Principles of Communication-Part 1
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Module 1
Lecture No 5
Modulation Property of Fourier Transform, Dirac Delta or Unit Impulse Function-Definition and Fourier Transform

Hello, welcome to another module in this massive open online course, alright. So let us continue our discussion on the Fourier transform, correct. Let us now look at the properties of the Fourier transform, let us look at the modulation property of the Fourier transform, alright. So let us look at the modulation that is what happens to the Fourier transform of a signal which is modulated, alright. So this is the modulation property of Fourier, transform.

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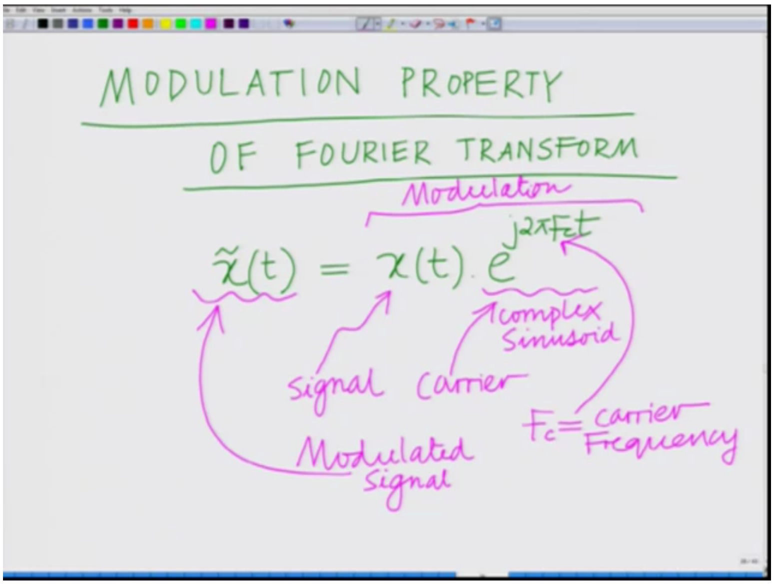

What we mean by modulation? Modulation simply refers to the operation that is x tilde t, which is x(t) multiplied by a complex a Sinusoid with a very high frequency that is multiplied by a Sinusoid at a very high frequency, in fact, this is 1 of the most fundamental principles of communication, that is this is a complex Sinusoid. This is which is termed as the carrier and this Fc that is this quantity Fc this is the carrier, this quantity Fc is the carrier frequency, alright. So we multiply with the Sinusoid this quantity Fc this denotes the carrier frequency and x(t) is your signal and x tilde t is termed as the modulated signal, this is termed as the modulated signal, this is this process is modulation.

Now in general when we modulated because here we are employing a complex Sinusoid in general we employ a real signal that is either a cosine are a sine signal, alright but in general so but this (prop) but in principle this can be the presented as x(t) multiplied by this complex Sinusoid e raise to j 2 Pi FcT is a complex Sinusoid at the carrier frequency Fc, alright. E raise to j 2 Pi FcT is the complex carrier Fc is the carrier frequency and this carrier frequency is much higher that is in communication system, this carrier frequency is much higher typically much higher than the maximum frequency orders of magnitude higher than the maximum frequency of the or maximum much higher than the bandwidth of the signal x(t), alright and this process is termed as modulation.

Of course, we are going to keep encountering this frequently, right? As we go through the various modules in this course principles of modulation in various communication system. We are going to keep encountering this very frequently and this is a very key aspect a very important step in any communications system, alright. Now what happens to the Fourier transform under modulation? That is if I consider a signal x(t) which has Fourier transform that is Fourier transform pair with that is x(t) which has Fourier transform FF X(F).

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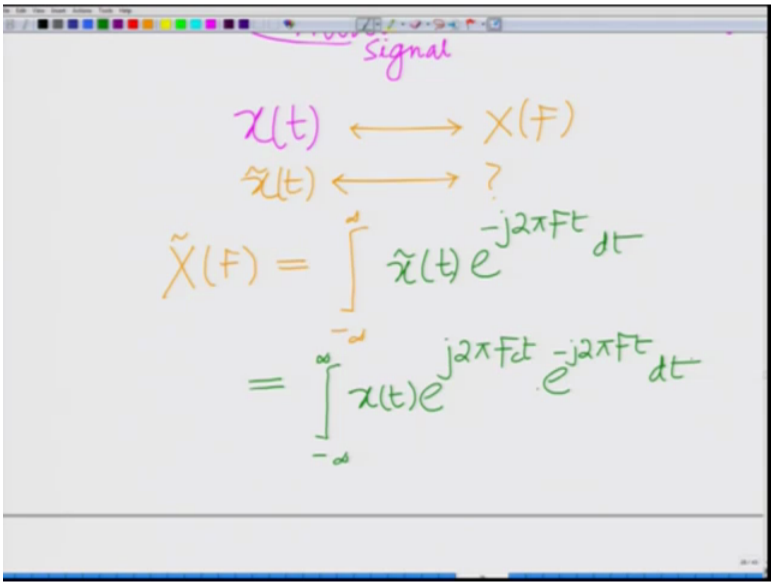

Now what is the Fourier transform of the modulated signal x tilde t, well x tilde F equals to minus infinity to infinity, I can write this as x(t) or x tilde T e to the power of - j 2 Pi Ft dt, which is substituting the expressions for x tilde t that is x(t) e to the power of j 2 Pi Fct, e to the power of - j 2 Pi Ft dt which is equal.

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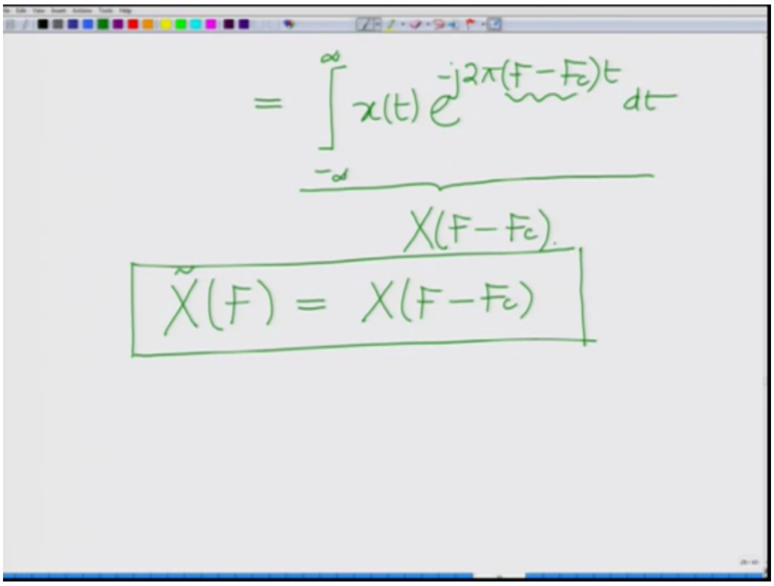

Now you can write this as x(t) e to the power of - j 2 Pi F - Fc dt and now you can see this is nothing but the Fourier transform of X evaluated at the frequency F - Fc. So what we have, we have this interesting result we have x tilde F the Fourier transform of the modulated signal is simply X(F) - Fc, it is a very interesting result and as I said since modulation is a very important step or a very important aspect of any communication system, this principle is used very frequently is very important in communication that is what we have seen is x tilde F is X(F) that is x tilde F the Fourier transform of the modulated signal is X(F) - Fc that is, it is simply a shifted version.

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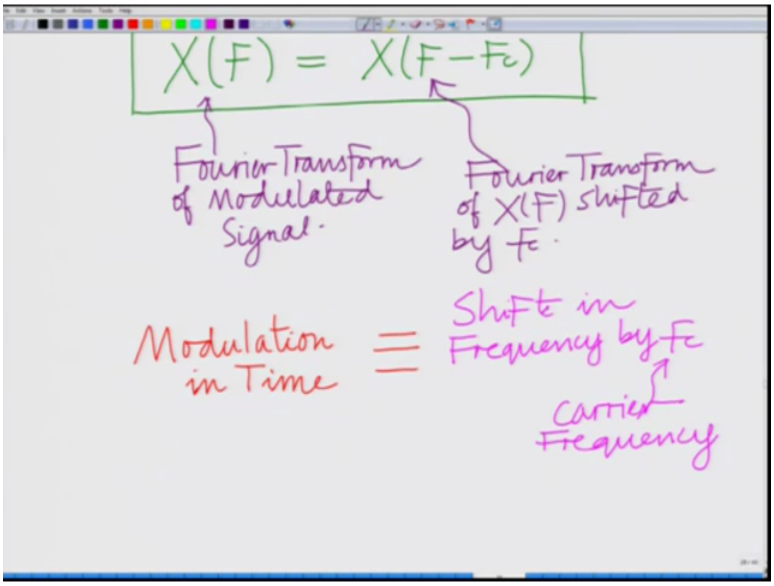

That is X(F) - Fc is the Fourier transform of X(F) shifted to Fc that is shifted by Fc, correct. So what we have is x tilde F this is the Fourier transform this is simply the Fourier transform of X(F) shifted by Fc. That is, so modulation in time so the principle is modulation in time. If you modulate modulation in time, this is equivalent to shift in frequency by Fc, where Fc we know is the carrier frequency. So modulation in time so if you modulate the signal in time that is simply that can be vision as take the original spectrum and shift it the frequency domain by Fc, where Fc is the carrier frequency.

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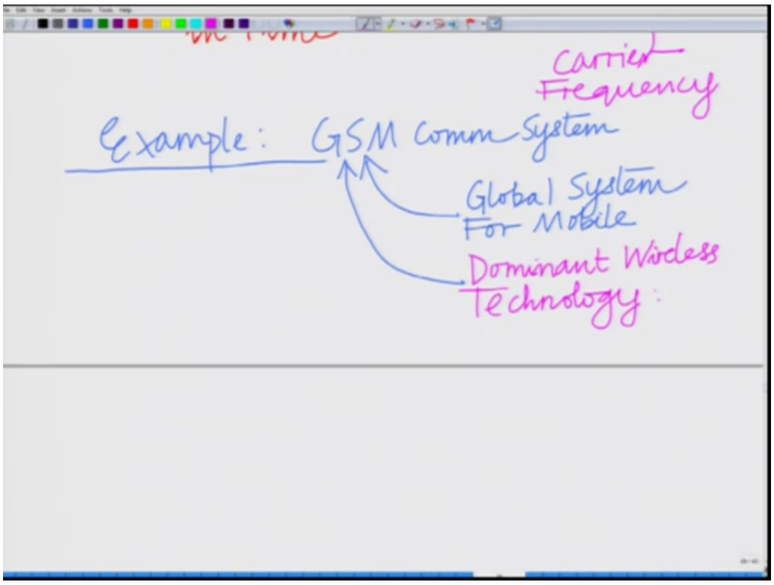

And this is an important principle in the communication systems as I have already said for instance let us take a simple example to understand this in a practical communication system, correct. Let us consider a GSM communication system as you all must be familiar, GSM is a wireless standard it stands for Global System For Mobile Communication and in fact it is one of the most popular wireless system wireless technology, wireless standards one of the most popular wireless technology which is used by a large number of wireless cellular user, alright, so GSM is the global for mobile and it is a very dominant wireless

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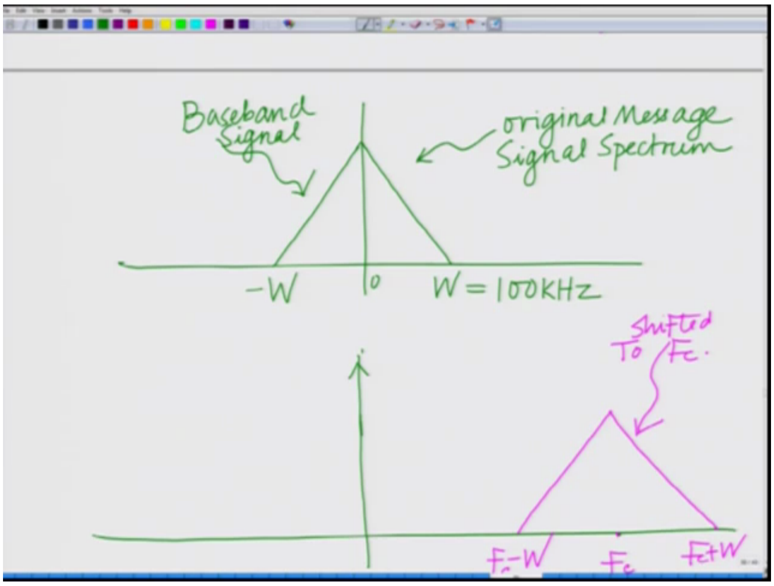

For instance, let us take a simple GSM, let us GSM if you look at the original signal, correct. What is termed it as a bandwidth of W equals to 100 kilohertz centered at frequency 0, so this is the original message spectrum or the original message; this is the original message signal spectrum for the purpose of transmission this is modulated to a very high carrier frequency.

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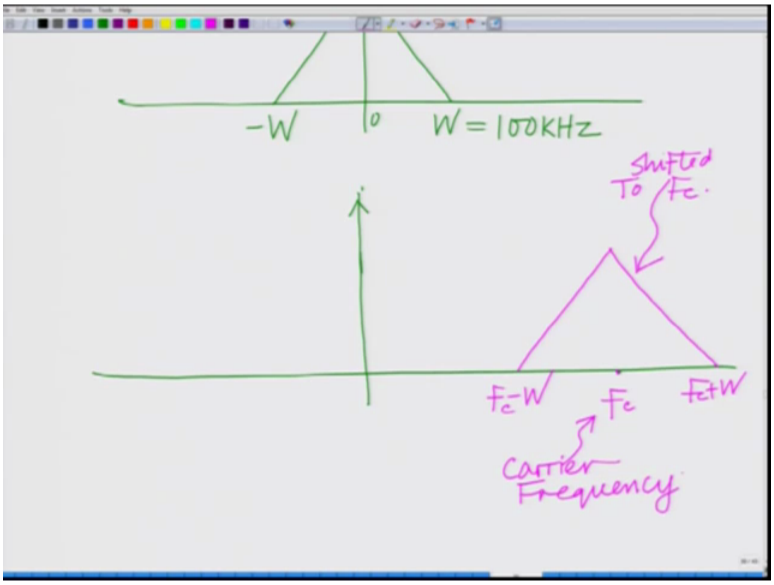

So this is modulated by a very high carrier frequency, I am not drawing it according to scale, alright. So it is modulated to a very high carrier frequency which is simply, so now it is shifted by Fc, so this will be Fc - w Fc + w, so this is shifted to Fc we have already seen Fc as the carrier frequency and in GSM typically Fc equals to either 900 megahertz or 1800 megahertz.

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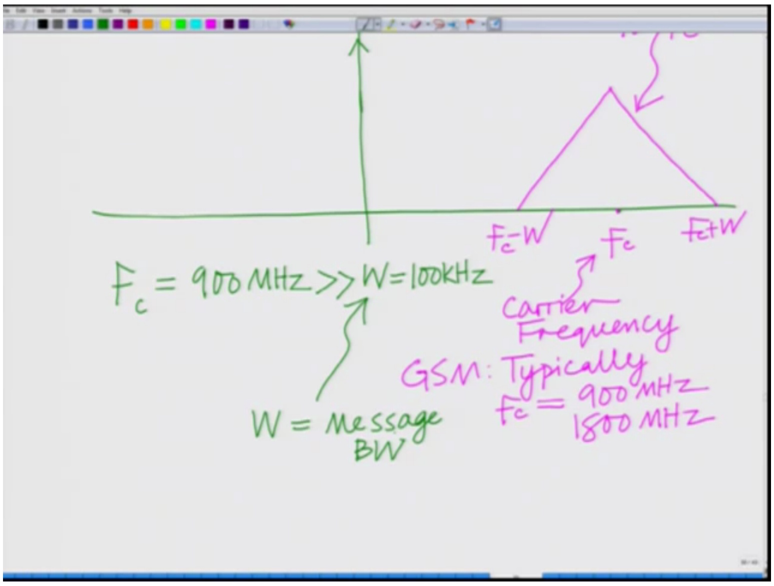

So what you are seeing here is that you are having an original GSM signal the message signal, which is of a bandwidth 100 kilohertz centered at frequency 0, correct? Now for the purpose of transmission I have to employ a very high carrier frequency in GSM employs a very high carrier frequency, which is 900 megahertz are typically around 1800 Megahertz and therefore this message spectrum is modulated with the carrier at 900 megahertz, which means original spectrum which is centered at 0 is shifted to the carrier frequency that is at 900 megahertz or 1800 megahertz depending on the carrier frequency.

Now the other thing that you can observe is the signal bandwidth is 100 kilohertz, right, which is of the order of 10 to the power of 3, correct or 10 to the power of 5, 100 kilohertz that is 100 into 10 to the power of 3, so 10 to the power of 5, but the carrier frequency is 900 megahertz which is 900 into 10 to the power you can roughly say 1000 megahertz, 1000 megahertz that is 1 gigahertz that is 10 to the power of 9. So your original message is of bandwidth 10 to the power around 10 to the power of 5 and when you translate it the transmitted symbol that is a modulated signal or the carrier has a frequency that is order of 10 to the power of 9.

So you can see that the carrier frequency Fc is much larger than the message bandwidth and this has a reason we are going to explain it in greater detail once we come to the modulation aspect, but you can see the modulating frequency the carrier frequency is much higher than the message bandwidth and this is a very important principle in communication. So we can see basically that Fc, so what you can see is that your Fc equals to 900 Megahertz is much larger than w which is equals to hundred kilohertz and this w is equals to the message bandwidth.

And this is also termed as the, this original message signal is also termed as the baseband signal this baseband signal, alright be centered at 0 frequency modulated to a very high carrier frequency and transmitted over the wireless radio channel and this is the principal of modulation, modulation means basically transmitting physically multiplying by a Sinusoid of a very high carrier frequency for the purpose of transmission, okay alright, so this is an important principle of communication. Now let us look at a very special function in both signal processing and communication for which we want to compute the Fourier transform and this is basically the unit impulse function.

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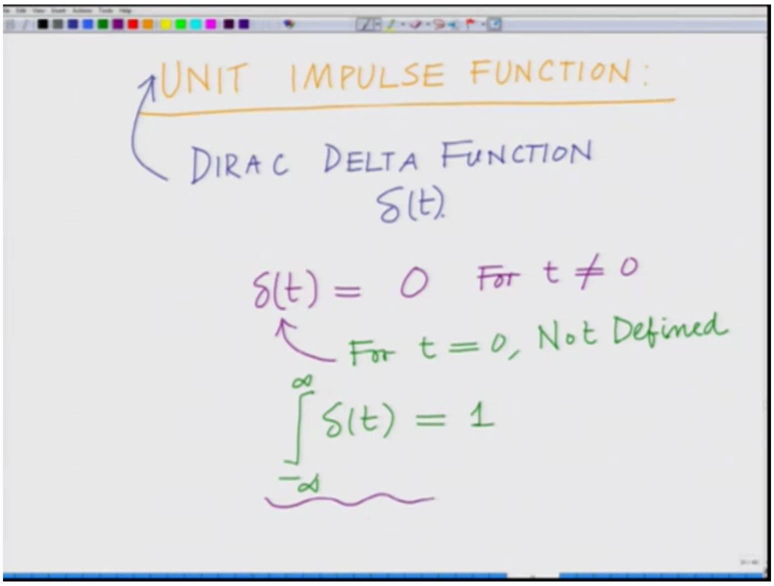

So let us start looking at another very interesting and important function, which is the unit impulse function. Now, this unit impulse function this is also known as the Dirac Delta function denoted by delta (t), this is the Greek symbol for Delta and this is represented in the time domain. Well, let us first define this Delta T, this direct Delta function is defined as 0 for all t0 equals to 0 , and for t equals to 0 it is not really defined that delta function for t equals to 0 it is not really defined not particularly defined. However, the integral - infinity to infinity of delta (t) this is equals to unity that is the area under this function that is if I look at this function the area under this, okay. The area under this Direct Delta function is always unity.

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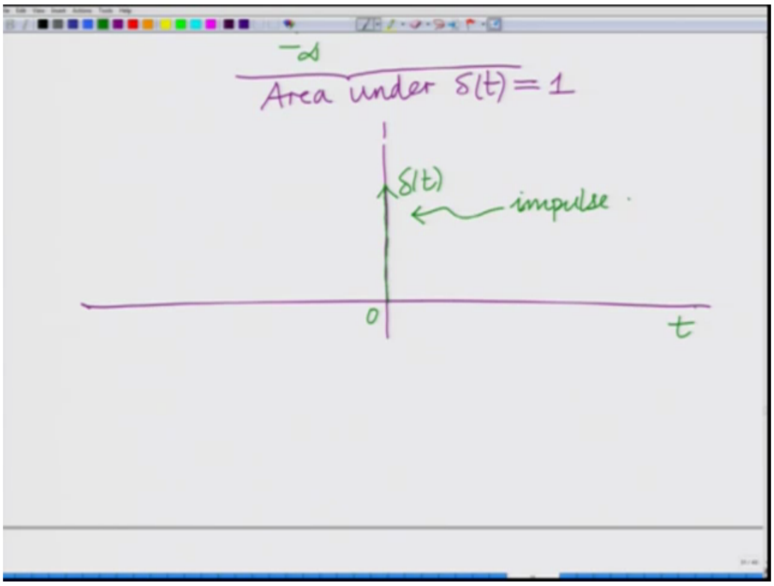

That is it is 0 everywhere, so value at 0 is not defined however, the area under this that is integral from - infinity to infinity delta (t) that is basically unity that is basically always a constant, that is 1. And it is represented as follows the special function this Delta function is represented as follows that is I have 0 everywhere and at 0 it is represented using a an impulse. Our unit this arrow this line with arrow pointing upwards is known as an impulse. That is it a 0 everywhere and at 0 its value is not defined, so it is shown with an arrow which is pointing upwards is also termed as an impulse for as we said the unit impulse function typically also termed simply as an impulse its meaning is evident from the (con) context. And the way to understand this impulse is, it cannot be described it cannot be visualize intuitively.

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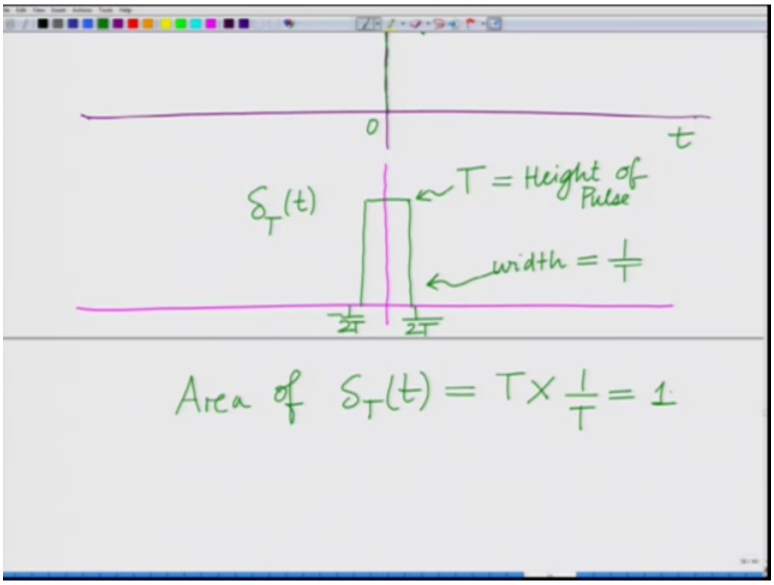

The way to understand this is basically, think of a pulse that is the way to understand this special function which is the impulse is to think about a pulse which is of height T of width 1 over T. So this so it expands 1 over 2 T - 1 over 2 T to 1 over 2 T, so width equals to 1 over T, alright. So area of the pulse, let us denote this pulse by delta T of t delta of sub capital T of t, Area, now you can see the area for any capital T, the area of delta T (t) is basically its height, which is equals to the area is basically the height which is equals to T into the width which is 1 over T which is equals to unity.

So we can think of a height, we impulse with height T, width 1 over T spanning from - 1 over 2 T to 1, so that the area is basically always capital T into 1 over T, which is basically 1 this is unity, alright. Now as the limit T tends to 0, now as a limit T tends to infinity, which means the height keeps increasing and the width keeps shrinking yet the area under it is a constant in the limit capital T tends to infinity and becomes the unit impulse function.

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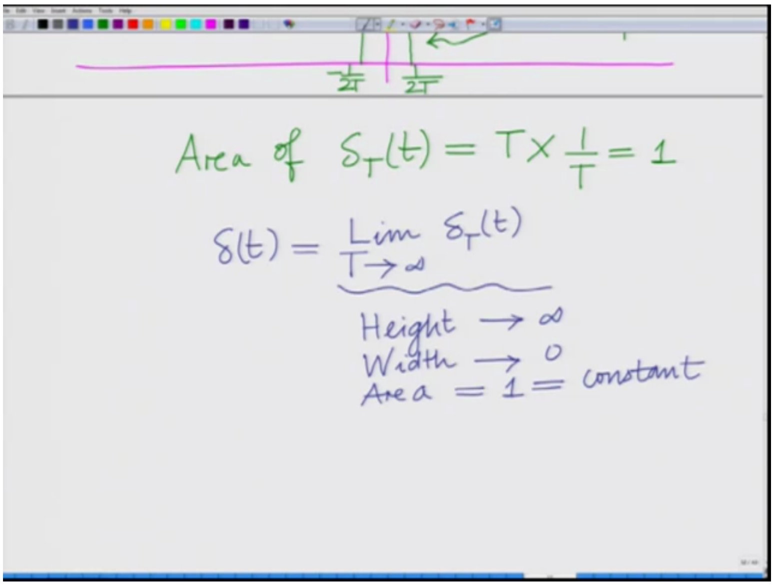

So delta T can be understood as the limit, one way to understand delta T is the limit as T tends to infinity of delta T which means that is this means height of the pulse tends to infinity, width tends to 0, and however area is 1 which is a constant. So this is basically 1 way to sort of intuit sort of visualize this unit impulse function which otherwise is very difficult to visualize or understand intuitively, alright.

Think of it as a pulse width 1 over T height capital T as the limit T tends to infinity, so that the width shrinks to 0, the height really becomes the height really becomes infinity, alright. And this impulse has several interesting properties, alright, so let us look at and it is important to understand these properties and this is important to use and we are going to use this function very frequently to study or to define the various aspects and study the various properties of the medication systems, alright.

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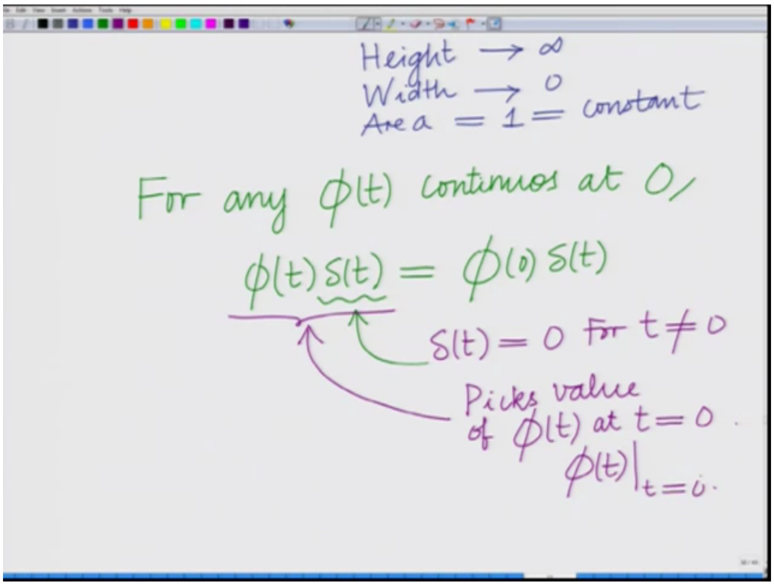

So the impulse has several interesting properties, alright. For instance, for any function and a continuous function Phi of t, for any continuous function Phi of t for any function Phi of t continuous at 0, alright. Need not be continuous in general for any function Phi of t continuous at 0 Phi t delta t is equals to Phi of 0 delta t. And this is natural since delta t equals to 0 for any t0 equals to, so delta t is 0 for any t0 equals to 0, so Phi by t Phi of t into delta t even Phi of t is continuous at 0. It is simply Phi of 0 times delta. So delta t is sort of picking the value of this function Phi of t at t equals to 0. So sort of picks or if you can put it over here picks of this operation, this operation picks so this picks value of Phi of t at equals to 0. Picks value of Phi of t at t equals to 0 that is Phi of t at evaluated at t equals to 0.

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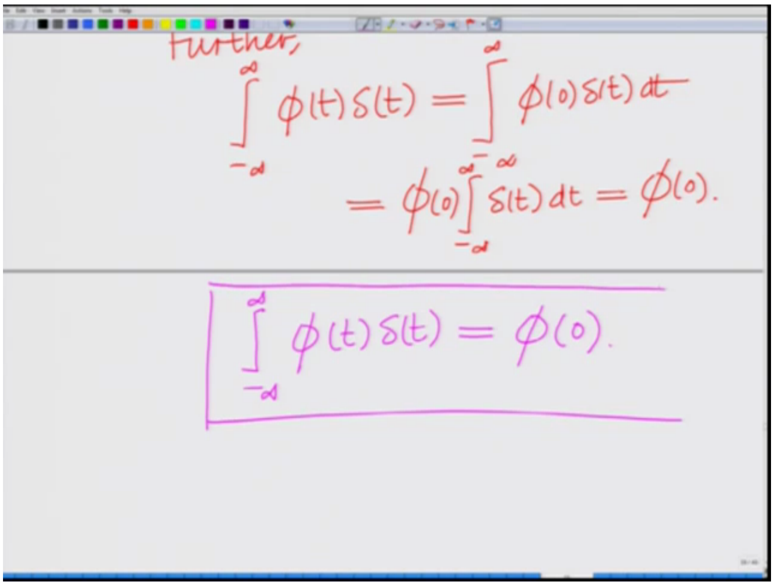

Now naturally further integral - t to - infinity to infinity Phi of t delta t by the o property is integral Phi of 0 delta t dt, Phi of 0 is constant, so it comes out of the integral so this is nothing by Phi of 0 integral - infinity to infinity delta t dt this is equals to Phi of 0, alright. So what we have is basically integral - infinity to infinity, Phi of t dt equals to Phi of 0, okay. So that is for any function Phi which is continuous at t equals to continuous at t equals to. Now let us consider a shifted version of this impulse that is shifted to t0 that is delta t - t0.

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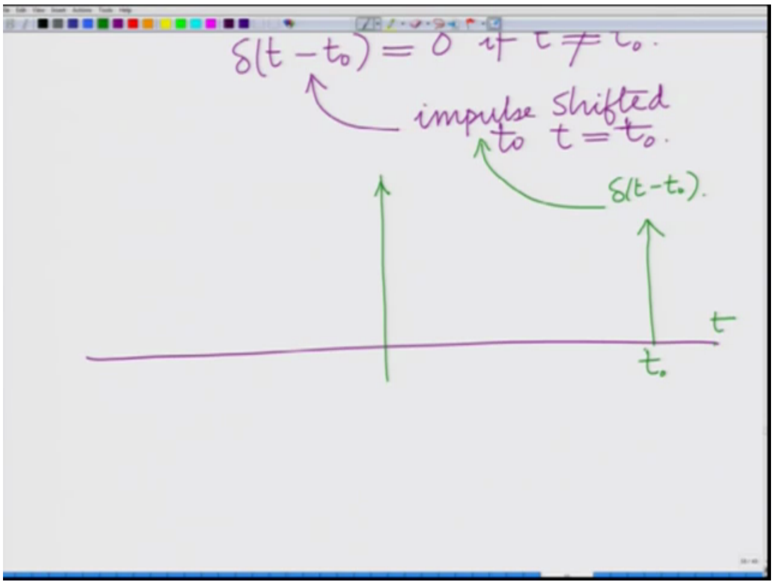

So let us consider delta t - t0 naturally this is 0 for every t0 equals to t nod, correct, what is this? This is impulse shifted to t equals to t0 therefore you have your access, correct, your time axis, this is your time and this denotes the impulse shifted to t0, alright. So this is basically your delta t - t0, alright this is the time axis, so this is your impulse shifted to t equals to t0.

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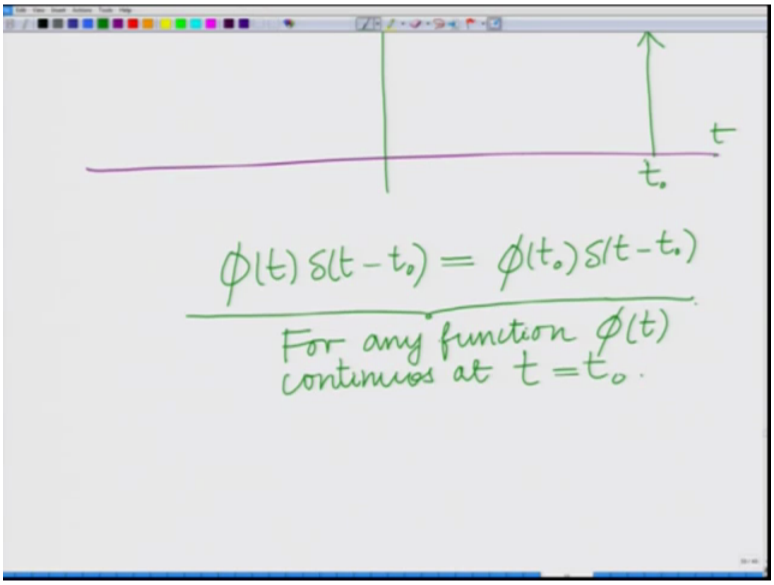

Naturally, Phi of t again Phi of t into delta t - t nod, since delta t - t0 is 0, for all t0 equals to t0, this is simply Phi of t0 into delta t - t0, correct, so this is obviously for any function Phi of t, continuous at t equals to t nod. Now I have to qualify the statement, this is for any function of Phi of t continuous at t equals to to t0.

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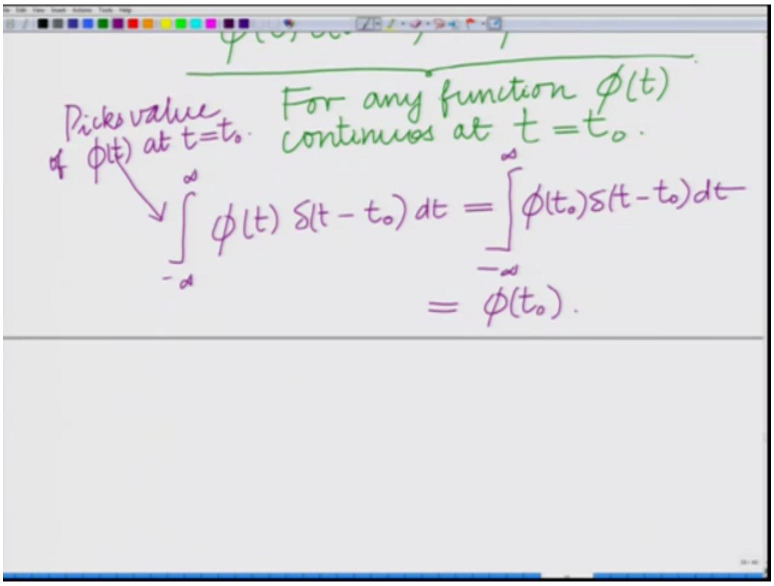

Further, if I integrate this that is integral - infinity to infinity Phi of t delta t - t0 dt this is naturally integral of Phi of t0 delta t - t0 dt, which is Phi of t0, okay. So this is basically picks the value or picks the value of Phi of t that is integral Phi of t delta t - t0 dt from - infinity to infinity is basically Phi of t0, alright. Basically, it picks the value of Phi of t the function Phi of t at t equals to t0, provided Phi of t is continuous at t equals to t0, alright. So this is an important constraint on the qualification of this argument or this result to keep in mind, okay alright.

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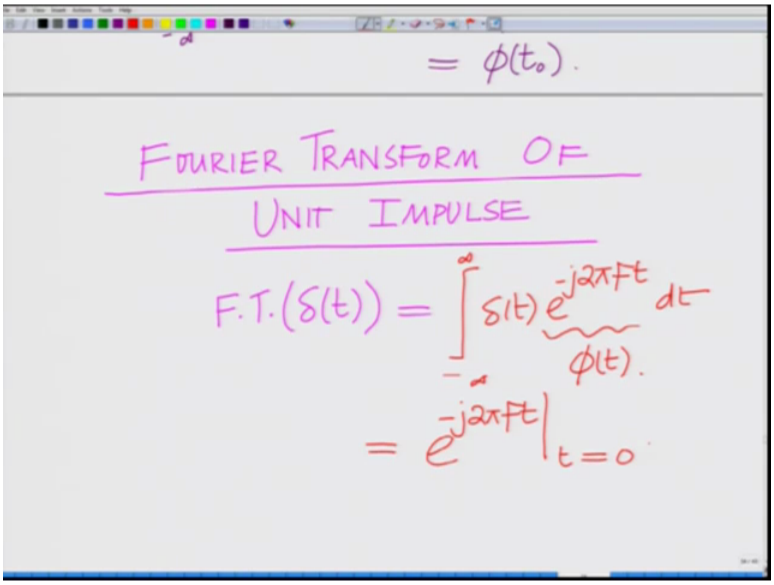

Now let us look at the Fourier transform of this unit impulse which is going to be very important, the Fourier transform of the unit impulse that is easy to evaluate that is your Fourier transform of delta t, this is equals to - infinity to infinity delta t e power of - j 2 Pi Ft dt alright, this is a standard expression for the Fourier transform and now you can treat this function Phi of t and by the property that we have shown before that is integral - infinity to infinity delta t Phi of t dt is nothing but Phi of t evaluated at t equals to 0. So this becomes e power - j 2 Pi Ft evaluated at t equals to 0 and e power - j F 2 Pi Ft evaluated at t equals to 0 is 1, so this is simply basically just 1 at all frequencies the frequency domain, alright.

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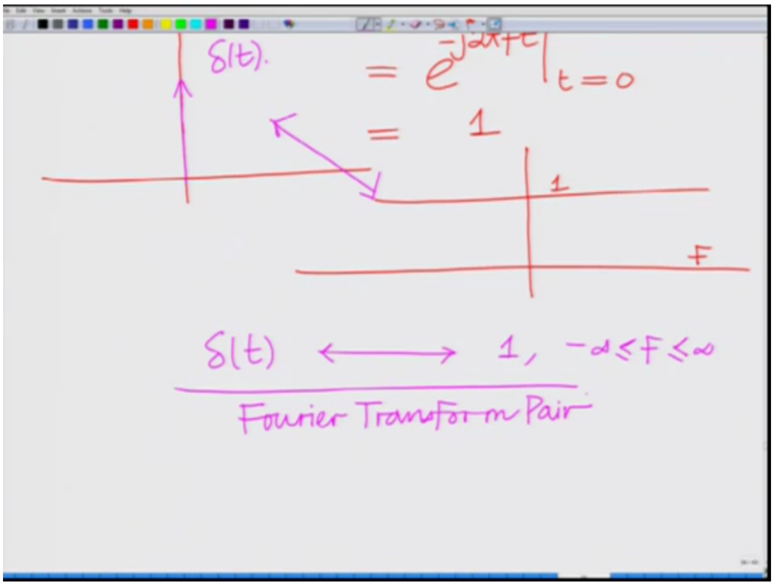

And this is the Fourier transform of this function which is the impulse, so these 2 forms a Fourier transform pair. So delta t the Fourier transform is simply unity for - infinity less than or equals to that is for all frequencies - infinity less than equals to infinity, this form a Fourier transform this form a Fourier transform pair.

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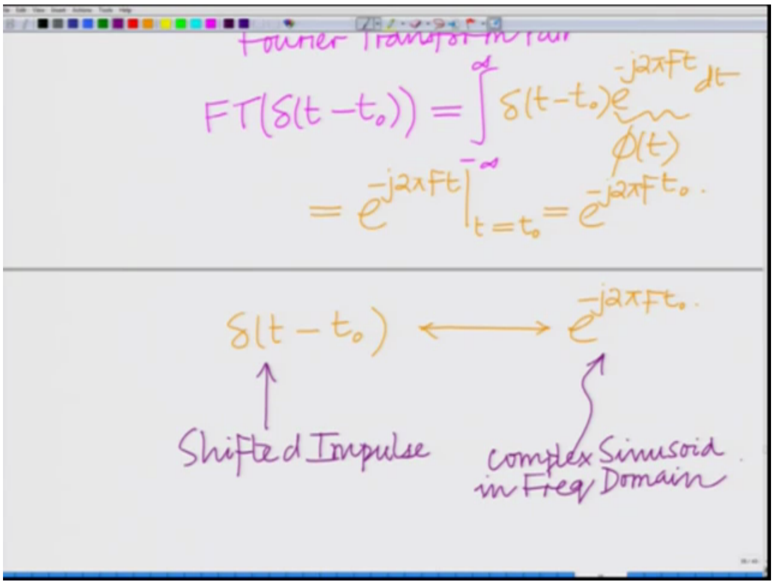

Against similarly, you can find the Fourier transform, the shifted impulse delta t - t0, correct. The shifted impulse of FT of delta t - t0 equals to - infinity to infinity delta t - t0 e power - j 2 Pi dt, once again this is your Phi t from the previous properties. So this is e to the power of - j 2 Pi Ft evaluated at equals to t0 that is e to the power of - j 2 Pi F Ft0. So delta t - t0that is the shifted impulse is basically the Fourier transform pair is delta t - t0 is e to the power of - j 2 Pi

Ft and notice that this is well, of course we know this is the shifted impulse, this is your shifted impulse and this e to the power of - j 2 Pi F0 in the frequency, remember this is in the frequency domain not the time domain, so this is a complex Sinusoid in the frequency domain, correct.

Usually we have the Sinusoid in the time domain which is e to the e power - j 2 Pi F0 t, which is frequency F0 which is a function of time at frequency F0. Now we have the analog in the frequency domain e power - j 2 Pi F t0, alright. So this is a complex Sinusoid in the frequency domain this is defined for all frequencies - infinity to infinity less than equals to F less than equals to infinity. So this is a complex Sinusoid in the frequency domain.

So the shifted impulse delta t - t0 has basically the Fourier transform which is a complex Sinusoid in the frequency domain, alright. So basically we will have to stop this module here, we have looked at the modulation property, alright. The modulation is a very important; we said modulation is a very important step in any communication system, alright. And subsequently we looked at this very interesting and very fundamental function which is the unit impulse function which is also known as the direct delta function denoted by delta t, looked at it is various properties and also the Fourier transform of delta t and delta t - T0, which is the shifted impulse or the shifted direct delta, alright. So let us stop here and continue with other aspects in the subsequent modules, thank you.