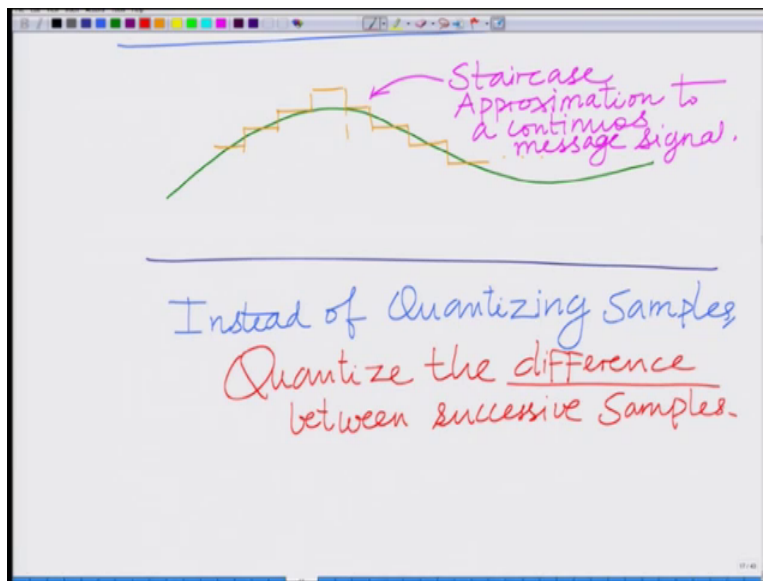


**Course on Principles of Communication Systems – Part 1**  
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**Lecture 45**  
**Module 8**  
**Introduction to Delta Modulation, One-bit Quantizer**

Hello, welcome to another module in this massive open online course. So we are looking at different quantization schemes, alright we have looked at the uniform quantization, the Lloyd-max quantization and we have also (use) looked at companding in different compressors. in this module let us look at delta modulation, alright which is a different firm of quantization, alright that is converting an continuous valued analog signal, right? Sampling it converting the continuous valued analog samples into a set of that is mapping them to a set of levels belonging to a discrete set, alright. So at that is a different quantization scheme which is termed as delta modulation, okay.

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So we are going to look at a different quantization scheme or a different modulation scheme delta modulation modulation and in the sense pulse amplitude here also we have pulses whose amplitudes are modulated. But we will see there is a an interesting difference so what we have is we have a continuous message signal and in delta modulation what we try to do is basically I

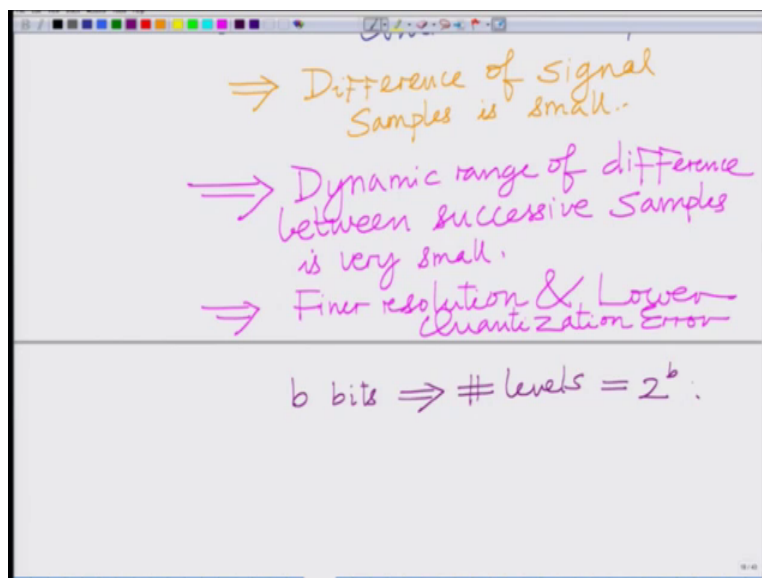
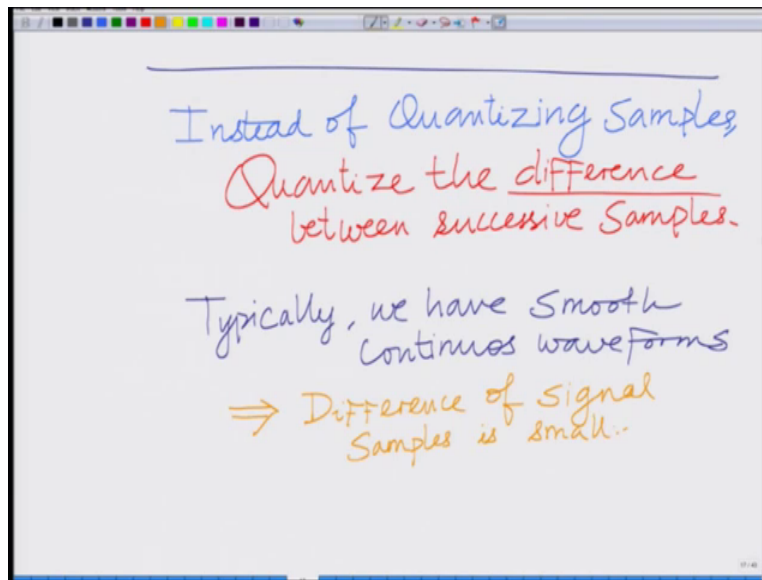
have to explain it intuitively we build a stair case approximation to this continuous message signal so we try to (ex) try to express it as you can see basically as a staircase kind of, right?

So we express it, right so this what we are trying to do is we are trying to build a staircase kind of approximation if I remove this thing, right? What we are trying to do is we are trying to build a staircase approximation to a continuous message signal, okay and how do we do it? Instead of quantizing the message signal instead of quantizing the samples of the original message signal we quantize the differences of the samples of the message signal and we are going to illustrate this shortly.

So the key principle in delta modulation is instead of quantizing the samples, quantize the differences between the samples instead of quantizing the samples quantize the difference between successive samples. Now what is the advantage of that? The advantage is remember the signals that we are trying to quantize sample and quantize are smooth signals, alright so the signal is varying slowly, right?

So the signal sample might be large, but the differences between successive samples is small, so we are trying to quantize so if we have quantizer of fixed resolution that is a fixed number of bits when we quantize the differences which are small the dynamic range of the differences is small, right? So naturally the quantization intervals can be very much more finer the quantization intervals are smaller which means the quantization is much more finer.

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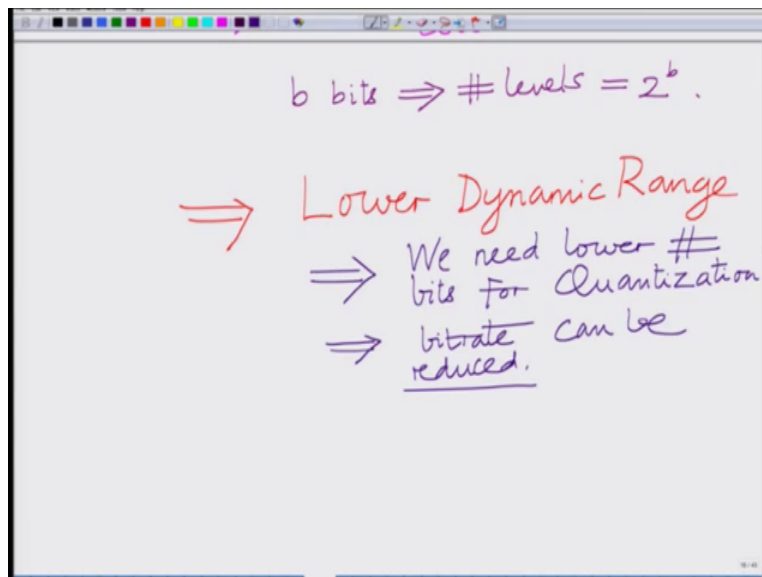
Therefore the quantization error is lower which leads to an improved performance, alright so let us note that so what we have when we have a continuous signals see we have continuous typically we have smooth continuous waveforms which implies that the difference of signal samples which implies that the difference of the signal sample is small. Now that implies that the dynamic range of the difference the dynamic range of the difference between successive samples is very small, okay this implies that we have a fine resolution and lower quantization error, right?

See if you have a quantizer with resolution of  $b$  bits number of levels is 2 to the power of  $b$ , right? So  $b$  bit resolution implies number of levels equals 2 to the power of  $b$ . So the idea is if the

dynamic range is large, so the number of levels is same so if the dynamic range is large then the quantization intervals are going to be wider that is 2 to the power of b levels in this (qua) 2 to the power of b intervals in this large dynamic range.

If the dynamic rang is small, then we have the same number of intervals so since the dynamic range is small the average width of each interval is going to be smaller, which means the quantization is finer, right? The quantization is finer and therefore the quantization error is lower that is the principle of delta modulation. Or one can also say that if we keep the quantization error constant if we lower the dynamic range, right? For the same quantization error we can lower the number quantization levels. In fact, we can lower the resolution of the quantizer, right? If we lower the number of quantization levels basically we are lowering the level of resolution of the quantizer, alright.

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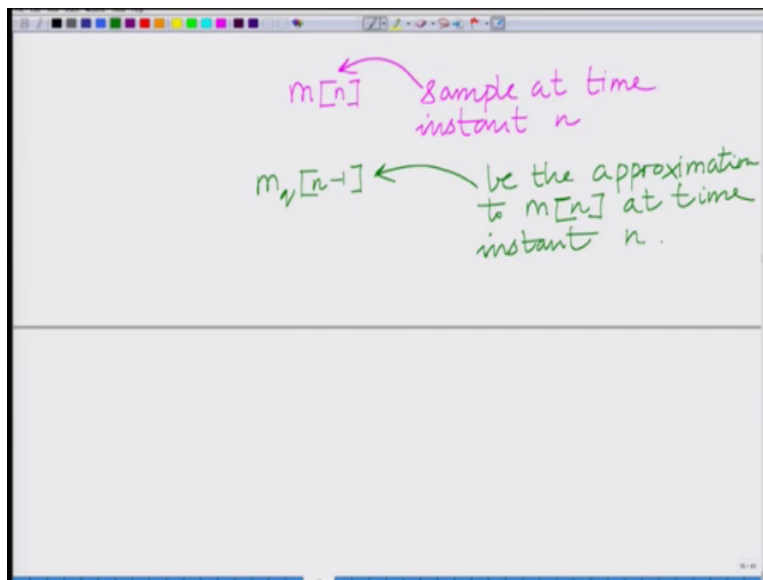
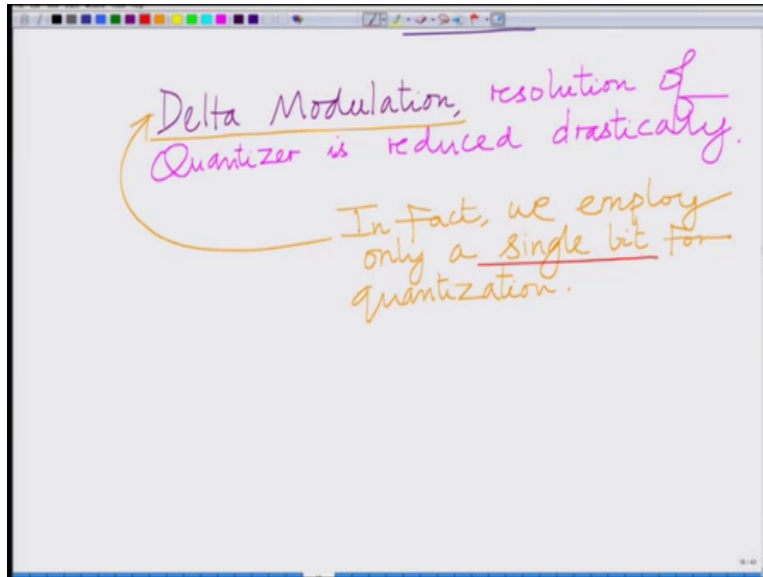


So basically since the dynamic range is small so the other way to look at it is lower dynamic range so this also implies that lower dynamic range implies well implies we need lower number of bits for quantization which implies that the bit rate implies that the bit load implies bit rate can be reduced, bit rate meaning the number of bits output per sample.

So bit rate can be reduced drastically, right? So bit rate can be reduced for the same (fo) while not increasing the quantization error, alright and in fact in delta modulation very specifically we are going to show that we are going to employ only a signal bit for the quantization that is the

most important aspect of delta modulation delta modulation is efficient because the quantizer resolution needed is only a single bit and in fact delta modulation reduces this bit resolution required drastically.

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Handwritten whiteboard notes showing the definition of error signal  $e[n]$  and the quantized error signal  $e_q[n]$ .

At the top, the error signal is defined as  $e[n] = m[n] - m_q[n-1]$ . The word "error" is written in red above  $e[n]$ , and "instant n." is written in green above  $m[n]$ . An arrow labeled "Quantize" points from  $m[n]$  to  $m_q[n-1]$ .

Below, the quantized error signal is defined as  $e_q[n] = \Delta \operatorname{sgn}(e[n])$ . The word "Quantized Error" is written in purple to the left of  $e_q[n]$ . The signum function is defined as  $\operatorname{sgn}(x) = \begin{cases} +1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$ . A note below states: "If  $e[n] > 0 \Rightarrow e_q[n] = +1$ ".

Handwritten whiteboard notes showing the definition of error signal  $e[n]$  and the quantized error signal  $e_q[n]$ , including the case for negative error.

The quantized error signal is defined as  $e_q[n] = \Delta \operatorname{sgn}(e[n])$ . The word "Quantized Error" is written in purple to the left of  $e_q[n]$ . The signum function is defined as  $\operatorname{sgn}(x) = \begin{cases} +1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$ . A note below states: "If  $e[n] > 0 \Rightarrow e_q[n] = +\Delta$ ". A second note below states: " $e[n] < 0 \Rightarrow e_q[n] = -\Delta$ ".

So in delta modulation resolution of quantizer is resolution of the quantizer is  $(\Delta)$ (9:44) in fact we need only need a single bit as we are going to show in fact we employ only a single bit this is the single bit for in fact we employ only a single bit for quantization that is the important aspect of delta modulation, okay. Now how do we do it in delta modulation, okay let us say  $m_q[n]$  minus so let us start with  $m[n]$  we know  $m[n]$   $m[n]$  is the sample at time instant  $n$  this is a sample at time instant  $n$ .

Now let  $m_q[n-1]$ , okay so  $m_q[n-1]$  this be the approximation now we are going to show how we are going to derive this  $m_q[n-1]$  so  $m[n]$  we get from the sampling  $m_q[n-1]$  be the approximation at time instant  $n$  approximation to  $m[n]$  at  $(\Delta)$ (11:36). Therefore the

error, so we have  $m[n]$  which is the sample value at time instant  $n$  and we have  $m_q[n-1]$  which is approximation, alright the quantized approximation to  $m[n]$  at time instant  $n$ , okay.

And therefore what we have is we have the error  $e[n]$  will be equal to this will be equal to well  $m[n]$  minus  $m_q[n-1]$  this is the error, right? Sample minus the approximation, now we are going to quantize  $e[n]$  and we are going to do that with 1 bit so now we are going to quantize observe that we are going to quantize  $e[n]$ . So therefore what we are going to have is we are going to have  $e_q[n]$  that is a quantized error equals delta times sgn of  $e[n]$ , okay.

So  $e_q[n]$  notice that  $e_q$  of  $n$  this is the quantized error, so we are quantizing not the sample but we are quantizing the error quantized error, sgn function is basically let us use the simplified version of sgn function of  $x$  equals well let us say this is equal to  $x$  if well this is equal to plus 1 if  $x$  greater than 0 minus 1 if  $x$  less than 0 of course at 0 it is 0 but we do not care about that single value, so which means this can only take depending on the sgn of  $e[n]$  if  $e[n]$  greater than 0 implies  $e[n]$  greater than 0 this implies basically that you can see that  $e_q[n]$  equals plus delta.

On the other hand,  $e[n]$  less than 0 the naturally sgn of  $e[n]$  is minus 1 less than 0 implies  $e_q[n]$  is minus delta. So now you can see  $e_q[n]$  can only take 2 values either plus delta or minus delta, alright only two values therefore the number of bits required is 1, okay.

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$e[n] < 0 \Rightarrow e_q[n] = -\Delta$

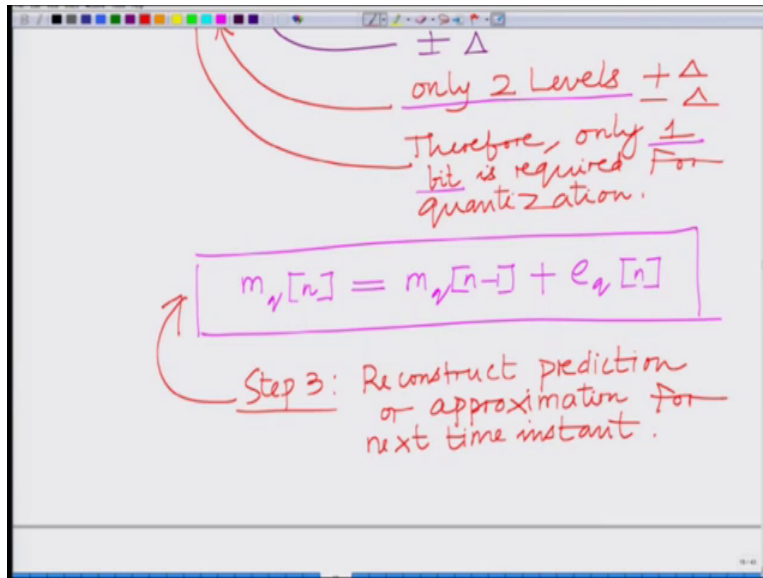
$e_q[n]$  can only be  $+\Delta$  or  $-\Delta$

$\pm \Delta$

only 2 Levels  $+\Delta$   
 $-\Delta$

Therefore, only 1 bit is required for quantization.

$$m_q[n] = m_q[n-1] + e_q[n]$$



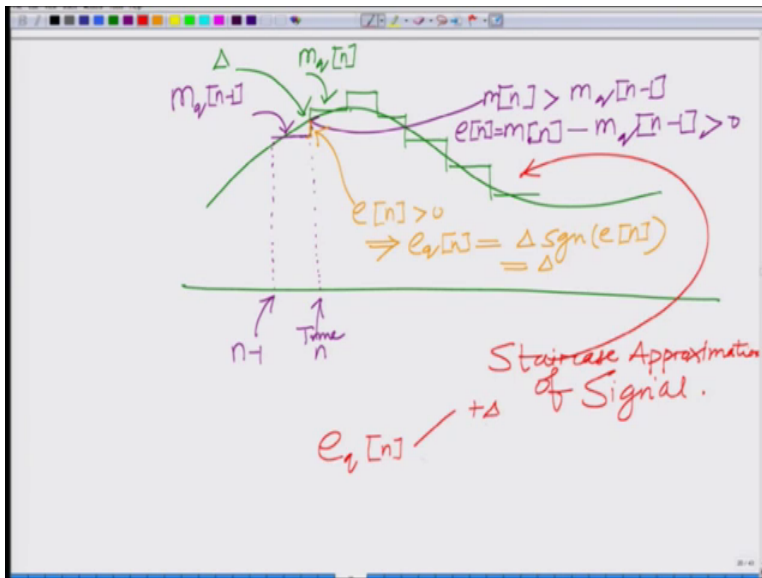
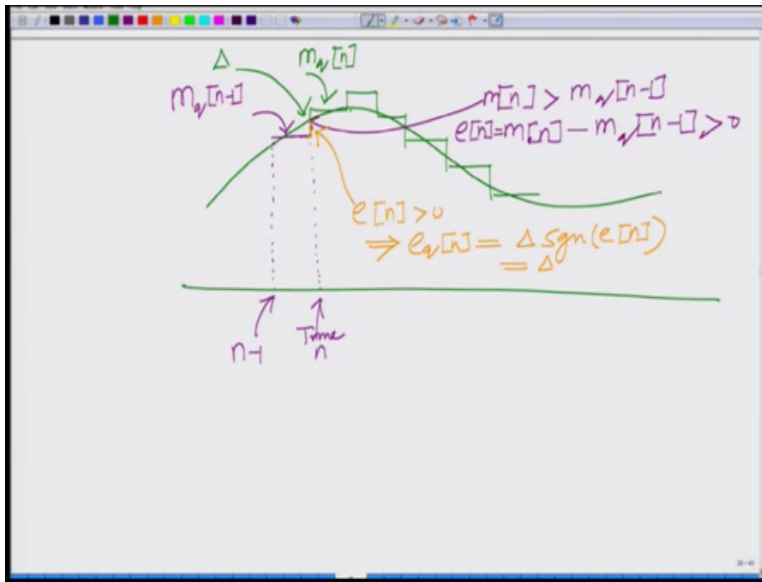
So let us note that so  $e[n]$  can take only two values  $e_q[n]$  which is what we are quantizing not  $e[n]$  remember  $e_q[n]$  so  $e[n]$  is the error and we are quantizing the difference that is the difference we are quantizing the difference  $e_q[n]$  can only be plus delta or minus so this is basically your plus or minus delta hence only implies only two levels plus delta minus delta.

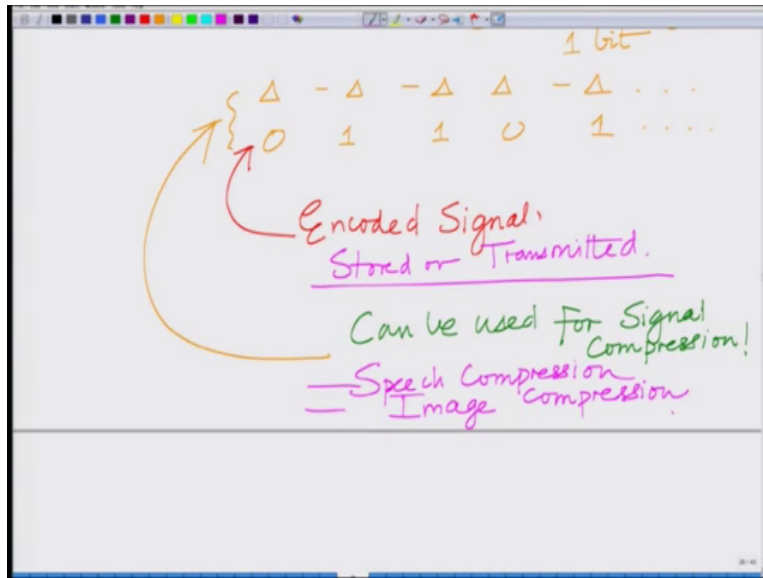
And therefore, only one bit and therefore only one bit only a single bit only one bit is required for quantizer since we are using only two levels we are using only two levels, therefore only one bit only a single bit is required for quantization, okay so this is your quantization and now how do we reconstruct the approximation now we have  $m_q[n]$  which is the approximation at time  $n$  which is also in fact if you think about it the reconstruction at time  $n$  this is  $m_q[n]$  minus 1  $m_q[n]$  plus  $e_q$  of  $e_q[n]$ , okay so this is the step and in fact this is the key step this is the quantization so in fact we can number the steps let us number the step this is step 1 we reconstruct the difference, this is step 2 quantize the difference, okay.

So step one is basically difference, step 2 is basically quantize the difference and step 3 is basically what we have in step 3 is basically reconstruct the prediction which is  $m_q[n]$  which is a reconstruction but you can also say (reconstru) because this forms the prediction or approximation for the next time (inst) approximation (())(17:48) and then this process will continue in the loop, alright this process will continue in the loop, alright.



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So let us see how this works if I have a signal  $m[n]$  at time instant  $n$  I have an approximation let say I have a  $m_q[n-1]$  approximation at time instant  $n-1$  I have an approximation, okay so this is basically at time instant let say  $n-1$  time instant  $n-1$  now this is let us say time instant  $n$ , okay this is time instant  $n$  now at time instant  $n$  this level is  $m_q[n-1]$  let me write it clearly this is  $m_q[n-1]$  which is the reconstruction at time instant  $n-1$  and which forms the approximation at time instant  $n$ .

Now if you can see  $m[n]$  is greater than  $m_q[n-1]$  greater than  $m_q[n-1]$  which means the error  $m[n] - m_q[n-1]$  is greater than 0 remember this is your  $e[n]$  this is the error so this is greater than 0 remember this if you look at this part here this part is your  $e[n]$ . So  $e[n]$  is greater than 0 which implies  $e_q[n]$  equals  $\Delta \cdot \text{sgn}(e[n])$ , okay now since  $e[n]$  is greater than 0  $\text{sgn}(e[n])$  is positive so this is  $\Delta$ .

So  $e_q[n]$  is  $\Delta$  so that is basically your  $\Delta$  that is basically your  $\Delta$  so this is of height  $\Delta$  and so  $m_q[n-1] + \Delta$  that gives the reconstruction, so that will be your  $m_q[n]$  that gives the reconstruction. So this is how you obtain the staircase approximation and then subsequently we have the difference we do that at every time instant you see we can (approx). Now obviously if  $e[n]$  is negative if the error that is  $m[n] - m_q[n-1]$  is negative then  $\Delta \cdot \text{sgn}(e[n])$  so  $e[n]$  is negative  $\Delta \cdot \text{sgn}(e[n])$   $e[n]$  will be minus  $\Delta$ , so obviously the staircase will go down when  $\Delta$  is positive the staircase will go up.

So you (approx) so you get a staircase approximation of the signal, so this is how you are getting a staircase approximation of the signal, so this is how we are getting a this is your basic staircase approximation of the signal this is your staircase approximation of the original signal, okay. And now you can see what is being quantized? Only the error is being quantized in only 1 bit  $e[n]$  is as a plus delta or minus delta, okay so  $e[n]$  is either plus delta or minus delta.

So this is the difference is encoded in 1 bit, okay so this difference is encoded using 1 bit this is so either we have let say plus delta corresponds to plus delta corresponds to 0 the level minus delta corresponds to information bit 1. Now corresponding to for instance now we have this difference sequence delta minus delta minus delta delta minus delta corresponding to this we can have an encoded bits sequence so delta corresponds to 0 0, 1, 1, 0, 1 so on so on.

So this is the encoded this is the encoded signal and this encoded signal is either stored, it has storage device as a stored or transmitted. And for the purpose of storage you can say that we are now achieving compression because we are able to reuse only 1 bit to characterize the signal so remember this also gives us compression in fact there are lot of application even in application such as signal compression that is for instance speech compression or image compression.

So this gives us an efficient way not just to quantize represent the signal but also to compress the signal since we are using only 1 bit. So this can also so interestingly in fact this can also be used for this encoding can also be used for can be used for realizes can also be used for signal compression such as speech compression or image compression etcetera, okay. So we have the signal, error error is quantized to 1 bit and it is either stored or that is encoded into a single bit that is plus delta as 1 bit plus delta at 0 or for example is a plus delta as 0 and minus delta as 1 and is encoded bit stream is then either stored or transmitted, that is the delta modulation, alright.

And in subsequent module we look at how to reconstruct this signal, how to reconstruct the original signal from this encoded bit stream, alright so we will stop here thank you.