Principles of Communication- Part I
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Module No 2
Lecture 06: Duality Property of Fourier Transform and Introduction to Linear Time Invariant (LTI) Systems

Hello welcome to another module in this massive open online course, alright. So let us continue our discussion on the properties of Fourier transform. Let us look at the duality property of Fourier transform which is a very important and interesting property.

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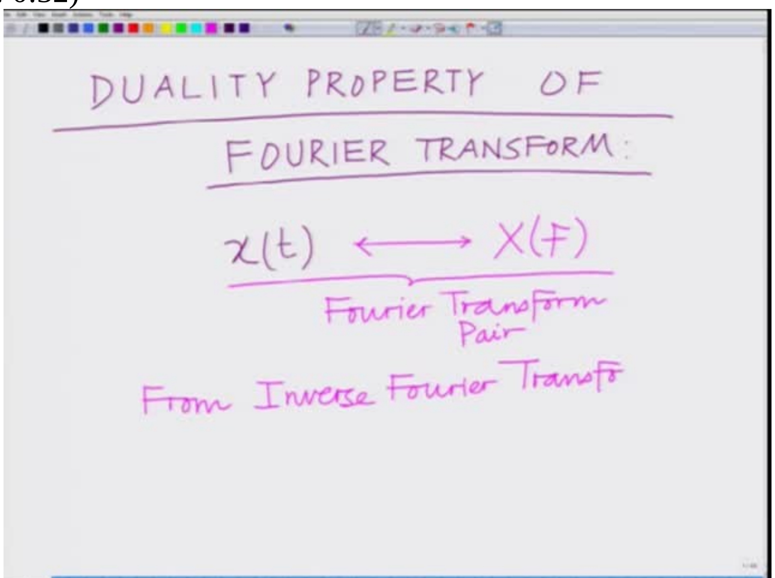

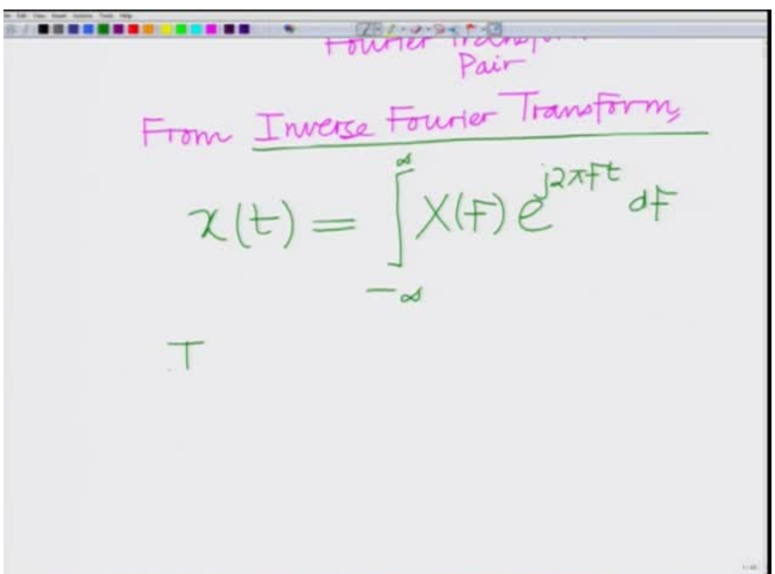

So what we want to look at today is the duality, we have to look at the duality property of the Fourier transform, okay. So let us say x(t) and X(F) form a Fourier transform pair. So we have x(t) and the Fourier transform is denoted by capital X(F) , okay. So this is our Fourier transform pair , alright. Now using the inverse Fourier transform or from the inverse Fourier transform what we have is we have well capital X(t) is given as the inverse Fourier transform of X(F) since a form a Fourier transform pair e to the power of j 2 pi Ft dF. This is your inverse Fourier transform, correct?

This is the relationship we have from the inverse Fourier transform, there is small x(t) is - x - infinity to infinity capital X(F) e to the power of j 2 pi Ft DF. Now we what we are going to do is we are going to do something interesting, we are going to interchange these variables t, remember t and F are simply variables, alright. So what we are going to do, we are going to replace t by F and F by t, so we are going to interchange t and F, okay. Because this integral, remember it holds for all t and all F.

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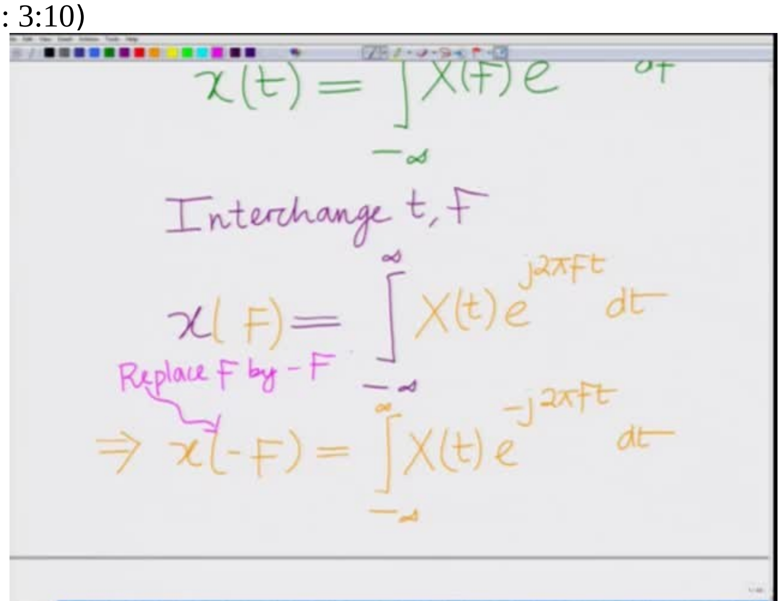

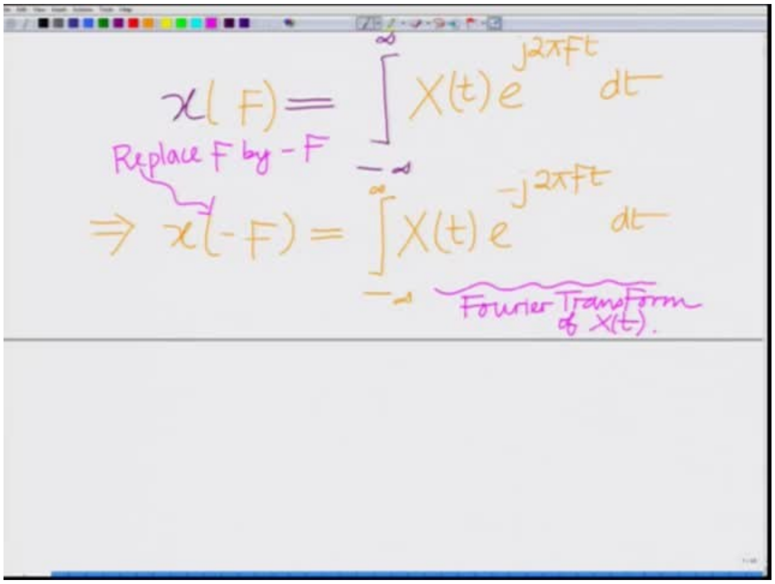

So now what we are going to do is we are going to interchange or let me just write it more clearly we are going to interchange when we interchange t and F what we have is X(F) equals - infinity to infinity capital X(t) e to the power of j 2 pi Ft dt.

Now what I am going to do is I am going to replace F pi - F. So what I will have is e to the power of j 2 pi Ft will becomes e to the power of - j 2 pi Ft this becomes X of - F. So what I have is x of or let us write another step this implies that X of - F equals integral - infinity to infinity x(t) e to the power of - j 2 pi Ft dt, alright. So what we have done here is we have replaced F by - F.

And now if you look at this integral capital X(t) e to the power of - j 2 pi Ft dt integral - infinity to infinity this is nothing but the Fourier transform of capital F(t), alright correct? So we have small x(t) with Fourier transform capital X(F). Now what we are saying is that capital X(t), correct? That is if we replaced F by t that is instead of the Fourier transform in the frequency domain if you consider that as a time function capital X(t) it is equivalent Fourier transform becomes small x of - F.

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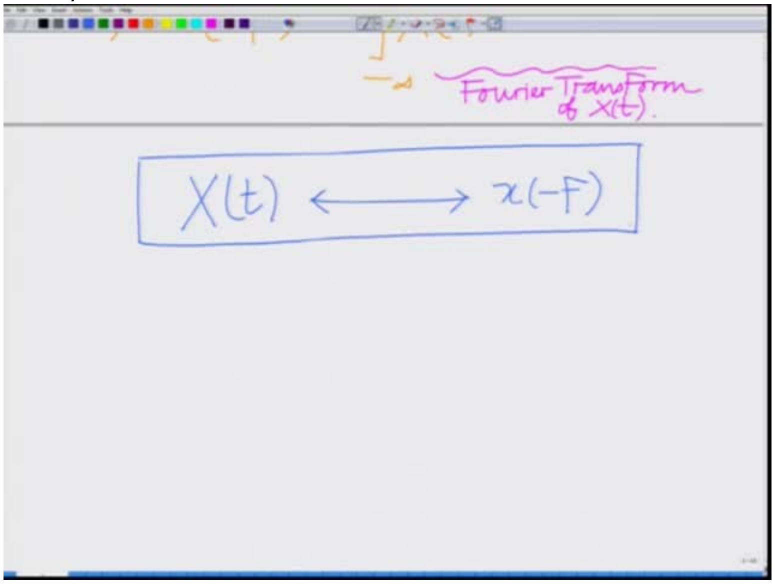

That is what was previously the time function if you replace t by F or replace t by - F that becomes the Fourier transform capital X(t) and that is the interesting result that we have that X(t) that is capital X(t) is Fourier transform pair with small x of - F and this is the principle of duality, Fourier transform duality of the Fourier transform.

So this is a principle of duality, this is your principle or property of duality of the Fourier transform that is if small x(t) and capital X(F) form a Fourier transform pair, then capital X(F) and capital X(t) and small x of - F also form a Fourier transform pair, okay. So this is the property of duality. So if we know the Fourier transform of a signal capital and small x(t), correct? Capital it is given by capital X(F) we can alternatively also find, right, using the principle of duality the Fourier transform of the time domain signal capital X(t).

For instance let us illustrate you this using an example, alright.

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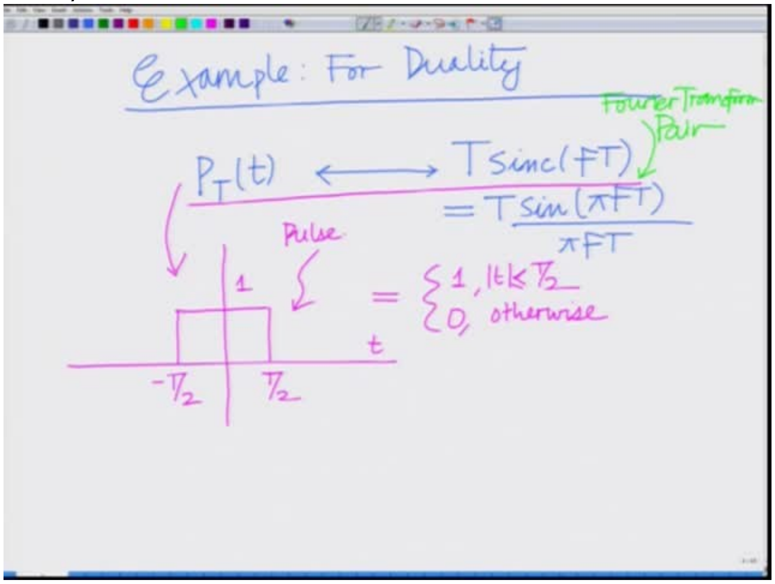

So let us illustrate this example for duality property of the Fourier transform, correct alright and we already know that if we have a pulse PT of t its Fourier transform is given by T sinc FT where sinc x is sin pi x by pi x, so this is T sin pi FT by pi Ft and we know this pulse, is a square pulse which is unity which has unit height for T between - T by 2 to T by 2. So this is a pulse or rather rectangular pulse, correct?

This is equal to 1 for magnitude t less than or equal to T by 2 and zero otherwise, correct this forms a Fourier transform pair.

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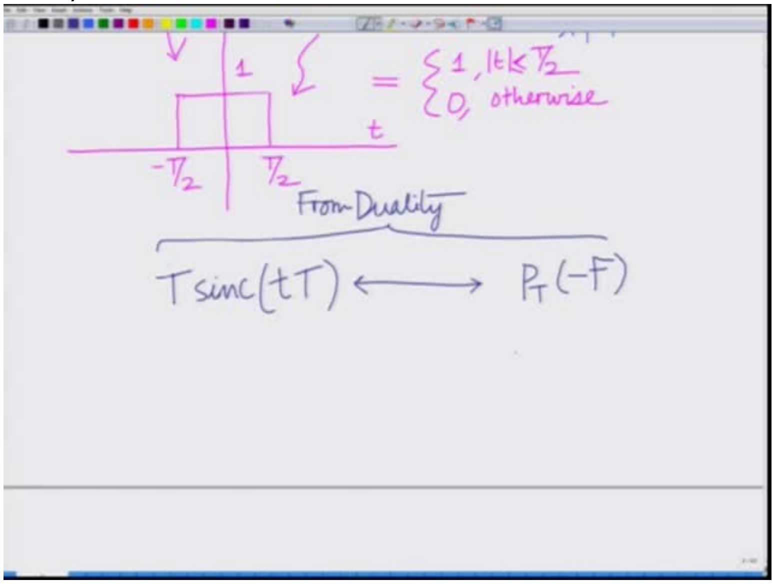

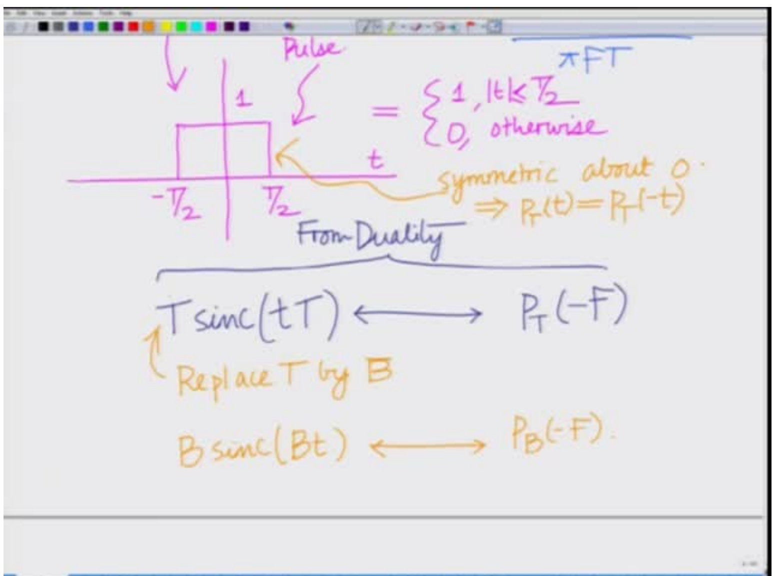

Now what we are going to do is this is our x(t) and this is our capital X(F). So this forms out Fourier transform. Now we are going to consider capital X(t) that is by F we replaced t. So t sinc FT replace F by t, so that becomes what does that become? That becomes T sinc that is the time domain T sinc t of t, T sinc FT becomes T sinc (tt) and the Fourier transform of this is given by PT of replace PT of - F. That is Fourier transform of this is given by x of - F replace t by - F that is PT of - F, okay.

And this is what we get employing the principle of that is from duality tells us that T sinc of t times capital T has PT of - F. Now what we are going to do is just for convenience, replace T replace the constant T by B then what we are going to have? B that is capital T is a constant, I am going to simply denote it by B. So T sinc Bt has or B sinc Bt has a Fourier transform PB of - F. Now this pulse PB remember this pulse if you look at this pulse which is 1 from - T by 2 to T by 2, it is symmetric about the positive and negative that it has an even symmetry, right?

It is that is PT of - t is equal PT of t, so therefore PT of - F is equal to PT of F, correct? Because it is symmetric about origin, so this pulse notice that this pulse is symmetric about zero. This is symmetric about zero, this implies PT of t equals P capital T of - t.

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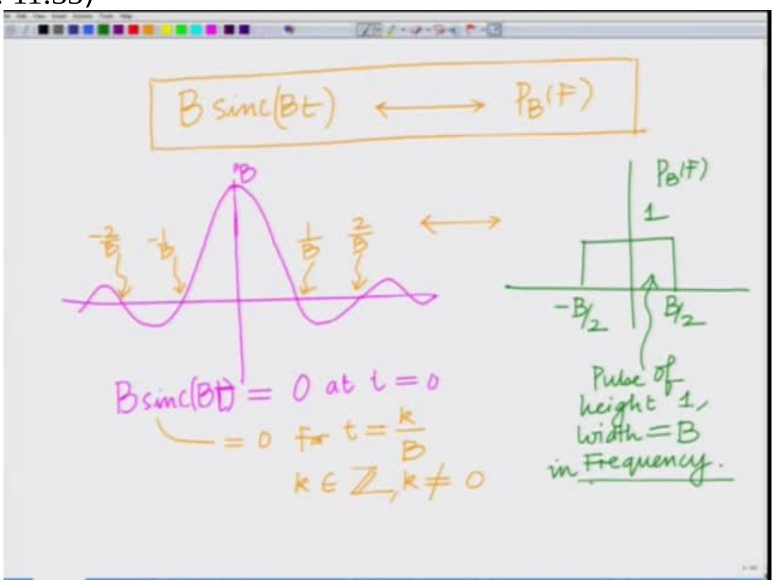

Therefore PB of - F equals PB of F and this PB of F remember and therefore finally we can write this as, to just write it completely B sinc Bt has Fourier transform PB of F. And therefore now we have an interesting result, now you can see that in the time domain you have B sinc of Bt that is B sinc of Bt remember equals B at t equal to zero. So at t equal to zero this is equal to B and then it has a sin that is it is a wave with decreasing amplitude and remember the zeroes occur at multiples of B that is 1 over B, 2 over B, - 1 over B, - 2 over B.

So this is equal to zero for t equals k over B where k element of any integer, k is not equal to to zero, alright. So we have B sinc Bt, remember at t equal to zero this is capital B except at that position or all integer multiples of 1 over B, correct? That is t equal to k over B, alright. This is equal to zero because remember we said the sine this is the numerator we have sine of pi Bt and sin is zero at all integer multiples of pi. Therefore at T equal to any integer k over B this will be zero.

And in the frequency domain what this result says in the frequency domain it is Fourier transform is given by PB of F. that is now if you look at the Fourier transform will be, remember PB of F is a pulse from, correct? of height 1 from - B by 2 to B by 2 this is your P of b of F. So this is a pulse height 1 width equal to B in the frequency domain , correct?

So now we have used duality, so what is interesting is previously we have seen pulse in time has a Fourier transform which is sinc in the frequency, now what we are saying is a sinc analogously the sinc in the time domain has a Fourier transform which is given by pulse or a window in the frequency domain and this is very interesting because it shows that this signal that is sinc in the time has a Fourier transform which is limited in the frequency domain, that is it is nonzero only in - B over 2 to B over 2, correct?

That it is bandwidth based by bandwidth of the w equal to B over 2 and it is zero everywhere and this is therefore very relevant in the context of communication.

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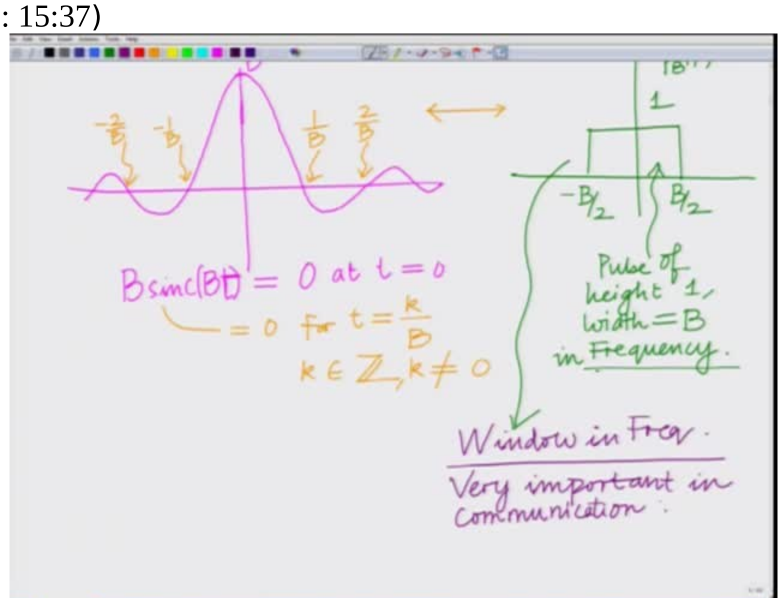

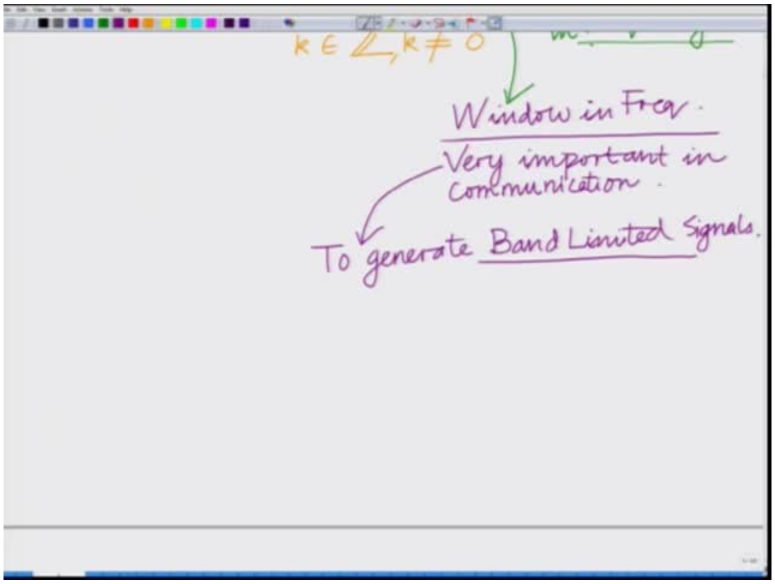

The reason being in the communication that is if you look at the communication, so this if you look at this pulse or window in frequency, this is very important in communication. For instance, alright for instance very important communication to generate band limited signals. This is the important idea to generate band limited, what do we mean by band limited signals?

That is each for instance each cellular operator has certain bandwidth over the which the operator which the which the cellular operator has purchased and over which the cellular operator is allowed to transmit, alright. So the the cellular operator can transmit the signal that is only is spans that particular band, alright? He is not allowed to, that is a particular cellular operator is not allowed to transmit signals on a bands which are outside that frequency band.

This so therefore it any communication any wireless communication 1 needs to restrict the transmitted signals to certain a certain band a certain possible a certain band that is allowed for transmission, the certain band that is licensed to that particular cellular operator or certain that certain band for instance in Wi-Fi there is an open band that is legal for that that in which 1 is allowed to legally transmit the Wi-Fi signal only it is 1 has to generate a band limited signals and this sinc function in the time domain helps us generate the band limited signal, alright.

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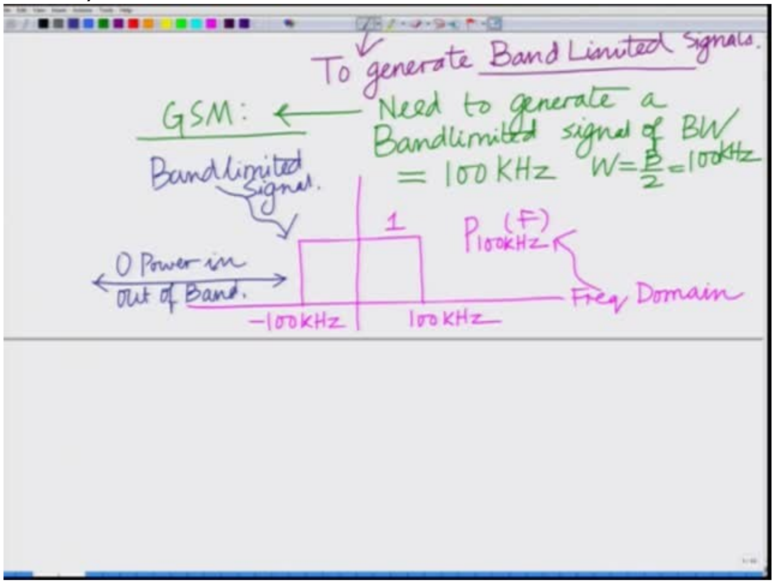

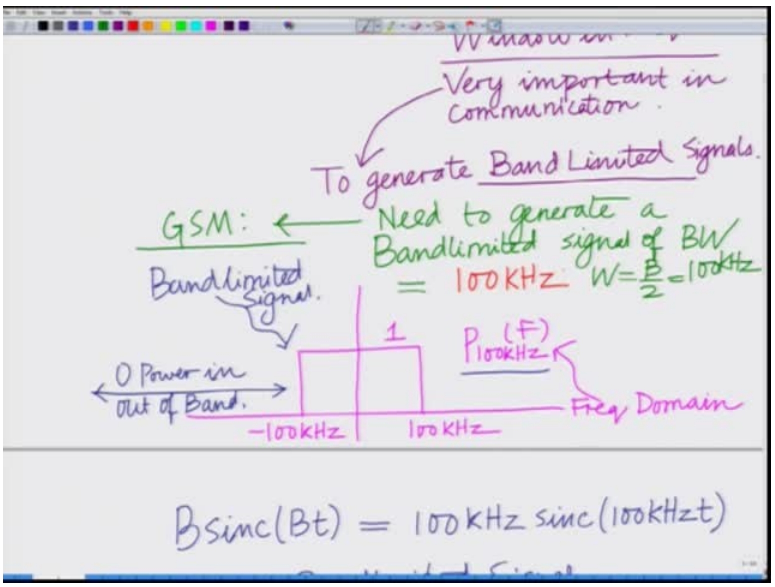

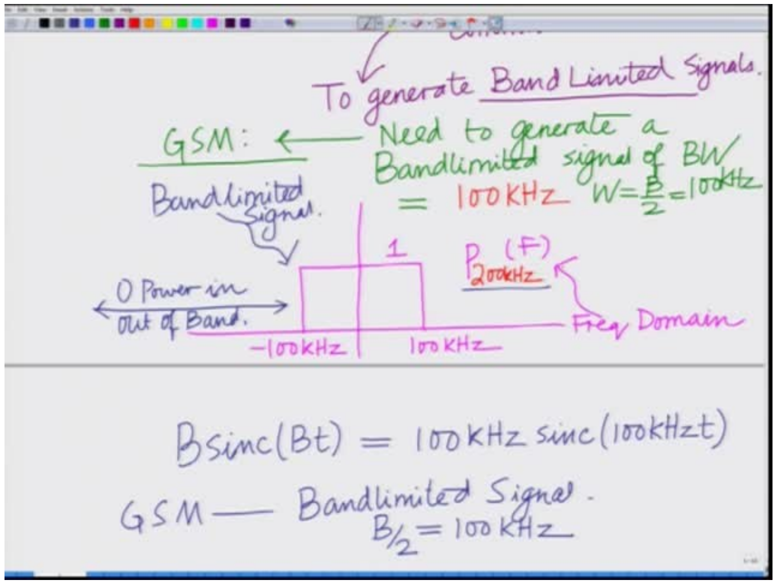

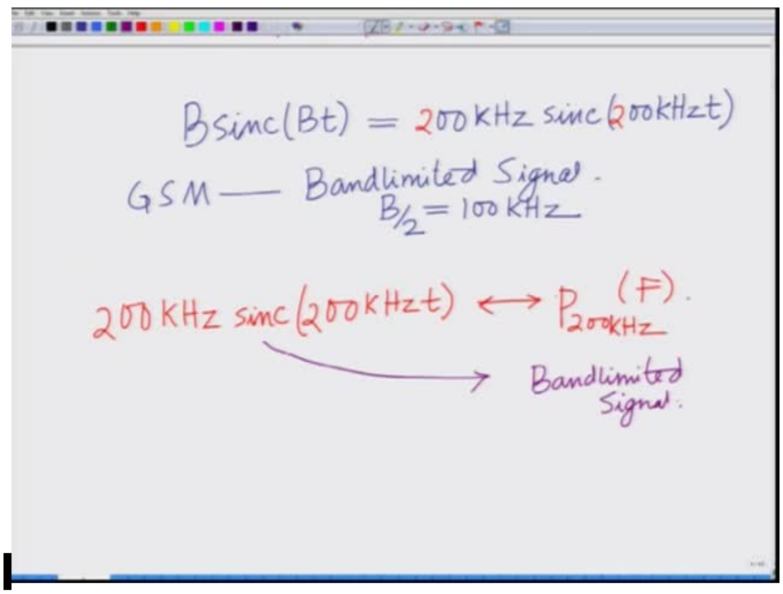

So let us look an application of this in GSM for instance let us look at the context of GSM say for example this is an example In GSM, 1 needs to generate is need to generate a band limited signal of bandwidth equals 100 kilohertz that is W equals B by 2 equals 100 kilohertz and therefore what we have is we have the frequency domain signal is spans from 100 kilo - 100 kilohertz to 100 kilohertz. Let us say this amplitude is unity. So we have P of D equals 100 kilohertz of F, alright.

So this is the frequency in the and this is what we call as a band limited signal, remember this is what we are calling as a band limited signal because the signal is limited to the band, - 100 kilohertz to 100 kilohertz. The energy or the power that is radiated in the band outside that is out of band radiation that is a power that is radiated or the power transmitted in the band outside it is - 100 kilohertz to 100 kilohertz is zero.

Therefore it does not cause interference to other wireless system or other communication system or the signals of other cellular operators in bands that are outside this specified band. So the out of band radiation is zero. So this is the band limited signal, alright. So zero power in out of band. And what we have shown is that this signal if you want to generate such a band limited signal then required time domain signal is B sinc B of t which is basically your 100 kilohertz sinc 100 kilohertz times t.

So now for the GSM if you want to generate bandlimited signals, correct? With B by 2 equals 100 kilohertz now I can conveniently employ that duality property at this signal which is 100 kilohertz times sinc B sinc Bt this will give me the band limited signal which is band limited to 100 kilohertz that is PB, I am sorry this has to be P, I am sorry this has to be PB so this B is basically your 200 200 kilohertz, correct? So B is equal to 200 kilohertz bandwidth W equal to 200 kilohertz .

So band limited signal of bandwidth , correct? 100 kilohertz. So what we have is B equals 200 kilohertz. So this is PB of 200 kilohertz so this will be your B sinc Bt which is basically 200 kilohertz it is (()) (22:15) sinc 200 kilohertz, B by 2 is 100 kilohertz. So again 200 kilohertz sinc 200 kilohertz that will give me this P 200 kilohertz of F, alright.

So this is the signal that is band limited this is your band limited. So this is basically the band limited, that is basically what we are saying is the transmitter power or radiated power belongs to a certain band and it zero outside the band so that it does not cause interference to the other devices or other operators. And therefore it is very important in the context of communication, alright. It is the sinc signal in the time domain which is a band limited signal, alright okay.

And so that is an example for basically the duality property the duality property of the Fourier transform, so that is an important application of the duality property of the Fourier transform. And now what we can what we are going to do is look at another very important principle of communication that is the transmission of a signal through a linear system.

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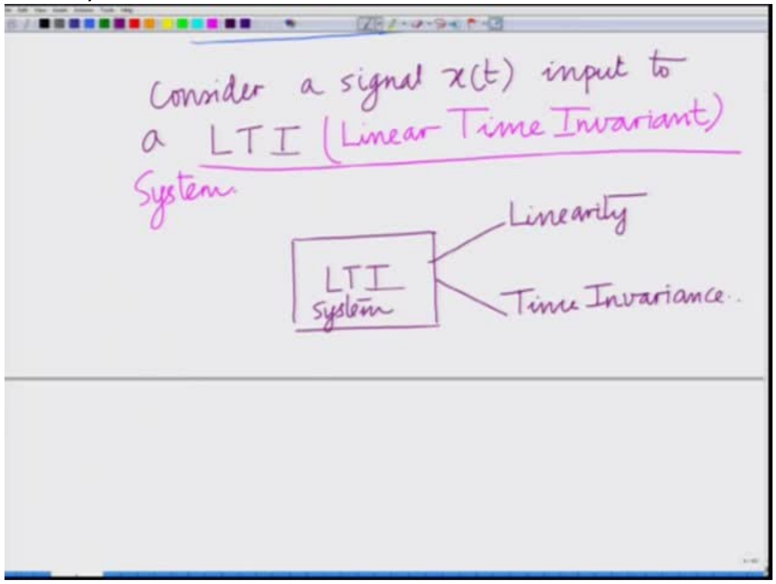

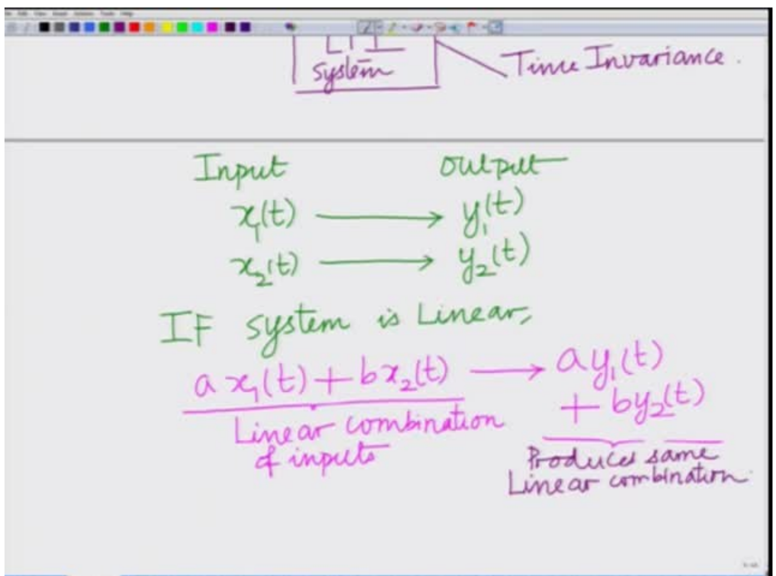

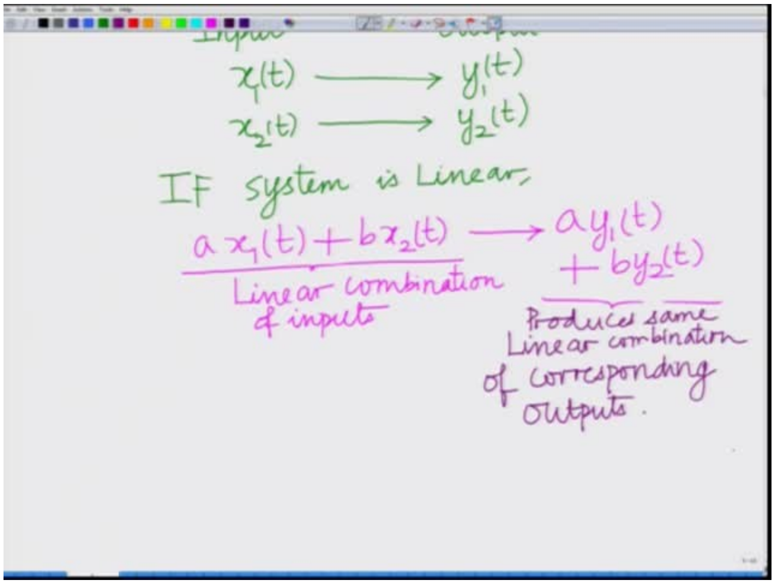

So what I would like to do is look at the transmission of a signal through a linear system, alright. So what we have is basically, let us consider a signal that is considered signal x(t) given as an input to a LTI where LTI denotes basically a linear time invariant system. This system this system has is very relevant and very important as for signal processing and communication, alright. So let me just briefly describe what is an LTI system?

An LTI system follows 2 properties that is linearity and as the name implies time invariance, correct? What is the linearity property? That is if x(t) gives output y(t) that is x1 t gives output y1 t that is your input x2 t or x1 gives the output y1 t, x2 t gives out put y1 t then if the system is linear then a times x1 t plus b times y2 t gives b times x2 t gives an output a times y1 t plus b times y2 t.

That is linear combination of inputs produces the same linear combination of outputs of the corresponding outputs that is if input x1 t produces output y1 t input x2 t produces output y2 t then if I give a linear combination of the inputs that is a times x1 t plus b times x2 t then the your observed the output is given by the corresponding linear combination of the outputs that is a times y1 t plus b times y2 t this is the linearity property, alright.

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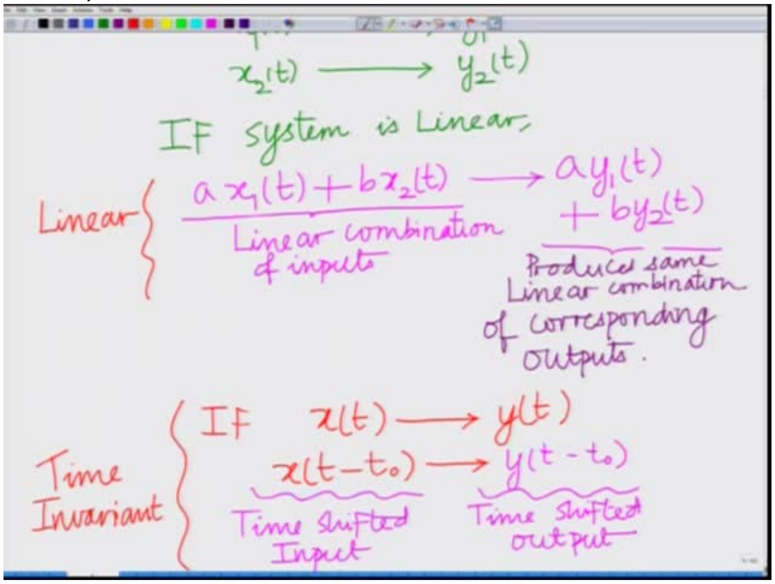

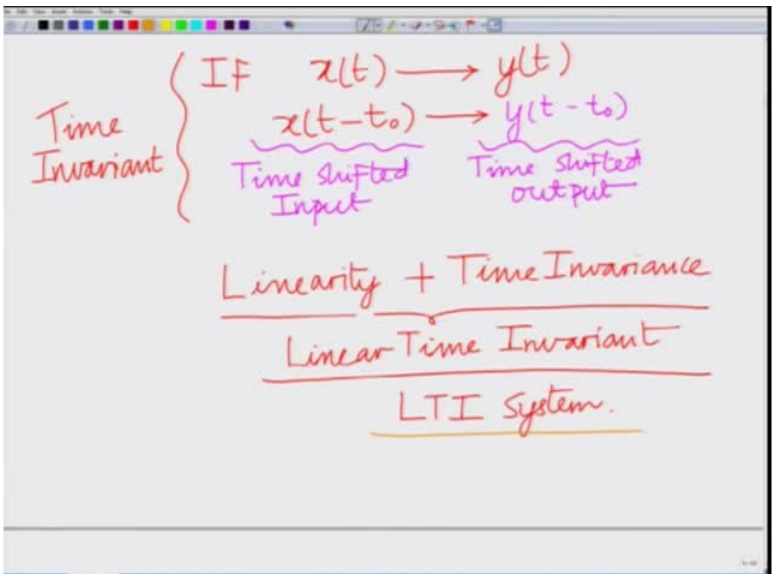

And then the time invariance which is basically if x(t) gives output y(t) then x(t) - x(t) not that is time shifted input gives that is if you have the time shifted input gives time shifted output. That is output is shifted by that is the output is shifted by t not. That is if the input is time shifted by x(t) - y(t) not, correct? It can be delayed by t not or it can be advanced that is if t not is negative than it is an advanced in time correct?

Then the output is similarly time shifted that is why t - t not and that is for every x(t) that is if x(t) is an output for any input x(t) the output is y(t). If the input is shifted by t not that is x(t) - t not the output is y(t) - t not, right? The output is simply a time shifted version of the output corresponding to the corresponding to x(t) then this system is known as a time invariant system. Such a system is known as a time invariant system. This is known as the, this is known as a time invariant system, alright.

So this system if this (prop) system follows this property that is ax1 t plus bx2 t gives rise to a y1 t gives output ay2 t, ay1 t plus by 2 t and x(t) by t not gives output y(t) - t not that is if the system follows linearity and time invariance that is linear follows the linearity property such a system is known as, such a system is known as a linear time invariant or LTI system. That is if the system follows the linearity and time invariance property then this system is known as a an LTi system or a linear time invariant system, alright.

So what we have done in this module is we have looked at the duality property of Fourier transform, alright. We also looked at an example of that application of the duality property of the Fourier transform into a simple application in generating band limited signal in a communication in a practical communication system and also we have started our discussion on linear time invariant system. So we will stop this module here and continue with other aspects in subsequent modules, thank you.