

An Introduction to Coding Theory
Professor Adrish Banerji
Department of Electrical Engineering
Indian Institute of Technology, Kanpur
Module 05
Lecture Number 21
Problem Solving Session-IV

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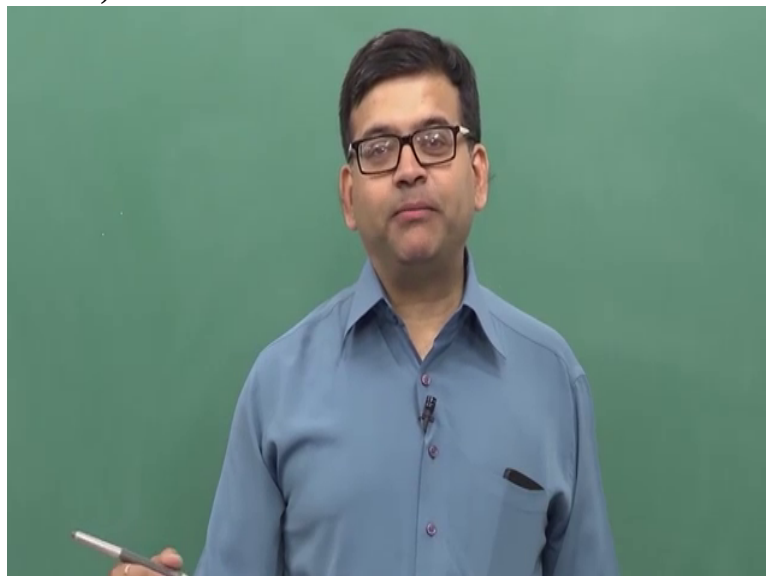


Lecture #11B: Problem solving session-IV



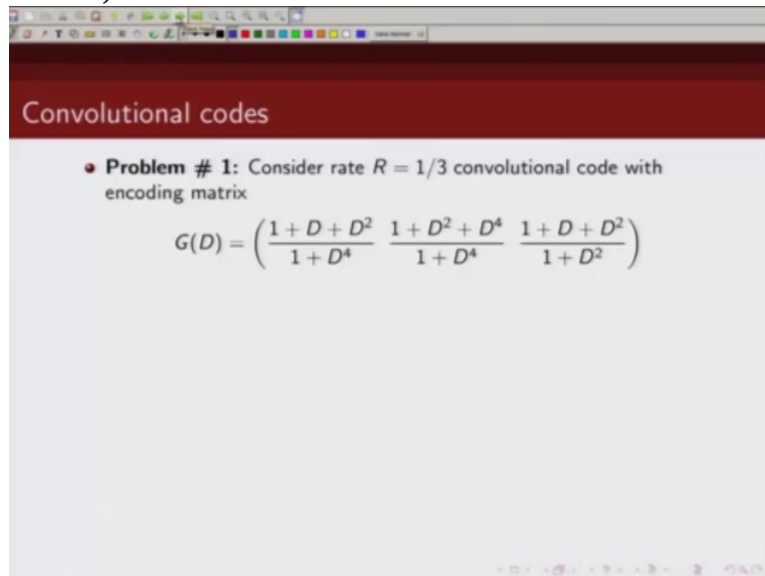
So before we go to concatenated codes, let us spend

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some time solving some problems.

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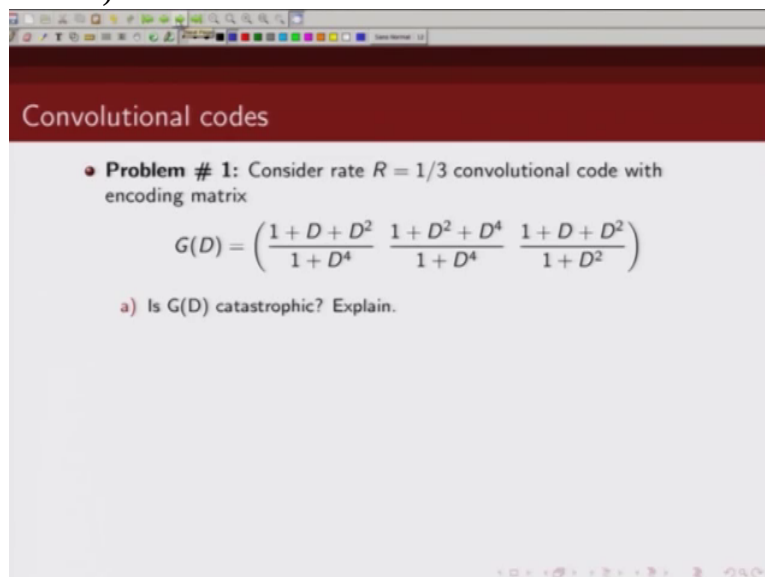
Convolutional codes

- **Problem # 1:** Consider rate $R = 1/3$ convolutional code with encoding matrix

$$G(D) = \begin{pmatrix} \frac{1+D+D^2}{1+D^4} & \frac{1+D^2+D^4}{1+D^4} & \frac{1+D+D^2}{1+D^2} \end{pmatrix}$$

So the first question is you are given a rate one third convolutional code with generator matrix G of D which is given by this.

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Convolutional codes

- **Problem # 1:** Consider rate $R = 1/3$ convolutional code with encoding matrix

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a) Is $G(D)$ catastrophic? Explain.

The first question is, is this a catastrophic encoder? Will an encoder which has a generator matrix like this; will this result in a catastrophic encoder? So if

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Convolutional codes

- **Problem # 1:** Consider rate $R = 1/3$ convolutional code with encoding matrix

$$G(D) = \left(\frac{1+D+D^2}{1+D^4} \quad \frac{1+D^2+D^4}{1+D^4} \quad \frac{1+D+D^2}{1+D^2} \right)$$

- a) Is $G(D)$ catastrophic? Explain.
- **Solution:** Yes, $G(D)$ can be equivalently written as

$$G(D) = \frac{1}{1+D^2} \left[\frac{1+D+D^2}{1+D^2} \quad \frac{1+D^2+D^4}{1+D^2} \quad 1+D+D^2 \right]$$

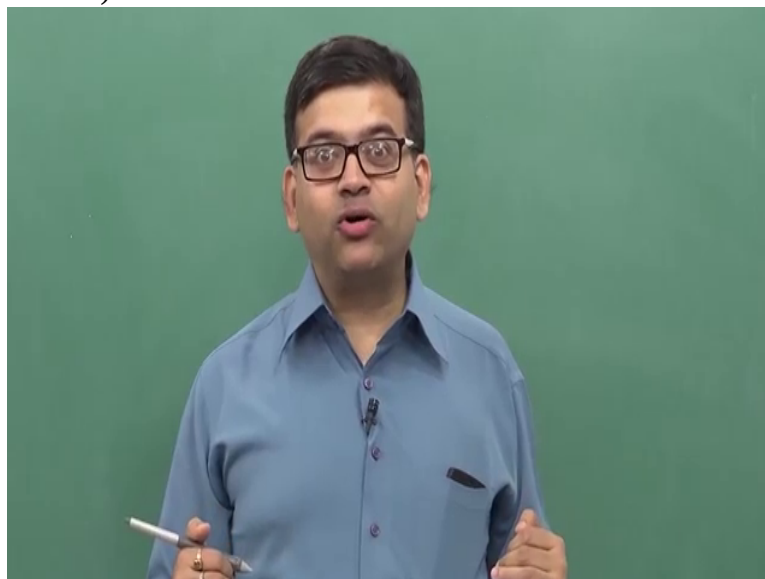
or equivalently in the form

$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \quad 1+D+D^2 \quad 1+D^2]$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

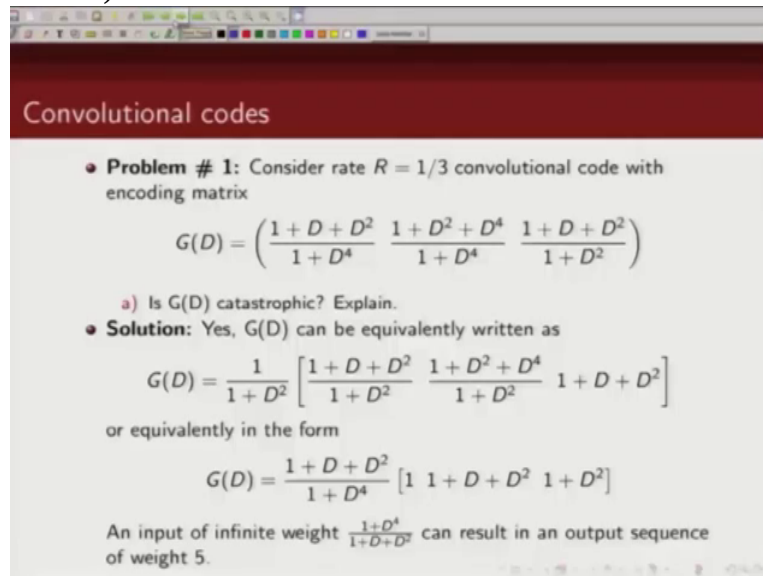
you recall what is a catastrophic encoder, a catastrophic

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encoder generates a finite weight output corresponding to an infinite weight input sequence. Now if we try to look it in terms of state diagram, in a state away from all zero state, there is a self loop around a state where a non-zero input results in all zero output, right. Now let's look at this generator matrix

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Convolutional codes

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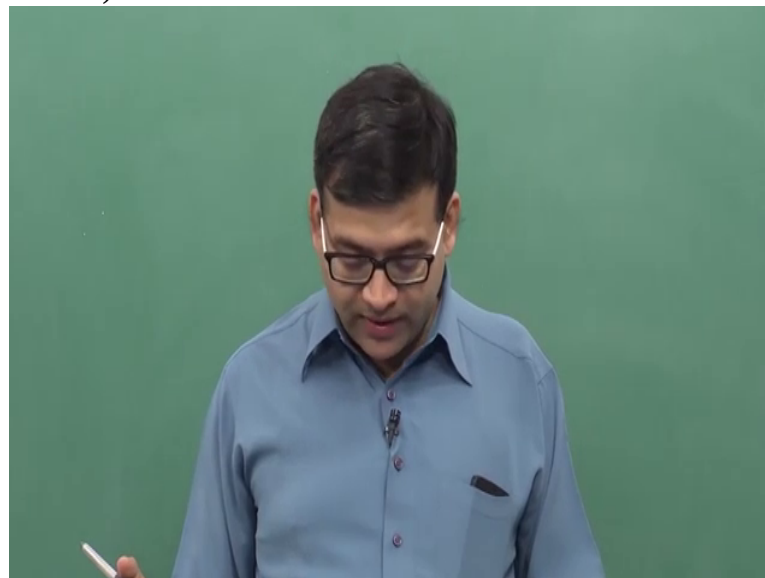
or equivalently in the form

$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \quad 1+D+D^2 \quad 1+D^2]$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

and let us try to simplify, put it in a minimal form. So we can see the denominator, one plus D square is common. So if we take that out, we get here 1 plus D plus D square and this is 1 plus D square, this is 1 plus D square plus D four divided by 1 plus D square and this is 1 plus D plus D square. Similarly we see in the numerator there is a common term 1 plus D plus D square. If we take that out, what we get here is then this is 1, 1 plus D plus D square and 1 plus D square. Now how do we know whether this will result in a catastrophic

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encoder or not. So look at

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Convolutional codes

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$$G(D) = \frac{1}{1+D^2} \begin{bmatrix} 1+D+D^2 & 1+D^2+D^4 & 1+D+D^2 \end{bmatrix}$$
 or equivalently in the form

$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \ 1+D+D^2 \ 1+D^2]$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

this particular generator matrix. Now what is my output sequence? My output sequence $v(D)$ is $u(D)g(D)$.

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Convolutional codes

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$$G(D) = \frac{1}{1+D^2} \begin{bmatrix} 1+D+D^2 & 1+D^2+D^4 & 1+D+D^2 \end{bmatrix}$$
 or equivalently in the form $v(D) = u(D)G(D)$

$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \ 1+D+D^2 \ 1+D^2]$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

Now is there any input sequence which is of infinite weight but can result in a finite output weight for $v(D)$? If you pay close attention to $g(D)$ we notice that if our input $u(D)$ is chosen as $1 + D^4 + 1 + D + D^2$,

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Convolutional codes

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or equivalently in the form

$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \ 1+D+D^2 \ 1+D^2]$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

Handwritten notes:
 $v(D) = \frac{u(D)}{1+D^2} G(D)$
 $v(D) = \frac{1+D^4}{1+D+D^2}$

if our input is chosen in this particular fashion, then what would be the corresponding output $v(D)$? If the input is chosen this way then output will be $u(D)$ times $G(D)$ so this term will cancel this term so what

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Convolutional codes

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or equivalently in the form

$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \ 1+D+D^2 \ 1+D^2]$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

Handwritten notes:
 $v(D) = \frac{u(D)}{1+D^2} G(D)$
 $v(D) = \frac{1+D^4}{1+D+D^2}$

you will be left with is this. So your $v(D)$ would be $1, 1+D+D^2$ and $1+D^2$. And what

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Convolutional codes

- **Problem # 1:** Consider rate $R = 1/3$ convolutional code with encoding matrix

$$G(D) = \begin{pmatrix} \frac{1+D+D^2}{1+D^4} & \frac{1+D^2+D^4}{1+D^4} & \frac{1+D+D^2}{1+D^2} \end{pmatrix}$$

- a) Is $G(D)$ catastrophic? Explain.
- **Solution:** Yes, $G(D)$ can be equivalently written as $\begin{bmatrix} 1 & 1+D+D^2 & 1+D^2 \end{bmatrix}$

$$G(D) = \frac{1}{1+D^2} \begin{bmatrix} 1+D+D^2 & 1+D^2+D^4 & 1+D+D^2 \end{bmatrix}$$

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$$G(D) = \frac{1+D+D^2}{1+D^4} \begin{bmatrix} 1 & 1+D+D^2 & 1+D^2 \end{bmatrix}$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

is the weight of this? (Delete kar diya, woh ganda)

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Convolutional codes

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or equivalently in the form

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An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

So note here, so the input that will cause this output is given by 1 plus D four by 1 plus D plus D square,

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Convolutional codes

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$$G(D) = \begin{pmatrix} \frac{1+D+D^2}{1+D^4} & \frac{1+D^2+D^4}{1+D^4} & \frac{1+D+D^2}{1+D^2} \end{pmatrix}$$

- a) Is $G(D)$ catastrophic? Explain.
- **Solution:** Yes, $G(D)$ can be equivalently written as $\frac{v(D)}{1+D+D^2}$ where $v(D) = [1 \quad 1+D+D^2 \quad 1+D^2]$

$$G(D) = \frac{1}{1+D^2} \begin{bmatrix} 1+D+D^2 & 1+D^2+D^4 & 1+D+D^2 \end{bmatrix}$$

or equivalently in the form $v(D) = \frac{1+D^4}{1+D+D^2}$

$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \quad 1+D+D^2 \quad 1+D^2]$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

right? Now we can expand this. So let's say $1 + D^4$, this is $1 + D + D^2$. So let's just take

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Convolutional codes

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or equivalently in the form $v(D) = \frac{1+D^4}{1+D+D^2}$

$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \quad 1+D+D^2 \quad 1+D^2] \frac{1+D^4}{1+D+D^2}$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

1. $1 + D + D^2$, this will be $D + D^2 + D^4$, now

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Convolutional codes

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$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \quad 1+D+D^2 \quad 1+D^2]$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

this will be plus D, this will be D plus D square plus D cube, then this will be D cube plus D 4, you can write D square, so like that basically we, we can see that this is a infinite series. The input is an infinite series, 1 plus D plus D square is essentially an infinite series. We can

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Convolutional codes

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- Solution:** Yes, $G(D)$ can be equivalently written as $v(D) = [1 \quad 1+D+D^2 \quad 1+D^2]$

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or equivalently in the form

$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \quad 1+D+D^2 \quad 1+D^2]$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

expand it like that where as output is a finite series, it is just 1, 1 plus D plus D square, and the third bit is 1 plus D square. So you can see, input has lots of 1's in it but the output has finite 1s. So this is a case of catastrophic

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encoder.

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Convolutional codes

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An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

Handwritten notes on the slide:
 $v(D) = \frac{1+D^4}{1+D+D^2} = 1+D$
 $\frac{1+D^4}{1+D+D^2} = 1+D + \frac{1+D^4 - (1+D+D^2)(1+D)}{1+D+D^2}$
 $\frac{1+D^4}{1+D+D^2} = 1+D + \frac{1+D^4 - (1+D+D^2)(1+D)}{1+D+D^2}$
 $\frac{1+D^4}{1+D+D^2} = 1+D + \frac{1+D^4 - (1+D+D^2)(1+D)}{1+D+D^2}$
 $\frac{1+D^4}{1+D+D^2} = 1+D + \frac{1+D^4 - (1+D+D^2)(1+D)}{1+D+D^2}$

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Convolutional codes

b) Find a minimal encoder whose encoding matrix is equivalent to $G(D)$.

Solution: An equivalent minimal encoder is given by

$$G(D) = [1 \ 1 + D + D^2 \ 1 + D^2]$$

Now the second question is what would be the minimal encoding matrix for the generator matrix given in the previous example.

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Convolutional codes

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a) Is $G(D)$ catastrophic? Explain.

• **Solution:** Yes, $G(D)$ can be equivalently written as $v(D) = [1 \ 1+D+D^2 \ 1+D^2]$

$$G(D) = \frac{1}{1+D^2} \begin{bmatrix} 1+D+D^2 & 1+D^2+D^4 & 1+D+D^2 \end{bmatrix}$$

or equivalently in the form

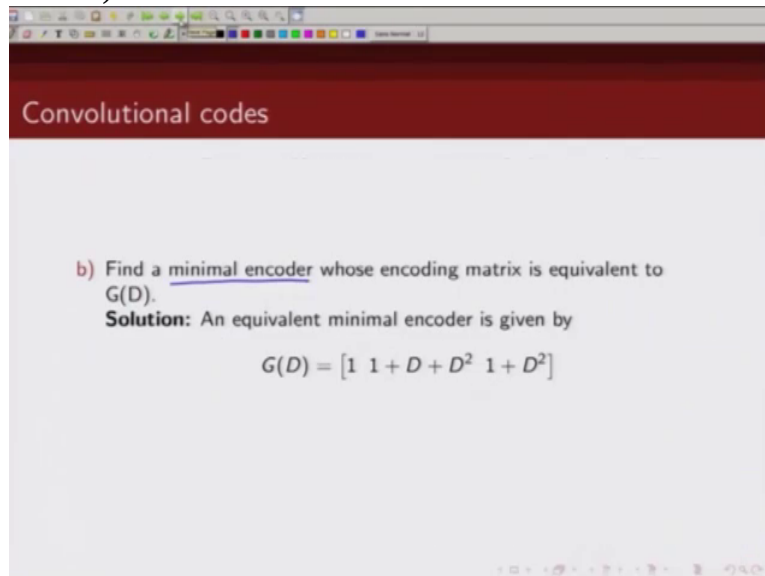
$$G(D) = \frac{1+D+D^2}{1+D^4} [1 \ 1+D+D^2 \ 1+D^2]$$

An input of infinite weight $\frac{1+D^4}{1+D+D^2}$ can result in an output sequence of weight 5.

Handwritten notes:
 $v(D) = \frac{1+D+D^2}{1+D+D^2} = 1+D+D^2$
 $\frac{1+D+D^2}{1+D+D^2} = 1+D+D^2$
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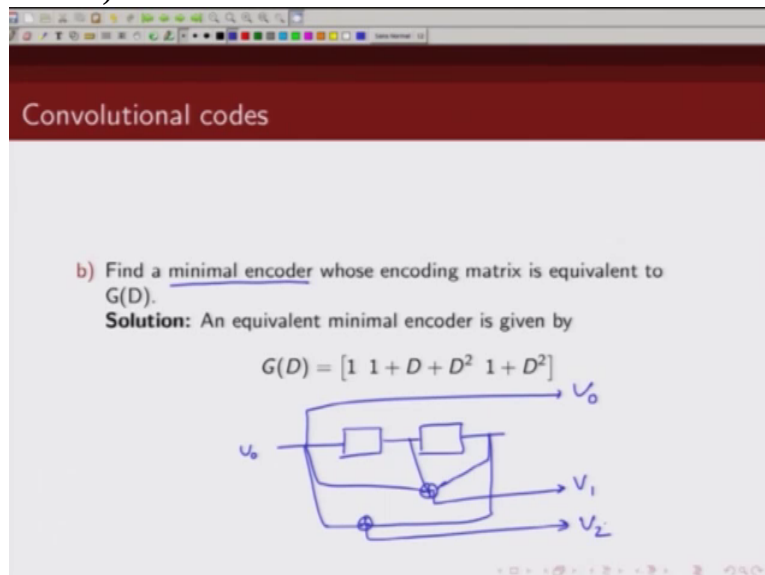
If I ask you, find out the minimal encoding matrix for this encoder. So what do we do? We take out all the common factors. If we take out common factors, then we basically what we get is like this is our minimal encoding

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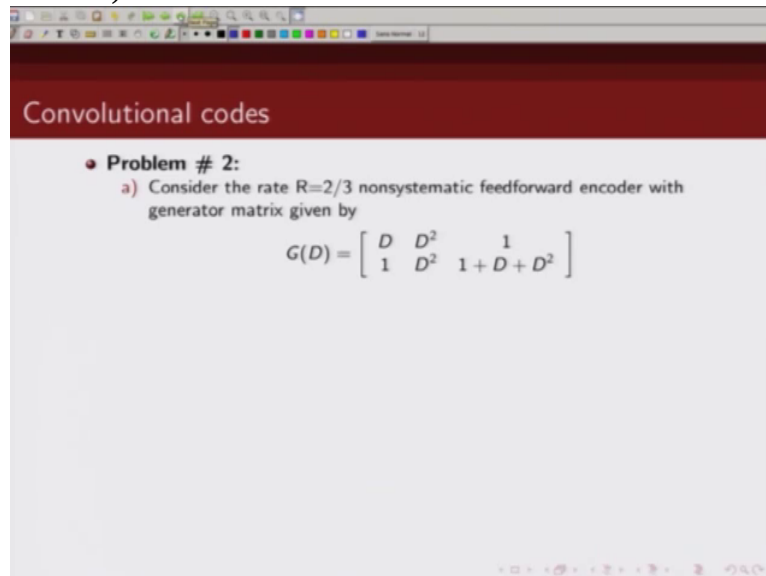
matrix and if we can write, if I ask you to draw this encoder, we can, this is, k is 1, n is 3, the maximum memory is 2 so I am drawing 2 memory elements here. The first coded bit is just 1, so this is the information sequence that goes in. Second one is 1 plus D plus D square. So that's your, let's call it v_0 , this is v_1 , this is u_0 , and the third bit, coded bit is 1, and D square this is your v_2 ,

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Ok. So this is the minimal encoder for the same generator matrix given in the previous example.

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Convolutional codes

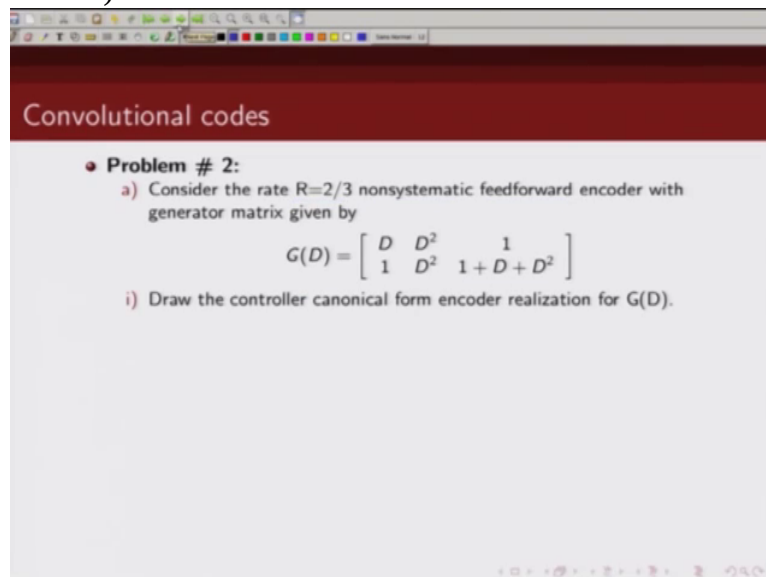
• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

So consider a rate two third non systematic feed forward encoder. So this is a generator matrix for a non systematic code, rate two third and it is a feed forward encoder. There are no feedback polynomials here.

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Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

i) Draw the controller canonical form encoder realization for $G(D)$.

The first question is draw the controller canonical form realization for this generator matrix. Now in controller canonical form realization we have 1 set

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of shift register for input. Now how many inputs do we have here? k is 2

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A slide titled "Convolutional codes" with a red header. The content includes a problem statement and a generator matrix. The matrix is a 2x3 matrix with elements D , D^2 , and 1 in the first row, and 1 , D^2 , and $1 + D + D^2$ in the second row. A handwritten blue " $k=$ " is next to the matrix. The problem asks to draw the controller canonical form encoder realization for $G(D)$.

so we will have 2 sets of shift registers for this. One for this and second

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Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1+D+D^2 \end{bmatrix} \quad k=2$$

i) Draw the controller canonical form encoder realization for $G(D)$.

set of shift registers for this. Now what

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Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1+D+D^2 \end{bmatrix} \quad k=2$$

i) Draw the controller canonical form encoder realization for $G(D)$.

should be the maximum memory for each of these shift registers? You can see here the maximum power of d is 2. So we should have 2 memory elements for the first input. Similarly for the second input also we should have 2 memory elements. Let's call it u_0 and u_1 .

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Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1+D+D^2 \end{bmatrix} \quad k=2$$

i) Draw the controller canonical form encoder realization for $G(D)$.

U_0 → [] → []

U_1 → [] → []

Now there are 3 outputs. So the outputs are, this is 1 output, D times

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Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1+D+D^2 \end{bmatrix} \quad k=2$$

i) Draw the controller canonical form encoder realization for $G(D)$.

U_0 → [] → []

U_1 → [] → []

the first and one times u_1 , so D times the first input is this, and one times second input so that is this. So this is your first coded bit. Let's call it v_0 .

(Refer Slide Time 08:28)

Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1+D+D^2 \end{bmatrix} \quad k=2$$

i) Draw the controller canonical form encoder realization for $G(D)$.

Now what's the second coded bit? This is this term,

(Refer Slide Time 08:33)

Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1+D+D^2 \end{bmatrix} \quad k=2$$

i) Draw the controller canonical form encoder realization for $G(D)$.

$D^2 u_0$ is this term and $D^2 u_1$ is this term, so this is your v_1

(Refer Slide Time 08:44)

Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1+D+D^2 \end{bmatrix} \quad k=2$$

i) Draw the controller canonical form encoder realization for $G(D)$.

and the third output is this.

(Refer Slide Time 08:49)

Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1+D+D^2 \end{bmatrix} \quad k=2$$

i) Draw the controller canonical form encoder realization for $G(D)$.

So this is, just a minute, u 1 and one D term and D square term.

(Refer Slide Time 09:12)

Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix} \quad k=2$$

i) Draw the controller canonical form encoder realization for $G(D)$.

So this is your controller canonical form realization for this generator matrix. So this is

(Refer Slide Time 09:20)

Convolutional codes

• **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

i) Draw the controller canonical form encoder realization for $G(D)$.

• **Solutions:**

i) Controller canonical form encoder realization is given in Figure 1

Figure: Canonical Form encoder realization

precisely what we have here. You can see, so this shift register is for

(Refer Slide Time 09:30)

Convolutional codes

- **Problem # 2:**
 - a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by
$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$
 - i) Draw the controller canonical form encoder realization for $G(D)$.
- **Solutions:**
 - i) Controller canonical form encoder realization is given in Figure 1

Figure: Canonical Form encoder realization

this input and this

(Refer Slide Time 09:34)

Convolutional codes

- **Problem # 2:**
 - a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by
$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$
 - i) Draw the controller canonical form encoder realization for $G(D)$.
- **Solutions:**
 - i) Controller canonical form encoder realization is given in Figure 1

Figure: Canonical Form encoder realization

this shift register is for

(Refer Slide Time 09:37)

Convolutional codes

● **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

i) Draw the controller canonical form encoder realization for $G(D)$.

● **Solutions:**

i) Controller canonical form encoder realization is given in Figure 1

Figure: Canonical Form encoder realization

this input.

(Refer Slide Time 09:40)

Convolutional codes

● **Problem # 2:**

a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

i) Draw the controller canonical form encoder realization for $G(D)$.

● **Solutions:**

i) Controller canonical form encoder realization is given in Figure 1

Figure: Canonical Form encoder realization

Maximum memory element for the first one is 2, second one also 2 and we can see now, the first output is D times u_1 which is this plus u_2 times this.

(Refer Slide Time 10:00)

Convolutional codes

● **Problem # 2:**
 a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

i) Draw the controller canonical form encoder realization for $G(D)$.

● **Solutions:**
 i) Controller canonical form encoder realization is given in Figure 1

Figure: Canonical Form encoder realization

The second output is $D^2 u_1$ and $D^2 u_2$, so that is this.

(Refer Slide Time 10:11)

Convolutional codes

● **Problem # 2:**
 a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

i) Draw the controller canonical form encoder realization for $G(D)$.

● **Solutions:**
 i) Controller canonical form encoder realization is given in Figure 1

Figure: Canonical Form encoder realization

And the third output is u_1 which is this and this is $u_1 D$ times u_1 , D^2 times u_1 , so that's your third output,

(Refer Slide Time 10:27)

Convolutional codes

- **Problem # 2:**
 - a) Consider the rate $R=2/3$ nonsystematic feedforward encoder with generator matrix given by
$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$
 - i) Draw the controller canonical form encoder realization for $G(D)$.
- **Solutions:**
 - i) Controller canonical form encoder realization is given in Figure 1

Figure: Canonical Form encoder realization

Ok.

(Refer Slide Time 10:31)

Convolutional codes

- ii) Find the generator matrix $G'(D)$ of the equivalent systematic feedback encoder.

- ii) **Solutions:** We have
$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

Now this was a non systematic encoder. Can we find an equivalent systematic encoder or systematic encoding matrix for this generator matrix, the answer is yes. So how do we find a systematic encoding matrix? So this has to be put in the form like this; $1 \ 0 \ 0 \ 1$ and some matrix here let's call it a $1 \ D$ times a $2 \ D$ and $b \ 1 \ D$ times $b \ 2 \ D$. So we will have to bring

(Refer Slide Time 11:10)

Convolutional codes

ii) Find the generator matrix $G(D)$ of the equivalent systematic feedback encoder.

ii) **Solutions:** We have

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

this matrix in this particular form. So we have to get this to 1, this to, this has to be changed to 1, this has to be brought to 0,

(Refer Slide Time 11:22)

Convolutional codes

ii) Find the generator matrix $G(D)$ of the equivalent systematic feedback encoder.

ii) **Solutions:** We have

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

, this has, this we have to brought to 1 and this we have to bring to

(Refer Slide Time 11:28)

Convolutional codes

ii) Find the generator matrix $G'(D)$ of the equivalent systematic feedback encoder.

ii) **Solutions:** We have

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{a_1(D)}{a_2(D)} \\ 0 & 1 & \frac{b_1(D)}{b_2(D)} \\ \frac{D}{1} & \frac{D^2}{1} & 0 \\ 1 & D^2 & 1 + D + D^2 \\ 0 & 1 & 1 \end{bmatrix}$$

0. Now we will do elementary row operation to get an identity matrix here. So let's do that.

(Refer Slide Time 11:37)

Convolutional codes

ii) Find the generator matrix $G'(D)$ of the equivalent systematic feedback encoder.

ii) **Solutions:** We have

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

• Now applying Row 1 \leftarrow (Row 1)/D, we get

$$G(D) = \begin{bmatrix} 1 & D & \frac{1}{D} \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

So first thing that we do is, we make this a 1. How do we make this a 1? We do this transformation that row 1 is, row 1 by D. So we divide this whole thing by D, what we get is 1 D 1 by D. Next we would like to get a zero here.

(Refer Slide Time 12:04)

Convolutional codes

ii) Find the generator matrix $G(D)$ of the equivalent systematic feedback encoder.

ii) **Solutions:** We have

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1 + D + D^2 \end{bmatrix}$$

• Now applying $\text{Row 1} \leftarrow (\text{Row 1})/D$, we get

$$G(D) = \begin{bmatrix} \frac{1}{D} & D^2 & 1 + D + D^2 \end{bmatrix}$$

Here we would like to get a zero. How can we get a zero here?

(Refer Slide Time 12:09)

Convolutional codes

• Now applying $\text{Row 2} \leftarrow \text{Row 1} + \text{Row 2}$, we get

$$G(D) = \begin{bmatrix} 1 & D & \frac{1}{D} \\ 0 & D + D^2 & \frac{1 + D + D^2 + D^3}{D} \end{bmatrix}$$

• Now applying $\text{Row 2} \leftarrow (\text{Row 2})/(D + D^2)$, we get

$$G(D) = \begin{bmatrix} 1 & D & \frac{1}{D} \\ 0 & 1 & \frac{1 + D^2}{D^2} \end{bmatrix}$$

• Now applying $\text{Row 1} \leftarrow \text{Row 1} + D (\text{Row 2})$, we get

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{D}{D} \\ 0 & 1 & \frac{1 + D^2}{D^2} \end{bmatrix}$$

So we will do this transformation, row 2 is row 1 plus row 2. If we do that, so we

(Refer Slide Time 12:21)

Convolutional codes

ii) Find the generator matrix $G(D)$ of the equivalent systematic feedback encoder.

ii) **Solutions:** We have

$$G(D) = \begin{bmatrix} D & D^2 & 1 \\ 1 & D^2 & 1+D+D^2 \end{bmatrix}$$

• Now applying $\text{Row 1} \leftarrow (\text{Row 1})/D$, we get

$$G(D) = \begin{bmatrix} \frac{1}{D} & D & \frac{1}{D} \\ 1 & D^2 & 1+D+D^2 \end{bmatrix}$$

add these two, this will become 0, this will become D plus D square and what we will get is

(Refer Slide Time 12:28)

Convolutional codes

• Now applying $\text{Row 2} \leftarrow \text{Row 1} + \text{Row 2}$, we get

$$G(D) = \begin{bmatrix} \frac{1}{D} & D & \frac{1}{D} \\ 0 & D+D^2 & 1+\frac{D+D^2}{D} \end{bmatrix}$$

• Now applying $\text{Row 2} \leftarrow (\text{Row 2})/(D+D^2)$, we get

$$G(D) = \begin{bmatrix} \frac{1}{D} & D & \frac{1}{D} \\ 0 & 1 & \frac{1+D^2}{D+D^2} \end{bmatrix}$$

• Now applying $\text{Row 1} \leftarrow \text{Row 1} + D(\text{Row 2})$, we get

$$G(D) = \begin{bmatrix} 1 & 0 & D \\ 0 & 1 & \frac{1+D^2}{D+D^2} \end{bmatrix}$$

this. Next we would like to get a 1 here,

(Refer Slide Time 12:36)

Convolutional codes

- Now applying $\text{Row 2} \leftarrow \text{Row 1} + \text{Row 2}$, we get

$$G(D) = \begin{bmatrix} 1 & D & \frac{1}{D} \\ 0 & D + D^2 & 1 + \frac{D + D^2}{D} \end{bmatrix}$$
- Now applying $\text{Row 2} \leftarrow (\text{Row 2}) / (D + D^2)$, we get

$$G(D) = \begin{bmatrix} 1 & D & \frac{1}{D} \\ 0 & 1 & \frac{1 + D^2}{D^2} \end{bmatrix}$$

like to get a 1 here. How can we get a 1 here? We divide row 2 by this. So we do this transformation that row 2 is row 2 divided by D plus D square and once we do that, we get this. Next we would like to get a zero here.

(Refer Slide Time 12:58)

Convolutional codes

- Now applying $\text{Row 2} \leftarrow \text{Row 1} + \text{Row 2}$, we get

$$G(D) = \begin{bmatrix} 1 & D & \frac{1}{D} \\ 0 & D + D^2 & 1 + \frac{D + D^2}{D} \end{bmatrix}$$
- Now applying $\text{Row 2} \leftarrow (\text{Row 2}) / (D + D^2)$, we get

$$G(D) = \begin{bmatrix} 1 & D & \frac{1}{D} \\ 0 & 1 & \frac{1 + D^2}{D^2} \end{bmatrix}$$
- Now applying $\text{Row 1} \leftarrow \text{Row 1} + D (\text{Row 2})$, we get

$$G(D) = \begin{bmatrix} 1 & 0 & D \\ 0 & 1 & \frac{1 + D^2}{D^2} \end{bmatrix}$$

How do we get a zero here? We multiply row 2 by D and add it to row 1. So we do this transformation that row 1 is row 1 plus D times row 2. And when we do that, we get this. So this is our equivalent systematic encoder for the

(Refer Slide Time 13:27)

Convolutional codes

ii) Find the generator matrix $G'(D)$ of the equivalent systematic feedback encoder.

ii) **Solutions:** We have

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{a_1(D)}{a_2(D)} \\ 0 & 1 & \frac{b_1(D)}{b_2(D)} \\ \frac{D}{1+D+D^2} & \frac{D^2}{1+D+D^2} & 1 \end{bmatrix}$$

generator matrix this, Ok.

(Refer Slide Time 13:33)

Convolutional codes

iii) Is $G'(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row } 2 \leftarrow (\text{Row } 2)(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D \\ 0 & D^2 & 1+D^2 \end{bmatrix}$$

Next. Is this equivalent systematic generator matrix, is it realizable? If it is not, find out an equivalent realizable generator matrix and draw its corresponding minimal encoder realization. Now note here

(Refer Slide Time 13:52)

Convolutional codes

- Now applying $\text{Row 2} \leftarrow \text{Row 1} + \text{Row 2}$, we get

$$G(D) = \begin{bmatrix} 1 & -D & \frac{1}{D} \\ 0 & D + D^2 & 1 + D + D^2 + D^3 \end{bmatrix}$$

- Now applying $\text{Row 2} \leftarrow (\text{Row 2}) / (D + D^2)$, we get

$$G(D) = \begin{bmatrix} 1 & -D & \frac{1}{D} \\ 0 & 1 & \frac{1 + D^2}{D} \end{bmatrix}$$

- Now applying $\text{Row 1} \leftarrow \text{Row 1} + D (\text{Row 2})$, we get

$$G(D) = \begin{bmatrix} 1 & 0 & D \\ 0 & 1 & \frac{1 + D^2}{D} \end{bmatrix}$$

this generator matrix has a term 1 plus

(Refer Slide Time 13:58)

Convolutional codes

- Now applying $\text{Row 2} \leftarrow \text{Row 1} + \text{Row 2}$, we get

$$G(D) = \begin{bmatrix} 1 & -D & \frac{1}{D} \\ 0 & D + D^2 & 1 + D + D^2 + D^3 \end{bmatrix}$$

- Now applying $\text{Row 2} \leftarrow (\text{Row 2}) / (D + D^2)$, we get

$$G(D) = \begin{bmatrix} 1 & -D & \frac{1}{D} \\ 0 & 1 & \frac{1 + D^2}{D} \end{bmatrix}$$

- Now applying $\text{Row 1} \leftarrow \text{Row 1} + D (\text{Row 2})$, we get

$$G(D) = \begin{bmatrix} 1 & 0 & D \\ 0 & 1 & \frac{1 + D^2}{D} \end{bmatrix}$$

D square in the denominator. Now this cannot be realizable. So any denominator term that we have, it has to be of the form 1 plus some polynomial here

(Refer Slide Time 14:13)

Convolutional codes

- Now applying $\text{Row 2} \leftarrow \text{Row 1} + \text{Row 2}$, we get

$$G(D) = \begin{bmatrix} 1 & -D & \frac{1}{D} \\ 0 & D + D^2 & 1 + \frac{D + D^2}{D} \end{bmatrix}$$
- Now applying $\text{Row 2} \leftarrow (\text{Row 2}) / (D + D^2)$, we get

$$G(D) = \begin{bmatrix} 1 & -D & \frac{1}{D} \\ 0 & 1 & \frac{1 + D + D^2}{D^2} \end{bmatrix}$$
- Now applying $\text{Row 1} \leftarrow \text{Row 1} + D (\text{Row 2})$, we get

$$G(D) = \begin{bmatrix} 1 & 0 & D \\ 0 & 1 & \frac{1 + D + D^2}{D^2} \end{bmatrix} \quad 1 + f(D)$$

but here this 1 is not here. So we cannot realize a rational function of this form using our shift register. So this particular equivalent systematic encoder is not realizable. However if we multiply this by D square, what we

(Refer Slide Time 14:33)

Convolutional codes

- iii) Is $G'(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.
- iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row 2} \leftarrow (\text{Row 2})(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

get if we do this transformation, what we get is this. This is no longer, so what we are getting now is basically a new equivalent encoder which is in the feed forward form and it is realizable.

(Refer Slide Time 14:54)

Convolutional codes

iii) Is $G(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row } 2 \leftarrow (\text{Row } 2)(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

• Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

So how do we realize it? Again if we using controller canonical form realization we will have one set of shift registers for this

(Refer Slide Time 15:02)

Convolutional codes

iii) Is $G(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row } 2 \leftarrow (\text{Row } 2)(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

• Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

input, another set of shift registers for this input.

(Refer Slide Time 15:06)

Convolutional codes

iii) Is $G(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row } 2 \leftarrow (\text{Row } 2)(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

- Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

What's the maximum memory for the first row? The maximum power of D is 1. So we will have 1 memory element for the first input and what's the maximum power of D for the second? That's 2 here, 2 here, 2 here

(Refer Slide Time 15:22)

Convolutional codes

iii) Is $G(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row } 2 \leftarrow (\text{Row } 2)(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D^1 \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

- Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

so we will use 2 memory element

(Refer Slide Time 15:25)

Convolutional codes

iii) Is $G(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row } 2 \leftarrow (\text{Row } 2)(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D^1 \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

- Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

for the second input. And again what are our outputs? There are 3 outputs. The first output is this,

(Refer Slide Time 15:34)

Convolutional codes

iii) Is $G(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row } 2 \leftarrow (\text{Row } 2)(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D^1 \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

- Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

this is u_1 times, this is just u_1 so this is this, second one is D^2

(Refer Slide Time 15:42)

Convolutional codes

iii) Is $G(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying Row 2 \leftarrow (Row 2) (D^2) . We get

$$G(D) = \begin{bmatrix} 1 & 0 & D^{-1} \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

- Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

of u_2 . So D^2 of u_2 is just this term.

(Refer Slide Time 15:49)

Convolutional codes

iii) Is $G(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying Row 2 \leftarrow (Row 2) (D^2) . We get

$$G(D) = \begin{bmatrix} 1 & 0 & D^{-1} \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

- Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

So this is my second output and the third

(Refer Slide Time 15:53)

Convolutional codes

iii) Is $G'(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row } 2 \leftarrow (\text{Row } 2)(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D^{-1} \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

- Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

output is this,

(Refer Slide Time 15:56)

Convolutional codes

iii) Is $G'(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row } 2 \leftarrow (\text{Row } 2)(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D^{-1} \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

- Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

D times u_1 D which is this one and one times u_2 D and D square times u_2 D , so that's this.
 This is our third

(Refer Slide Time 16:12)

Convolutional codes

iii) Is $G(D)$ realizable? If not, find an equivalent realizable generator matrix and draw the corresponding minimal encoder realization.

iii) Equivalent systematic encoder is not realizable, so we can make it realizable by applying $\text{Row } 2 \leftarrow (\text{Row } 2)(D^2)$. We get

$$G(D) = \begin{bmatrix} 1 & 0 & D^{-1} \\ 0 & D^2 & 1 + D^2 \end{bmatrix}$$

• Controller canonical form encoder realization of the equivalent encoder is given in Figure 2

Figure: Canonical Form encoder realization of the equivalent encoder

output. Now given

(Refer Slide Time 16:15)

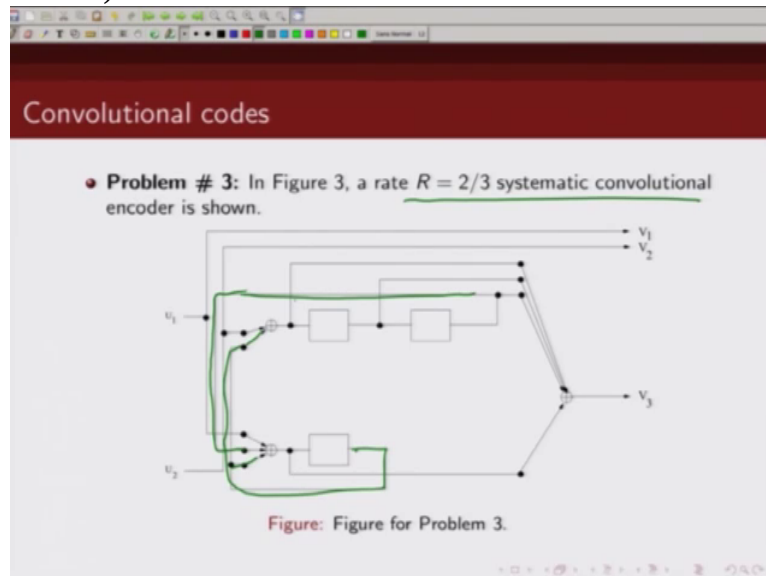
Convolutional codes

• **Problem # 3:** In Figure 3, a rate $R = 2/3$ systematic convolutional encoder is shown.

Figure: Figure for Problem 3.

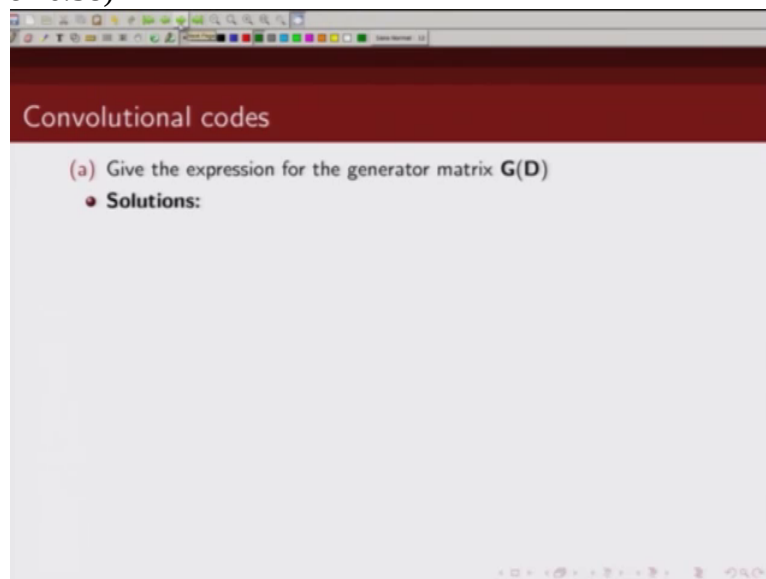
below is a rate two third systematic convolutional encoder. Please note this is neither in the controller canonical form realization or in the observer canonical form realization. Note here the feedback terms that are coming here are not only coming from the same encoders like this, feedback is not only, so if you look at the feedback, feedback from this is going to this encoder and feedback from here is going to this encoder. So not only feedback is coming to the same encoder but it is also going to the other encoder.

(Refer Slide Time 16:55)



So this realization is

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a very compact realization. The question that has been asked is can you find out the generator matrix corresponding to this encoder? So

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Convolutional codes

(a) Give the expression for the generator matrix $\mathbf{G(D)}$

- Solutions:
- We can write

$$v_1(D) = u_1(D)$$
$$v_2(D) = u_2(D)$$
$$v_3(D) = (1 + D + D^2)x(D) + y(D)$$

how do we find the generator matrix? We know this is a relation between the input and the output. So how these

(Refer Slide Time 17:20)

Convolutional codes

(a) Give the expression for the generator matrix $\mathbf{G(D)}$

- Solutions:
- We can write

$$v_1(D) = u_1(D)$$
$$v_2(D) = u_2(D)$$
$$v_3(D) = (1 + D + D^2)x(D) + y(D)$$

$v(D) = \underline{u(D)} G(D)$

inputs are getting mapped to the output, that is governed by this generator matrix.

(Refer Slide Time 17:26)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- **Solutions:**
- We can write

$$v_1(D) = u_1(D)$$

$$v_2(D) = u_2(D)$$

$$v_3(D) = (1 + D + D^2)x(D) + y(D)$$

$v(D) = u(D)G(D)$

So what we are going to do is we are going to write the output $v(D)$ in terms of input $u(D)$. And then that would give us our generator matrix. So our objective is to write v_1 , v_2 , v_3 in terms of u_1 and u_2 ,

(Refer Slide Time 17:50)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- **Solutions:**
- We can write

$$v_1(D) = u_1(D)$$

$$v_2(D) = u_2(D)$$

$$v_3(D) = (1 + D + D^2)x(D) + y(D)$$

$v(D) = u(D)G(D)$

fine. We use some auxiliary variables; x and y which basically will help us find the contents here. So if this is x of D , this term will be D times x of D and this will be D square times x times D .

(Refer Slide Time 18:11)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$v_1(D) = u_1(D)$$

$$v_2(D) = u_2(D)$$

$$v_3(D) = (1 + D + D^2)x(D) + y(D)$$

$v(D) = u(D) G(D)$

Similarly if this is y, this term will be D times y of D.

(Refer Slide Time 18:16)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$v_1(D) = u_1(D)$$

$$v_2(D) = u_2(D)$$

$$v_3(D) = (1 + D + D^2)x(D) + y(D)$$

$v(D) = u(D) G(D)$

So what is v 1 of D? v 1 of D is u 1 of D, you can see u directly goes, this input directly goes here. So v 1

(Refer Slide Time 18:30)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$v_1(D) = u_1(D)$$

$$v_2(D) = u_2(D)$$

$$v_3(D) = (1 + D + D^2)x(D) + y(D)$$

$v(D) = u(D) G(D)$

of D is u_1 of D . Similarly this input u_2 directly goes to the output here.

(Refer Slide Time 18:43)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$v_1(D) = u_1(D)$$

$$v_2(D) = u_2(D)$$

$$v_3(D) = (1 + D + D^2)x(D) + y(D)$$

$v(D) = u(D) G(D)$

So v_2 of D is u_2 of D . Now what is v_3 of D ? v_3 of D is this term which is x of D , this term D times x of D and this term which is D^2 times x of D . So this is this term plus this term. So it is these 3 terms. Now what is this term? This is y of D . So we have

(Refer Slide Time 19:17)

Convolutional codes

(a) Give the expression for the generator matrix $\mathbf{G(D)}$

- Solutions:
- We can write

$$\begin{aligned} v_1(D) &= u_1(D) \\ v_2(D) &= u_2(D) \\ v_3(D) &= (1 + D + D^2)x(D) + y(D) \end{aligned}$$

$v(D) = u(D) G(D)$

written v_1 of D , v_2 of D , v_3 of D in terms of u_1 , u_2 , x of D and y of D . Now note we need to get rid of x of D and

(Refer Slide Time 19:30)

Convolutional codes

(a) Give the expression for the generator matrix $\mathbf{G(D)}$

- Solutions:
- We can write

$$\begin{aligned} v_1(D) &= u_1(D) \\ v_2(D) &= u_2(D) \\ v_3(D) &= (1 + D + D^2)x(D) + y(D) \end{aligned}$$

$v(D) = u(D) G(D)$

y of D , and we have to

(Refer Slide Time 19:32)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$v_1(D) = u_1(D)$$

$$v_2(D) = u_2(D)$$

$$v_3(D) = (1 + D + D^2)x(D) + y(D)$$

$$v(D) = u(D) G(D)$$

write these in terms of u_1 and u_2 . Now what is x of D ? x of D is this and this. Similarly what is y of D ?

(Refer Slide Time 19:44)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$v_1(D) = u_1(D)$$

$$v_2(D) = u_2(D)$$

$$v_3(D) = (1 + D + D^2)x(D) + y(D)$$

$$v(D) = u(D) G(D)$$

y of D is this term, this, this term sorry this term and this term, Ok. So we can write

(Refer Slide Time 20:01)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\begin{aligned} v_1(D) &= u_1(D) \\ v_2(D) &= u_2(D) \\ v_3(D) &= (1 + D + D^2)x(D) + y(D) \end{aligned}$$

$v(D) = u(D) G(D)$

2 more equations

(Refer Slide Time 20:02)

Convolutional codes

- Also,

$$\begin{aligned} y(D) &= u_1(D) + D^2x(D) + Dy(D) \\ x(D) &= u_2(D) + Dy(D) \end{aligned}$$

- Solving for $x(D)$ and $y(D)$, we get

$$\begin{aligned} y(D) &= \frac{1}{1 + D + D^3} u_1(D) + \frac{D^2}{1 + D + D^3} u_2(D) \\ x(D) &= \frac{D}{1 + D + D^3} u_1(D) + \frac{1 + D}{1 + D + D^3} u_2(D) \end{aligned}$$

- Hence, $v_3(D)$ is

$$y(D) = \frac{1 + D + D^2 + D^3}{1 + D + D^3} u_1(D) + \frac{1 + D^2 + D^3}{1 + D + D^3} u_2(D)$$

for x of D and y of D. So again y of D as I said is u 1 of D,

(Refer Slide Time 20:10)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\frac{v_1(D)}{4} = \frac{u_1(D)}{4} \quad \frac{v_2(D)}{4} = \frac{u_2(D)}{4} \quad \frac{v_3(D)}{4} = \frac{(1+D+D^2)x(D) + y(D)}{4}$$

y of D is u 1 of D which is this one, this is u 1 of D

(Refer Slide Time 20:21)

Convolutional codes

- Also,

$$\frac{y(D)}{4} = \frac{u_1(D)}{4} + \frac{D^2x(D)}{4} + \frac{Dy(D)}{4}$$

$$\frac{x(D)}{4} = \frac{u_2(D)}{4} + \frac{Dy(D)}{4}$$

- Solving for $x(D)$ and $y(D)$, we get

$$y(D) = \frac{1}{1+D+D^3}u_1(D) + \frac{D^2}{1+D+D^3}u_2(D)$$

$$x(D) = \frac{D}{1+D+D^3}u_1(D) + \frac{1+D}{1+D+D^3}u_2(D)$$

- Hence, $v_3(D)$ is

$$y(D) = \frac{1+D+D^2+D^3}{1+D+D^3}u_1(D) + \frac{1+D^2+D^3}{1+D+D^3}u_2(D)$$

plus D square x of D.

(Refer Slide Time 20:24)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\frac{v_1(D)}{4} = \frac{u_1(D)}{4} \boxed{G(D)}$$
$$\frac{v_2(D)}{4} = \frac{u_2(D)}{4}$$
$$\frac{v_3(D)}{4} = \frac{(1 + D + D^2)x(D) + y(D)}{4}$$

D square x of D is this term, D square of x of D is this term which is coming here, this term and there is another term

(Refer Slide Time 20:36)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\frac{v_1(D)}{4} = \frac{u_1(D)}{4} \boxed{G(D)}$$
$$\frac{v_2(D)}{4} = \frac{u_2(D)}{4}$$
$$\frac{v_3(D)}{4} = \frac{(1 + D + D^2)x(D) + y(D)}{4}$$

here

(Refer Slide Time 20:38)

Convolutional codes

- Also,

$$\begin{aligned} y(D) &= u_1(D) + D^2x(D) + Dy(D) \\ x(D) &= u_2(D) + Dy(D) \end{aligned}$$

- Solving for $x(D)$ and $y(D)$, we get

$$\begin{aligned} y(D) &= \frac{1}{1+D+D^3}u_1(D) + \frac{D^2}{1+D+D^3}u_2(D) \\ x(D) &= \frac{D}{1+D+D^3}u_1(D) + \frac{1+D}{1+D+D^3}u_2(D) \end{aligned}$$

- Hence, $v_3(D)$ is

$$y(D) = \frac{1+D+D^2+D^3}{1+D+D^3}u_1(D) + \frac{1+D^2+D^3}{1+D+D^3}u_2(D)$$

which is D times y of D. So D

(Refer Slide Time 20:41)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\begin{aligned} v_1(D) &= u_1(D) \\ v_2(D) &= u_2(D) \\ v_3(D) &= (1+D+D^2)x(D) + y(D) \end{aligned}$$

$v(D) = u(D) G(D)$

times y of D, note here, the third input here is this one which is D times y

(Refer Slide Time 20:47)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\begin{aligned} v_1(D) &= u_1(D) \\ v_2(D) &= u_2(D) \\ v_3(D) &= (1 + D + D^2)x(D) + y(D) \end{aligned}$$

$v(D) = u(D) G(D)$

of D. Similarly x of D is, first one is this term which is u of D, so this is u

(Refer Slide Time 20:58)

Convolutional codes

- Also,

$$\begin{aligned} y(D) &= u_1(D) + D^2x(D) + Dy(D) \\ x(D) &= u_2(D) + Dy(D) \end{aligned}$$

- Solving for x(D) and y(D), we get

$$\begin{aligned} y(D) &= \frac{1}{1 + D + D^3} u_1(D) + \frac{D^2}{1 + D + D^3} u_2(D) \\ x(D) &= \frac{D}{1 + D + D^3} u_1(D) + \frac{1 + D}{1 + D + D^3} u_2(D) \end{aligned}$$

- Hence, $v_3(D)$ is

$$y(D) = \frac{1 + D + D^2 + D^3}{1 + D + D^3} u_1(D) + \frac{1 + D^2 + D^3}{1 + D + D^3} u_2(D)$$

of D and the second term is

(Refer Slide Time 21:01)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\frac{v_1(D)}{4} = \frac{u_1(D)}{4} \quad \frac{v_2(D)}{4} = \frac{u_2(D)}{4} \quad \boxed{G(D)}$$

$$\frac{v_1(D)}{4} = \frac{u_1(D)}{4}$$

$$\frac{v_2(D)}{4} = \frac{u_2(D)}{4}$$

$$\frac{v_3(D)}{4} = \frac{(1 + D + D^2)x(D) + y(D)}{4}$$

this term which is D times y of D,

(Refer Slide Time 21:06)

Convolutional codes

- Also,

$$\frac{y(D)}{4} = \frac{u_1(D)}{4} + \frac{D^2x(D)}{4} + \frac{Dy(D)}{4}$$

$$\frac{x(D)}{4} = \frac{u_2(D)}{4} + \frac{Dy(D)}{4}$$

- Solving for $x(D)$ and $y(D)$, we get

$$y(D) = \frac{1}{1 + D + D^3}u_1(D) + \frac{D^2}{1 + D + D^3}u_2(D)$$

$$x(D) = \frac{D}{1 + D + D^3}u_1(D) + \frac{1 + D}{1 + D + D^3}u_2(D)$$

- Hence, $v_3(D)$ is

$$y(D) = \frac{1 + D + D^2 + D^3}{1 + D + D^3}u_1(D) + \frac{1 + D^2 + D^3}{1 + D + D^3}u_2(D)$$

this one, Ok. So now we have got equations of y of D, x of D in terms of u 1 D and u 2 D. So let's write, bring y of D at one side and x of D at one side and write them in terms of, y of D and x of D in terms of u 1 D and u 2 D. So if we solve this, what we get is y of D is given by this and x of D is given by this. Now we plug these values of y of D and x of D given by this into

(Refer Slide Time 21:46)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\underline{v_1(D)} = \underline{u_1(D)}$$

$$\underline{v_2(D)} = \underline{u_2(D)}$$

$$\underline{v_3(D)} = \underline{(1 + D + D^2)x(D) + y(D)}$$

$$\underline{v(D)} = \underline{u(D)} G(D)$$

here, into this expression of

(Refer Slide Time 21:51)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\underline{v_1(D)} = \underline{u_1(D)}$$

$$\underline{v_2(D)} = \underline{u_2(D)}$$

$$\underline{v_3(D)} = \underline{(1 + D + D^2)x(D) + y(D)}$$

$$\underline{v(D)} = \underline{u(D)} G(D)$$

y 3 of D. So we plug this value of x D and y D which we just computed,

(Refer Slide Time 21:58)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\frac{v_1(D)}{4} = \frac{u_1(D)}{4} \quad \frac{v_2(D)}{4} = \frac{u_2(D)}{4} \quad G(D)$$

$$\frac{v_1(D)}{4} = \frac{u_1(D)}{4}$$

$$\frac{v_2(D)}{4} = \frac{u_2(D)}{4}$$

$$\frac{v_3(D)}{4} = \frac{(1 + D + D^2)x(D) + y(D)}{4}$$

we plug those values in here. If we do that, we

(Refer Slide Time 22:03)

Convolutional codes

- Also,

$$\frac{y(D)}{4} = \frac{u_1(D)}{4} + \frac{D^2 x(D)}{4} + \frac{Dy(D)}{4}$$

$$\frac{x(D)}{4} = \frac{u_2(D)}{4} + \frac{Dy(D)}{4}$$

- Solving for $x(D)$ and $y(D)$, we get

$$y(D) = \frac{1}{1 + D + D^3} u_1(D) + \frac{D^2}{1 + D + D^3} u_2(D) \quad \checkmark$$

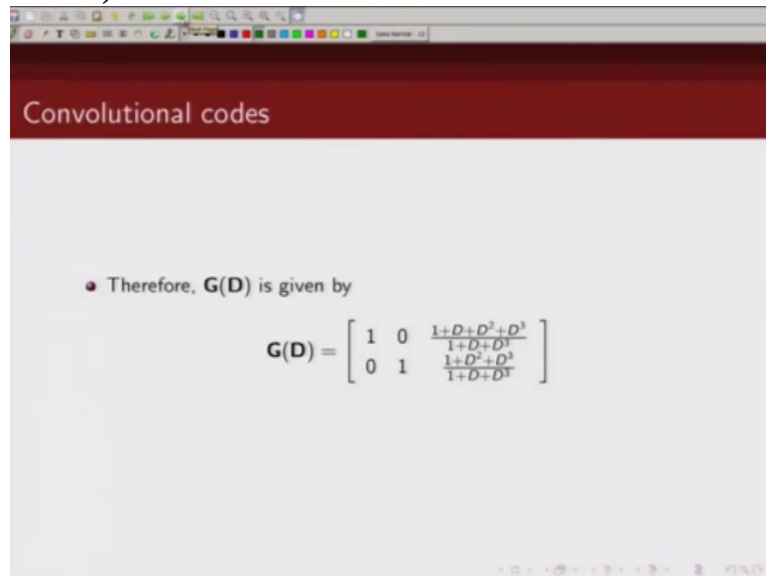
$$x(D) = \frac{D}{1 + D + D^3} u_1(D) + \frac{1 + D}{1 + D + D^3} u_2(D) \quad \checkmark$$

- Hence, $v_3(D)$ is

$$y(D) = \frac{1 + D + D^2 + D^3}{1 + D + D^3} u_1(D) + \frac{1 + D^2 + D^3}{1 + D + D^3} u_2(D)$$

will get the expression of y_3 of D , Ok. Now,

(Refer Slide Time 22:11)



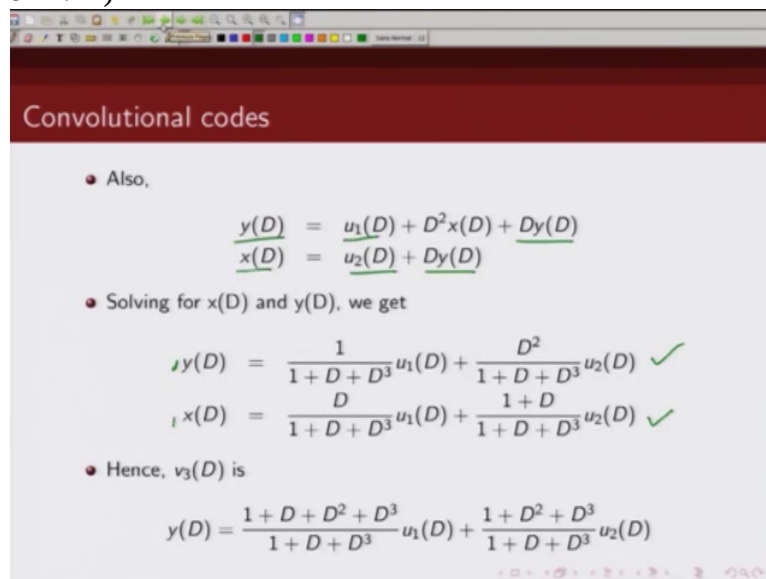
Convolutional codes

- Therefore, $\mathbf{G(D)}$ is given by

$$\mathbf{G(D)} = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

so if we do that finally

(Refer Slide Time 22:14)



Convolutional codes

- Also,

$$\begin{aligned} \underline{y(D)} &= \underline{u_1(D)} + D^2 \underline{x(D)} + \underline{Dy(D)} \\ \underline{x(D)} &= \underline{u_2(D)} + \underline{Dy(D)} \end{aligned}$$

- Solving for $x(D)$ and $y(D)$, we get

$$\begin{aligned} y(D) &= \frac{1}{1+D+D^3} u_1(D) + \frac{D^2}{1+D+D^3} u_2(D) \quad \checkmark \\ x(D) &= \frac{D}{1+D+D^3} u_1(D) + \frac{1+D}{1+D+D^3} u_2(D) \quad \checkmark \end{aligned}$$

- Hence, $v_3(D)$ is

$$y(D) = \frac{1+D+D^2+D^3}{1+D+D^3} u_1(D) + \frac{1+D^2+D^3}{1+D+D^3} u_2(D)$$

this is v_3 of D , so if we do that what we get is then v_3

(Refer Slide Time 22:21)

Convolutional codes

- Also,

$$\begin{aligned} \underline{y(D)} &= \underline{u_1(D)} + D^2 \underline{x(D)} + \underline{Dy(D)} \\ \underline{x(D)} &= \underline{u_2(D)} + \underline{Dy(D)} \end{aligned}$$

- Solving for $x(D)$ and $y(D)$, we get

$$\begin{aligned} y(D) &= \frac{1}{1+D+D^3} u_1(D) + \frac{D^2}{1+D+D^3} u_2(D) \quad \checkmark \\ x(D) &= \frac{D}{1+D+D^3} u_1(D) + \frac{1+D}{1+D+D^3} u_2(D) \quad \checkmark \end{aligned}$$

- Hence, $v_3(D)$ is

$$v_3(D) = \frac{1+D+D^2+D^3}{1+D+D^3} u_1(D) + \frac{1+D^2+D^3}{1+D+D^3} u_2(D)$$

of D is this times u 1 D plus this times u 2 of D. So now we are in a position to

(Refer Slide Time 22:30)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:**
- We can write

$$\begin{aligned} v_1(D) &= u_1(D) \\ v_2(D) &= u_2(D) \\ v_3(D) &= (1+D+D^2)x(D) + y(D) \end{aligned}$$

$v(D) = u(D) G(D)$

write the generator matrix. The first equation that we will require is this one. Second equation we will require is this one.

(Refer Slide Time 22:39)

Convolutional codes

(a) Give the expression for the generator matrix $G(D)$

- Solutions:
- We can write

$$\underline{v_1(D)} = \underline{u_1(D)} \quad \text{--- (1)}$$

$$\underline{v_2(D)} = \underline{u_2(D)} \quad \text{--- (2)}$$

$$\underline{v_3(D)} = \underline{(1 + D + D^2)x(D) + y(D)}$$

$$\underline{v(D)} = \underline{u(D)} \underline{G(D)}$$

And the third

(Refer Slide Time 22:40)

Convolutional codes

- Also,

$$\underline{y(D)} = \underline{u_1(D) + D^2x(D) + Dy(D)}$$

$$\underline{x(D)} = \underline{u_2(D) + Dy(D)}$$

- Solving for $x(D)$ and $y(D)$, we get

$$y(D) = \frac{1}{1 + D + D^3} u_1(D) + \frac{D^2}{1 + D + D^3} u_2(D) \quad \checkmark$$

$$x(D) = \frac{D}{1 + D + D^3} u_1(D) + \frac{1 + D}{1 + D + D^3} u_2(D) \quad \checkmark$$

- Hence, $v_3(D)$ is

$$\underline{v_3(D)} = \underline{\frac{1 + D + D^2 + D^3}{1 + D + D^3} u_1(D) + \frac{1 + D^2 + D^3}{1 + D + D^3} u_2(D)}$$

equation that we will require is this one,

(Refer Slide Time 22:43)

Convolutional codes

- Also,
$$\begin{aligned} \underline{y(D)} &= \underline{u_1(D)} + D^2 \underline{x(D)} + \underline{Dy(D)} \\ \underline{x(D)} &= \underline{u_2(D)} + \underline{Dy(D)} \end{aligned}$$
- Solving for $x(D)$ and $y(D)$, we get
$$\begin{aligned} y(D) &= \frac{1}{1+D+D^3} u_1(D) + \frac{D^2}{1+D+D^3} u_2(D) \quad \checkmark \\ x(D) &= \frac{D}{1+D+D^3} u_1(D) + \frac{1+D}{1+D+D^3} u_2(D) \quad \checkmark \end{aligned}$$
- Hence, $v_3(D)$ is
$$y_3(D) = \frac{1+D+D^2+D^3}{1+D+D^3} u_1(D) + \frac{1+D^2+D^3}{1+D+D^3} u_2(D) \quad \text{---} \textcircled{3}$$

right. So

(Refer Slide Time 22:44)

Convolutional codes

- Therefore, $\mathbf{G(D)}$ is given by
$$\mathbf{G(D)} = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

you can think of it as like this, so we have 3

(Refer Slide Time 22:49)

Convolutional codes

(a) Give the expression for the generator matrix $\mathbf{G(D)}$

- Solutions:
- We can write

$$\underline{v_1(D)} = \underline{u_1(D)} \quad \text{--- (1)}$$

$$\underline{v_2(D)} = \underline{u_2(D)} \quad \text{--- (2)}$$

$$\underline{v_3(D)} = \underline{(1 + D + D^2)x(D) + y(D)}$$

$$\underline{v(D)} = \underline{u(D)} \underline{G(D)}$$

output, v_1 , v_2 , v_3 , 2 input u_1 , u_2 , so we are writing

(Refer Slide Time 22:56)

Convolutional codes

- Therefore, $\mathbf{G(D)}$ is given by

$$\mathbf{G(D)} = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

$v_1(D)$, $v_2(D)$, $v_3(D)$ in terms of $u_1(D)$, $u_2(D)$ and this G matrix.

(Refer Slide Time 23:13)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

So what is $v_1(D)$? $v_1(D)$ is $u_1(D)$. So then our G matrix here, again G matrix is 2 cross 3, so $v_1(D)$ is $u_1(D)$, so we get 1 0. $v_2(D)$ is $u_2(D)$, so we get 0 1, and what is $v_3(D)$, $v_3(D)$ is this, this times

(Refer Slide Time 23:42)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

$u_1(D)$ and this times $u_2(D)$.

(Refer Slide Time 23:46)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$
$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

So this will be our final generator matrix corresponding to the encoder that is shown in this

(Refer Slide Time 23:58)

Convolutional codes

- Problem # 3:** In Figure 3, a rate $R = 2/3$ systematic convolutional encoder is shown.

Figure: Figure for Problem 3.

figure, Ok. Now

(Refer Slide Time 24:03)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

the next question is can we realize this encoder in the controller canonical form? So the answer is yes, we can realize it.

(Refer Slide Time 24:14)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [v_1(D) \ v_2(D)] G$$

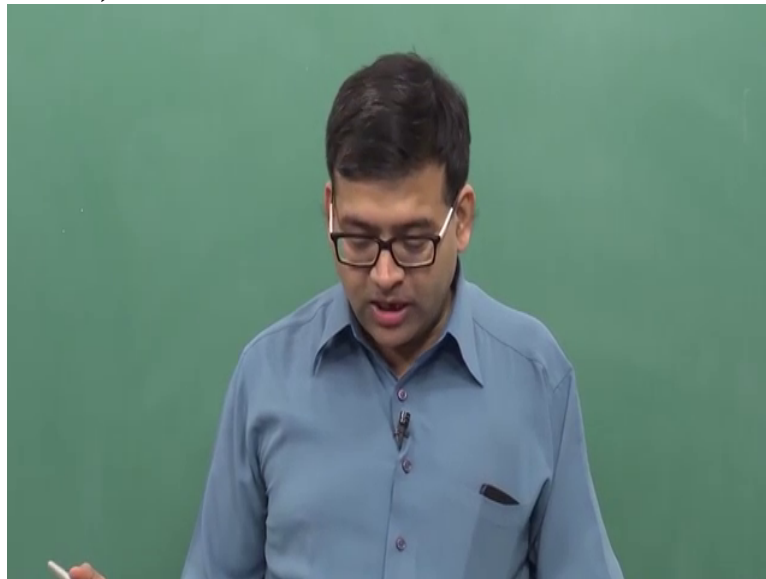
$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

We can have the expression for generator matrix. So to realize it in controller

(Refer Slide Time 24:22)



canonical form again, so there is one

(Refer Slide Time 24:25)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$
$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

• Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

set of shift registers for each input, so this is one input, this is second input right. Please note this is a feedback polynomial so we would require a feedback polynomial and now maximum degree here is 3, maximum degree here also is 3,so

(Refer Slide Time 24:46)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

we will require 2 set of shift registers, first one is this one. Please note this as

(Refer Slide Time 24:58)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

3 memory elements and similarly second shift register, this also has 3 memory elements.

(Refer Slide Time 25:05)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

That is because the maximum degree

(Refer Slide Time 25:08)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^4}{1+D+D^3} \end{bmatrix}$$

of this rational function is 3 and similarly maximum degree of this rational function is 3. And we just implement this.

(Refer Slide Time 25:20)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.




Figure: Answer to Problem 3(b)

So v_1 , v_2 is just u_1 , so that's just this.

(Refer Slide Time 25:26)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

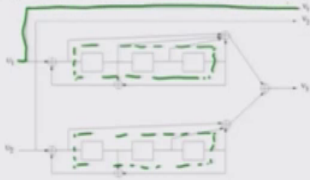


Figure: Answer to Problem 3(b)

v_2 is u_2 , that's just this.

(Refer Slide Time 25:32)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

What is

(Refer Slide Time 25:35)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

$v_3(D)$? $v_3(D)$ is $1 + D + D^2 + D^3$ divided by $1 + D + D^3$ u $1 + D$, plus $1 + D^2 + D^3$ u $2 + D$,

(Refer Slide Time 25:59)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

$$v_3(D) = \frac{1+D+D^2+D^3}{1+D+D^3} u_1(D) + \frac{1+D^2+D^3}{1+D+D^3} u_2(D)$$

right? So relationship between v_3 and u_1 D is given by this. So let's

(Refer Slide Time 26:07)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

$$v_3(D) = \frac{1+D+D^2+D^3}{1+D+D^3} u_1(D) + \frac{1+D^2+D^3}{1+D+D^3} u_2(D)$$

implement this. So numerator has 1 plus D plus D square plus D cube. So you can see here,

(Refer Slide Time 26:14)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

this is my 1, this is my D, this is my D square, this is my D cube.

(Refer Slide Time 26:24)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

And similarly the denominator has 1

(Refer Slide Time 26:26)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

$$v_3(D) = \frac{1+D+D^2+D^3}{1+D+D^3} u_1(D) + \frac{1+D^2+D^3}{1+D+D^3} u_2(D)$$

plus D plus D cube. So

(Refer Slide Time 26:31)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

the denominator, this is the 1 term, this is the D term; this is the D cube term. So this part is implemented.

(Refer Slide Time 26:43)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

Next is this. Following the same procedure

(Refer Slide Time 26:52)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

$$v_3(D) = \frac{1+D+D^2+D^3}{1+D+D^3} u_1(D) + \frac{1+D^2+D^3}{1+D+D^3} u_2(D)$$

we can find out the mapping between $u_2(D)$ and $v_3(D)$. The feed forward connections are $1, D^2$ and D^3 . So then this is

(Refer Slide Time 27:03)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

1, this is D no connection, D square is this and D cube is this.

(Refer Slide Time 27:11)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

(Refer Slide Time 27:12)

Convolutional codes

$$[v_1(D) \ v_2(D) \ v_3(D)] = [u_1(D) \ u_2(D)] G$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Therefore, $G(D)$ is given by

$$G(D) = \begin{bmatrix} 1 & 0 & \frac{1+D+D^2+D^3}{1+D+D^3} \\ 0 & 1 & \frac{1+D+D^2+D^3}{1+D+D^3} \end{bmatrix}$$

$$v_3(D) = \frac{1+D+D^2+D^3}{1+D+D^3} u_1(D) + \frac{1+D+D^2+D^3}{1+D+D^3} u_2(D)$$

Similarly the feedback connections are 1, D and D cube. So

(Refer Slide Time 27:18)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

the feedback connections; this is the 1, this is D and this is D cube. And

(Refer Slide Time 27:28)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

v 3 is the combination of these 2. So this is my v 3.

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Convolutional codes

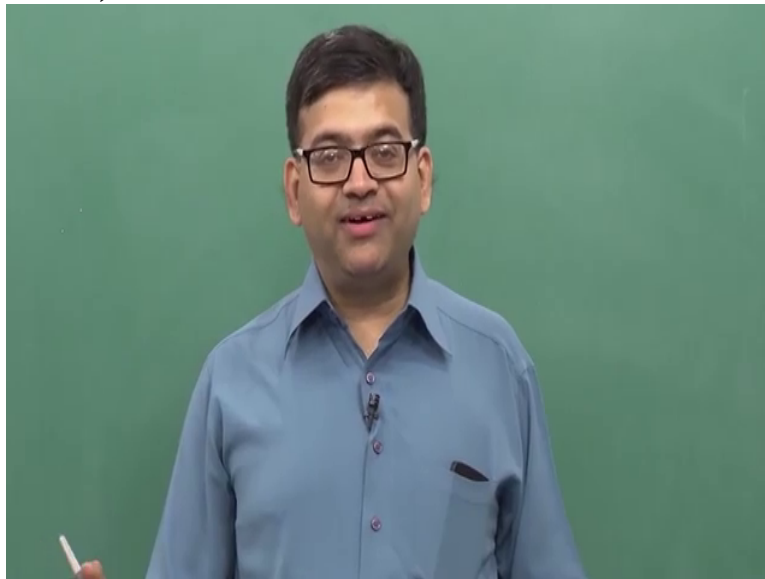
(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

So I hope this is clear how we can realize

(Refer Slide Time 27:39)



this encoder using controller canonical form realization.

Now the next

(Refer Slide Time 27:45)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

The diagram shows a convolutional encoder in controller canonical form. It consists of two parallel stages, each with two delay elements (represented by boxes with 'z' inside). The top stage has two inputs, x_1 and x_2 , and two outputs, y_1 and y_2 . The bottom stage has two inputs, x_1 and x_2 , and two outputs, y_1 and y_2 . The diagram is drawn with green lines.

Figure: Answer to Problem 3(b)

question is how many termination bits are required to bring this encoder back to all zero state? Now what does termination means? Termination means

(Refer Slide Time 27:55)



we are bringing this encoder back to all zero state. So no matter what the state is, if you want to bring them back

(Refer Slide Time 28:06)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

Figure: Answer to Problem 3(b)

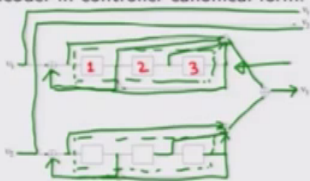
to the all zero state, the number of termination bits required is equal to how many memory elements we have. So in the controller canonical form realization to bring this shift register, the first shift register you want to bring it to all zero state, we would require 3 bits because we have 3 memory elements here, 1, 2 and 3. Similarly

(Refer Slide Time 28:32)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.



The diagram shows a convolutional encoder in controller canonical form. It consists of two parallel shift registers. The top shift register has three stages labeled 1, 2, and 3. The bottom shift register has three stages labeled 4, 5, and 6. The input x_1 is fed into both shift registers. The output y_1 is the sum of the outputs of the two shift registers. The output y_2 is the output of the top shift register. The output y_3 is the output of the bottom shift register. The diagram is annotated with green lines and arrows.

Figure: Answer to Problem 3(b)

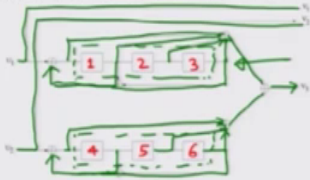
for this shift register we require additional 3 bits, so 4, 5, 6 so

(Refer Slide Time 28:40)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.



The diagram shows a convolutional encoder in controller canonical form. It consists of two parallel shift registers. The top shift register has three stages labeled 1, 2, and 3. The bottom shift register has three stages labeled 4, 5, and 6. The input x_1 is fed into both shift registers. The output y_1 is the sum of the outputs of the two shift registers. The output y_2 is the output of the top shift register. The output y_3 is the output of the bottom shift register. The diagram is annotated with green lines and arrows.

Figure: Answer to Problem 3(b)

total we require 6 termination bits, 3 to terminate this encoder and 3 to terminate this encoder.
So we require

(Refer Slide Time 28:51)

Convolutional codes

(b) Realize the convolution encoder in controller canonical form. How many termination bits are required to terminate the encoder for controller canonical form realization.

- Convolutional encoder in controller canonical form.

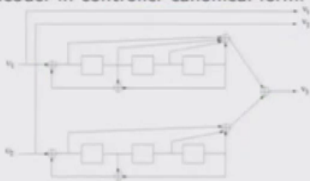


Figure: Answer to Problem 3(b)

- The controller canonical form realization of this encoder uses 6 memory elements, so the encoder will require 6 termination bits to return to all-zero state.

6 termination bits.

(Refer Slide Time 28:54)

Convolutional codes

- Problem # 4:**
 - Show that the a-priori probability can be written in this form
$$P(u_i = \pm 1) = A_i \exp^{u_i L_i(u_i)/2}$$
where $L_i(\cdot)$ is the a-priori L-values of the information bits.

Finally let's come to the B C J R algorithm that we talked about. So the first question is can you write

(Refer Slide Time 29:03)

Convolutional codes

• **Problem # 4:**

(a) Show that the a-priori probability can be written in this form

$$P(u_i = \pm 1) = A_i \exp^{u_i L_s(u_i)/2}$$

where $L_s(\cdot)$ is the a-priori L-values of the information bits.

(b) Using this result, show that the branch metrics $\gamma^*(s, s')$ for a continuous-output AWGN channel (in log-domain) can be written as

$$\gamma^*(s, s') = \frac{u_i L_s(u_i)}{2} + L_c r_i \cdot v_i$$

where $L_c = 4E_s/N_0$ is the channel reliability factor. Notations are the same as used in class lectures.

the a priori probability in this particular form, and also the branch metric in log domain, can it be written in this particular form? Now u_i is my input, L_s is the a p p value for the a priori inputs, L_c is the reliability factor which is given by 4 times E_s by N_0 ,

(Refer Slide Time 29:28)

Convolutional codes

• **Problem # 4:**

(a) Show that the a-priori probability can be written in this form

$$P(u_i = \pm 1) = A_i \exp^{u_i L_s(u_i)/2}$$

where $L_s(\cdot)$ is the a-priori L-values of the information bits.

(b) Using this result, show that the branch metrics $\gamma^*(s, s')$ for a continuous-output AWGN channel (in log-domain) can be written as

$$\gamma^*(s, s') = \frac{u_i L_s(u_i)}{2} + L_c r_i \cdot v_i$$

where $L_c = 4E_s/N_0$ is the channel reliability factor. Notations are the same as used in class lectures.

other notations are same as which are used in the lecture. v is codeword, r is received sequence so can we write these in terms like this. So let's look at it.

(Refer Slide Time 29:46)

Convolutional codes

- **Solutions:** We can write

$$\begin{aligned}
 P(u_l = \pm 1) &= \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}} \\
 &= \frac{e^{\pm L_s(u_l)}}{\{1 + e^{\pm L_s(u_l)}\}} \\
 &= \frac{e^{-L_s(u_l)/2}}{\{1 + e^{-L_s(u_l)}\}} e^{u_l L_s(u_l)/2} \\
 &= A_l e^{u_l L_s(u_l)/2},
 \end{aligned}$$

So what's the probability of u being plus 1 or minus 1? Let's take like plus 1, let's say what's the prob, what's the probability that u l is plus 1? Now this can be written as this by 1.

(Refer Slide Time 29:59)

Convolutional codes

- **Solutions:** We can write

$$\begin{aligned}
 \underline{P(u_l = \pm 1)} &= \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}} \\
 \underline{P(u_l = +1)} &= \frac{e^{\pm L_s(u_l)}}{\{1 + e^{\pm L_s(u_l)}\}} \\
 &= \frac{e^{-L_s(u_l)/2}}{\{1 + e^{-L_s(u_l)}\}} e^{u_l L_s(u_l)/2} \\
 &= A_l e^{u_l L_s(u_l)/2},
 \end{aligned}$$

So I can write as probability of u l being plus 1 divided by probability u l is plus 1 plus probability u l is minus 1, I can write it

(Refer Slide Time 30:13)

Convolutional codes

• **Solutions:** We can write

$$\begin{aligned}
 \underline{P(u_l = \pm 1)} &= \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}} \\
 \frac{P(u_l = +1)}{1} &= \frac{e^{\pm L_s(u)}}{\{1 + e^{\pm L_s(u)}\}} \\
 \frac{P(u_l = +1)}{P(u_l = +1) + P(u_l = -1)} &= \frac{e^{-L_s(u)/2}}{\{1 + e^{-L_s(u)}\}} e^{u_l L_s(u)/2} \\
 &= A_l e^{u_l L_s(u)/2},
 \end{aligned}$$

this way, right? And if I divide by probability of u l being minus 1, then what I get is probability being plus 1 by probability of u l being minus 1, 1 plus probability of u l being plus 1, probability of u l minus 1.

(Refer Slide Time 30:44)

Convolutional codes

• **Solutions:** We can write

$$\begin{aligned}
 \underline{P(u_l = \pm 1)} &= \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}} \\
 \frac{P(u_l = +1)}{1} &= \frac{e^{\pm L_s(u)}}{\{1 + e^{\pm L_s(u)}\}} \\
 \frac{P(u_l = +1)}{P(u_l = +1) + P(u_l = -1)} &= \frac{e^{-L_s(u)/2}}{\{1 + e^{-L_s(u)}\}} e^{u_l L_s(u)/2} \\
 &= A_l e^{u_l L_s(u)/2},
 \end{aligned}$$

So this is what I will get of the form here. You can see here, the form for, when u l is plus 1, I get probability of u l being plus 1, I get in this particular form. Now let's look at what's the probability that u l is minus 1. Again I can follow the same procedure. I can write this same as this by 1 or I can write the probability of u l is equal to minus 1 divided by probability of u l being plus 1 plus probability of u l being minus 1 and

(Refer Slide Time 31:23)

Convolutional codes

• Solutions: We can write

$$P(u_l = \pm 1) = \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L_s(u_l)}}{\{1 + e^{\pm L_s(u_l)}\}}$$

$$= \frac{e^{-L_s(u_l)/2}}{\{1 + e^{-L_s(u_l)}\}} e^{u_l L_s(u_l)/2}$$

$$= A_l e^{u_l L_s(u_l)/2}$$

Handwritten notes on the slide include:

- $\frac{P(u_l = +1)}{1}$
- $\frac{P(u_l = +1)}{P(u_l = +1) + P(u_l = -1)}$
- $\frac{P(u_l = +1)}{P(u_l = -1)}$
- $1 + \frac{P(u_l = +1)}{P(u_l = -1)}$
- $\frac{P(u_l = -1)}{1} = \frac{P(u_l = -1)}{P(u_l = +1) + P(u_l = -1)}$

I can divide this by probability of u l being 1, so this will be probability of u l equal to minus 1 by probability of u l being plus 1, 1 plus probability of u l being minus 1, probability of u l being plus 1, right?

(Refer Slide Time 31:49)

Convolutional codes

• Solutions: We can write

$$P(u_l = \pm 1) = \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L_s(u_l)}}{\{1 + e^{\pm L_s(u_l)}\}}$$

$$= \frac{e^{-L_s(u_l)/2}}{\{1 + e^{-L_s(u_l)}\}} e^{u_l L_s(u_l)/2}$$

$$= A_l e^{u_l L_s(u_l)/2}$$

Handwritten notes on the slide include:

- $\frac{P(u_l = +1)}{1}$
- $\frac{P(u_l = +1)}{P(u_l = +1) + P(u_l = -1)}$
- $\frac{P(u_l = +1)}{P(u_l = -1)}$
- $1 + \frac{P(u_l = +1)}{P(u_l = -1)}$
- $\frac{P(u_l = -1)}{1} = \frac{P(u_l = -1)}{P(u_l = +1) + P(u_l = -1)}$
- $\frac{P(u_l = -1)}{P(u_l = +1)}$
- $1 + \frac{P(u_l = -1)}{P(u_l = +1)}$

So this I can also write as this is equal to probability of u l being plus 1 by probability of u l being minus 1 raised to power minus 1 and this is 1 plus the same thing, raised to power minus 1. So if I combine this and this what I get

(Refer Slide Time 32:15)

Convolutional codes

• Solutions: We can write

$$P(u_l = \pm 1) = \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L_s(u_l)}}{\{1 + e^{\pm L_s(u_l)}\}}$$

$$= \frac{e^{-L_s(u_l)/2}}{\{1 + e^{-L_s(u_l)}\}} e^{u_l L_s(u_l)/2}$$

$$= A_l e^{u_l L_s(u_l)/2}$$

$$= \left[\frac{P(u_k = +1)}{P(u_k = -1)} \right]^{-1}$$

$$= \frac{1}{1 + \left[\frac{P(u_k = +1)}{P(u_k = -1)} \right]^{-1}}$$

Handwritten notes on the slide include:

- $\frac{P(u_1 = +1)}{1}$
- $\frac{P(u_1 = +1)}{P(u_2 = +1) + P(u_2 = -1)}$
- $\frac{P(u_k = +1)}{P(u_k = -1)}$
- $1 + \frac{P(u_k = +1)}{P(u_k = -1)}$
- $\frac{P(u_k = -1)}{1}$
- $\frac{P(u_k = -1)}{P(u_k = +1) + P(u_k = -1)}$
- $1 + \frac{P(u_k = -1)}{P(u_k = +1)}$

is the first step here, Ok.

(Refer Slide Time 32:20)

Convolutional codes

• Solutions: We can write

$$P(u_l = \pm 1) = \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L_s(u_l)}}{\{1 + e^{\pm L_s(u_l)}\}}$$

$$= \frac{e^{-L_s(u_l)/2}}{\{1 + e^{-L_s(u_l)}\}} e^{u_l L_s(u_l)/2}$$

$$= A_l e^{u_l L_s(u_l)/2}$$

$$= \left[\frac{P(u_k = +1)}{P(u_k = -1)} \right]^{-1}$$

$$= \frac{1}{1 + \left[\frac{P(u_k = +1)}{P(u_k = -1)} \right]^{-1}}$$

Handwritten notes on the slide include:

- $\frac{P(u_1 = +1)}{1}$
- $\frac{P(u_1 = +1)}{P(u_2 = +1) + P(u_2 = -1)}$
- $\frac{P(u_k = +1)}{P(u_k = -1)}$
- $1 + \frac{P(u_k = +1)}{P(u_k = -1)}$
- $\frac{P(u_k = -1)}{1}$
- $\frac{P(u_k = -1)}{P(u_k = +1) + P(u_k = -1)}$
- $1 + \frac{P(u_k = -1)}{P(u_k = +1)}$

I can write by combining this and this, I will get this. Now if I write these ratios of probabilities in terms of L values, so what is this L value of u l, this is log of probability of u l being plus 1 by probability of u l being minus 1. So this can be then written as

(Refer Slide Time 32:50)

Convolutional codes

$$L_a(u_i) = \log \frac{P(u_i = +1)}{P(u_i = -1)}$$

• Solutions: We can write

$$P(u_i = \pm 1) = \frac{[P(u_i = +1)/P(u_i = -1)]^{\pm 1}}{\{1 + [P(u_i = +1)/P(u_i = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L_a(u_i)}}{\{1 + e^{\pm L_a(u_i)}\}}$$

$$= \frac{e^{-L_a(u_i)/2}}{\{1 + e^{-L_a(u_i)}\}} e^{u_i L_a(u_i)/2} = A_i e^{u_i L_a(u_i)/2}$$

$$= \frac{[P(u_i = +1)]^{-1}}{1 + [P(u_i = +1)/P(u_i = -1)]^{-1}}$$

Handwritten notes on the slide include:

- $\frac{P(u_i = +1)}{1}$
- $\frac{P(u_i = +1)}{P(u_i = -1)}$
- $\frac{P(u_i = +1) + P(u_i = -1)}{1 + \frac{P(u_i = +1)}{P(u_i = -1)}}$
- $\frac{P(u_i = -1)}{1}$
- $\frac{P(u_i = -1)}{P(u_i = +1)}$
- $\frac{P(u_i = -1) + P(u_i = +1)}{1 + \frac{P(u_i = -1)}{P(u_i = +1)}}$

e raised to power L a u i. If I do that, if I plug this in

(Refer Slide Time 33:01)

Convolutional codes

$$L_a(u_i) = \log \frac{P(u_i = +1)}{P(u_i = -1)}$$

• Solutions: We can write

$$P(u_i = \pm 1) = \frac{[P(u_i = +1)/P(u_i = -1)]^{\pm 1}}{\{1 + [P(u_i = +1)/P(u_i = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L_a(u_i)}}{\{1 + e^{\pm L_a(u_i)}\}}$$

$$= \frac{e^{-L_a(u_i)/2}}{\{1 + e^{-L_a(u_i)}\}} e^{u_i L_a(u_i)/2} = A_i e^{u_i L_a(u_i)/2}$$

$$= \frac{[P(u_i = +1)]^{-1}}{1 + [P(u_i = +1)/P(u_i = -1)]^{-1}}$$

Handwritten notes on the slide include:

- $\frac{P(u_i = +1)}{1}$
- $\frac{P(u_i = +1)}{P(u_i = -1)}$
- $\frac{P(u_i = +1) + P(u_i = -1)}{1 + \frac{P(u_i = +1)}{P(u_i = -1)}}$
- $\frac{P(u_i = -1)}{1}$
- $\frac{P(u_i = -1)}{P(u_i = +1)}$
- $\frac{P(u_i = -1) + P(u_i = +1)}{1 + \frac{P(u_i = -1)}{P(u_i = +1)}}$

first line what I get here is this term, Ok.

(Refer Slide Time 33:06)

Convolutional codes

$L_x(u_i) = \log \frac{P(u_i = +1)}{P(u_i = -1)}$

• Solutions: We can write

$$P(u_l = \pm 1) = \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L_x(u_l)}}{\{1 + e^{\pm L_x(u_l)}\}}$$

$$= \frac{e^{-L_x(u_l)/2}}{\{1 + e^{-L_x(u_l)}\}} e^{u_l L_x(u_l)/2} = A_l e^{u_l L_x(u_l)/2}$$

$$= \frac{[P(u_l = +1)]^{-1}}{1 + [P(u_l = +1)]^{-1}}$$

Handwritten notes on the slide include:

- $\frac{P(u_l = +1)}{1}$
- $\frac{P(u_l = +1)}{P(u_l = -1)}$
- $\frac{P(u_l = +1) + P(u_l = -1)}{1 + \frac{P(u_l = +1)}{P(u_l = -1)}}$
- $\frac{P(u_l = -1)}{1}$
- $\frac{P(u_l = -1)}{P(u_l = +1) + P(u_l = -1)}$
- $\frac{P(u_l = -1)}{1 + \frac{P(u_l = -1)}{P(u_l = +1)}}$

Now note I can further simplify this into this expression.

(Refer Slide Time 33:16)

Convolutional codes

$L_x(u_i) = \log \frac{P(u_i = +1)}{P(u_i = -1)}$

• Solutions: We can write

$$P(u_l = \pm 1) = \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L_x(u_l)}}{\{1 + e^{\pm L_x(u_l)}\}}$$

$$= \frac{e^{-L_x(u_l)/2}}{\{1 + e^{-L_x(u_l)}\}} e^{u_l L_x(u_l)/2} = A_l e^{u_l L_x(u_l)/2}$$

$$= \frac{[P(u_l = +1)]^{-1}}{1 + [P(u_l = +1)]^{-1}}$$

Handwritten notes on the slide include:

- $\frac{P(u_l = +1)}{1}$
- $\frac{P(u_l = +1)}{P(u_l = -1)}$
- $\frac{P(u_l = +1) + P(u_l = -1)}{1 + \frac{P(u_l = +1)}{P(u_l = -1)}}$
- $\frac{P(u_l = -1)}{1}$
- $\frac{P(u_l = -1)}{P(u_l = +1) + P(u_l = -1)}$
- $\frac{P(u_l = -1)}{1 + \frac{P(u_l = -1)}{P(u_l = +1)}}$

You can see when u l is plus 1, when u l is plus 1, what do we get? When u l is plus 1,

(Refer Slide Time 33:26)

Convolutional codes

$u_k = +1$

$$L_a(u_k) = \log \frac{P(u_k = +1)}{P(u_k = -1)}$$

• Solutions: We can write

$$P(u_l = \pm 1) = \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}}$$

$$\frac{P(u_l = +1)}{1} = \frac{e^{\pm L_a(u_l)}}{\{1 + e^{\pm L_a(u_l)}\}}$$

$$\frac{P(u_l = +1)}{P(u_l = +1) + P(u_l = -1)} = \frac{e^{-L_a(u_l)/2}}{\{1 + e^{-L_a(u_l)}\}} e^{u_l L_a(u_l)/2}$$

$$= A_l e^{u_l L_a(u_l)/2}$$

$$= \frac{[P(u_k = +1)]^{-1}}{1 + [P(u_k = +1)]^{-1}}$$

$$= \frac{P(u_k = -1)}{1 + \frac{P(u_k = -1)}{P(u_k = +1)}}$$

this is e raised to power L a u l by 2 and e raised to power minus L a by 2. So this will be basically 1 so this will be 1 times 1 plus e raised to power minus L a u which can be

(Refer Slide Time 33:44)

Convolutional codes

$u_k = +1$

$$L_a(u_k) = \log \frac{P(u_k = +1)}{P(u_k = -1)}$$

• Solutions: We can write

$$P(u_l = \pm 1) = \frac{[P(u_l = +1)/P(u_l = -1)]^{\pm 1}}{\{1 + [P(u_l = +1)/P(u_l = -1)]^{\pm 1}\}}$$

$$\frac{P(u_l = +1)}{1} = \frac{e^{\pm L_a(u_l)}}{\{1 + e^{\pm L_a(u_l)}\}}$$

$$\frac{P(u_l = +1)}{P(u_l = +1) + P(u_l = -1)} = \frac{e^{-L_a(u_l)/2}}{\{1 + e^{-L_a(u_l)}\}} e^{u_l L_a(u_l)/2}$$

$$= A_l e^{u_l L_a(u_l)/2}$$

$$= \frac{[P(u_k = +1)]^{-1}}{1 + [P(u_k = +1)]^{-1}}$$

$$= \frac{P(u_k = -1)}{1 + \frac{P(u_k = -1)}{P(u_k = +1)}}$$

written as e L a u 1 plus e L a u. This is

(Refer Slide Time 33:52)

Convolutional codes

$u_k = +1 \Rightarrow \frac{1}{1 + e^{-L(u)}}$
 $= \frac{e^{L(u)}}{1 + e^{L(u)}}$

• Solutions: We can write

$$P(u_i = \pm 1) = \frac{[P(u_i = +1)/P(u_i = -1)]^{\pm 1}}{\{1 + [P(u_i = +1)/P(u_i = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L_s(u)}}{\{1 + e^{\pm L_s(u)}\}}$$

$$= \frac{e^{-L_s(u)/2}}{\{1 + e^{-L_s(u)}\}} e^{u_i L_s(u)/2}$$

$$= A_i e^{u_i L_s(u)/2}$$

Additional notes and equations:

- $L_s(u) = \log \frac{P(u_k = +1)}{P(u_k = -1)}$
- $\frac{P(u_i = +1)}{P(u_i = -1)} = \frac{P(u_k = -1)}{P(u_k = +1) + P(u_k = -1)}$
- $\frac{P(u_k = +1)}{P(u_k = -1)}$
- $1 + \frac{P(u_k = +1)}{P(u_k = -1)}$
- $\frac{P(u_k = +1)}{P(u_k = -1)}$
- $\frac{P(u_k = -1)}{P(u_k = +1)}$
- $1 + \frac{P(u_k = -1)}{P(u_k = +1)}$
- $\left[\frac{P(u_k = +1)}{P(u_k = -1)} \right]^{-1}$
- $1 + \left[\right]^{-1}$

precisely what I have written here. And if u_i is minus 1, this will be e raised to power minus $L_s(u)$ by 2 and e raised to power $L_s(u)$ by 2. So this term will become, in that case, e raised to power minus $L_s(u)$ by 2 plus e raised to power $L_s(u)$ by 2. So this term can be written in terms of this, right? And what is this term? What is this term? This I can simplify this term, let's make some space. I can simplify this term as e raised to power minus $L_s(u)$ by 2 and I have e raised to power $L_s(u)$ by 2, this is e raised to power $L_s(u)$ by 2 plus e raised to power minus $L_s(u)$ by 2.

(Refer Slide Time 35:03)

Convolutional codes

$u_k = +1 \Rightarrow \frac{1}{1 + e^{-L(u)}}$
 $= \frac{e^{L(u)}}{1 + e^{L(u)}}$

• Solutions: We can write

$$P(u_i = \pm 1) = \frac{[P(u_i = +1)/P(u_i = -1)]^{\pm 1}}{\{1 + [P(u_i = +1)/P(u_i = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L_s(u)}}{\{1 + e^{\pm L_s(u)}\}}$$

$$= \frac{e^{-L_s(u)/2}}{\{1 + e^{-L_s(u)}\}} e^{u_i L_s(u)/2}$$

$$= A_i e^{u_i L_s(u)/2}$$

Additional notes and equations:

- $L_s(u) = \log \frac{P(u_k = +1)}{P(u_k = -1)}$
- $\frac{P(u_i = +1)}{P(u_i = -1)} = \frac{P(u_k = -1)}{P(u_k = +1) + P(u_k = -1)}$
- $\frac{P(u_k = +1)}{P(u_k = -1)}$
- $1 + \frac{P(u_k = +1)}{P(u_k = -1)}$
- $\frac{P(u_k = -1)}{P(u_k = +1)}$
- $1 + \frac{P(u_k = -1)}{P(u_k = +1)}$
- $\left[\frac{P(u_k = +1)}{P(u_k = -1)} \right]^{-1}$
- $1 + \left[\right]^{-1}$

So what I am doing here is I am writing this particular term. So this I can write as this and this. So this cancels out.

(Refer Slide Time 35:18)

Convolutional codes

$u_i = +1 \Rightarrow \frac{1}{1 + e^{-L(u_i)}} = e^{-L(u_i)}$

$L(u_i) = \log \frac{P(u_i = +1)}{P(u_i = -1)}$

• Solutions: We can write

$$P(u_i = \pm 1) = \frac{[P(u_i = +1)/P(u_i = -1)]^{\pm 1}}{\{1 + [P(u_i = +1)/P(u_i = -1)]^{\pm 1}\}}$$

$$= \frac{e^{\pm L(u_i)}}{\{1 + e^{\pm L(u_i)}\}}$$

$$= \frac{e^{-L(u_i)/2}}{\{1 + e^{-L(u_i)}\}} e^{u_i L(u_i)/2}$$

$$= A_i e^{u_i L(u_i)/2}$$

Handwritten notes on the slide include: $P(u_i = -1) = \frac{P(u_i = -1)}{P(u_i = +1) + P(u_i = -1)}$ and $1 + \left[\frac{P(u_i = +1)}{P(u_i = -1)} \right]^{-1}$.

And this is e raised to power x plus e raised to power minus x. This will be cosh of x and that's the symmetric function, so that does not depend on sign of u l whether u l is plus 1 or minus 1, it does not depend on that. I can write this in terms of this expression.

(Refer Slide Time 35:41)

Convolutional codes

• Also,

$$\begin{aligned} \gamma_i(s', s) &= A_i e^{u_i L(u_i)/2} e^{-(E_s/N_0) \|r_i - v_i\|^2} \\ &= A_i e^{u_i L(u_i)/2} e^{(2E_s/N_0)(r_i \cdot v_i) - (E_s/N_0)(\|r_i\|^2 + \|v_i\|^2)} \\ &= A_i e^{-(E_s/N_0)(\|r_i\|^2 + \|v_i\|^2)} e^{u_i L(u_i)/2} e^{(L_c/2)(r_i \cdot v_i)} \\ &= A_i B_i e^{u_i L(u_i)/2} e^{(L_c/2)(r_i \cdot v_i)} \end{aligned}$$

So we will use the expression that we derived in the previous slide for a priori value which was u l being plus 1 or minus 1 as a L e raised to power u l L a u l by 2. We will use this

(Refer Slide Time 36:03)

Convolutional codes

$$p(u_k = \pm 1) = A_k e^{-\frac{u_k L_c(u_k)}{2}}$$

- Also,

$$\begin{aligned} \gamma_l(s', s) &= A_l e^{u_l L_s(u_l)/2} e^{-(E_s/N_0) \|r_l - v_l\|^2}, \\ &= A_l e^{u_l L_s(u_l)/2} e^{(2E_s/N_0)(r_l \cdot v_l) - (E_s/N_0)(\|r_l\|^2 + \|v_l\|^2)} \\ &= A_l e^{-(E_s/N_0)(\|r_l\|^2 + \|v_l\|^2)} e^{u_l L_s(u_l)/2} e^{(L_c/2)(r_l \cdot v_l)} \\ &= A_l B_l e^{u_l L_s(u_l)/2} e^{(L_c/2)(r_l \cdot v_l)} \end{aligned}$$

expression to simplify the expression for our branch metric for our b c j r algorithm. Now note if you recall we have written the expression for branch metrics as a priori probability u_l and then we had, for a w g n channel we had this expression and

(Refer Slide Time 36:30)

Convolutional codes

$$p(u_k = \pm 1) = A_k e^{-\frac{u_k L_c(u_k)}{2}}$$

- Also,

$$\begin{aligned} \gamma_l(s', s) &= A_l e^{u_l L_s(u_l)/2} \boxed{e^{-(E_s/N_0) \|r_l - v_l\|^2}} \\ &= A_l e^{u_l L_s(u_l)/2} e^{(2E_s/N_0)(r_l \cdot v_l) - (E_s/N_0)(\|r_l\|^2 + \|v_l\|^2)} \\ &= A_l e^{-(E_s/N_0)(\|r_l\|^2 + \|v_l\|^2)} e^{u_l L_s(u_l)/2} e^{(L_c/2)(r_l \cdot v_l)} \\ &= A_l B_l e^{u_l L_s(u_l)/2} e^{(L_c/2)(r_l \cdot v_l)} \end{aligned}$$

of course there was some constant factor which did not depend on u_l ,

(Refer Slide Time 36:37)

Convolutional codes

$$P(U_k = \pm 1) = A_k e^{\frac{u_k L_c(u_k)}{2}}$$

• Also,

$$\begin{aligned} \gamma_l(s', s) &= \frac{A_k P(u_k)}{A_l e^{u_l L_c(u_l)/2}} e^{-(E_s/N_0) \|r_l - v_l\|^2} \\ &= \frac{A_k e^{u_k L_c(u_k)/2}}{A_l e^{u_l L_c(u_l)/2}} e^{(2E_s/N_0)(r_l \cdot v_l) - (E_s/N_0)(\|r_l\|^2 + \|v_l\|^2)} \\ &= A_l e^{-(E_s/N_0)(\|r_l\|^2 + \|v_l\|^2)} e^{u_l L_c(u_l)/2} e^{(L_c/2)(r_l \cdot v_l)} \\ &= A_l B_l e^{u_l L_c(u_l)/2} e^{(L_c/2)(r_l \cdot v_l)} \end{aligned}$$

right. So what we did just now was we derived that this a priori probability can be written in this particular fashion, right. Let's further simplify the expression for branch metric. So this we can expand as r square plus v l square plus 2 times dot product or r and v l. Now this does not depend on choice of v of l . And if v of l is mapped to plus 1 and minus 1, v l square will be 1. So this also will be a constant term. So then this term will then not depend on choice of v of l . So what then will we be left with is, so this term we can just take out as some sort of constant which does not

(Refer Slide Time 37:32)

Convolutional codes

$$P(U_k = \pm 1) = A_k e^{\frac{u_k L_c(u_k)}{2}}$$

• Also,

$$\begin{aligned} \gamma_l(s', s) &= \frac{A_k P(u_k)}{A_l e^{u_l L_c(u_l)/2}} e^{-(E_s/N_0) \|r_l - v_l\|^2} \\ &= \frac{A_k e^{u_k L_c(u_k)/2}}{A_l e^{u_l L_c(u_l)/2}} e^{(2E_s/N_0)(r_l \cdot v_l) - (E_s/N_0)(\|r_l\|^2 + \|v_l\|^2)} \\ &= \frac{A_l e^{-(E_s/N_0)(\|r_l\|^2 + \|v_l\|^2)}}{A_l e^{u_l L_c(u_l)/2}} e^{(L_c/2)(r_l \cdot v_l)} \\ &= A_l B_l e^{u_l L_c(u_l)/2} e^{(L_c/2)(r_l \cdot v_l)} \end{aligned}$$

depend on choice of v of l and what will be left is this term which we are

(Refer Slide Time 37:44)

Convolutional codes

$$P(U_k = \pm 1) = A_k e^{-\frac{U_k L_c(u)}{2}}$$

• Also,

$$\begin{aligned} \gamma_I(s', s) &= \frac{A'_k P(U_k)}{A_I e^{u L_s(u)/2} e^{-(E_s/N_0) \|\mathbf{r}_I - \mathbf{v}_I\|^2}} \\ &= \frac{A'_k e^{u L_s(u)/2} e^{-(E_s/N_0) \|\mathbf{r}_I - \mathbf{v}_I\|^2}}{A_I e^{u L_s(u)/2} e^{(2E_s/N_0)(\mathbf{r}_I \cdot \mathbf{v}_I) - (E_s/N_0)(\|\mathbf{r}_I\|^2 + \|\mathbf{v}_I\|^2)}} \\ &= \frac{A'_k e^{-(E_s/N_0)(\|\mathbf{r}_I\|^2 + \|\mathbf{v}_I\|^2)} e^{u L_s(u)/2}}{A_I e^{(2E_s/N_0)(\mathbf{r}_I \cdot \mathbf{v}_I) - (E_s/N_0)(\|\mathbf{r}_I\|^2 + \|\mathbf{v}_I\|^2)}} \\ &= A_I B_I e^{u L_s(u)/2} e^{(L_c/2)(\mathbf{r}_I \cdot \mathbf{v}_I)} \end{aligned}$$

writing here, which we are writing here and

(Refer Slide Time 37:51)

Convolutional codes

$$P(U_k = \pm 1) = A_k e^{-\frac{U_k L_c(u)}{2}}$$

• Also,

$$\begin{aligned} \gamma_I(s', s) &= \frac{A'_k P(U_k)}{A_I e^{u L_s(u)/2} e^{-(E_s/N_0) \|\mathbf{r}_I - \mathbf{v}_I\|^2}} \\ &= \frac{A'_k e^{u L_s(u)/2} e^{-(E_s/N_0) \|\mathbf{r}_I - \mathbf{v}_I\|^2}}{A_I e^{u L_s(u)/2} e^{(2E_s/N_0)(\mathbf{r}_I \cdot \mathbf{v}_I) - (E_s/N_0)(\|\mathbf{r}_I\|^2 + \|\mathbf{v}_I\|^2)}} \\ &= \frac{A'_k e^{-(E_s/N_0)(\|\mathbf{r}_I\|^2 + \|\mathbf{v}_I\|^2)} e^{u L_s(u)/2}}{A_I e^{(2E_s/N_0)(\mathbf{r}_I \cdot \mathbf{v}_I) - (E_s/N_0)(\|\mathbf{r}_I\|^2 + \|\mathbf{v}_I\|^2)}} \\ &= A_I B_I e^{u L_s(u)/2} e^{(L_c/2)(\mathbf{r}_I \cdot \mathbf{v}_I)} \end{aligned}$$

the next term that will be left is this term which we are writing

(Refer Slide Time 37:58)

Convolutional codes

$P(u_k = \pm 1) = A_k e^{\frac{u_k L_c(u_k)}{2}}$

• Also,

$$\begin{aligned} \gamma_I(s', s) &= \frac{A_k' P(u_k)}{A_k e^{u_k L_c(u_k)/2} e^{-(E_s/N_0)||r_1-v_1||^2}} \\ &= \frac{A_k e^{u_k L_c(u_k)/2} e^{(2E_s/N_0)(r_1 \cdot v_1) - (E_s/N_0)(||r_1||^2 + ||v_1||^2)}}{A_k e^{u_k L_c(u_k)/2} e^{-(E_s/N_0)(||r_1||^2 + ||v_1||^2)}} \\ &= \frac{A_k e^{-(E_s/N_0)(||r_1||^2 + ||v_1||^2)} e^{u_k L_c(u_k)/2} e^{(L_c/2)(r_1 \cdot v_1)}}{A_k B_k e^{u_k L_c(u_k)/2} e^{(L_c/2)(r_1 \cdot v_1)}} \end{aligned}$$

here. Please note

(Refer Slide Time 38:02)

Convolutional codes

$P(u_k = \pm 1) = A_k e^{\frac{u_k L_c(u_k)}{2}}$

• Also,

$$\begin{aligned} \gamma_I(s', s) &= \frac{A_k' P(u_k)}{A_k e^{u_k L_c(u_k)/2} e^{-(E_s/N_0)||r_1-v_1||^2}} \\ &= \frac{A_k e^{u_k L_c(u_k)/2} e^{(2E_s/N_0)(r_1 \cdot v_1) - (E_s/N_0)(||r_1||^2 + ||v_1||^2)}}{A_k e^{u_k L_c(u_k)/2} e^{-(E_s/N_0)(||r_1||^2 + ||v_1||^2)}} \\ &= \frac{A_k e^{-(E_s/N_0)(||r_1||^2 + ||v_1||^2)} e^{u_k L_c(u_k)/2} e^{(L_c/2)(r_1 \cdot v_1)}}{A_k B_k e^{u_k L_c(u_k)/2} e^{(L_c/2)(r_1 \cdot v_1)}} \end{aligned}$$

L_c is 4 times E_s by N_{naught} . So that's why we are writing it as E raised to power L_c by 2 and dot product

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computation for our B C J R

(Refer Slide Time 38:44)

Convolutional codes

$$P(u_k = \pm 1) = A_k e^{-\frac{u_k L_c(u_k)}{2}}$$

• Also,

$$\begin{aligned} \gamma_l(s', s) &= \underbrace{A_k' p(u_k)}_{A_k' p(u_k)} \underbrace{A_l e^{u_l L_c(u_l)/2}}_{A_l e^{u_l L_c(u_l)/2}} \underbrace{e^{-(E_s/N_0) \|r - v_l\|^2}}_{e^{-(E_s/N_0) \|r - v_l\|^2}} \\ &= \underbrace{A_l e^{u_l L_c(u_l)/2}}_{A_l e^{u_l L_c(u_l)/2}} \underbrace{e^{-(2E_s/N_0)(r \cdot v_l)}}_{e^{-(2E_s/N_0)(r \cdot v_l)}} \underbrace{e^{-(E_s/N_0)(\|r\|^2 + \|v_l\|^2)}}_{e^{-(E_s/N_0)(\|r\|^2 + \|v_l\|^2)}} \\ &= \underbrace{A_l e^{-(E_s/N_0)(\|r\|^2 + \|v_l\|^2)}}_{A_l e^{-(E_s/N_0)(\|r\|^2 + \|v_l\|^2)}} \underbrace{e^{u_l L_c(u_l)/2}}_{e^{u_l L_c(u_l)/2}} \underbrace{e^{(L_c/2)(r \cdot v_l)}}_{e^{(L_c/2)(r \cdot v_l)}} \\ &= \underbrace{A_l B_l}_{A \text{-prior}} \underbrace{e^{u_l L_c(u_l)/2}}_{\text{Received channel values}} \underbrace{e^{(L_c/2)(r \cdot v_l)}}_{e^{(L_c/2)(r \cdot v_l)}} \quad L_c = 4 \frac{E_s}{N_0} \end{aligned}$$

algorithm if we are considering additive white Gaussian noise

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Convolutional codes

- Also,

$$\begin{aligned} \gamma_l(s', s) &= A_l e^{uL_d(u)/2} e^{-(E_s/N_0)\|\mathbf{r}_l - \mathbf{v}_l\|^2}, \\ &= A_l e^{uL_d(u)/2} e^{(2E_s/N_0)(\mathbf{r}_l \cdot \mathbf{v}_l) - (E_s/N_0)(\|\mathbf{r}_l\|^2 + \|\mathbf{v}_l\|^2)} \\ &= A_l e^{-(E_s/N_0)(\|\mathbf{r}_l\|^2 + \|\mathbf{v}_l\|^2)} e^{uL_d(u)/2} e^{(L_c/2)(\mathbf{r}_l \cdot \mathbf{v}_l)} \\ &= A_l B_l e^{uL_d(u)/2} e^{(L_c/2)(\mathbf{r}_l \cdot \mathbf{v}_l)} \end{aligned}$$
- Thus

$$\gamma_l^*(s', s) = \ln \gamma_l(s', s) = \frac{uL_d(u)}{2} + \frac{L_c}{2} \mathbf{r}_l \cdot \mathbf{v}_l$$

channel. Now if we consider branch metric in the log domain, then log of this term will be some sort of constant, we just

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Convolutional codes

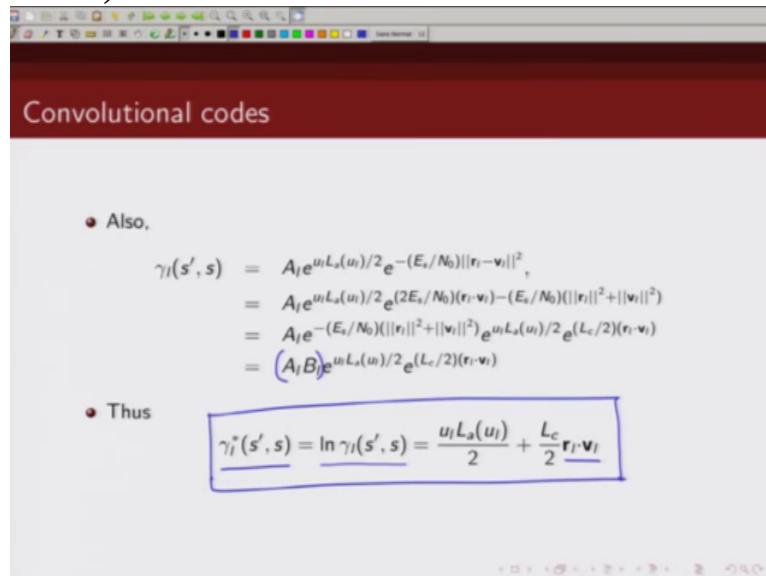
- Also,

$$\begin{aligned} \gamma_l(s', s) &= A_l e^{uL_d(u)/2} e^{-(E_s/N_0)\|\mathbf{r}_l - \mathbf{v}_l\|^2}, \\ &= A_l e^{uL_d(u)/2} e^{(2E_s/N_0)(\mathbf{r}_l \cdot \mathbf{v}_l) - (E_s/N_0)(\|\mathbf{r}_l\|^2 + \|\mathbf{v}_l\|^2)} \\ &= A_l e^{-(E_s/N_0)(\|\mathbf{r}_l\|^2 + \|\mathbf{v}_l\|^2)} e^{uL_d(u)/2} e^{(L_c/2)(\mathbf{r}_l \cdot \mathbf{v}_l)} \\ &= (A_l B_l) e^{uL_d(u)/2} e^{(L_c/2)(\mathbf{r}_l \cdot \mathbf{v}_l)} \end{aligned}$$
- Thus

$$\underline{\gamma_l^*(s', s) = \ln \gamma_l(s', s) = \frac{uL_d(u)}{2} + \frac{L_c}{2} \mathbf{r}_l \cdot \mathbf{v}_l}$$

ignore it because this does not depend on choice of v of l, u of l so this will become u l, L value, a priori L value by 2 plus L c by 2 and dot product between the received sequence and the transmitted codeword. So this will be then our simplified expression for

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Convolutional codes

- Also,
$$\begin{aligned}\gamma_l(s', s) &= A_l e^{uL_s(u)/2} e^{-(E_s/N_0)\|\mathbf{r}_l - \mathbf{v}_l\|^2}, \\ &= A_l e^{uL_s(u)/2} e^{(2E_s/N_0)(\mathbf{r}_l \cdot \mathbf{v}_l) - (E_s/N_0)(\|\mathbf{r}_l\|^2 + \|\mathbf{v}_l\|^2)} \\ &= A_l e^{-(E_s/N_0)(\|\mathbf{r}_l\|^2 + \|\mathbf{v}_l\|^2)} e^{uL_s(u)/2} e^{(L_c/2)(\mathbf{r}_l \cdot \mathbf{v}_l)} \\ &= (A_l B_l) e^{uL_s(u)/2} e^{(L_c/2)(\mathbf{r}_l \cdot \mathbf{v}_l)}\end{aligned}$$
- Thus
$$\gamma_l^*(s', s) = \ln \gamma_l(s', s) = \frac{uL_s(u)}{2} + \frac{L_c}{2} \mathbf{r}_l \cdot \mathbf{v}_l$$

for branch metric computation

(Refer Slide Time 39:30)



for B C J R algorithm over additive white Gaussian noise channel, thank you.