Course on Principles of Communication Systems – Part 1
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Lecture 46
Module 8
Signal Reconstruction in Delta Modulation, Schematic Diagrams, Slope Overload Distortion and Granular Noise

Hello, welcome to another module in this massive open online course. so we are looking at delta modulation and we have said that delta modulation as an efficient scheme for quantization because rather than quantizing the signal samples it quantizes the differences and further very interestingly the differences are represented only using a single information bit, so it is very efficient, alright quantizes the signal and as we said it can also be used for compression of the original signal that is instead of using a large number of bits per sample, only a single bit can be used to represent each sample and very efficiently, okay.

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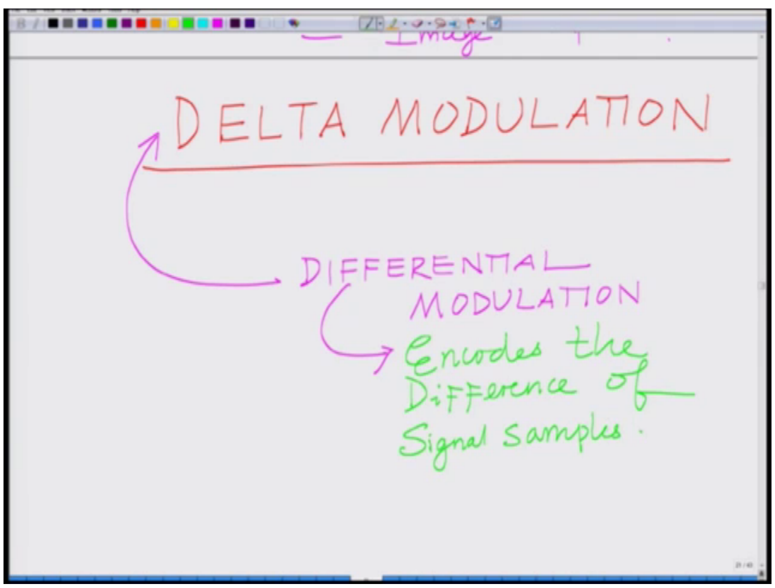

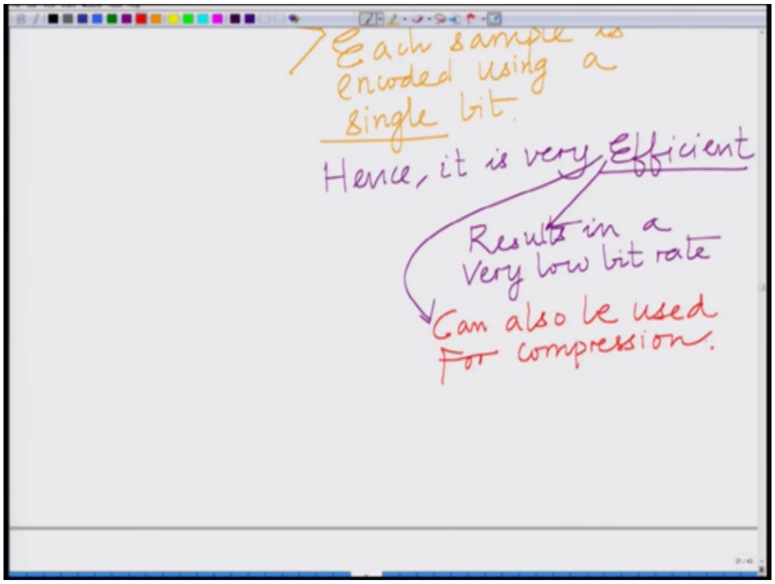

So let us note this so we are looking at delta modulation which is really a scheme for differential modulation because it encodes the differences, correct? It is a differential modulation scheme because it encodes the differences encodes the and this is important it encodes the difference of the signal samples, correct? Further if you look at delta modulation each bit or each sample is represented only using a simple each sample is represented only using a simple each sample is encoded or quantized encoded using a single bit.

Hence this is a very efficient scheme hence it results in very low bit rate, hence it is very well efficient in the sense that it results in a low bit rate and can also be used for compression, okay. Now, we also saw, right? We also saw the procedure for delta modulation, okay and the reconstruction of delta modulation which is carried at the receiver can be done as follows.

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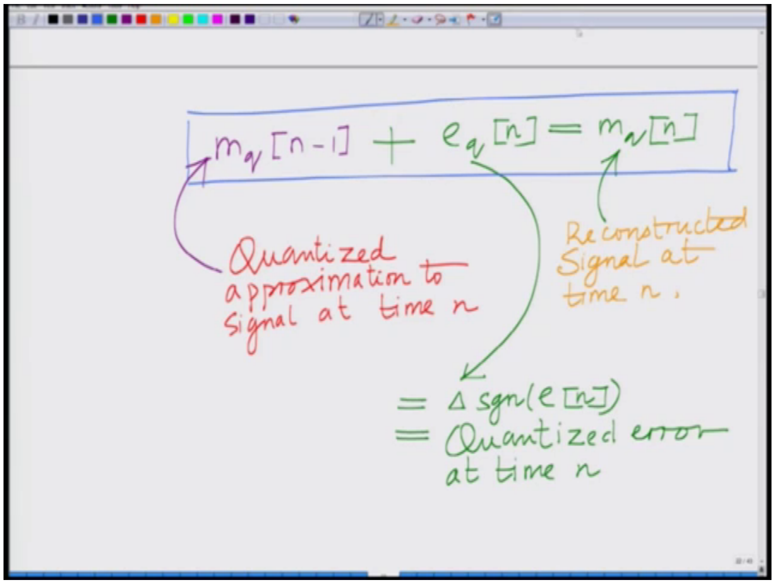

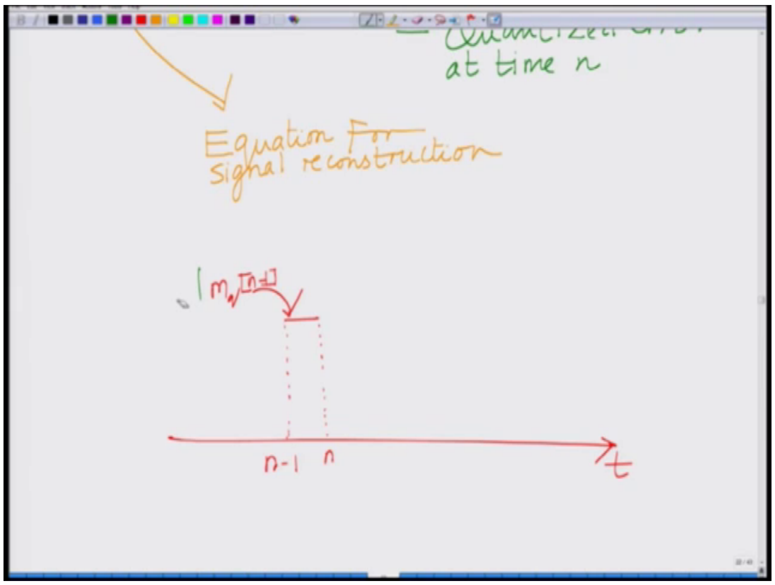

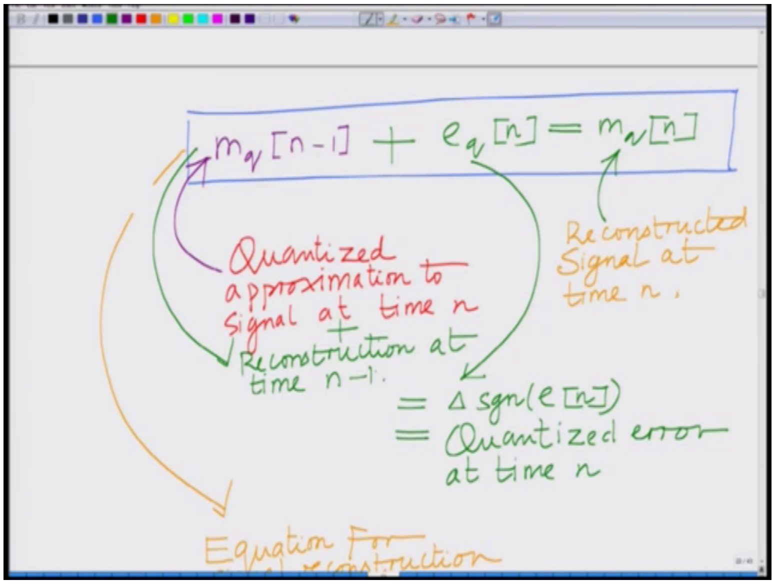

So we have mq[n] minus remember mq[n], right which is approximation mq[n] minus which is a approximation to the signal at time instant n, so I have mq[n] minus 1 remember I have mq[n] minus 1 which is the quantized approximation to the signal at time instant n minus 1 to this is the quantized approximation to the signal at time instant n this plus eq[n] remember eq[n], right? eq[n] is delta times sgn of e[n] this is the remember this is equal to delta times sgn of e[n] which is basically the quantized error, correct.

Which is basically quantized error at quantized error at time n therefore your mq[n minus 1] plus eq[n] equals mq[n], where mq[n] is the reconstructed signal at time instant n, okay mq[n] is the reconstructed signal mq[n] is the reconstructed signal at time. So we have mq[n] and that is very simple reconstruction equation this is our signal reconstruction equation this is our equation for signal reconstruction, okay let us note that this is our equation for signal reconstruction at the receiver, okay or wherever we are doing reconstruction, okay.

So this our equation for signal reconstruction, okay. So now let us see how it works whenever we are doing signal reconstruction we have time, okay and it is very simple as we have seen already the procedure itself is very simple, we have mq[n minus 1], correct? We have mq[n minus 1], okay so the reconstruction at n minus 1 this is mq[n minus 1] at this point we have time instant n so this is time instant n minus 1 time instant n.

So mq[n minus 1] this is the approximate this is the reconstruction at n minus 1 as well as the approximation at so mq[n minus 1] is the reconstruction so note that mq[n minus 1] is a quantized approximation to signal at time instant n as well as reconstruction at this is also the reconstruction of the signal at time n minus 1. That is the reconstructed level of the signal at time instant n minus 1 itself is an approximation to the signal at time instant n, alright.

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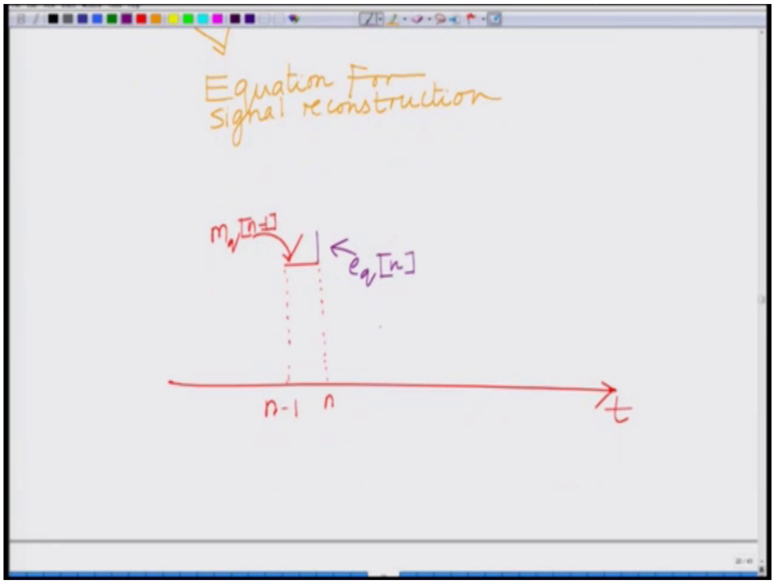

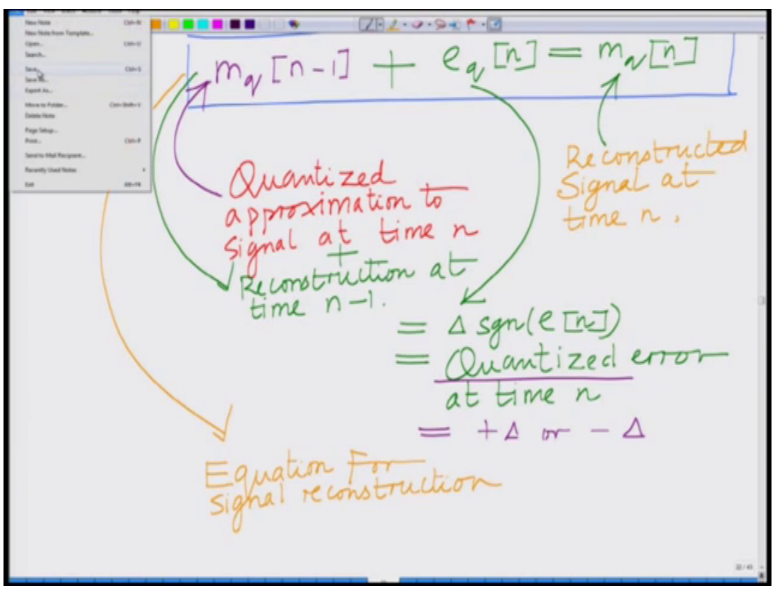

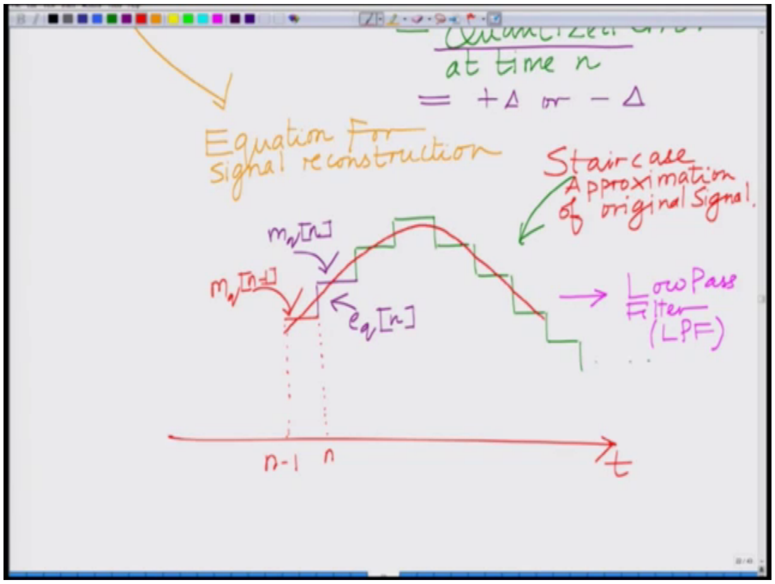

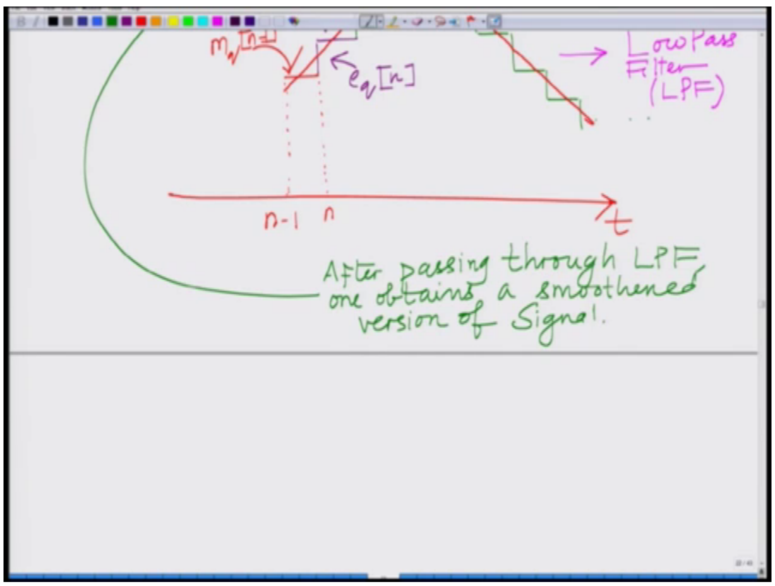

So naturally to this I add the quantized error, so this is your mq[n minus 1] to this you add eq[n] which is either remember this is either plus delta or minus delta eq[n] which is the quantized approximation to the error in delta modulation this is equal to plus delta or minus delta so this is also something that can be noted, this is either plus delta so eq[n] equals either plus delta or minus delta, okay this is the quantized error at time n which is either plus delta or minus delta eq[n] and that gives you finally the quantized so that is your mq[n] that is the quantized approximation of the signal at time instant n.

And similarly, one can again at n plus 1 you can add eq[n] plus 1 to obtain mq[n plus 1] and that is how we get the staircase approximation to the signal, so we get a staircase approximation to the original signal. So we get a so this is your staircase approximation to the as we said a delta modulation gives a this is staircase approximation of your original signal. Further, now observe that this staircase approximation is coarse, so I pass this to a low pass filter this is so because this is has sharp edges I pass it through a low pass filter (LPF) appropriately chosen bandwidth, and what that will do? That gives me a smoothened version of the signal.

So thus, LFP so this after passing through the LPF I get a after passing LPF one obtains a smoothened version of the signal one obtain a smoothen (aft) after passing through a low pass filter one basically obtains a smoothened version of the signal because remember in staircase approximation we have sharp rises, alright we have sharp corners sharp rises and sharp falls, alright so that gives rise to high frequency components.

So naturally to remove those things passing through a low pass filter of suitably chosen bandwidth, right? Naturally that has to be the bandwidth of the message signal, alright once you pass it through a low pass filter then obviously are going to get you are going to get a smoothened version of the signal, okay that is the smoothened version of the reconstructed signal, okay.

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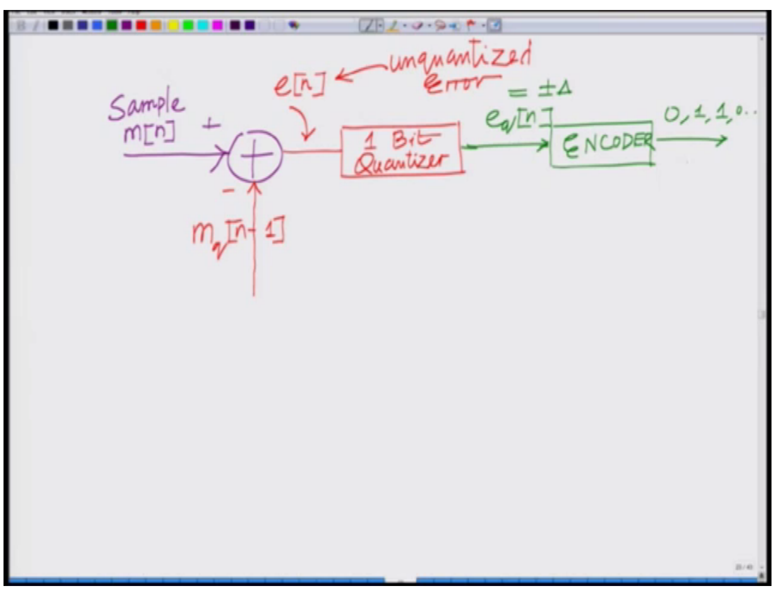

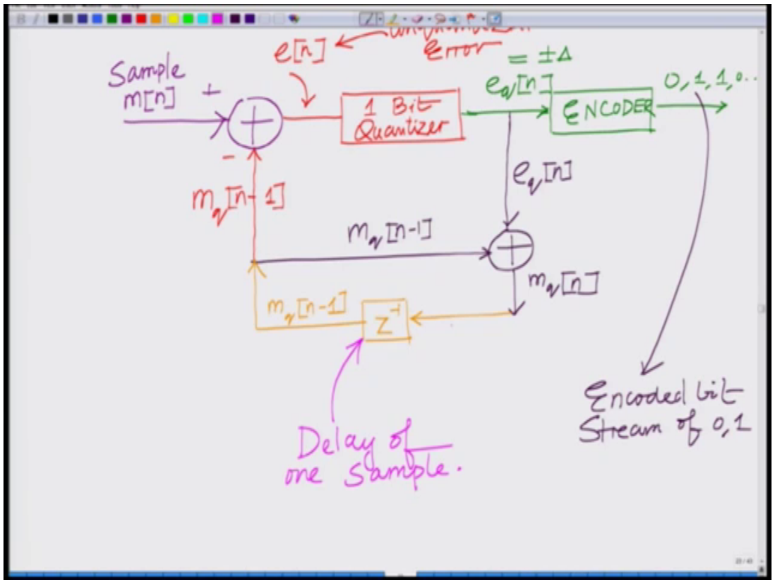

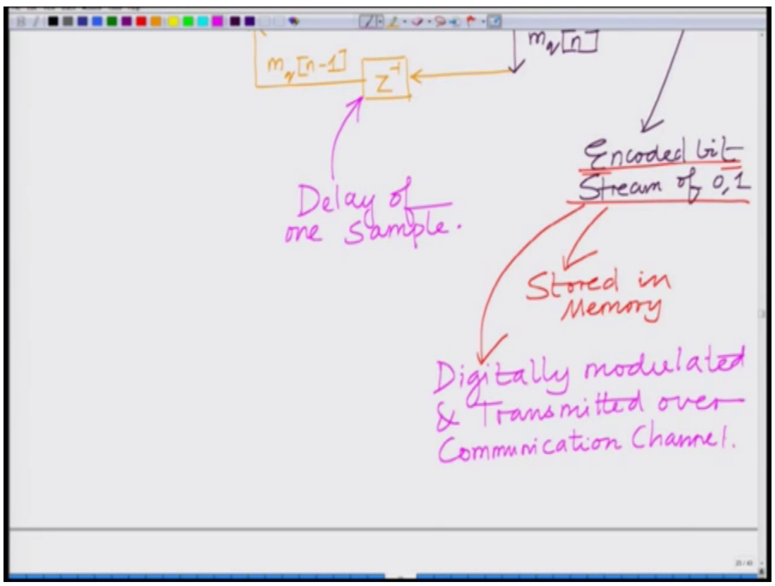

So now let us look at the schematic diagram for the delta modulator so I want to look at a schematic diagram of delta modulation.

Now if you look at a schematic diagram of delta modulation you are going to have sample mn, okay this is the sample at time instant n, okay this is your sample mn, okay. Now from this sample remember you are subtracting remember you are encoding the differences so you are subtracting mq[n minus 1] so from this sample you are subtracting mq[n minus 1] this is pass through your 1 bit quantizer that is delta time sgn of e[n], so this is pass through a 1 bit, so this mn minus mq[n minus 1] this is your e of the unquantized error e[n].

So this is the unquantized error this is your unquantized error e[n] you pass it through the 1 bit quantizer and then you get eq[n] once you pass it through 1 bit quantizer you get eq[n], okay. Now this eq[n] remember is equal to eq[n] equals plus or minus delta, so this has to be encoded, okay to obtain the information stream remember we said encoding is the process where you are mapping this to plus delta for instance let us say to 0 or for instance you can map plus delta to let say 0 and minus delta to 1, okay.

So you get an information bit stream of 0's and 1's, okay so that is the job of the encoder. So you get an encoded bit streams of 0's and 1's so this is basically your encoded bit stream of 0's and 1's, okay so let us note that down, so encoder this is your encoder this yields your encoded bit stream of 0 comma 1 encoded bit streams of 0's and 1's. and of course now this eq[n] we are not done with eq[n] remember eq[n] has to be added eq[n] has now we have eq[n], alright so we have eq[n] has to be added to mq[n minus 1] to give mq[n].

So we have eq[n] we have mq[n minus 1] we have eq[n] that gives us mq[n] this gives us mq[n]. Now this mq[n] you pass it through a delay, okay so you delay it by one sample this is your delay that gives us so mq[n] pass it through a delay of 1 sample that gives rise to so this zee inverse basically is a zee transform some of you might be familiar with it this zee inverse means it is a delay of one sample.

So mq[n] is delayed pass through this zee inverse module, alright you delay it by one sample you get mq[n minus 1]. Now mq[n minus 1] is subtracted from mq[m] to get e[n] that is unquantized error at the same time mq[n minus 1] is added to eq[n] to get mq[n], alright. Now mq[n] is basically going to be the reconstruction at time instant n it is also remember it is also the approximation for the closest approximation for the signal at time instant n because mq[n] is subtracted from m[n plus 1], alright.

So mn[n plus 1] minus mq[n] that gives us e[n plus 1] and so on, okay. And of course the encoder takes in eq[n] that is the plus or minus deltas, right the 1 bit quantizer produces plus or minus delta the encoder takes this, right? Plus or minus deltas and provide maps them into information bit stream that is 1's and 0's this can either be stored, alright in memory or this can be converted into an appropriate digital modulation (cons) mapped to again a digital modulation consolation, alright digitally modulated, right? And then transmitted over the channel.

So these 1's can bit can be stored or digitally modulated and transmitted over a digital communication channel. So that is how we are getting from a continuous signal that is for sampling it, quantizing it, converting into bit stream, digitally modulating it and transmitting to over the channel, okay. So this encoded bit of 0's and 1's so this is the encoded bit of 0's and 1's so this can be either stored in memory in an appropriate memory or this can also be digitally modulated and transmitted over a communication channel.

This can be digitally modulated and transmitted over a communication channel, so this is basically your schematic of a delta modulator, alright which basically as we said is very efficient because it basically quantizes the differences and quantizes them and represents them using only a single bit. So they are basically providing a single bit per every input sample, okay so that is basically the idea that is basically also the advantage of delta modulation.

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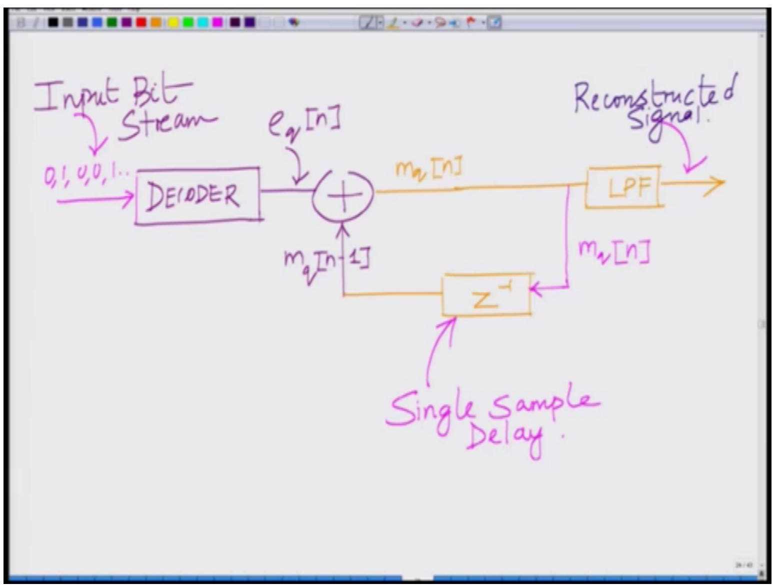

Now let us look at the schematic for reconstruction the schematic for reconstruction as we have seen yesterday that is very simple or as we have seen today that in this module is very simple, so we have the decoder we have the input bits, okay so you have the bits so this is your input bit stream we have the input bit stream, okay so this is your that is the encoded input bit stream naturally that has to be passed through a decoder the encoded bit stream has to be passed through a decoder this is then this is what we get we get eq[n] a bit stream you can get the quantized errors eq[n] is nothing but the quantized errors.

To this you add it is very simple you add mq[n minus 1] the reconstruction at time instant n minus 1 mq[n minus 1] you add mq[n minus 1], alright you add mq[n minus 1] now that gives you mq[n], okay that gives you mq[n] that is reconstructed signal at time instant n, okay now that is passed through your low pass filter to obtain the smoothened signal at the same time we have mq[n] itself forms the closest approximation at time instant n n plus 1.

So we have this is mq[n] again remember zee inverse is simply a single sample delay, okay zee inverse whenever we have zee inverse of course some of you might be familiar from the zee transform notation this is a single sample delay, okay and therefore you get mq[n] and this is the your reconstructed signal, okay this is your reconstructed signal, so this is basically a schematic which is a very simple schematic all you are taking is basically your reconstructed signal it is a recursive process, right? Your (re) reconstructed signal at 10 n minus 1 mq[n minus 1] that forms an approximation at time instant n.

So mq[n minus 1] plus the quantized error eq[n] that gives mq[n] which is the reconstructed signal at time instant n. So now you have this staircase approximation of the signal which has sharp edges passes through a low pass filter to get a smoothened version of the signal, alright so very simple scheme, alright. So in have delta modulation for differential encoding, right? Basically encodes the differences using a single bit and the optimal the appropriate reconstruction.

So let us now look at conditions of delta modulation or delta modulation the distortions various distortions that can occur in delta modulation, okay.

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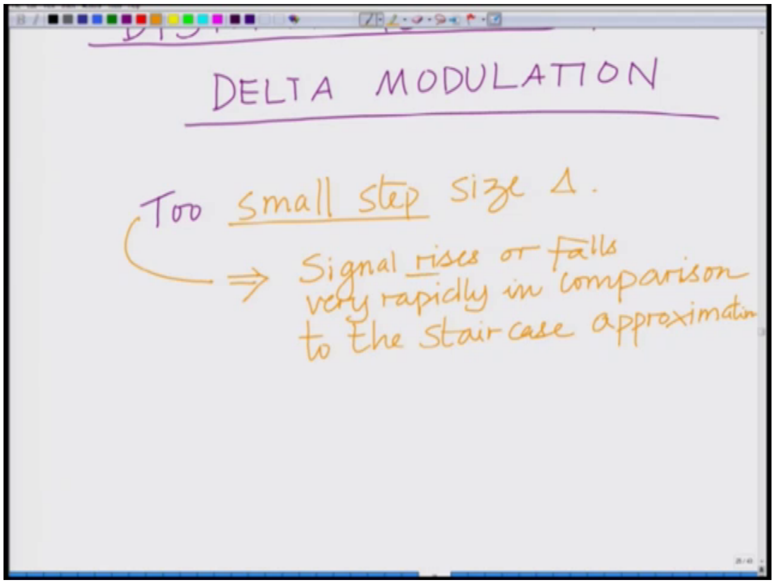

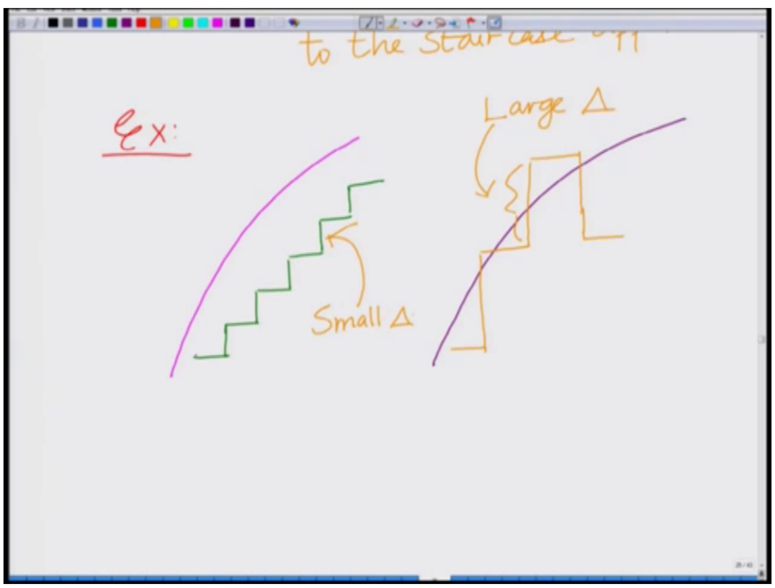

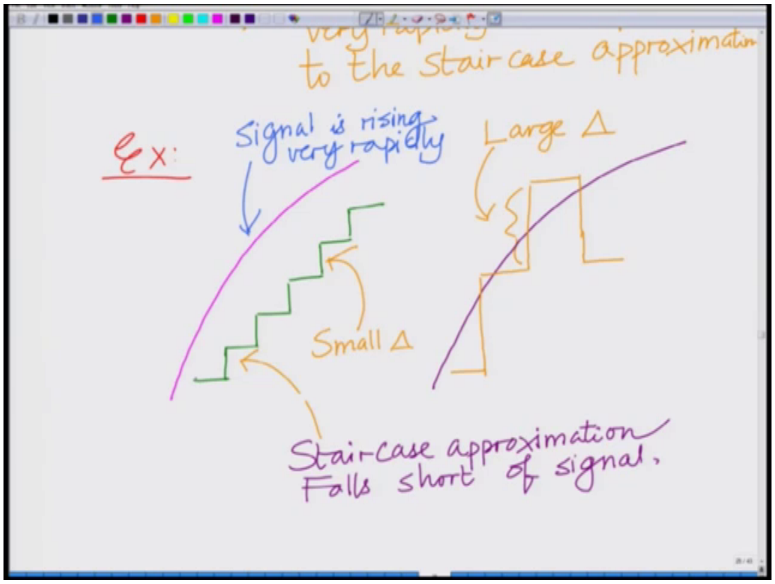

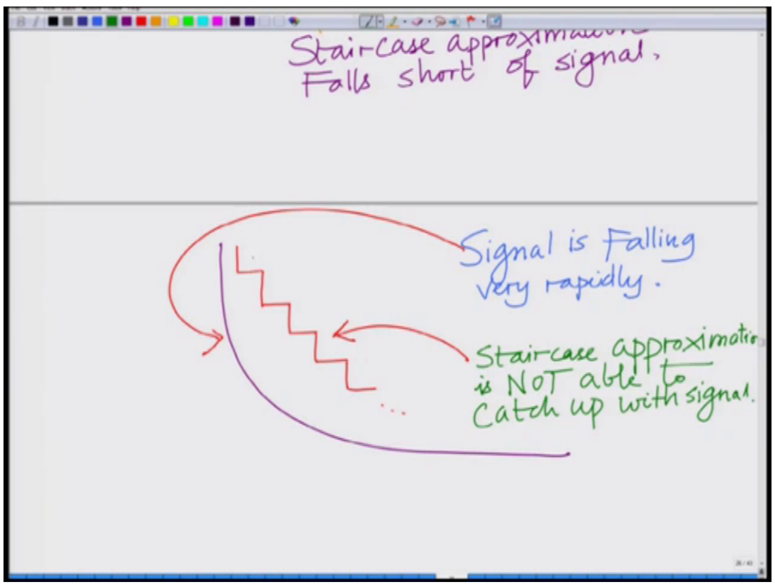

So we want to look at delta modulation or the distortions of the delta modulations or distortions in delta modulation and the distortions in delta modulation can occur as follows for instance if the steps wise is too small step size delta is too small and the signal empty that is a signal to be sampled rises very fast or it falls very fast, alright then the staircase approximation can fall short of the signal, okay.

So let us look at the first distortion too small step size delta occurs if that is we have a too small step size delta is too small this leads to the fact that if signal rises or falls very rapidly in comparison to the signal rises or falls very rapidly in comparison to the (steel case) staircase approximation we are going to have a problem, for example if we have a signal the signal is rising very rapidly but my staircase approximation because we have a small delta, correct the staircase approximation is never able to catch up is not able to (catc) is constantly falling.

So delta of course if delta is large, then at some point for instance if my delta on the other hand of this same signal if I have a large delta for instance like this delta is sufficiently large, right? Then my staircase approximation will be able to catch at some point so this is basically your large delta and this is basically your small delta and in this scenario what is happening staircase approximation is falling short of the signal, right? The staircase approximation falls short of the signal, okay so this is the problem with small.

Similarly, if the signal is falling rapidly on the other hand if the signal is falling rapidly so here signal is rising rapidly, okay now this is the difference signal is rising very rapidly, okay and here on the other hand signal is falling very rapidly and again once again because of the small step size, correct? Once again because of the once again the signal is not able to so if we have a delta approximation which is basically here at this point the signal is let us write it this way the signal is falling very rapidly and once again the delta approximation is not able because of the small delta is not able to catch up with the signal so on and so forth.

My signal is falling very rapidly and delta approximation the staircase approximation is not able to catch up with the signal let us put it that way. So in both scenarios when the signal is basically rising very rapidly or falling very rapidly or in other sense if the slope of the signal is too high, right? When the slope of the signal is too high such that the staircase approximation because of the small or limited delta size is not able to match, right? Is not able to match the rapid increase or rapid decrease in the signal that leads to slope overload distortion, that is there is a very high slope which is overloading your delta modulator so that leads to slope overload distortion.

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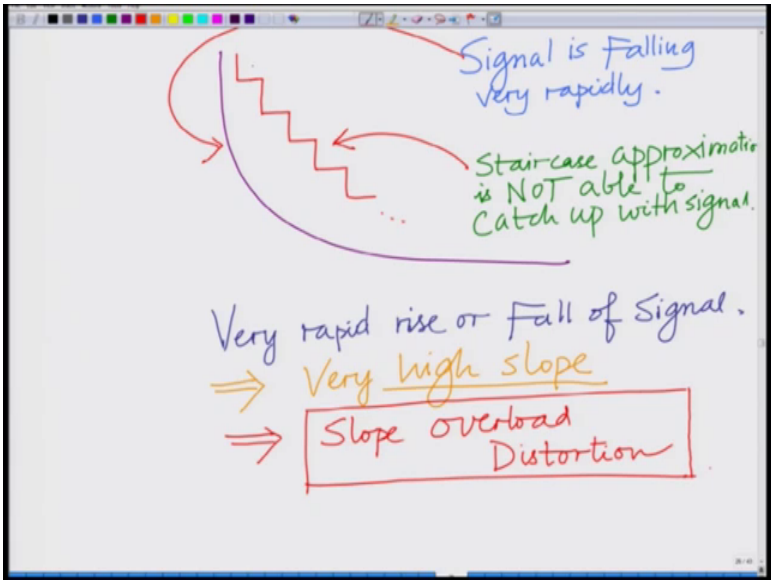

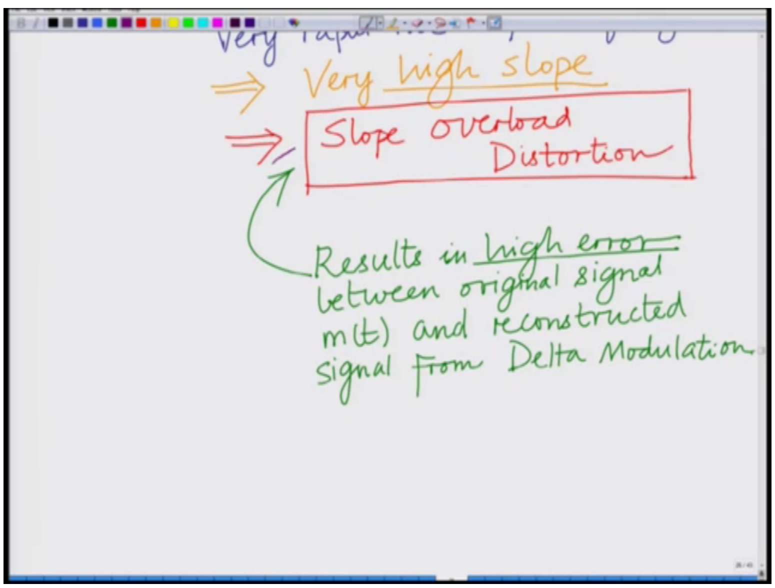

So very high very rapid rise or fall in the signal implies very high slope that overloads the delta modulator leads to this leads to slope this is termed as slope overload distortion.

As a result of slope overload distortion there is a high error between the staircase approximation and the actual quantizing. So these results in a high error original signal empty and reconstructed signal from the delta modulation reconstructed signal from delta modulation, okay reconstructed signal leads to a high error, correct. It leads to a very high error between the original signal and the signal reconstructed from the delta modulation.

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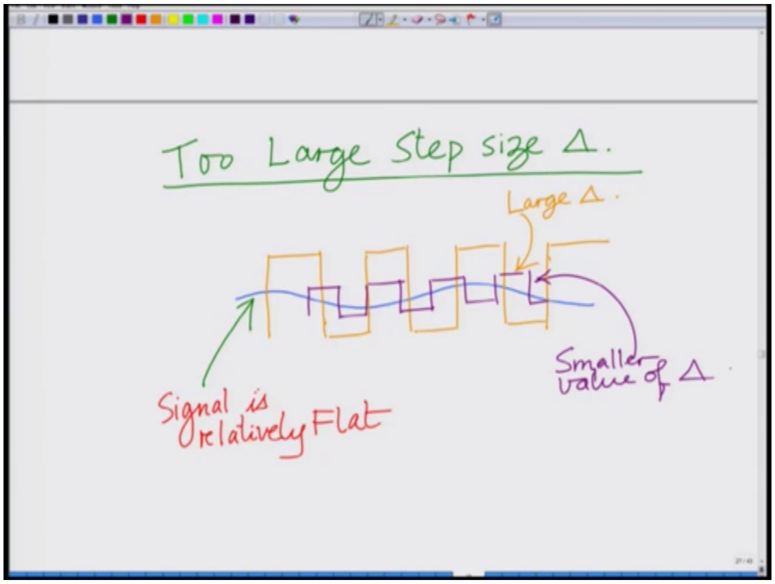

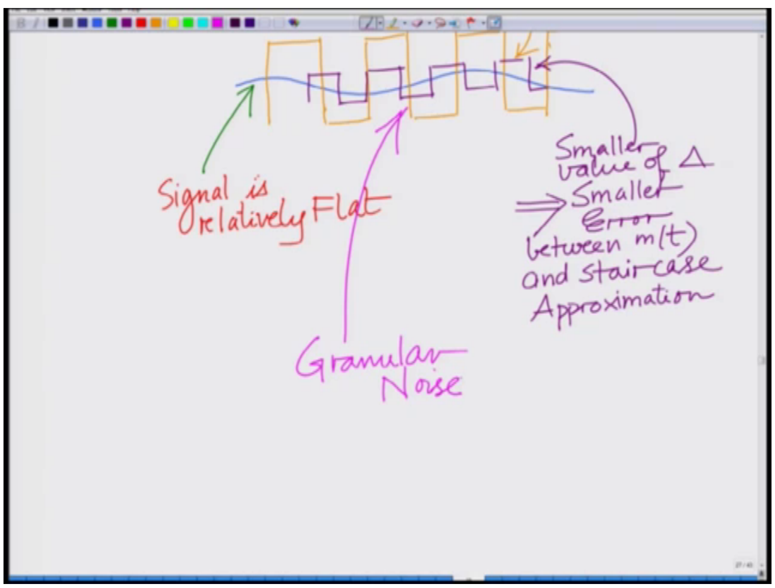

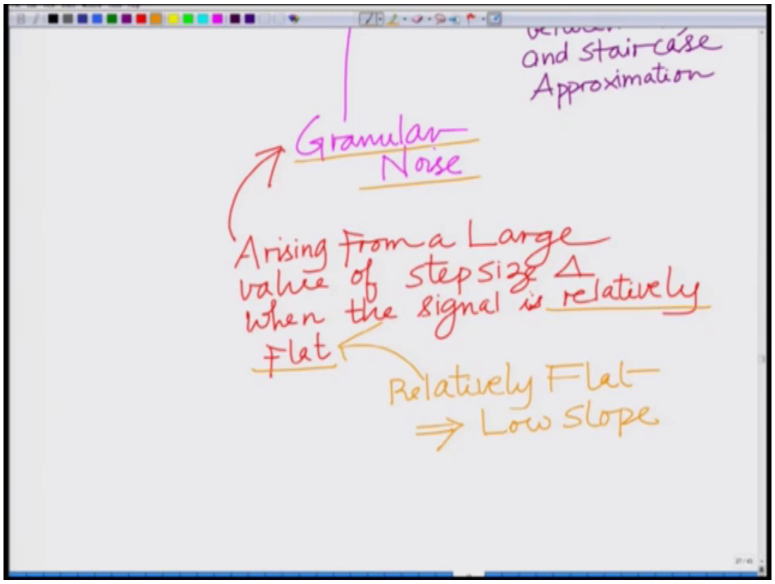

On the other hand, right? When the step size is too large now let us see what is the problem, so this slope overload distortion remember slope overload distortion this arises from ultimately from two small step sizes. Now when step size is too large, when we have too large step sizes then we have the other problem, if the signal is relatively flat, okay signal is relatively flat, then we have a so if the step size is very large, right? And the signal is relatively flat we have a large approximation error, okay.

So this is large delta, so this causes large delta, so the staircase (approxima) the error between the staircase approximation and the signal is relatively large in comparison to let us say we have a small value of delta, let us say we have a smaller value of delta which I am drawing with a different color over here, correct you can see that if you have a smaller value of delta smaller value of delta leads to this leads to smaller error between empty and the reconstructed signal that is our staircase approximation smaller error between the staircase approximation.

So while a large delta might be suitable to avoid slope overload distortion, the larger the delta that leads to a larger error when the signal is relatively flat because when the signal is relatively flat, we desire a smaller value of delta. Therefore, this distortion which arise is arising from the (lar) from a large value of delta when the signal is relatively flat this is termed as granular noise, okay. So and naturally this is termed as granular noise, alright and granular noise arising is arising from a large value of delta when the or the large value of step size delta when the signal is relatively flat.

Or when signal is relatively flat or whether signal is relatively flat implies that the slope is low relatively flat which implies low slope and that is causing this granular noise, okay. So we have these are basically the these are basically you can see a sought of the counter parts counter parts so we have large small delta if we have a small delta, then the staircase approximation is not able keep up with the signal when it is rising or falling rapidly that leads to slope overload distortion large delta we have large delta the approximation is very coarser especially when the signal is not rising or falling rapidly that lead to granular noise.

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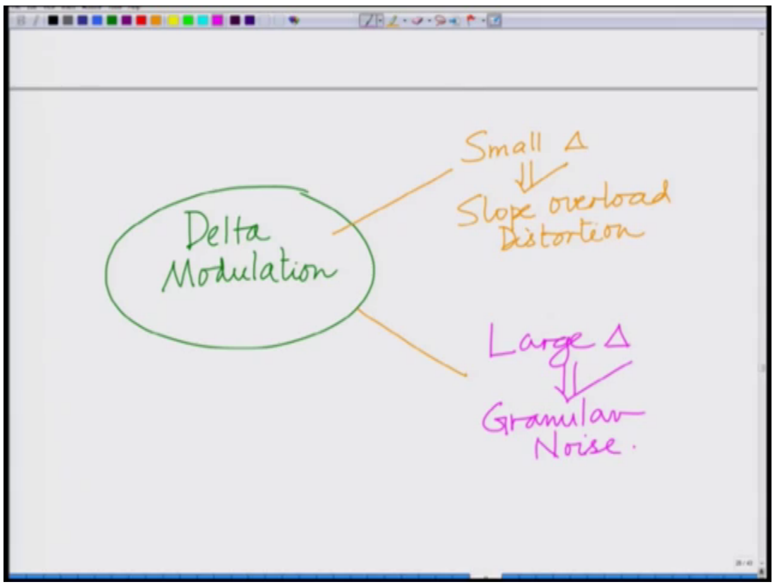

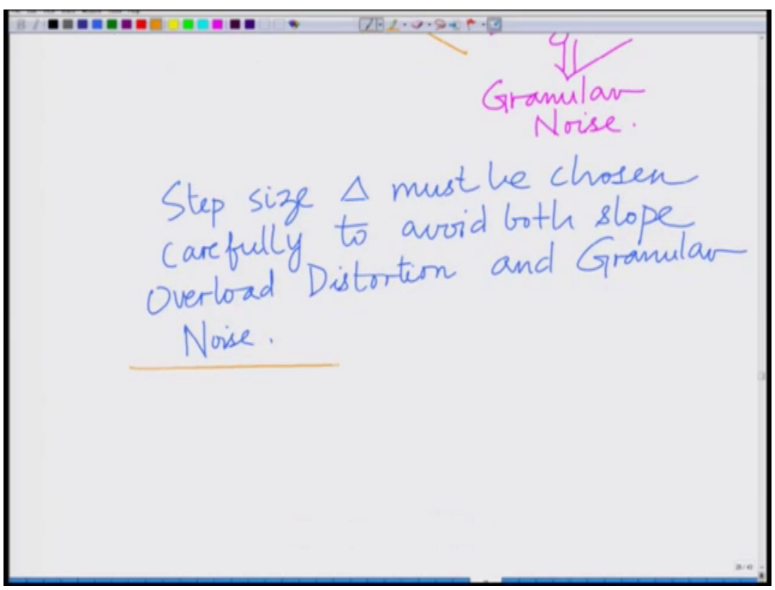

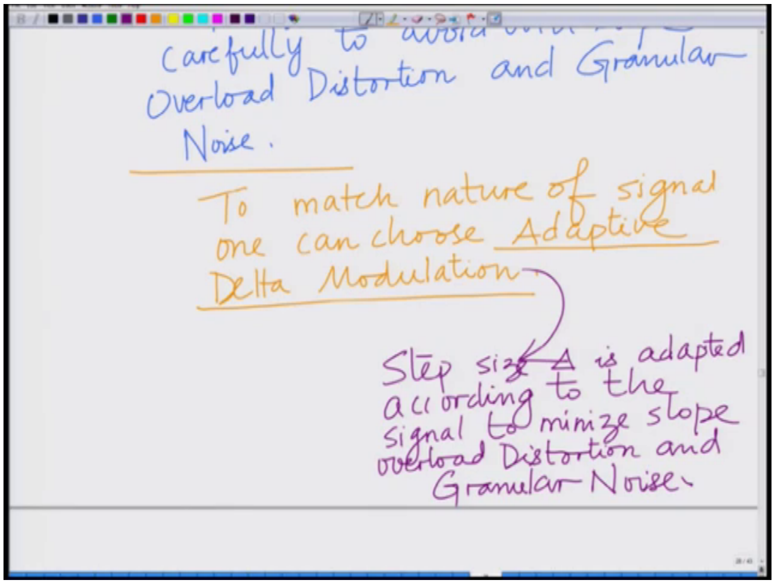

So we have two different kinds of distortions in delta modulation in delta modulation let me just summarize this when we have a large delta or when we have small delta and signal is rising or falling rapidly that implies we have slope overload distortion, when we have a large delta when the signal is relatively flat that leads to granular noise, so these are the two different kinds of distortions.

Now, if you have a large delta of course that also implies a wider dynamic range you can capture because the delta step sizes are large you can span a larger range, right. So which means you can capture a larger dynamic range very fast, alright. But however the slope overload distortion is also (()))(38:07), okay. Now if the delta is small then the approximation is very fine, right? The approximation is very fine so the noise is low when the signal is varying slowly, but if the signal is varying very fast then the noise is very high. So that so alright when you have large delta, obviously when the signal is varying very fast you can capture it, correct but if the signal is varying very slowly then it leads to granular noise, alright.

So these are the two so this is so one has to choose this step size delta intelligently, okay. So the step size delta has to be chosen intelligently, so there is a tradeoff of small delta versus large delta so the step size delta must be chosen with care I will say step size delta must be chosen optimally must be chosen carefully, okay it must be chosen careful to avoid both to our both slope overload distortion and granular noise.

Now one particular scheme which adaptively choses this delta size depending on the signal is adaptively modulation adaptive (mod) adaptive delta modulation adapts this delta size optimally so as to match the signal conditions, alright we will not going to look into this. So to match nature of signal one can choose adaptive delta modulation in adaptive delta modulation what happens is the step size delta is adaptively changed in this the step size delta is changed adaptively or it is adapted let me that is a better word it is adapted according to the to the signal to minimize both slope overload distortion and.

So it adapted, right? According to the it adapted according to the signal to minimize both the slope overload distortion and granular noise. Which means basically the signal is varying very rapidly is rising or falling very rapidly then obviously the delta must be made large, alright? On the other hand if signal is varying relatively slowly that is relatively flat, alright slope is low then the delta size can be made small, thereby basically your minimizing both error occurring due to slope overload distortion and also due to the granular noise, alright.

So that basically completes in a sense delta modulation as we have seen if it is an very efficient modulation scheme it is a differential modulation scheme which quantizes the differences between successive samples, alright and also it employs a single bit for quantization which makes it very efficient for both communication as well as compression purposes, thank you.