Principles of Communication-Part 1
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Module 1
Lecture No 4
Fourier Transform, Inverse Fourier Transform of Continuous Signals, Example of FT of Pulse and Sine Function

Hello, welcome to another module in this massive open online course. So in this module let us start looking at the Fourier transform, remembering we are looking at the spectra of signals and we said that spectra can be of 2 types, 1 is the discrete Fourier series, which is for a periodic signal and the other 1 is the Fourier transform which is defined for an Aperiodic continuous time signal x(t).

So let us start the Fourier transform, which we can abbreviate as F.T the Fourier transform and remember this is for the spectrum of an Aperiodic continuous time signal x(t), that is the signal x(t) you remember previous we looked at the spectrum that is the discrete Fourier series of a periodic signal x(t). Now we are looking at the Fourier transform, which gives the spectrum of a continuous Aperiodic times signal x(t), alright.

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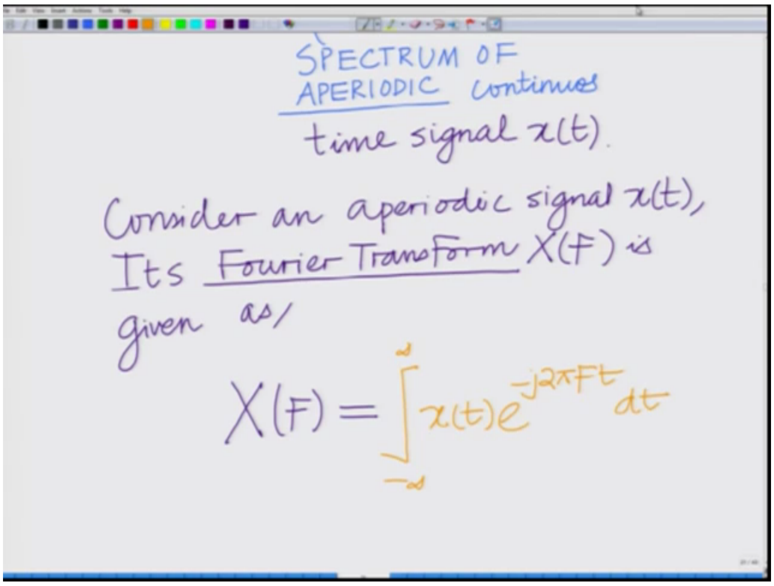

So let us consider a signal x(t) then its Fourier transform X(F), alright, so X(F) which is the Fourier transform of x(t) this is given as the integral, so let us start by writing consider an aperiodic signal x(t) it is Fourier transform or its Fourier transform X(F) is given as the Fourier transform X(F) equals to integral - infinity to infinity x(t) e power - j 2 Pi Ft dt, okay, so this is the signal this is the Fourier F is the frequency , okay alright.

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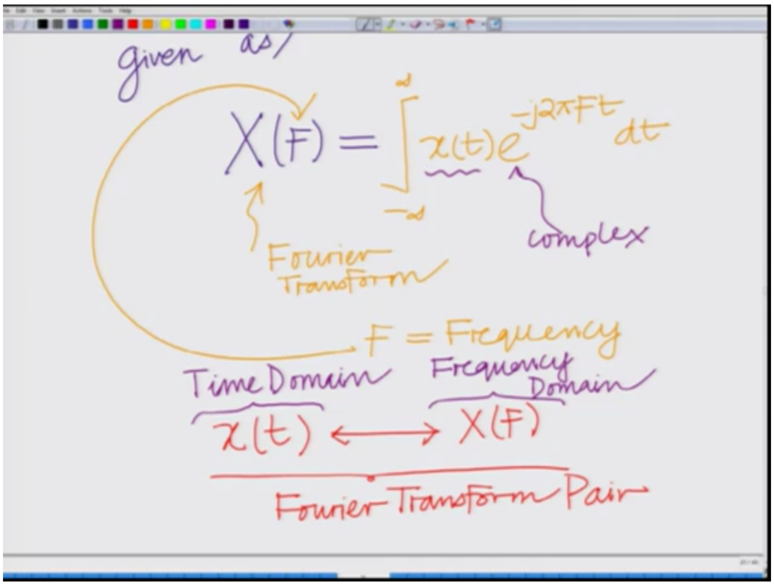

And we denote the signal x(t), that is the signal x(t) and its Fourier transform these form of Fourier transform pair and it is denoted by this symbol, so this forms the we call this x(t), signal x(t) and its Fourier transform as a Fourier transform pair. That is a signal x(t) and capital X(F) as the Fourier transform pair. Remember, the signal x(t) is in the time domain capital X(F) is in the frequency domain these are 2 totally different domains, alright.

So the signal x(t), let us also note that the signal x(t) this is in the time domain and this is in the frequency domain X(F) is in the, x(t) is in the time domain, X(F) is in the frequency domain, and note look at this even though x(t) might be real but this is a complex quantity. In fact, e to the power of - j 2 Pi Ft this is a complex Sinusoid, alright. So Fourier transform X(F) is in general a complex quantity so X(F) is in general complex, alright.

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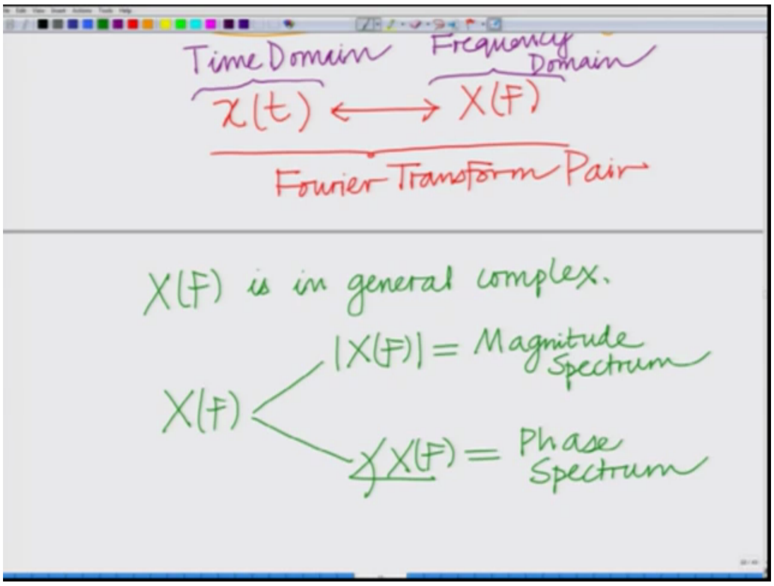

And therefore we have two components of X(F), 1 is the magnitude so I have the magnitude, this denotes the magnitude spectrum, then we have the angle that is the phase of X(F) which denotes the, so we have the magnitude spectrum and we have the phase spectrum that is we consider the magnitude X(F) is in general a complex quantity, the magnitude of X(F) denotes the magnitude spectrum, the phase of X(F) denotes the phase spectrum, okay alright. And of course, now what we have found so far is given x(t), how to find the Fourier transform X(F) in the frequency domain. Of course, we are also interested in the inverse problem that is given X(F), how to find the given the Fourier transform capital X(F), how to find the corresponding time domain signal and this is given through the corresponding inverse Fourier transform.

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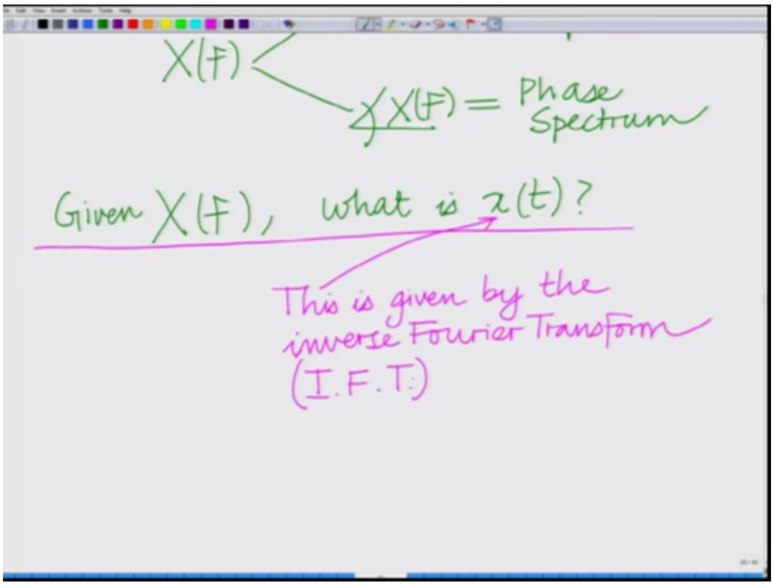

So what we have shown is given x(t), how to find X(F)? And now what we are interested in given X(F) what is the corresponding time domain signal x(t) and we are saying this is given by the inverse Fourier transform the IFT, the inverse Fourier transform can be defined as follows that is the signal x(t) corresponding to X(F) is given as integral - infinity to infinity X(F) e to the power of j 2 Pi Ft dt and this is simply known as the expression.

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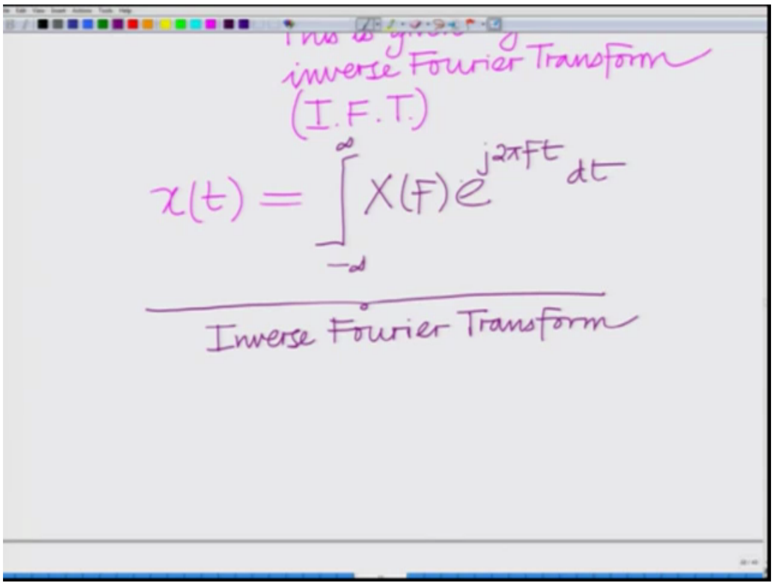

So notice that for the Fourier transform we have an e to the power of - j 2 Pi Ft, for the inverse Fourier transform we have an e to the power of j 2 Pi Ft, this is the expression for the inverse. Fourier transform, okay. This what we have is we have basically we have defined Fourier transform, which gives the spectrum of a continuous time Aperiodic signal x(t) and we also define or in we have also shown how to derive the expression for the continuous time signal in the time domain given the Fourier transform in the frequency domain.

There is for the Fourier transform of the continuous time signal is given by the expression for the Fourier transform the time domain signal from the Fourier transform that is capital X(F) is given by the inverse Fourier transform, alright. So let us now do a simple example to compute the Fourier transform, okay.

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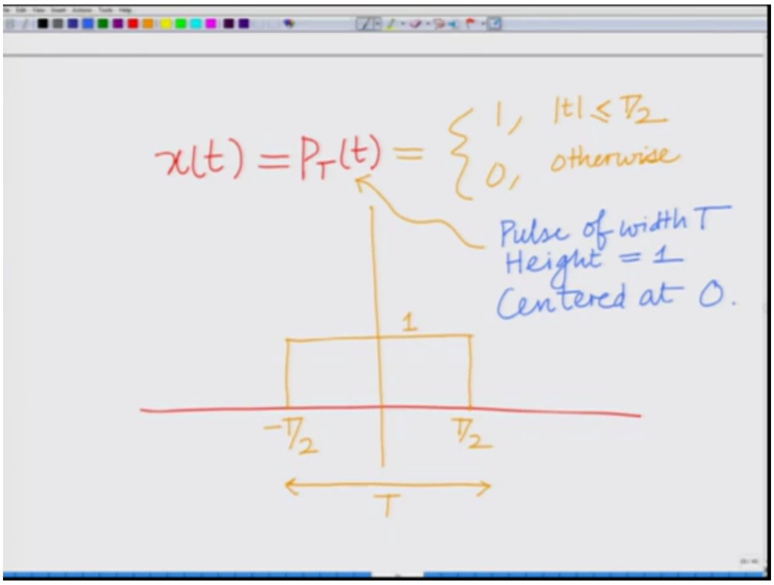

So let us now do a simple example to compute the Fourier transform, okay. Consider the pulse x(t), which is defined as, we have a pulse x(t) which is defined as P sub T of (t) which is equals to, let me first or the signal in the time domain. It is the pulse of width capital T which can be presented as follows that is the pulse of height unity from - T by 2 to T by 2. So the width of the pulse is capital T, so basically and this is outside of this window of size T this is 0, so this pulse P sub T of (t) is 1 for magnitude of t less than equals to T by 2 that is t lying between - T by 2 to capital T by 2 and 0 otherwise, alright.

So this is a pulse or rectangular window, we can also say that a pulse is nothing but a rectangular window. So this is a pulse of width T, height equals to 1 centered at, alright, so this is centered at 0, alright. So this is a pulse of width capital T or duration capital T height unity, alright and centered at 0 which means it expands - capital by 2 to capital T by 2, these are very simple signals and of course, it is of lot of relevance in the communication system.

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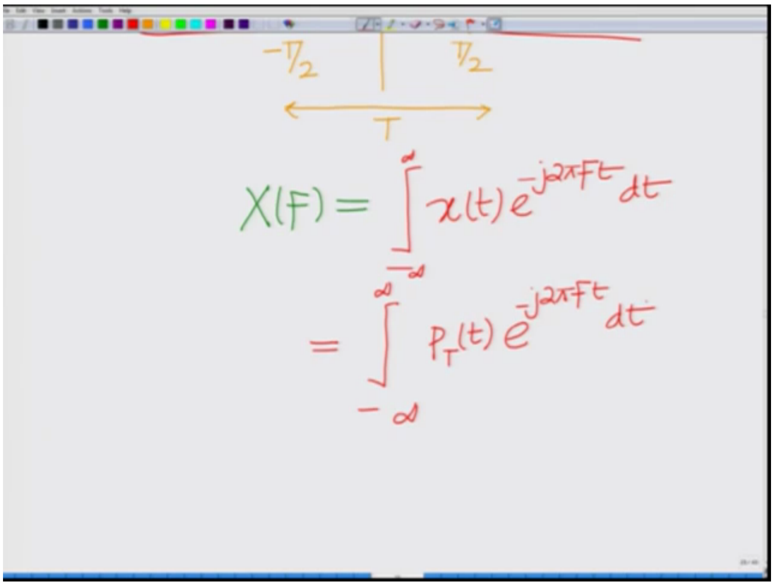

We are going to look at the relevance of this in the coming modules, alright. This is a very simple signal which arises very frequently, alright. This is a pulse of height 1 and with capital T, alright. Let us now fight the Fourier transform of this signal P sub capital T of t, okay. And we know the Fourier transform, we have shown just the expression earlier that is X(F) is given as - infinity to infinity x of (t) e to the power of - j 2 Pi Ft dt and x of (t) is nonzero only from - T by 2 to T by 2 in which it is 1, so let us write this as, let us first substitute P sub T of (t) e to the power of Ft dt than P sub T of t, remember this is nonzero only from - T over 2 to T over 2 in which it is 1 e to the power of - j 2 Pi Ft dt, okay.

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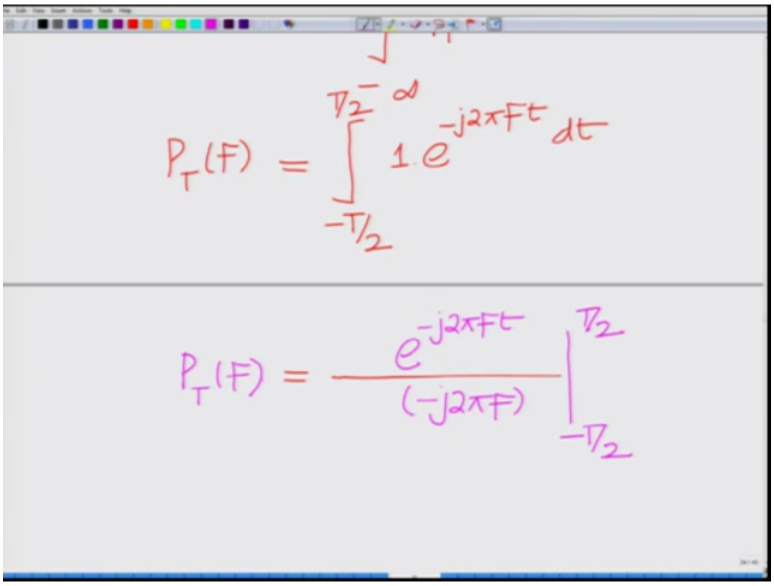

And this integral this is your you call this also as the Fourier transform of PT that is PTF and integral of P to the power of - j 2 Pi Ft, this is e to the power of - j 2 Pi Ft divided by - j 2 Pi F evaluated between the limits T by 2 to - T by 2, okay.

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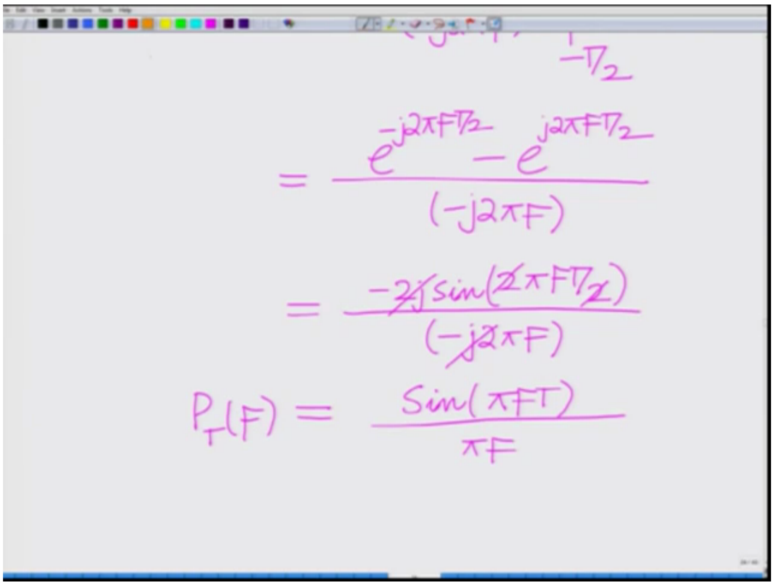

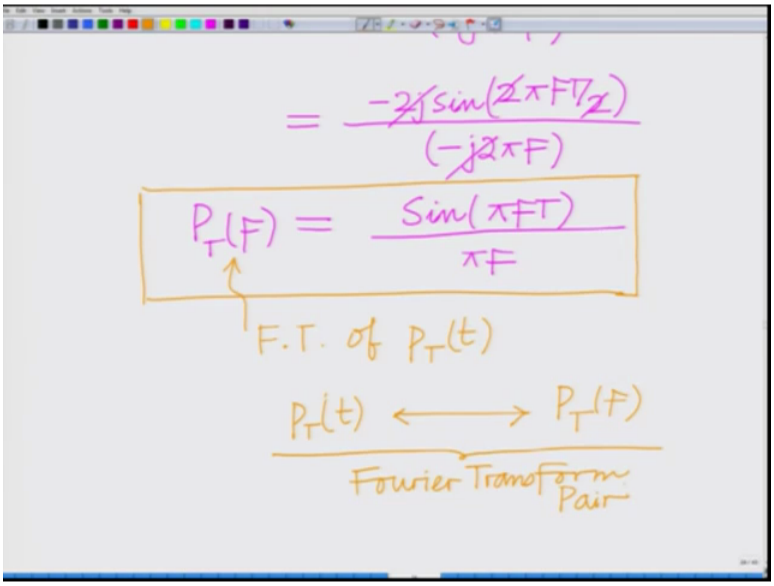

And now substituting the limits we have e to the power of - j 2 p F T by 2 - e to the power of j Pi F T by 2 by - j 2 Pi F and this is equals to, now e to the power of - j 2 Pi F T by 2 - e to the power of j 2 Pi F T by 2, this is basically - 2 j sin 2 Pi F T by 2, okay. So this is nothing but, so e this is as e to the power of I am sorry there is a 2 Pi missing over here this is - j 2 Pi F T by 2, so this will be - 2 j sine 2 Pi F T by 2 by - J 2 Pi F. The 2 j - 2 j - 2 j cancels and what we have and of course there is a 2 here goes. So what we have is sine Pi F T divided by Pi F, okay. So this is the expression for the, Fourier transform, this is Fourier transform of your pulse or rectangular window PT(t) and remember we can also say this as your PT(t) and it is spectrum PT(F), these form a Fourier transform pair .

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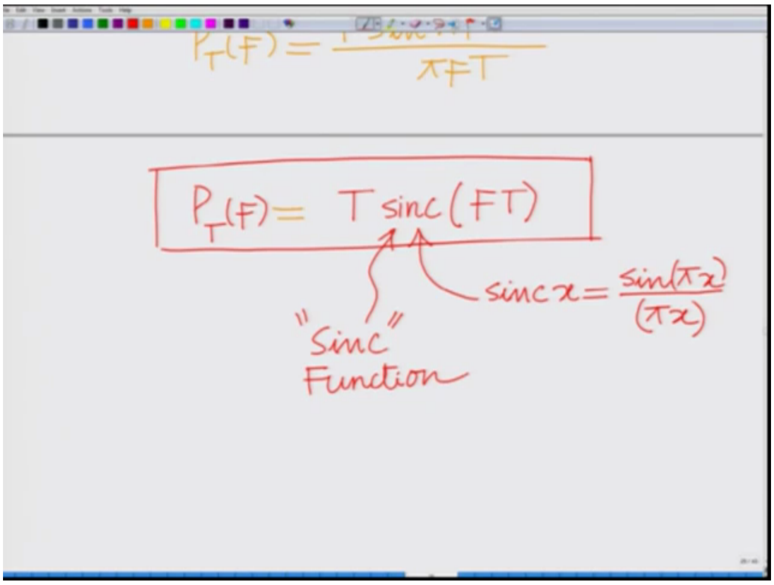

And in fact, now I can also write this PT(F) as follows that is PT(F) which is sine Pi FT by Pi F, I can multiply in the denominator by capital T numerator by capital T and I can now write this as T sinc (FT) where basically this function sinc of x this function and it is very interesting function and arises frequently in communication and signal cross sinc this is defined as sine Pi x over Pi x.

So I can conveniently write this using the sinc functions, this is called (()) (18:46) it is simply called as sinc function I can write this as capital T times sinc F capital T, alright where Capital T is the duration of the pulse, sinc denotes the sinc function, where sinc of x is sin of Pi x divided by Pi x, okay. So this is simply known as the sinc and it is useful to knowthe definition of this function because we are interested we frequently keep referring to this function, which is sinc function that is sine Pi x by Pi x, therefore it is useful for the definition of this function.

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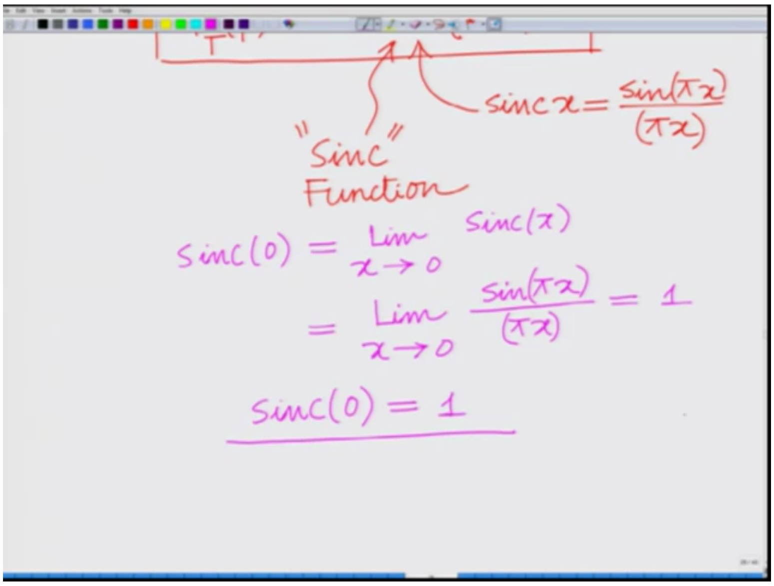

Now look at this, the sinc function, look at the properties of the sinc function, the sinc function at x equals to 0. If you take the limit extending to 0 for sine Pi x by Pi x or let us say limit extending to 0 of sinc x. Sinc of 0 is defined as limit extending to 0 of sinc of x, which is equals to the limit extending to 0 of sin Pi x by Pi x and this is equals to 1. Some people simply say that is sinc of 0 is simply set as equals to 1. So sinc of x at x equals to 0 is simply set as 1 because although it is not defined, strictly speaking it is not defined at 0 because sine x by x at x equals to 0 is undefined. So we consider the limit as extending to 0 and therefore the sinc of x is x tends to 0 is sinc of 0 is basically 1.

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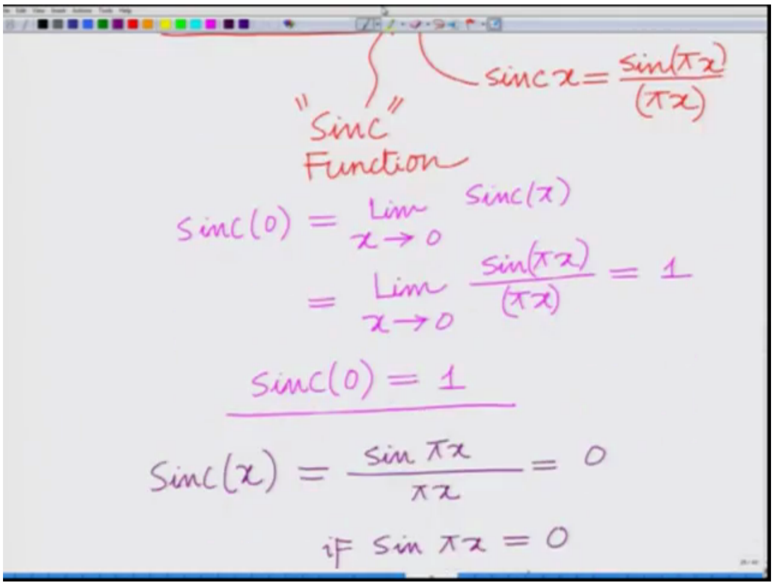

And further notice that sinc of x which is sine Pi x equals to 0 implies Pi x equals to any multiple of k times pie implies x equals to any integer k other than k not equals to 0. So sinc x at x equals to 0, so at x equals to 0 we have sinc of 0 equals to 1 except that at all other integers x equals to k sinc of k or sinc of k would be sine of k pie divided by k Pi by sine of k Pi for any integer k is 0, so this is 0. So for all integers k not 0 the sinc of x that is sinc k for k an integer is 0, okay.

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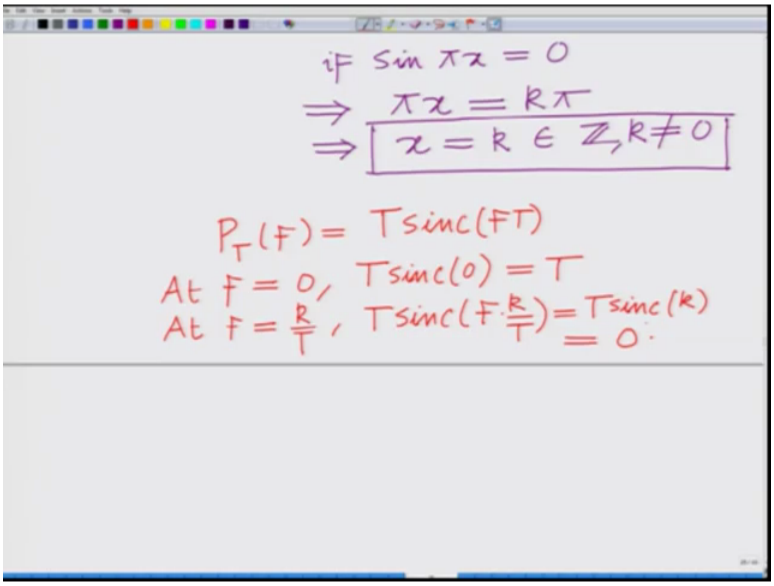

So therefore, now look at this we have our function PT(F), now let us go back using these properties your PT(F) equals to T sinc of (FT) if at F equals to 0, this becomes T times sinc of 0 this is equals to T at F equals to any k over T, we have T sinc F times k over T equals to T sinc k equals to 0. So at F equals to 0 the frequency responds T sinc F capital T is equals to capital T, at any other F that F equals to k over T that is multiples k of the inverse duration that is k over T where at T is the duration of the (()) (23:54) all multiples k at k over T, where k is any integer other than 0 the frequency response is 0.

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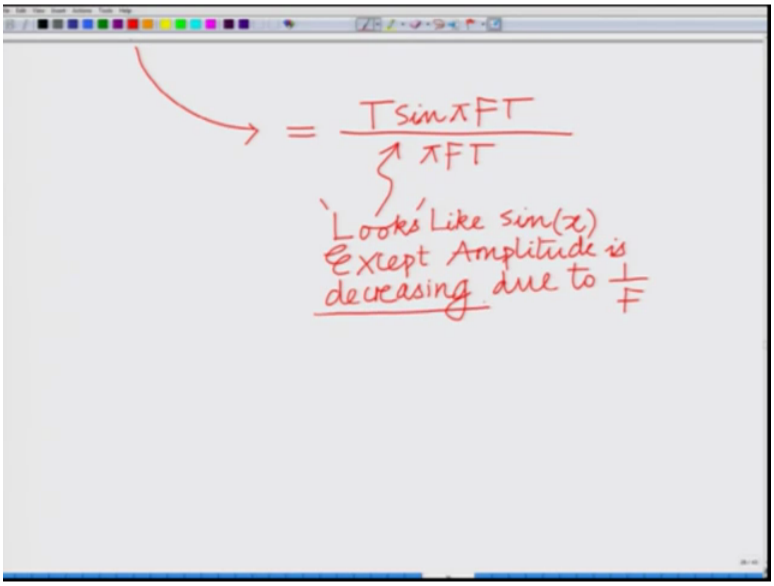

And if you can look at this, let us go back and look at this, this is T sin Pi FT by Pi FT. So this looks like sine x except amplitude is constantly decreasing due to this factor 1 over F, so amplitude is decreasing, so how does this look?

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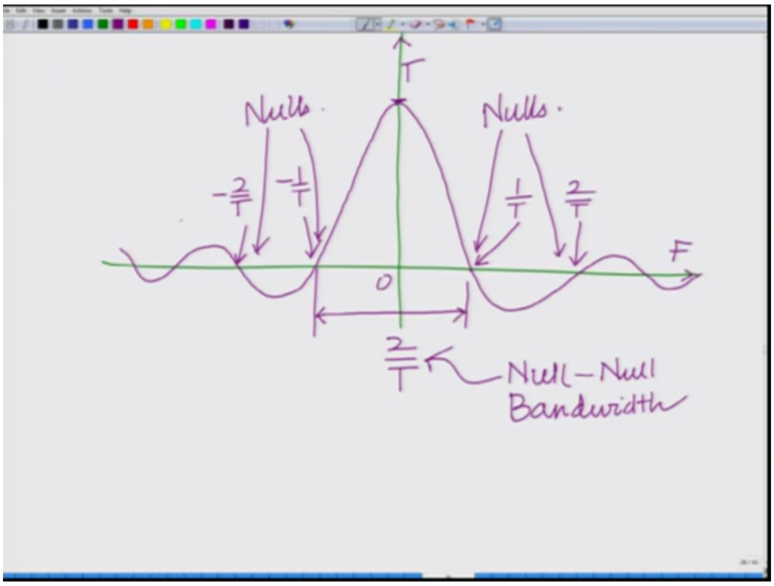

Let us try to plot this sinc function if I plot this function T sinc FT in the frequency domain this function looks as this is the frequency axis, this is amplitude and I will have at F equals to 0, this is T and then it will have an amplitude that is continuously decreasing and these points were here it is T, this is 0 and these points where it is 0 are integer multiple is of 1 over T, so this is 1 over T, 2 over T the first null on the negative axis is - 1 over T, - 2 over T, mind that is integer multiples of - 2 over T and therefore if you look at this sinc function, it has a decaying envelope and if you look at the null to null bandwidth, that will be - 1 by T to 1 over T.

So the spacing in the frequency domain, the spacing on the frequency axis between the nulls the nulls on the negative the first null on the negative frequency side first null on the positive frequency, this is known as the null to null bandwidth that is 2 over T, where T is the duration of the pulse, alright. So this is our frequency axis, this is our amplitude axis, 2 over T is the null to null, there are many notions of bandwidth this is known as the null to null bandwidth, alright. So this is the, so these are basically if you can see these are basically the nulls, okay. So these are the nulls, in fact, there are several nulls in fact every multiple of k over T, in fact, at every multiple integer multiple k, integer multiple of 1 over T that is k over T this is your except when k equals to 0 at F equals to 0 this is as we have already seen this is capital T, alright.

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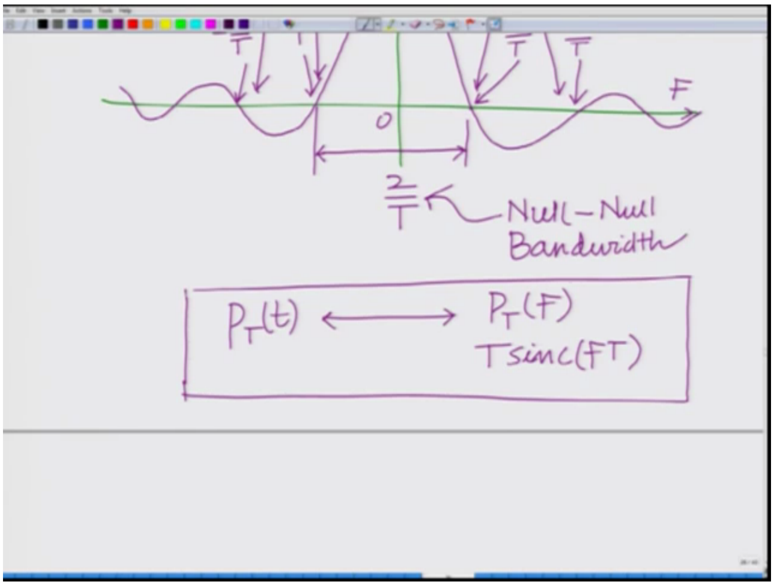

So this is T sinc FT and this is, so we can say our pulse PT(t) and we have now its Fourier transform pair, which is T sinc (FT), alright and this sinc function has very significance it has lot of applications in communications and it is very fundamental it is of fundamental importance in communication, alright. So this is basically this is given by the sinc function the Fourier transform the pulse as it (()) (28:31) a sinc function, alright.

So, in this module what we have done is basically we have defined Fourier transform of a continuous time Aperiodic signal, the inverse Fourier transform of given that is given Fourier transform, how to derive the time domain signal, and also illustrated the computation of the Fourier transform computation of the Fourier transform through an example.

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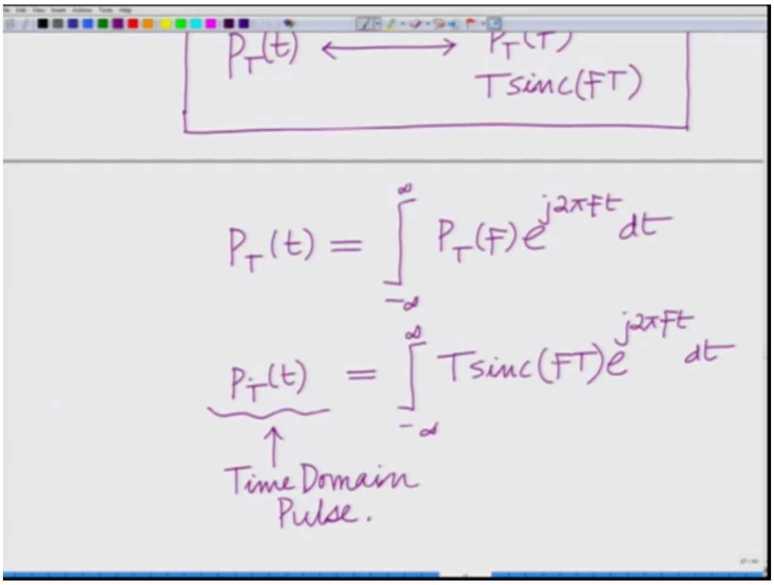

And just to conclude remember 1 can also now say that using the inverse Fourier transform we have PT(t) will be it must follow that PT(t) is the inverse Fourier transform of PT(F) and therefore you can say that integral - infinity to infinity T sinc (FT) e to the power of j 2 Pi Ft will be PT that is your time domain this gives back your, so this must give back the time domain pulse, alright. So if you take the Fourier transform PT(F) in the frequency domain computed, in inverse Fourier transform you must get back the time domain pulse, alright. So we will stop this module here and look other aspects subsequent modules, thank you.