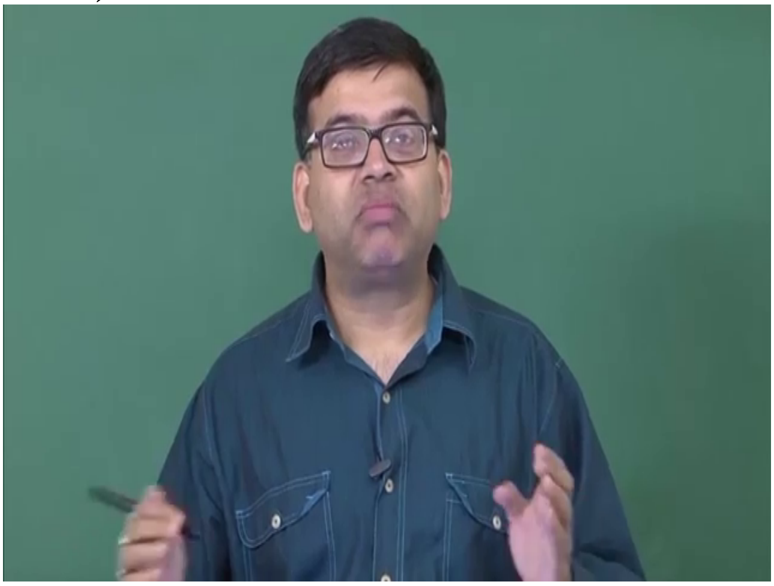

plotting these transfer characteristics on reverse axes and seeing whether a tunnel exists between them or not.

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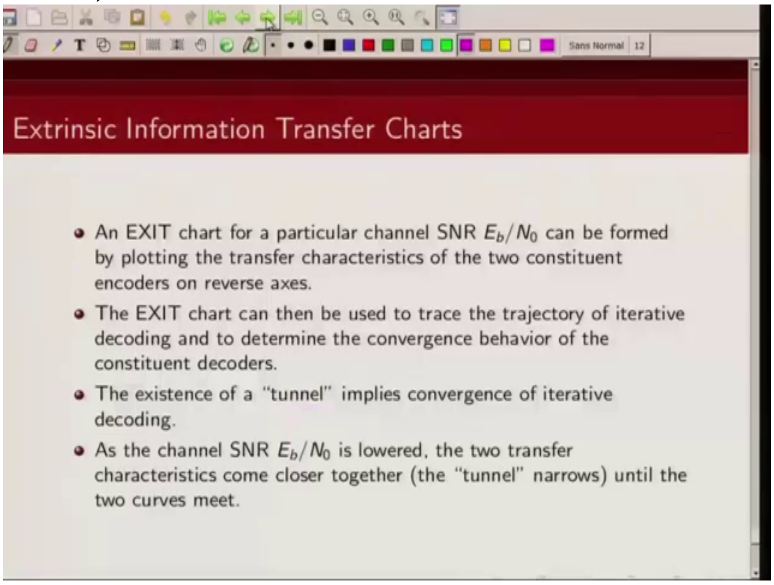

Now what happens if we reduce the channel operating S N R? If we reduce channel operating S N R, then these curves come closer until a point will come when they will barely touch or they will touch and cross each other. So the point, the minimum S N R

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where there still is a tunnel that's your threshold, decoding threshold.

So we have specified various methods for tracking the mutual information, tracking the extrinsic information and a priori information and this can be used to see how our constituent encoders will behave, how the turbo code, how the turbo decoder will behave under iterative decoding algorithm. Now what are the limitations of this analysis approach? Now this approach assumes that

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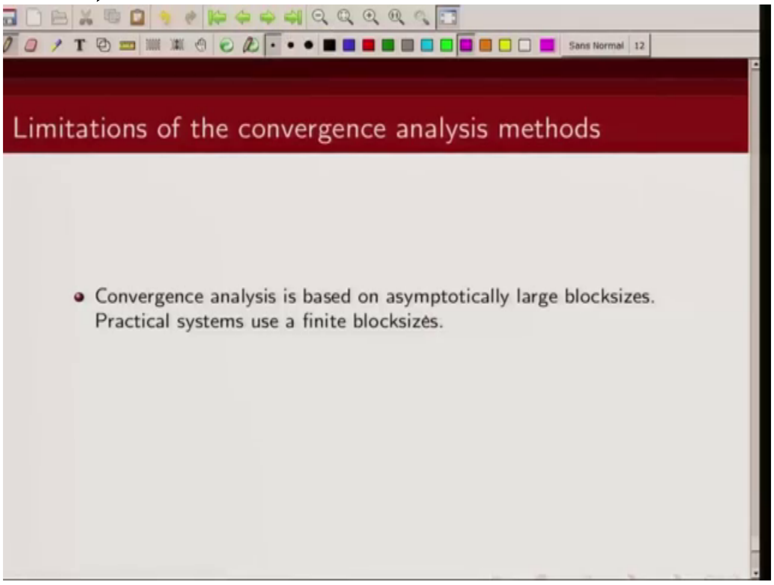

we have very large block sizes. So these convergence analysis results hold for very large block

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sizes but in practical systems we use small size block sizes so the thresholds predicted by this method may not be consistent when we use small block sizes and of course there are some assumptions, for example in El Gamal's technique we use Gaussian assumptions, we made assumption of consistency conditions. Those conditions may or may not hold, Ok. So with this I will conclude this discussion on convergence analysis of turbo codes, thank you.