

**An Introduction to Coding Theory**  
**Professor Adrish Banerji**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kanpur**  
**Module 01**  
**Lecture Number 01**  
**Introduction to Error Control Coding-I**

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Welcome the course on Coding Theory.

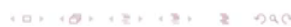
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An introduction to coding theory

Adrish Banerjee

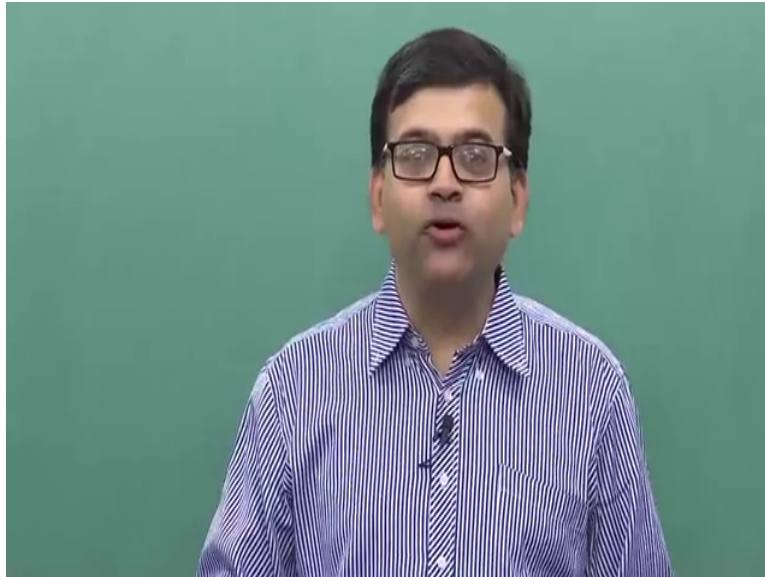
Department of Electrical Engineering  
Indian Institute of Technology Kanpur  
Kanpur, Uttar Pradesh  
India

Jan. 23, 2017



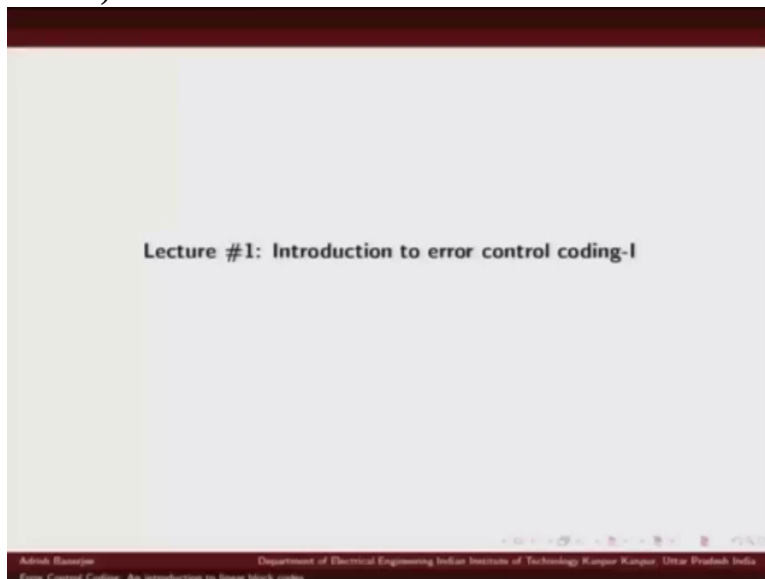
I am Adrish Banerji from I I T Kanpur. Today we are going to talk

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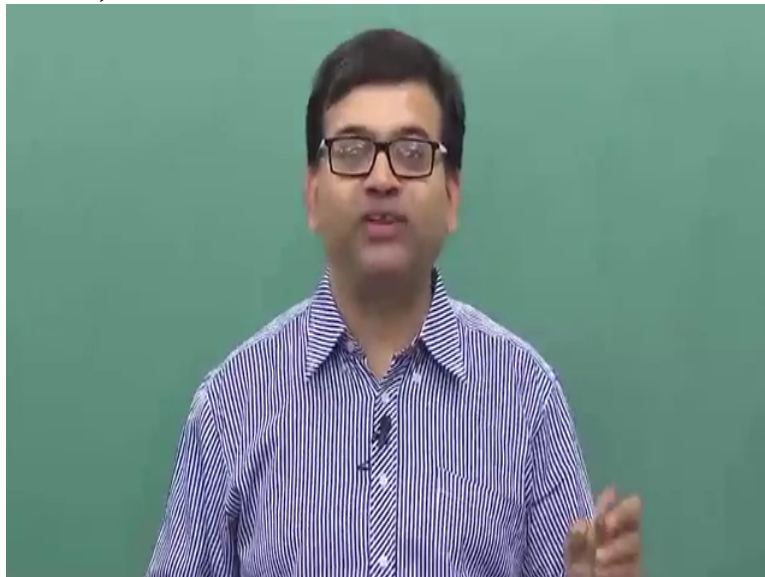
about basic introduction to what coding theory is all about So we will start our lecture.

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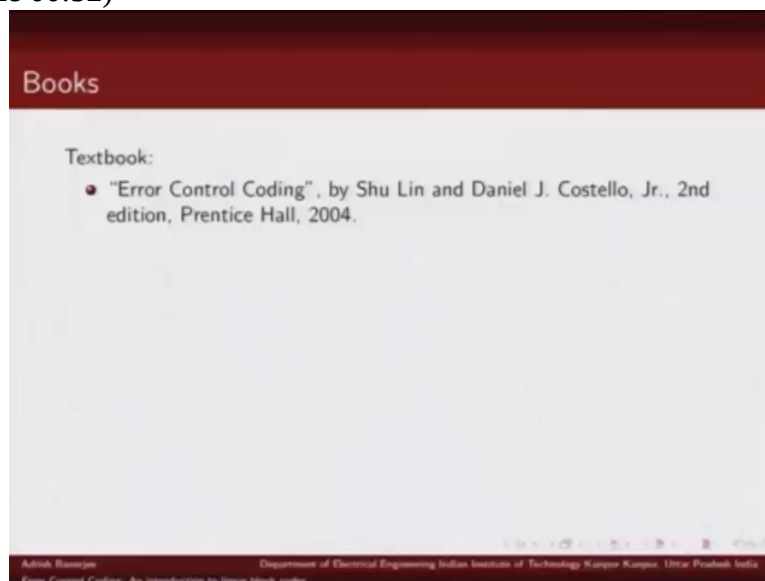
So in introduction, as I said, we will talk about what is coding theory, we will illustrate with a very

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simple example how error correcting codes can be used for error detection and error correction Before I start my lecture, I would like to talk about the books that we are going to use for this course, so we are going to

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follow this book Error Control Coding by Lin and Costello. It's a second edition of this book. We are going to follow this book as our textbook. And there are some very nice books which

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**Books**

Textbook:

- "Error Control Coding", by Shu Lin and Daniel J. Costello, Jr., 2nd edition, Prentice Hall, 2004.

References:

- F. J. MacWilliams, N. J. A. Sloane, "The Theory of Error-Correcting Codes", North-Holland, Amsterdam, 1977.
- R. E. Blahut, "Algebraic Codes for Data Transmission", 1st Edition, Cambridge University Press 2003.
- Todd K. Moon, "Error Correction Coding", 1st Edition, Wiley-Interscience, 2006.
- Cary W. Huffman, Vera Pless, "Fundamentals of Error-Correcting Codes", 1st Edition, Cambridge University Press, 2003.

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you can use as reference book, for example this book by Sloane and McWilliams is a very nice book on block codes. You could also follow this book by Blahut on Algebraic Codes for Data Transmission, this book Error Control Coding by Todd K. Moon, this also gives a very nice introduction to error correcting codes or you could use this book by Huffman and Pless called Fundamentals of Error-Control Codes.

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**Introduction**

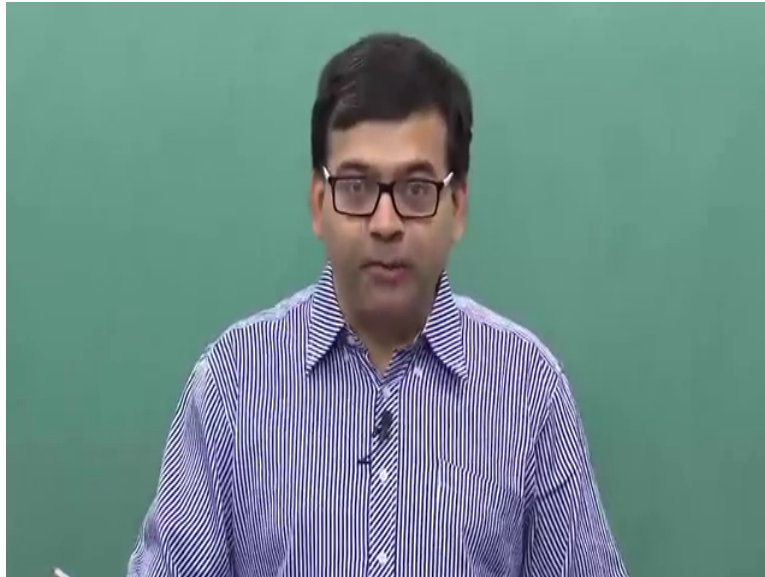
All communications involves three basic steps

- Encoding a message at its source.

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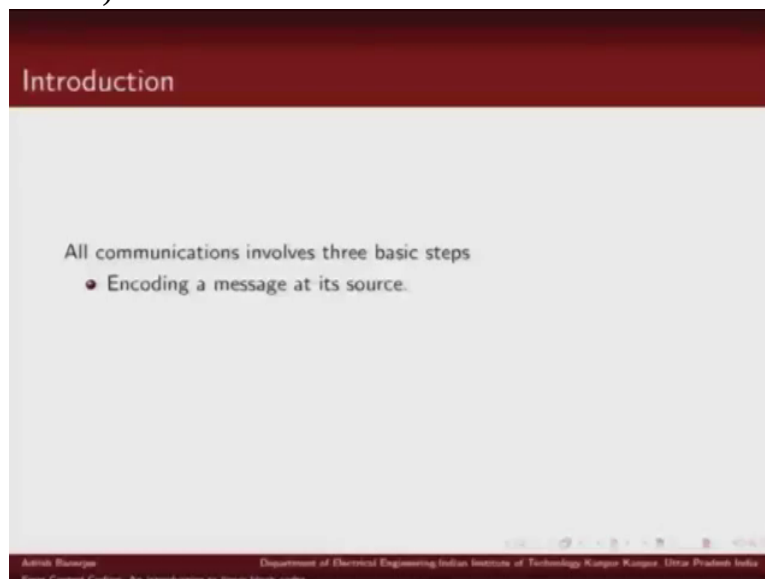
So when we talk about

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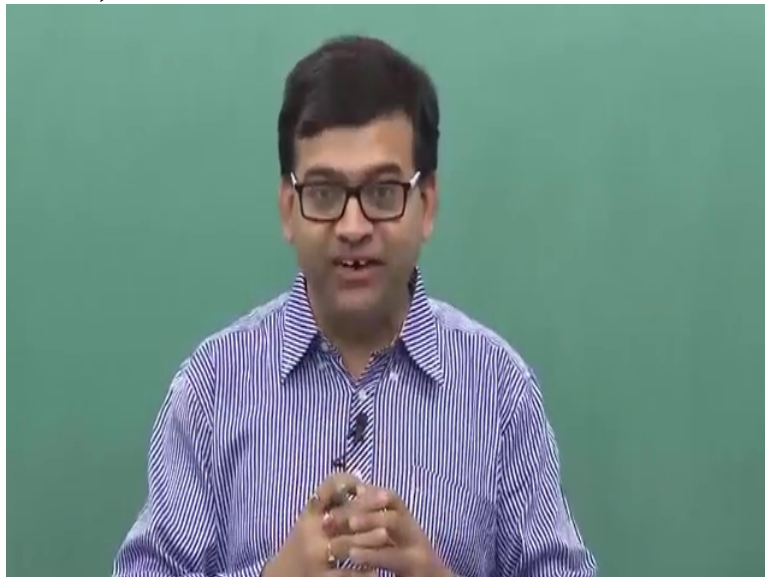
communications, communications basically involves three basic steps. The first is encoding a message. You have a message

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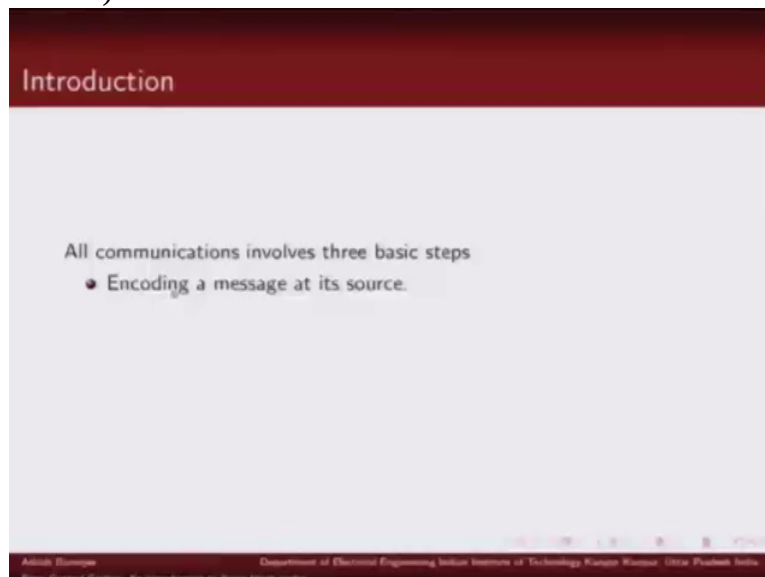
source that you want to represent efficiently

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Now you can for example consider a speech signal. Now if you want to transmit a speech signal you first have to convert analog signal to digital signal and then you need to get rid of useless redundancies. Why, because we want to transmit basically useful information. We want a, source inherently has lot of redundancy and when we try to represent a source, we would like to represent a source efficiently in minimum number of possible bits. So first step involved in any communication

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is basically encoding a message Second is, once you have

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The slide is titled "Introduction" in a dark red header. The main content area is light gray and contains the text "All communications involves three basic steps" followed by a bulleted list with two items: "Encoding a message at its source." and "Transmitting that message through a communication medium." At the bottom of the slide, there is a dark red footer containing the name "Adrish Banerjee", the affiliation "Department of Electrical Engineering Indian Institute of Technology Kanpur Kanpur, Uttar Pradesh India", and the course title "Power Control System: An introduction to power block codes".

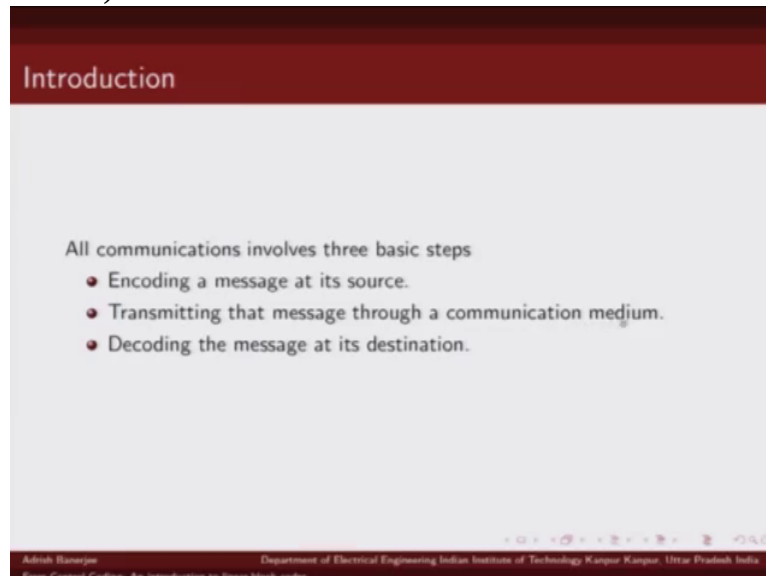
represented your source you want to transmit that source over a communication channel. So the second thing

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is transmission of the message through a communication

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channel and finally once the receiver received the message, it has to decode to find out what was the information that was transmitted. So broadly there are three steps

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involved in communication; encoding, transmission and then finally decoding So information theory basically



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**Introduction**

All communications involves three basic steps

- Encoding a message at its source.
- Transmitting that message through a communication medium.
- Decoding the message at its destination.

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gives us fundamental limits of what is a maximum limit of like what is the best compression, fundamental limit and compression that we can achieve. It also gives us fundamental limits on what is the maximum transmission rate possible over a communication channel.

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**Introduction**

- The transmission medium in communication is known as channel.

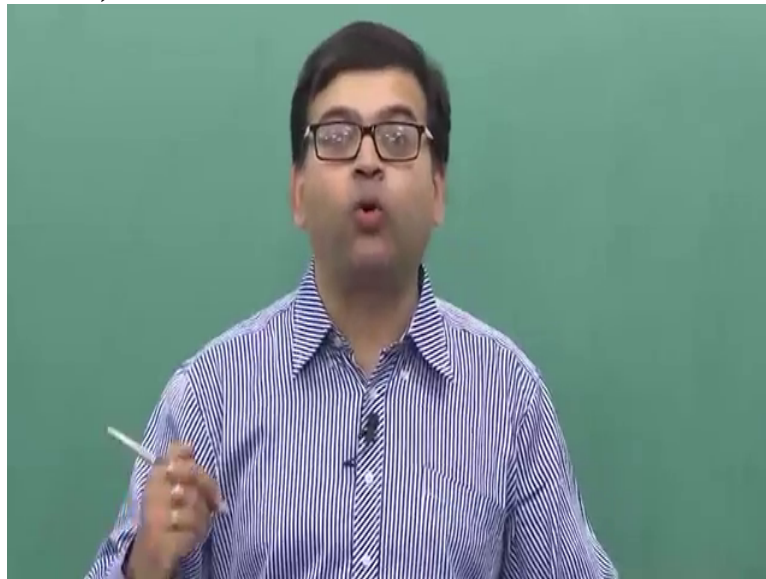
**Binary Symmetric Channel**

**Binary Erasure Channel**

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Error Control Codes: An introduction to linear block codes

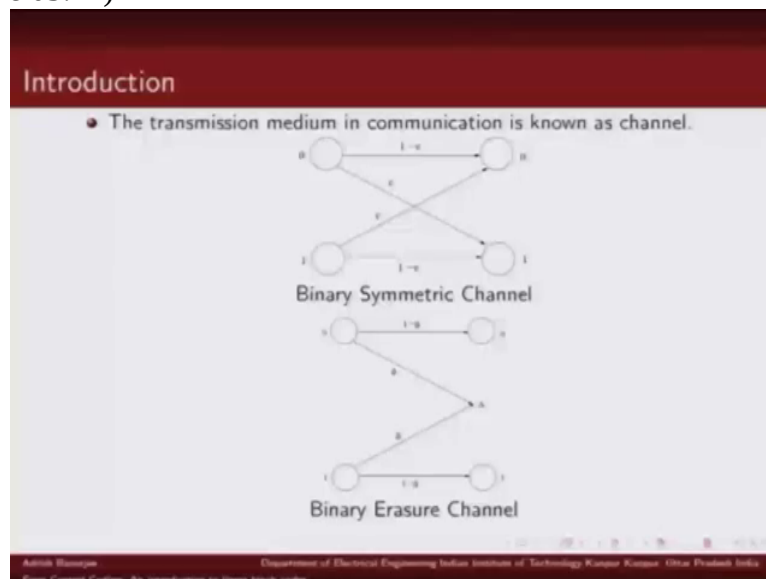
So let's spend some time on what is our transmission medium. So the transmission medium over which

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we want to send a packet, that is known a channel and here I have illustrated

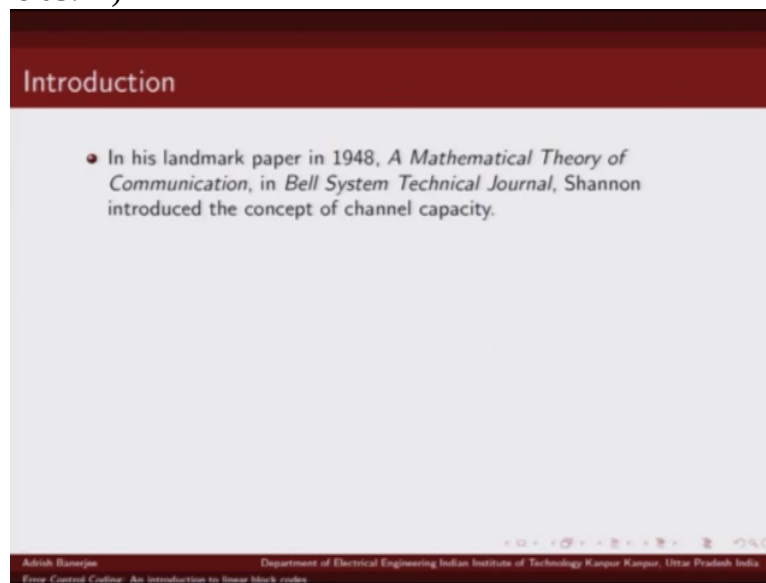
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two very simple channel models, the first which is binary symmetric channel; now you can see there are, it has binary inputs, zeros and one and similarly it has binary output 0 and 1. Now with probability  $1 - \epsilon$ , basically whatever you transmit is received correctly at the receiver. So this is a transmitter and this is a receiver side. So if you transmit 0 with probability  $1 - \epsilon$ , you will receive it correctly. Similarly if you transmit 1 with probability  $1 - \epsilon$  you will receive it correctly. And this crossover probability of error is basically given by  $\epsilon$ . So this is basically a symmetric channel and it's a binary channel because the binary input binary output, it's known as binary symmetric channel. Another channel which basically is very commonly used to model packet data networks is

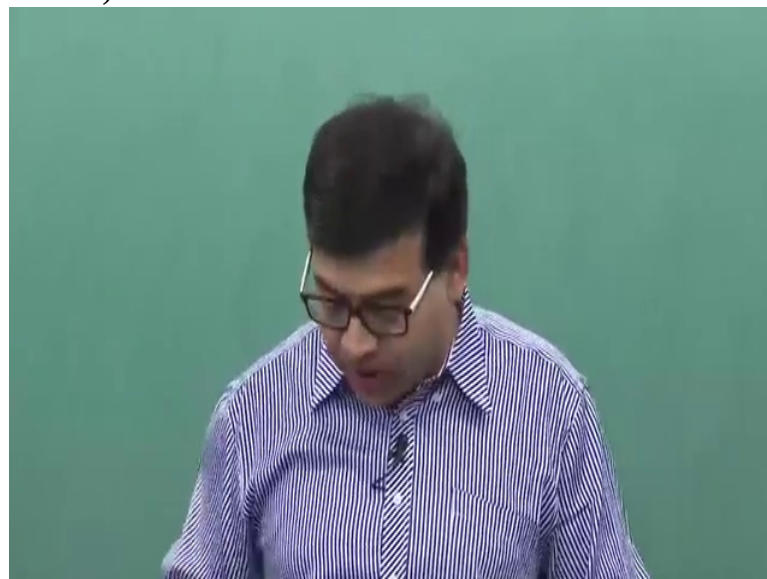
what is known as binary erasure channels. So they are binary inputs, 0s and 1s and the outputs are either you see whatever has been transmitted you receive it correctly or whatever you have transmitted is basically erased. So this delta that you see basically, we are denoting an erased bit using this symbol. So with probability  $1 - \delta$  you receive the bit correctly and with probability  $\delta$  the bit is erased or lost.

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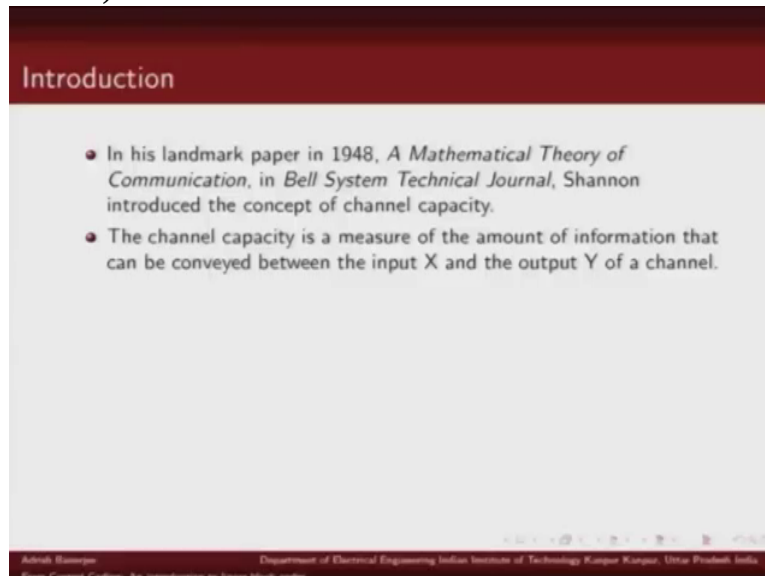
So in his landmark paper in 1948, Shannon introduced this concept of channel capacity, that what is the maximum rate at which we can communicate

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over a communication link; so a channel

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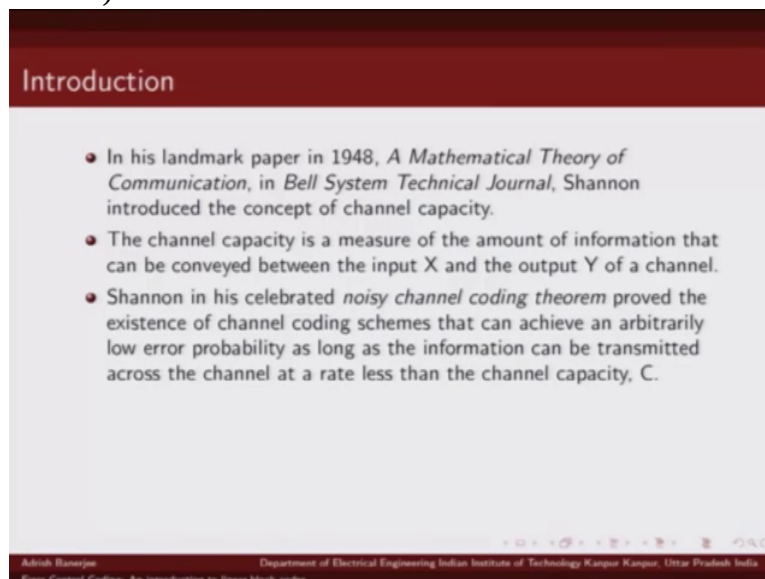
**Introduction**

- In his landmark paper in 1948, *A Mathematical Theory of Communication*, in *Bell System Technical Journal*, Shannon introduced the concept of channel capacity.
- The channel capacity is a measure of the amount of information that can be conveyed between the input  $X$  and the output  $Y$  of a channel.

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capacity is defined as the maximum amount of information that can be conveyed from the input to the output of a channel. Shannon

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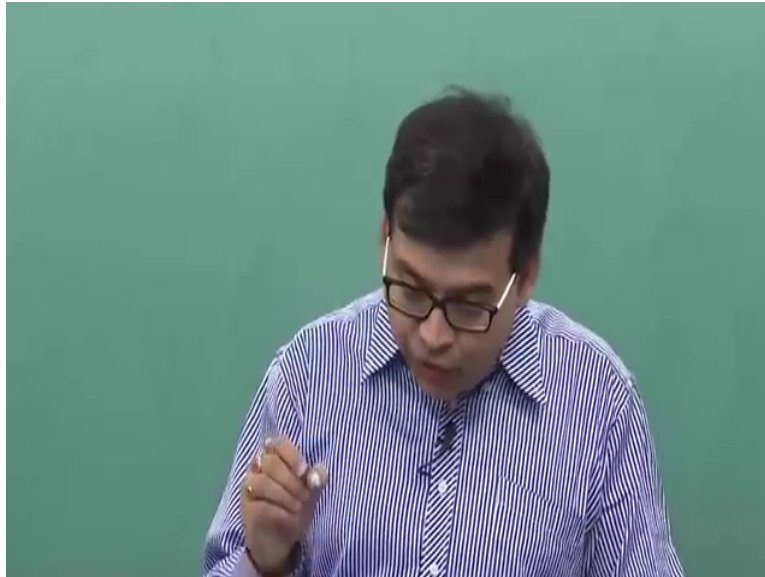
**Introduction**

- In his landmark paper in 1948, *A Mathematical Theory of Communication*, in *Bell System Technical Journal*, Shannon introduced the concept of channel capacity.
- The channel capacity is a measure of the amount of information that can be conveyed between the input  $X$  and the output  $Y$  of a channel.
- Shannon in his celebrated *noisy channel coding theorem* proved the existence of channel coding schemes that can achieve an arbitrarily low error probability as long as the information can be transmitted across the channel at a rate less than the channel capacity,  $C$ .

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in his theorem also proved that there exist channel coding schemes that can achieve very low, arbitrarily very low probability of error as long as the transmission rate is below channel capacity. So Shannon showed

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that there exist good channel codes as long as the transmission rate is below channel capacity we can achieve arbitrarily low probability of error. For example

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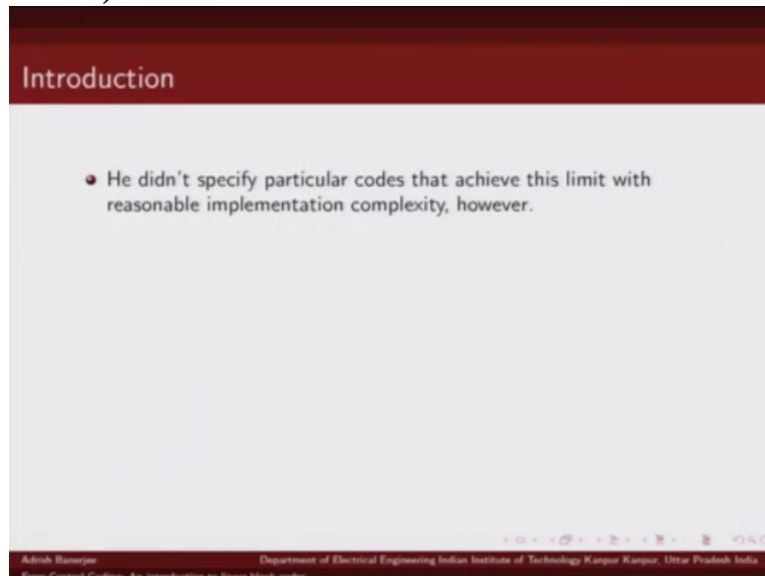
A presentation slide with a dark red header containing the word "Introduction" in white. The main content area is white with four bullet points in black text. At the bottom, there is a dark red footer with small white text.

- In his landmark paper in 1948, *A Mathematical Theory of Communication*, in *Bell System Technical Journal*, Shannon introduced the concept of channel capacity.
- The channel capacity is a measure of the amount of information that can be conveyed between the input  $X$  and the output  $Y$  of a channel.
- Shannon in his celebrated *noisy channel coding theorem* proved the existence of channel coding schemes that can achieve an arbitrarily low error probability as long as the information can be transmitted across the channel at a rate less than the channel capacity,  $C$ .
- Example: If the channel capacity of a particular communication link is (say) 2 Gbps. We can communicate over this channel at any desired rate less than 2 Gbps, and achieve arbitrary low error rates.

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if we talk of a channel capacity of a particular link to be two Giga bits per second then basically we should be able to communicate at rate, any rate up to two Giga bits over this communication link without basically, and can achieve very low probability of error at the decoder.

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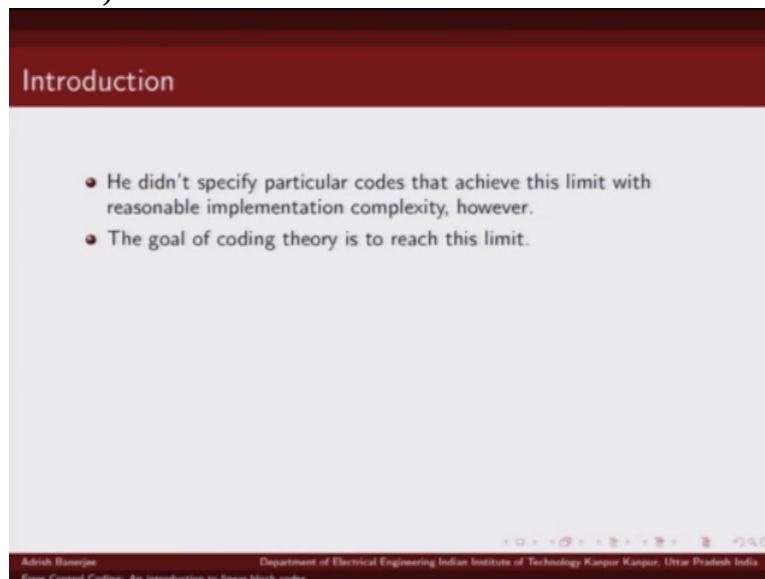
Introduction

- He didn't specify particular codes that achieve this limit with reasonable implementation complexity, however.

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Now in this theorem Shannon did not specify how to design such codes which have rate close to capacity and that's where basically error control coding comes into picture. So the goal of error correcting

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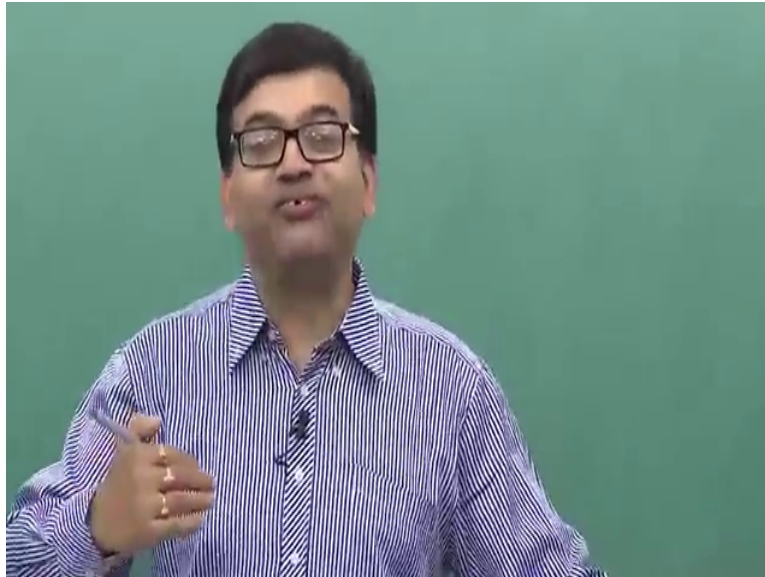
Introduction

- He didn't specify particular codes that achieve this limit with reasonable implementation complexity, however.
- The goal of coding theory is to reach this limit.

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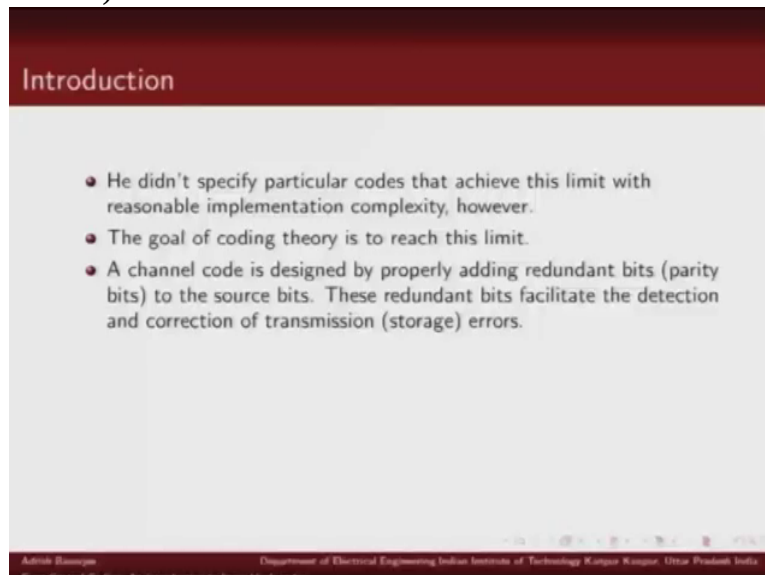
So the goal of error correcting coding theory is to achieve this, to design

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codes which can achieve this limit so basically Shannon has mentioned that we could transmit, we could design, as long as we design error correcting codes which have rate less than channel capacity, we can achieve arbitrarily low probability of error. So the goal of the coding theory or the error control coding is to design such error correcting codes with rates as close to capacity which can achieve arbitrarily low probability of error. And Shannon

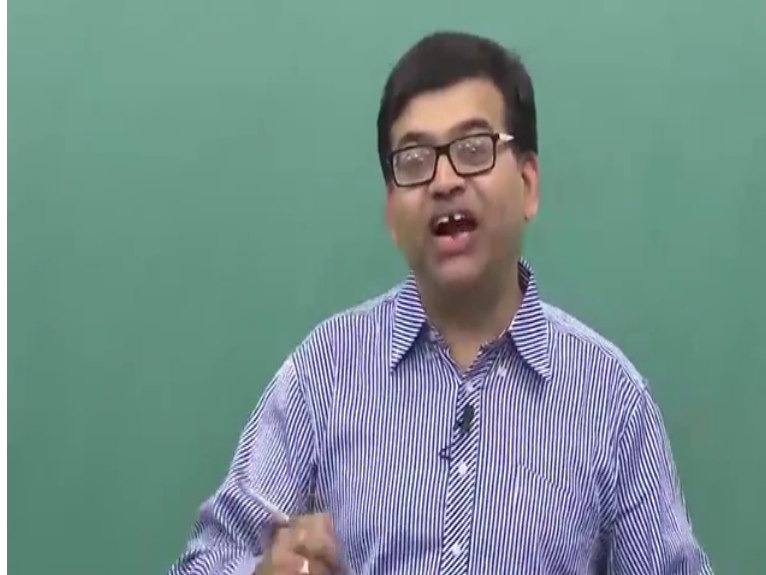
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did not specify how to design such code; so basically where coding theories come into picture. So how do we design an error correcting code? An error correcting code is designed by adding some redundant bits to the message bits. Message bits, we call them information bits and those additional redundant bits that we add, those are known as parity bits. So error correcting code is designed by properly adding some redundant bits to your message bit and

then send this coded message over a communication link. Now we use these additional redundant bits

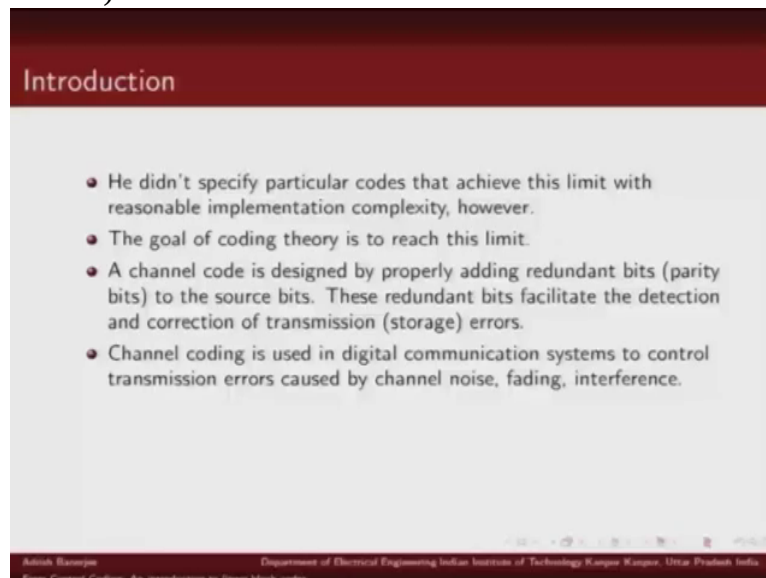
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to detect error and correct error

Error correcting

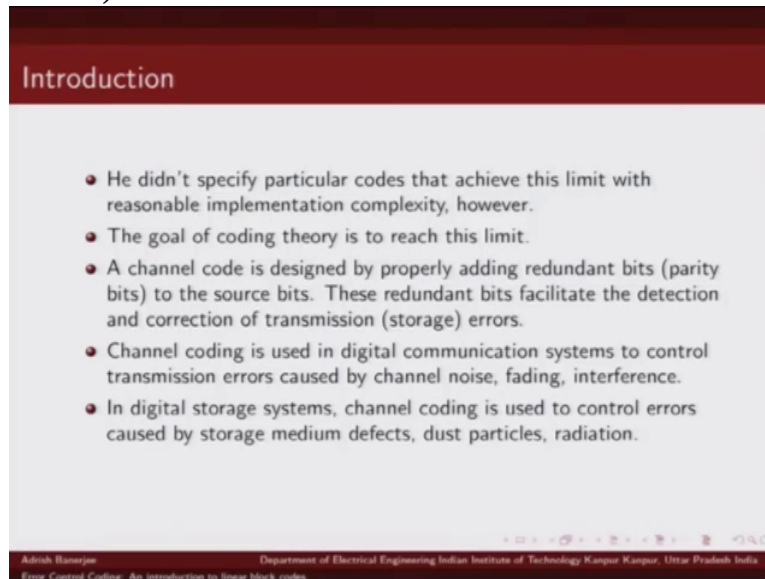
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code has wide range of applications in digital communications and storage. I have listed few of the uses, for example when we send a signal over communication link; it gets corrupted by noise, fading, interference. So to combat the interference of all these basically we use the error correcting codes to correct the errors. Similarly



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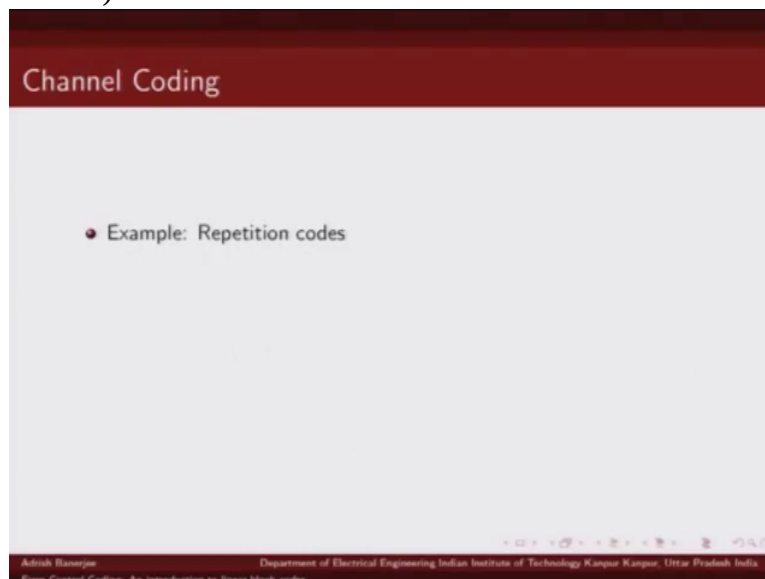
**Introduction**

- He didn't specify particular codes that achieve this limit with reasonable implementation complexity, however.
- The goal of coding theory is to reach this limit.
- A channel code is designed by properly adding redundant bits (parity bits) to the source bits. These redundant bits facilitate the detection and correction of transmission (storage) errors.
- Channel coding is used in digital communication systems to control transmission errors caused by channel noise, fading, interference.
- In digital storage systems, channel coding is used to control errors caused by storage medium defects, dust particles, radiation.

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in digital storage system you want to correct the error caused due to storage media defect, dust particles, radiations we use error correcting codes there.

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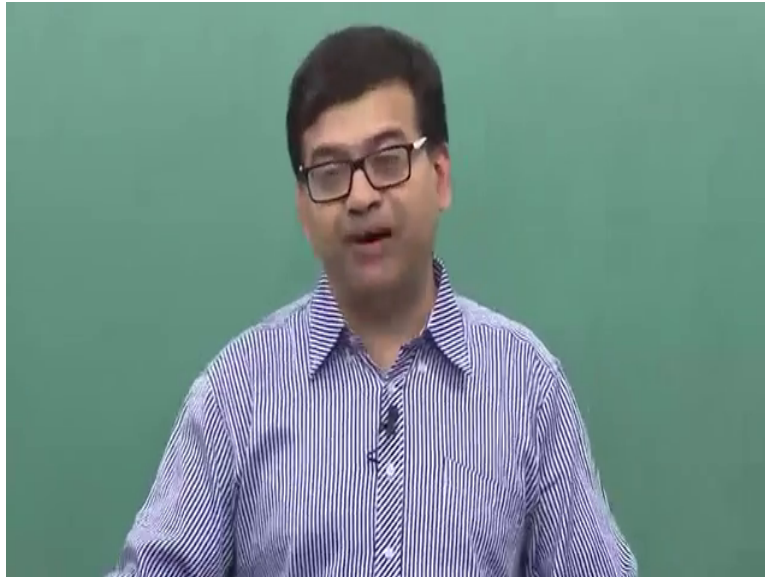
**Channel Coding**

- Example: Repetition codes

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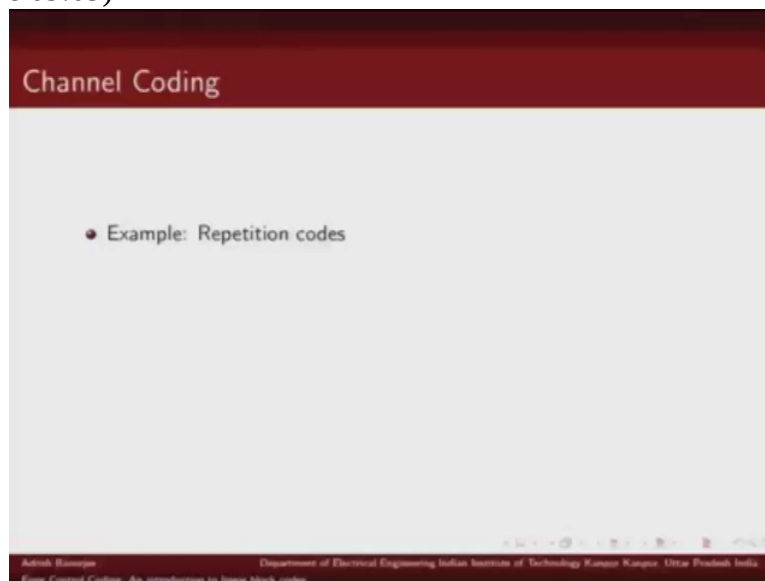
So let us take a very simple example of

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error correcting codes and illustrate

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how we can use error correcting codes to detect and correct errors. So example I am going to show you right now is of what is known as repetition code.

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Channel Coding

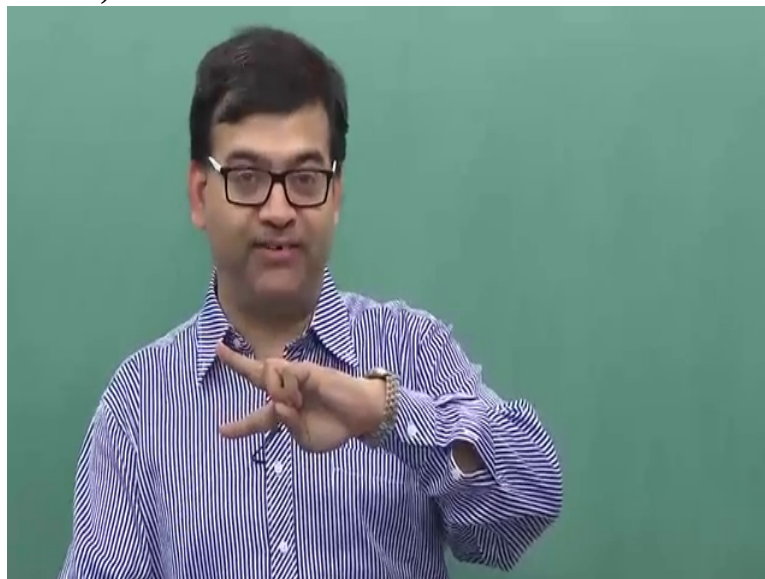
- Example: Repetition codes
- Rate  $R=1/2$  code

$0 \rightarrow 00$     $1 \rightarrow 11$

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So the rate is defined as the ratio of number of information bits to number of coded bits. So when I say rate one half code, I mean there is one information bit or one message bit and there are two coded bits.

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For example in a repetition code,

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Channel Coding

- Example: Repetition codes
- Rate  $R=1/2$  code

$0 \rightarrow 00$      $1 \rightarrow 11$

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in a binary repetition code basically we repeat whatever information bit is. So a rate of repetition code would be; would look like this. A binary rate of repetition code would look something like this, so for 0, we would be transmitting 0 0 and for 1, we would be transmitting 1 1. Similarly for a rate

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Channel Coding

- Example: Repetition codes
- Rate  $R=1/2$  code
- Rate  $R=1/3$  code

$0 \rightarrow 00$      $1 \rightarrow 11$

$0 \rightarrow 000$      $1 \rightarrow 111$

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one third repetition code, for 0, we will be transmitting 0 0 0, and for 1, we will be transmitting 1 1 1. So you can see here, in this, in rate one half is we are adding one additional redundant bit and for rate one third code basically we are adding two additional redundant bits. Now how we are going to make use of these redundant bits for error correction and error detection that will be explained in the next slide.

So let's take

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**Channel Coding**

- Example (contd.)

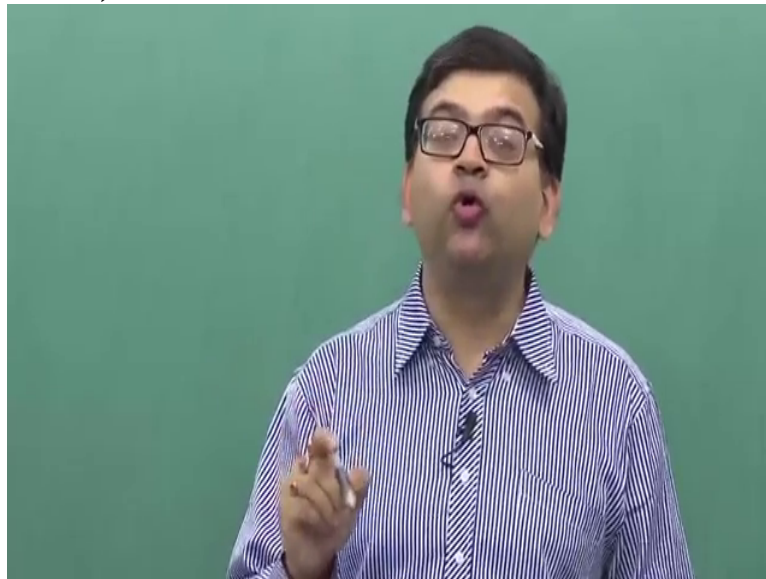
Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/2 Repetition codes:	00 00 11 11 00 11
Received coded bits (Single Error):	10 00 11 11 00 11
Received coded bits (Double Error):	11 00 11 11 00 11

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this example Let's say I want to transmit these set of bits. So I want to transmit 0 0 1 1 0 1. Now if I use a rate one half repetition code what would be my coded bits? For 0, I would be transmitting 0 0, for 0, I would be encoding as 0 0, for 1 I would be encoding them as 1 1, and for 1 I would be sending as 1 1, for 0 I will be sending as 0 0 and for 1 I will be sending as 1 1. So this will be my coded sequence, Ok. Now here I have illustrated one case where there is a single error. So this was basically the sequence which was transmitted. Think of it that this bit sequence has been transmitted over a binary symmetrical channel and this is what the received sequence I received. So you can see here, this is a case of a single error. The first bit which was transmitted 0 was received as 1. Now how can I use error correcting codes to detect error?

So since it is a rate one half code, for each information bit I am sending two coded bits. So at the receiver I will look at two bits at a time. So I will look at, first I will look at this 1 0. Now since it is a repetition code what do you expect? I expect that both the bits should be same, right? But here in this case first bit is 1; second bit is 0 which means there is a transmission error. So I am able to detect single error. How? Because these bits were encoded using rate half repetition code; I expect these two bits to be same. So I know there is an error in the first bit but I don't know whether this is bit 0 or bit 1. Let's look at other received bits, 0 0 this will be decoded as 0, 1 1 this will be decoded as 1, 1 1 this will be decoded as 1 there is no ambiguity, 0 0 this will be decoded as 0 again there is no ambiguity, and 1 1 this would be decoded as 1. So we can see that using one additional redundant bit we are able to detect

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single error

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**Channel Coding**

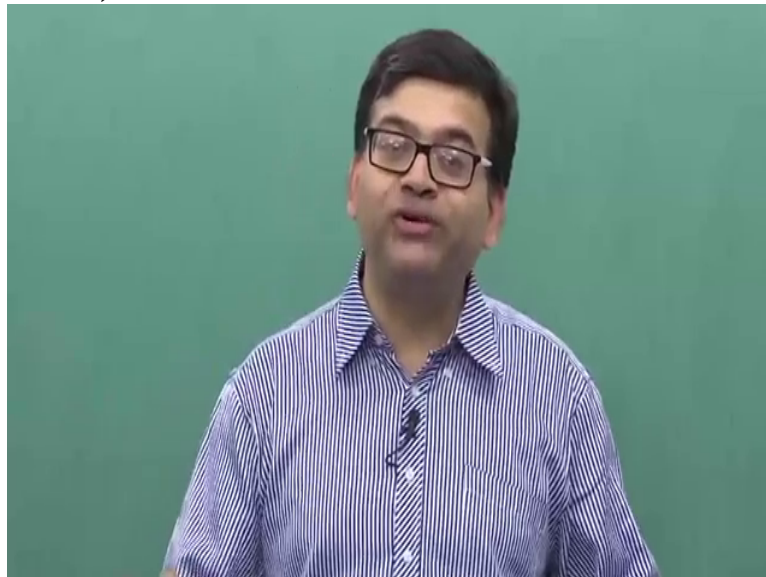
- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/2 Repetition codes:	00 00 11 11 00 11
Received coded bits (Single Error):	10 00 11 11 00 11
Received coded bits (Double Error):	11 00 11 11 00 11

Aditya Basu, Department of Electrical Engineering Indian Institute of Technology Kanpur, Kanpur, Uttar Pradesh, India  
Error Control Codes: An introduction to linear block codes.

Now let's look example for double error. So let's say the first and second bit are received in error. So what we have received is basically 1 1 0 0 1 1 1 1 0 0 1 1. So the first two received bits are in error 1 1. Now let's see whether we can detect using this rate half repetition code. So again we will follow the same logic for decoding. We will look at two bits at a time. So first two bits are 1 1; now since these bits are same, we will decode them as 1, but what was transmitted, it was 0. So we can see that this is a case of undetected error. Even those these two bits were received in error, the decoder is not able to detect this

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error So this kind of thing happens when the error pattern is such that it transforms one code word into some other code word. So since

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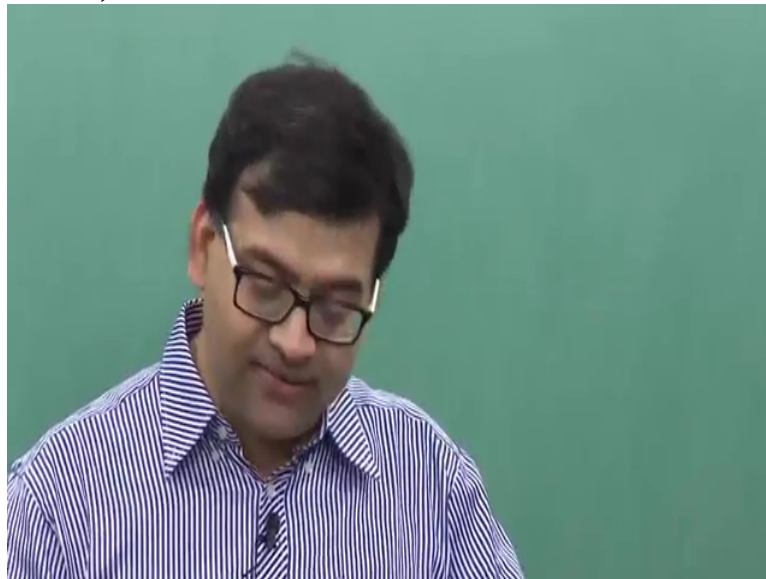
The slide is titled "Channel Coding" in a dark red header. Below the title, there is a section labeled "Example (contd.)". It lists four lines of bit sequences:

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/2 Repetition codes:	00 00 11 11 00 11
Received coded bits (Single Error):	10 00 11 11 00 11
Received coded bits (Double Error):	11 00 11 11 00 11

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1 1 is a valid code word for 1; the decoder is not able to detect this error. So this rate one half repetition is able to detect single error but it is not able to detect double errors.

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Now let's look at whether it can correct any errors. So let's look at

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**Channel Coding**

- Example (contd.)

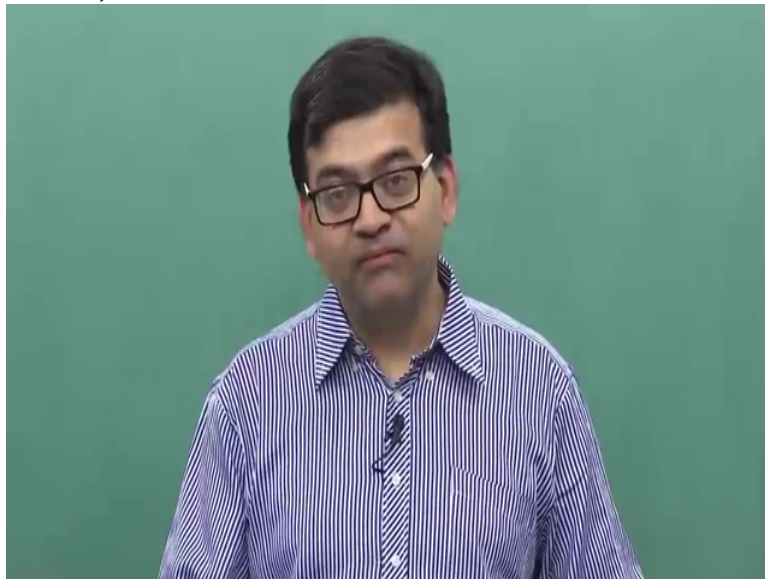
Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/2 Repetition codes:	00 00 11 11 00 11
Received coded bits (Single Error):	10 00 11 11 00 11
Received coded bits (Double Error):	11 00 11 11 00 11

Aditya Kumar, Department of Electrical Engineering Indian Institute of Technology Kanpur, Kanpur, Uttar Pradesh, India  
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this example when we had single error. So note here, so what we received was 1 0. So we were able to detect error that there was an error. But can we correct it? No



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we cannot. Why? It is equally likely that this

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**Channel Coding**

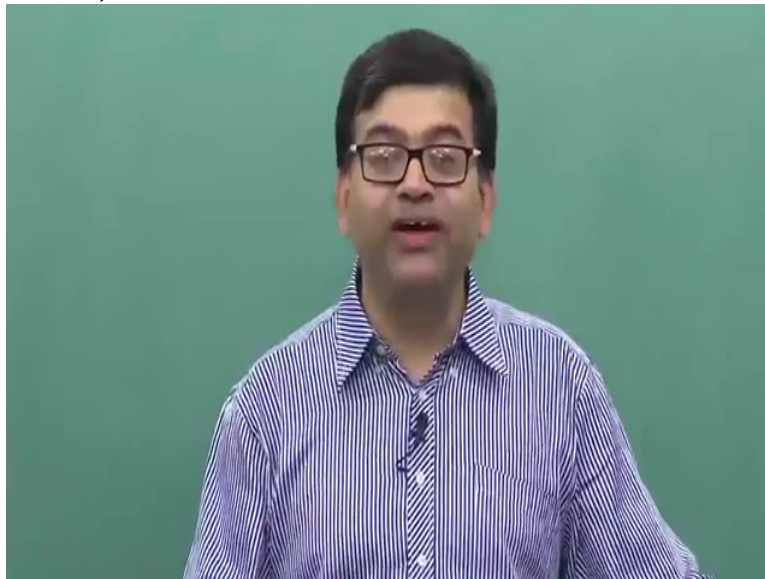
- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/2 Repetition codes:	00 00 11 11 00 11
Received coded bits (Single Error):	0 0 0 1 1 1 0 0 1 1
Received coded bits (Double Error):	1 1 0 0 1 1 1 0 0 1 1

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Error Control Coding: An Introduction to Stream-Based Codes

1 that we received was 0 or this 0 that we received was 1 if we are talking about binary symmetrical channel, right? So we do not know whether the first bit got flipped to 0, first bit got flipped to 1 instead of 0 or the second bit got flipped to 0 instead of being 1. So this particular rate half repetition code

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cannot correct any errors It can only detect single errors.

Now look at

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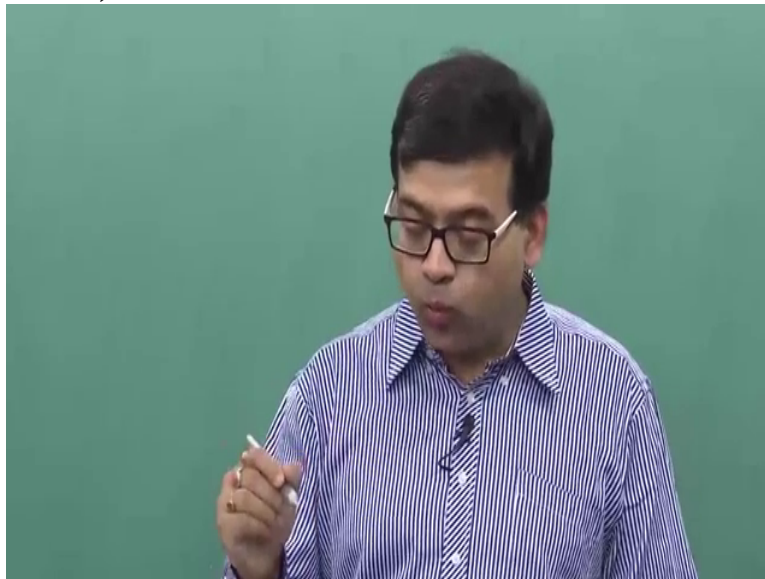
The slide is titled "Channel Coding" and contains the following text:

- Example (contd.)
- Information bits: 0 0 1 1 0 1
- Coded bits using Rate 1/2 Repetition codes: 00 00 11 11 00 11
- Received coded bits (Single Error): 10 00 11 11 00 11
- Received coded bits (Double Error): 11 00 11 11 00 11

At the bottom of the slide, there is a footer: "Adish Ranjan, Department of Electrical Engineering, Indian Institute of Technology Kanpur, Kanpur, Uttar Pradesh, India. Error Control Coding: An introduction to linear block codes."

another example This time we are considering a rate one third repetition code. So what does rate one third repetition code means? For each, and again we are considering binary code, so for each bit we are adding two parity bits and we are repeating the same bit. So for 0, we will be transmitting, we will be coding it as 0 0 0, for 1 we will be coding it as 1 1 1.

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So again we consider the same example of transmitting 0 0 1 1 0 1. So we are transmitting the same

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**Channel Coding**

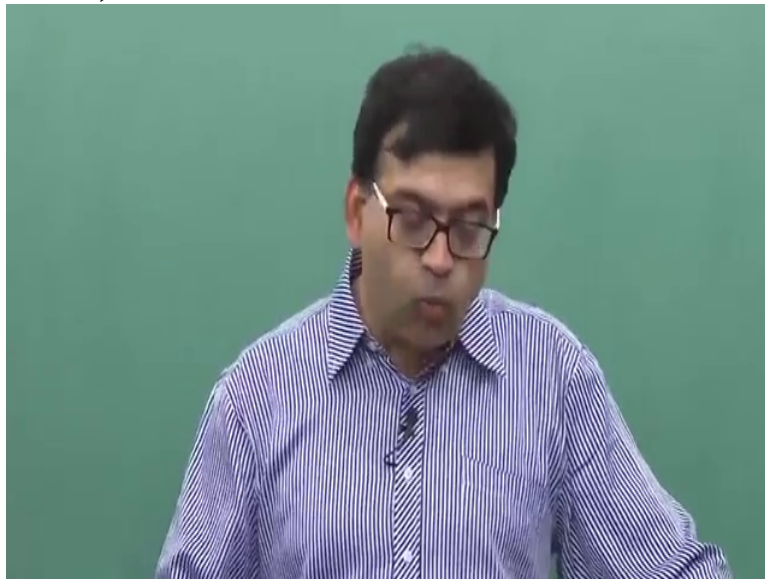
- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes:	000 000 111 111 000 111
Received coded bits (Single Error):	100 000 111 111 000 111
Received coded bits (Double Error):	110 000 111 111 000 111

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Error Control Coding - An introduction to Error Block codes

information sequence This time we are encoding them using rate one third repetition code so this 0 will be encoded as 0 0 0. Similarly 1 will be encoded as 1 1 1 so we will be transmitting this. So this information sequence will be coded in this particular way. Now we will again look at what happens

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when there are errors in the received sequence, like we did for rate one half repetition code. So let's again look at

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**Channel Coding**

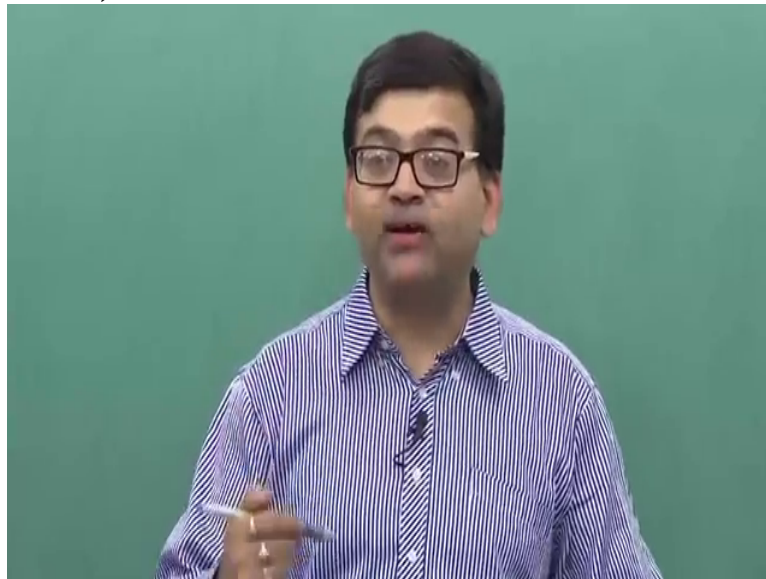
- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes:	000 000 111 111 000 111
Received coded bits (Single Error):	100 000 111 111 000 111
Received coded bits (Double Error):	110 000 111 111 000 111

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example for single error scenario So let's say the first bit was received in error. So instead of 0, we received a 1. Now let's see whether our rate one third repetition code can detect single error. So since it is rate one third code for each information bit we are sending 3 coded bits. So we are going at the receiver. We are going to look at 3 bits at a time. At the decoder we are going to look at 3 bits at a time. So we will first look at these 3 bits, 1 0 0. Now what do you expect? We expect, since we are using our repetition code, we expect

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all these 3 bits to be same But here in this case,

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**Channel Coding**

- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes:	000 000 111 111 000 111
Received coded bits (Single Error):	100 000 111 111 000 111
Received coded bits (Double Error):	110 000 111 111 000 111

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they are not, because what we have received is 1 0 0. Now what does that mean?

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That means there is a transmission error. So we are able to detect single error using a rate one third repetition code.

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**Channel Coding**

- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes:	000 000 111 111 000 111
Received coded bits (Single Error):	100 000 111 111 000 111
Received coded bits (Double Error):	110 000 111 111 000 111

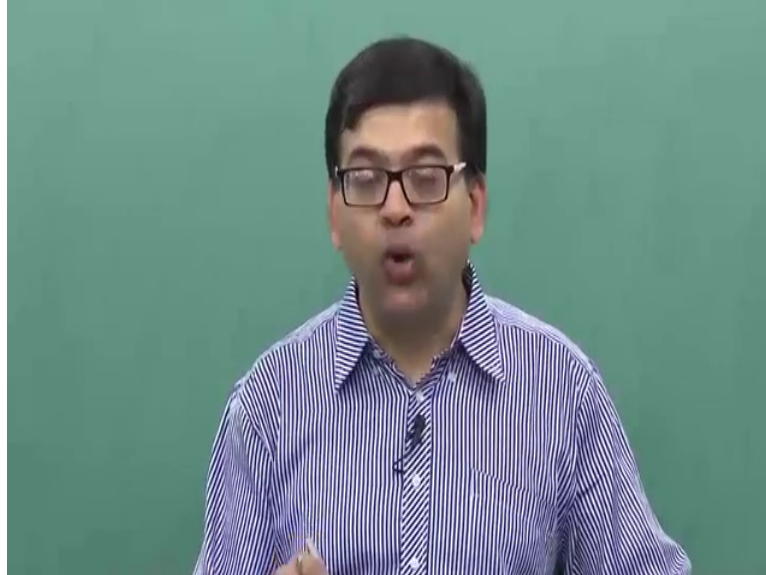
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If we look at other sets of bits 0 0 0, there is no error here; 1 1 1 no error, again no error here, no error here, no error here. So rate one half repetition code was able to detect single error, even rate one third code is also able to detect single error.

Now let's look at double error. So let's consider scenario when first two bits are received in error. So we have 1 1 and rest of the sequence is this, Ok. Now can we detect double error? We just look at; we again look at 3 bits at a time. So if you look at 3 bits at a time, the first 3

bits are 1 1 0. Now we could see that there is an error. Why? Because either this should have been 0 0 0 or 1 1 1

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but we received 1 1 0. So we are able to detect using rate one third repetition code

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**Channel Coding**

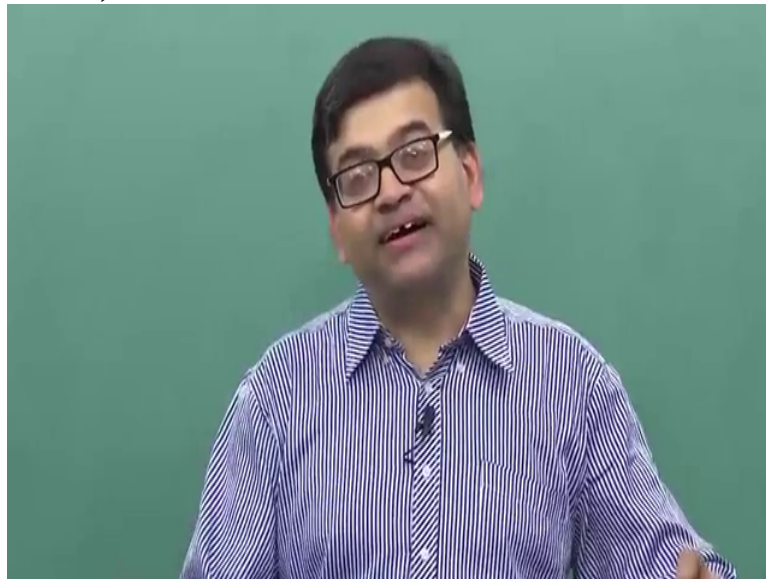
- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes:	000 000 111 111 000 111
Received coded bits (Single Error):	100 000 111 111 000 111
Received coded bits (Double Error):	110 000 111 111 000 111

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we are able to detect double errors as well which we were not able to detect using rate one half repetition code. Now let's look at the error correcting

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capability of this code So let's go back again and look at

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**Channel Coding**

- Example (contd.)

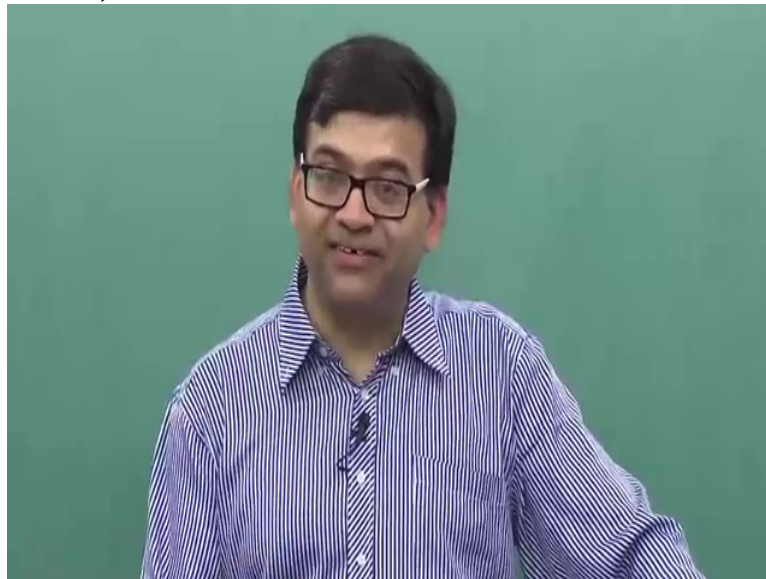
Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes:	000 000 111 111 000 111
Received coded bits (Single Error):	100 000 111 111 000 111
Received coded bits (Double Error):	110 000 111 111 000 111

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single error situation So when single error happens, so something like this; let's say one of the bit got flipped, 1 0 0. Now can we correct single errors?



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And the answer in this case is yes. Why? Because if you look at these 3 bits, two bits are already 0 and one bit is 1. So it is, and what are the possible outcomes?

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**Channel Coding**

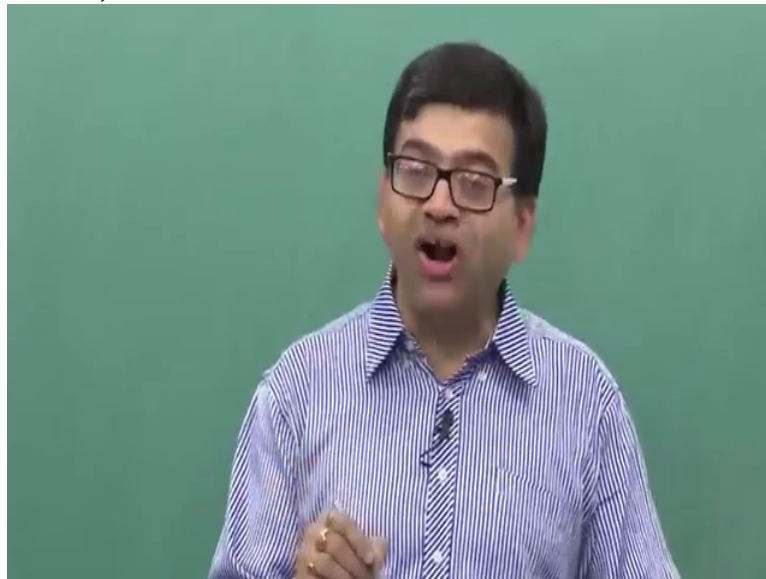
- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes:	000 000 111 111 000 111
Received coded bits (Single Error):	100 000 111 111 000 111
Received coded bits (Double Error):	110 000 111 111 000 111

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This could be either 0 0 0 or 1 1 1 and it is more likely

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that one bit got flipped. It is more likely that 0 got flipped to 1 rather than two 0s, two 1s getting flipped to 0. So it is

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**Channel Coding**

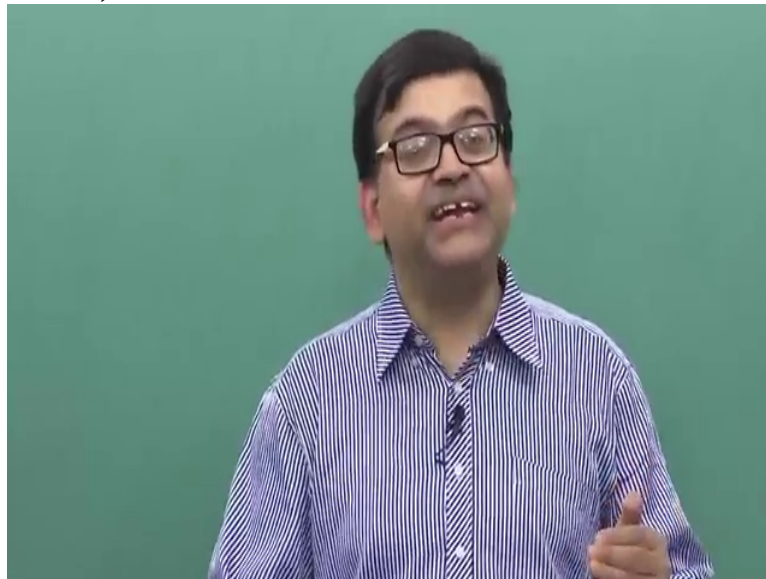
- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes:	000 000 111 111 000 111
Received coded bits (Single Error):	100 000 111 111 000 111
Received coded bits (Double Error):	110 000 111 111 000 111

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more likely that this bit got flipped from 0 to 1 instead of these two bits getting flipped from 1 to 0. So using majority logic, these majority of the bits are 0; we will decode this as 0. So we can see this rate one third repetition code can correct single error. This was not possible for rate one half repetition code. Now can it correct double errors? Now if you look at this 1 1 0, it will think that this particular bit got flipped from 1 to 0 so it will decode this as 1. So this cannot correct double errors. So to summarize, we saw that rate one half repetition code can detect single error but cannot correct single error.

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It cannot detect double errors whereas rate one third repetition code can correct single error

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**Channel Coding**

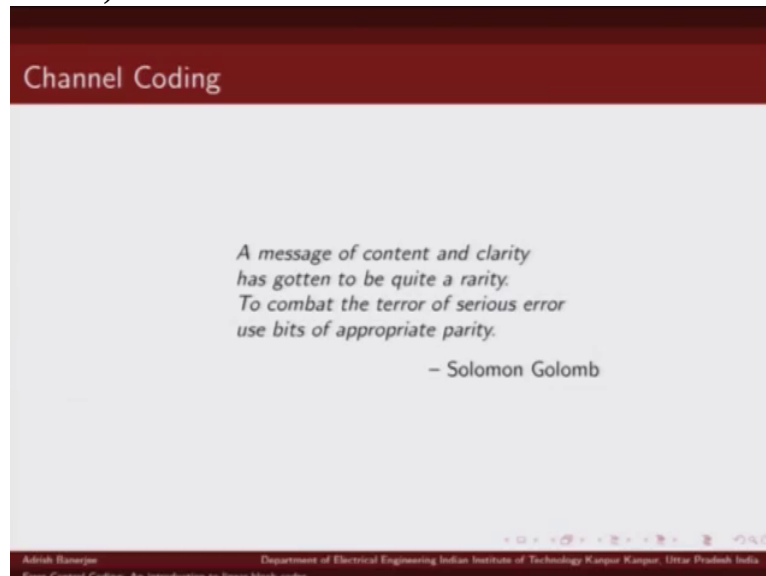
• Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes:	000 000 111 111 000 111
Received coded bits (Single Error):	100 000 111 111 000 111
Received coded bits (Double Error):	110 000 111 111 000 111

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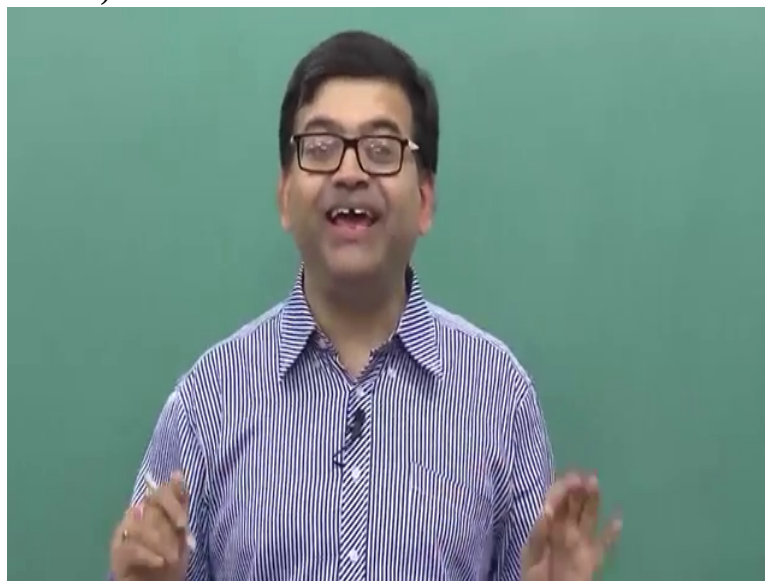
and can detect single error. It can detect double error but it cannot correct double errors. So why is this code better than the first code? It has certainly has better error detecting capability than the rate one half code. This we will discuss in subsequent lectures. It has to do with separation between, the distance separation between the code words and you can see basically in this particular code we are using two redundant bits and in the previous case we were just using one redundant bit. So the error correcting capability and error detecting capability of the code is depending, is dependent on the distance properties of the code and we will talk about in subsequent lectures. So to summarize it I think

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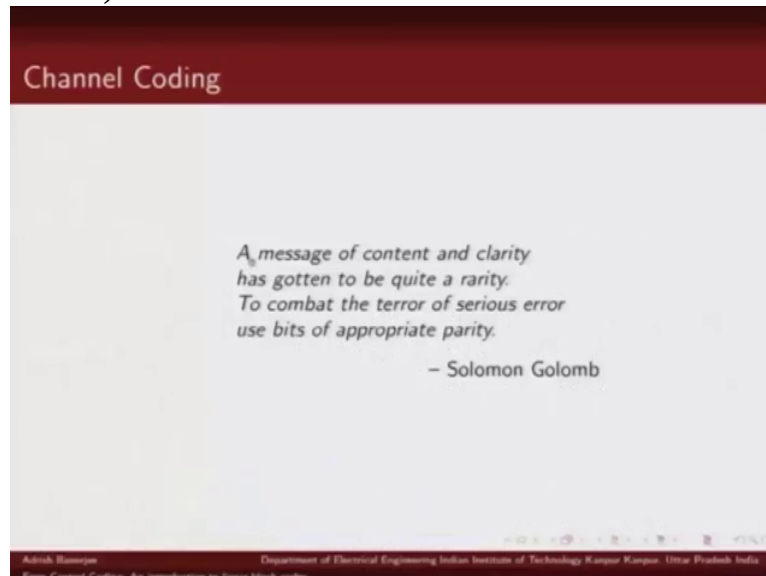
this quotations by Solomon Golomb rightly

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captures what error correcting code is all about So I will read it. A message

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The slide features a dark red header with the text "Channel Coding" in white. The main content area is light gray and contains a quote in black text: "A message of content and clarity has gotten to be quite a rarity. To combat the terror of serious error use bits of appropriate parity." Below the quote is the attribution "- Solomon Golomb". At the bottom of the slide, there is a dark red footer containing the name "Aditya Banerjee" and the affiliation "Department of Electrical Engineering, Indian Institute of Technology Kanpur, Kanpur, Uttar Pradesh, India".

of content and clarity has got to be quite a rarity. To combat the terror of serious error, use bits of appropriate parity. Thank you.