Principles of Communication- Part I
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Module No 7
Lecture 39
Pulse Amplitude Modulation (PAM), Spectrum of PAM Signal, Reconstruction of Original Signal from PAM Signal, Equalization

Hello welcome to another module in this massive open online course, so we are looking at pulse amplitude modulation, correct? Where the amplitude of a series of pulses is modulated using the sample the sampled values of a of the sampled signal, correct? So we are looking at pulse amplitude modulation, correct?

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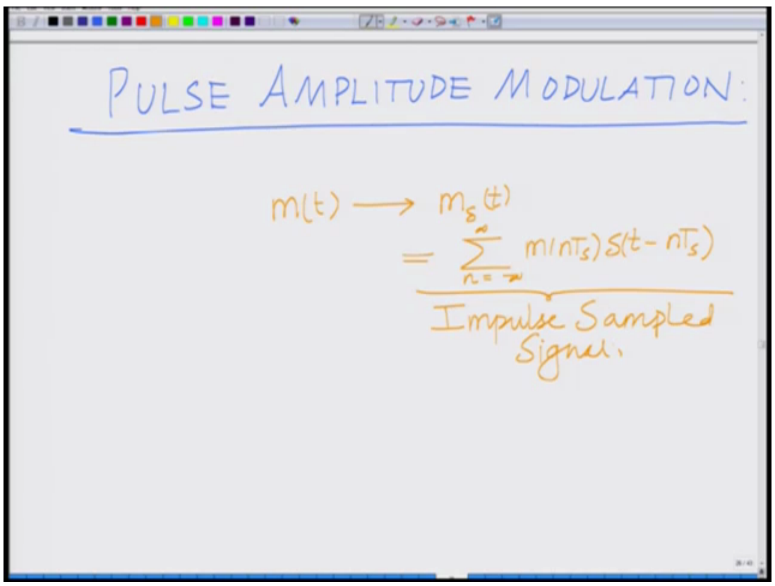

Okay there we have said that we have our original signal m of t it can be represented as the modulus sample it with the impulse that is your impulse sampling (sig) impulse sampled signal which is summation n equal to minus infinity to infinity m of nTs Delta t minus nTs this is our original impulse sampled signal, correct? This is our impulse sampled signal and what we want to do…

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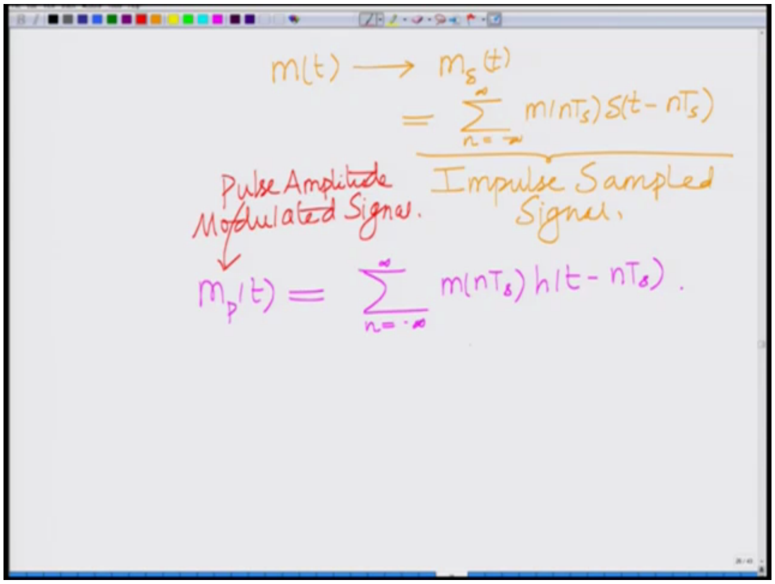

Now is our pulse amplitude modulated signal remember that is over nPt this is basically if you look at this this is your n equal to minus infinity to infinity mnTs h of t minus nTs where so this is my pulse amplitude modulated signal, so this is our pulse amplitude modulated signal which basically is basically this corresponds.

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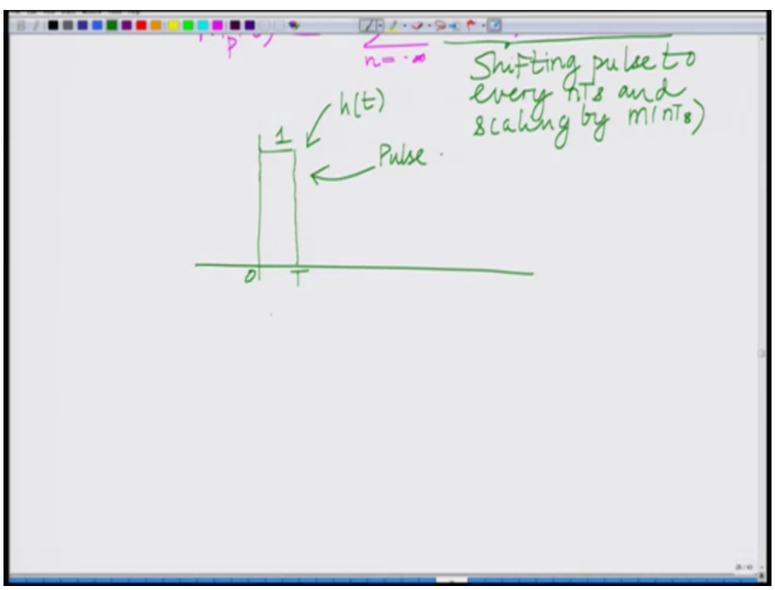

So my impulse is basically h of t that is my impulse remember, it is our I mean it is my pulse my pulse is basically of width 0 to T this is my pulse of height 1, correct? So this is my pulse and what we are doing is basically we are shifting the pulse to every Ts and scaling of basically modulating it by mnTs which is the value of the sample at the nth sampling instant that is nTs, okay. So this is basically shifting pulse nTs and scaling by m of nTs hence, I can also represent this as and therefore we have said that we can equivalently represent this this pulse amplitude modulated signal.

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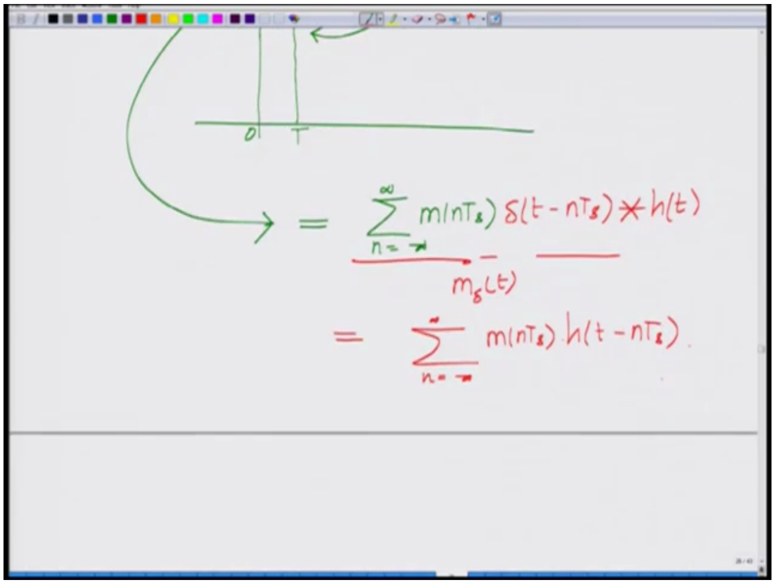

As if you remember this is your this is your impulse sampled signal that is mnTs n equal to minus infinity to infinity Delta t minus nTs this is your m delta t convolved with h of t and that gives me basically your pulse amplitude modulated signal, okay. That is mnTs impulse convolved impulse shifted impulse t minus nTs convolved with ht gives h of t minus nTs, okay. So this is your pulse amplitude modulated signal.

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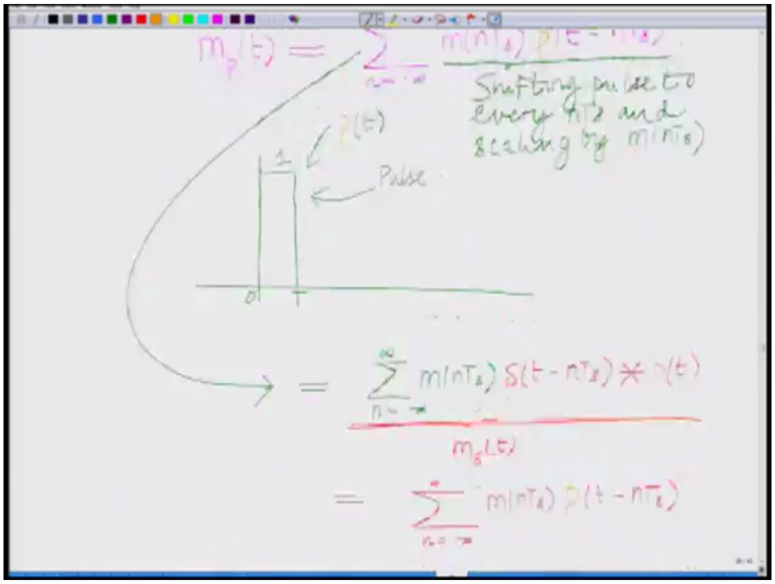

Now the interesting thing about this is basically if you look at this is basically m delta t your pulse amplitude modulated signal MPt equals m delta t convolved with h of t which means if I take the Fourier transform, now if I look at the Fourier transform of this pulse amplitude modulated signal which is denoted by MP of F naturally in the (Fouri) frequency domain convolution in the time domain becomes multiplication in the frequency domain therefore MP of F is given would be given as MP of F would be your Fourier transform of the impulse sampled signal times Hof F so M depth M delta F equals Fourier transform m delta t and HF equals Fourier transform.

I am denoting this by Pt not ht, so this will be Pt so the pulse shape is Pt so this will again be here it will be Pt, so Pt minus ts so this is Pt convolved with Pt, so this will be PF where Pf is the Fourier transform of the pulse shape Pt PF equal to Fourier transform of the pulse Pt, okay. So that is what we have that is a Fourier transform of the pulse Pt, correct? Where we are using Pt to denote the pulse shape now we already know the Fourier transform of m delta t we know that the Fourier transform m delta t which is Fourier transform M delta F, well that is simply your sum of shifted replicas of the original spectrum M of F, correct?

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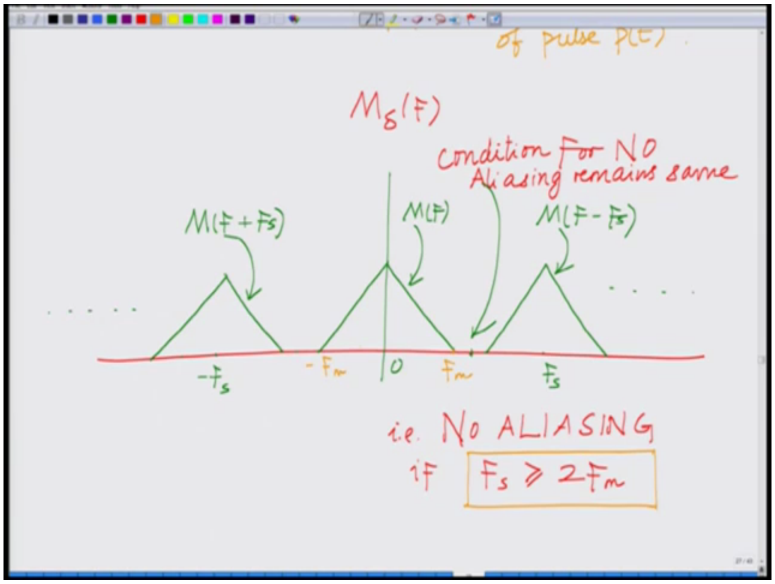

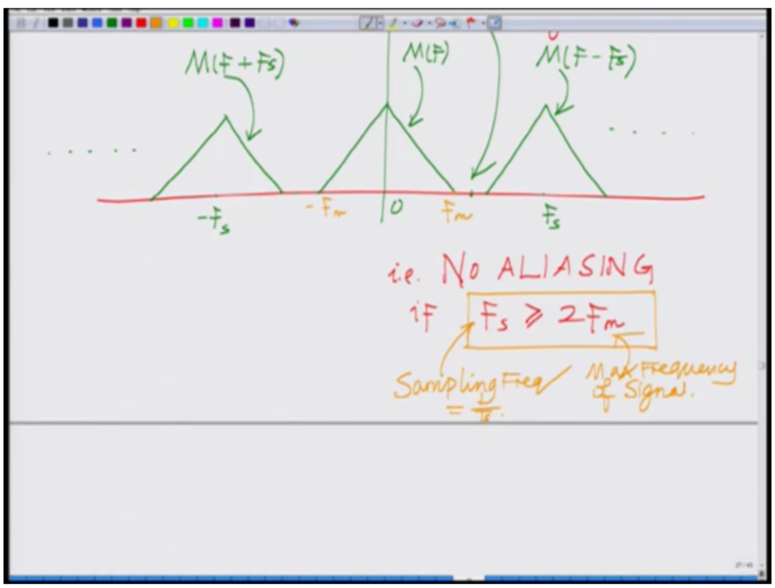

So this is M of F, so this is 0 this is Fs, this is minus Fs this is your M of F, this is your M of F minus Fs and so on this is your M of F plus Fs and so on M of F plus Fs and therefore the condition for aliasing that is condition for avoiding aliasing or condition for avoiding distortion remains the same, alright. So you are taking the spectrum m delta F and multiplying multiplying it by multiplying it by HF. So that does not cause overlapping of the signal (())(8:18), correct?

So the condition for avoiding aliasing still remains the same so the condition for avoiding remains same that is no aliasing if Fs greater than equal to 2 no aliasing if Fs is greater than or equal to, so this is your Fm which is the maximum frequency of the signal, okay. Where the sampling frequency Fs remember Fs is the sampling frequency equals 1 over your Ts Fm equals maximum frequency of signal basically also the bandwidth of the signal.

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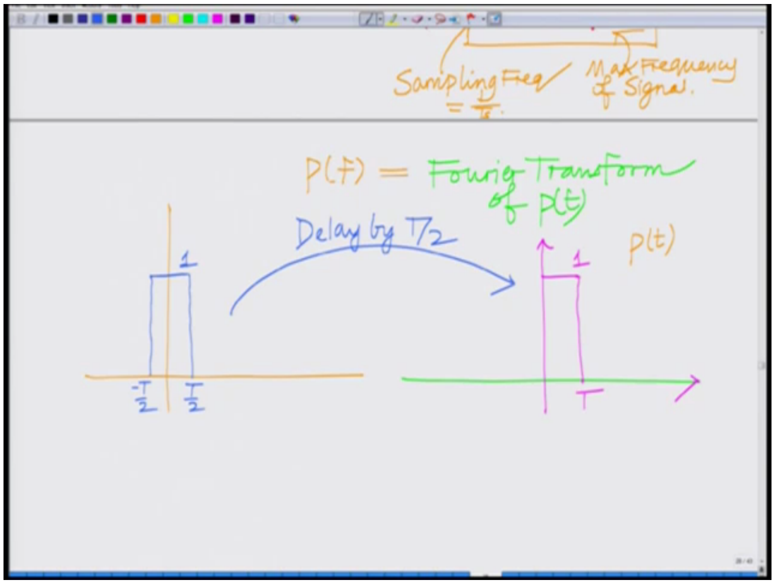

Now we need to find PF, PF is the Fourier transform so we need to find PF, PF is the Fourier transform of the pulse filter Pt, okay. So we need to find PF which is basically Fourier transform, remember PF which is basically Fourier transform of Pt we need to find this and remember is basically if you look at this Pt Pt is basically this pulse of height 1 pulse of height 1 this is Pt, now realize this can be obtained from this pulse it is a shifted version of this pulse of height 1 spread from so this can be obtained from this pulse or this can be obtained from this pulse which is basically your pulse from minus T by 2 to T by 2 of height 1. If I shift this by T if I shift this by rather T by 2, correct? If I shift this by T by 2 or delay by T by 2 rather I should say delay by T by 2. So what we have is the pulse of unit amplitude between minus T by 2 to T by 2. If I delay this by T by 2 then it will become then it becomes Pt which is the pulse of unit amplitude unit height between 0 and T, alright.

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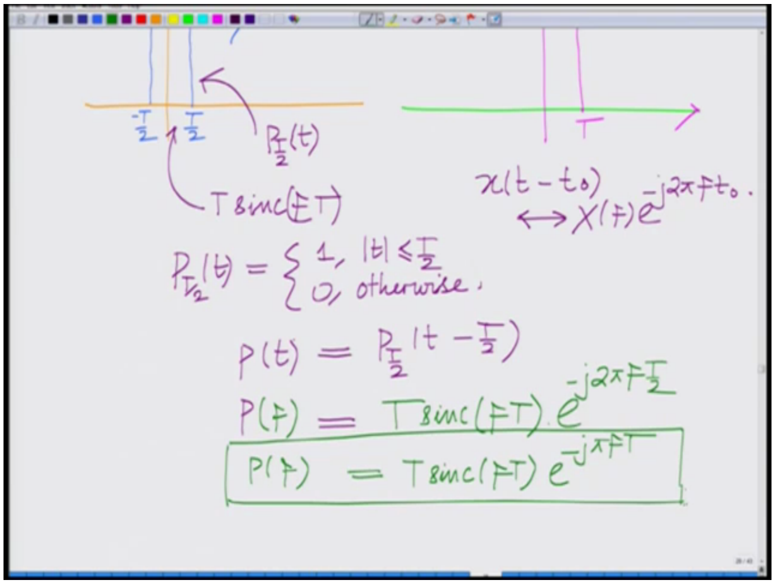

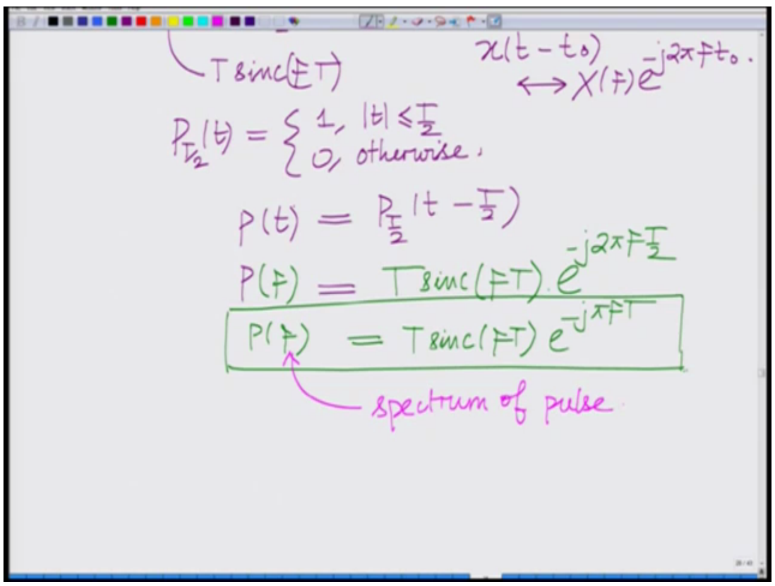

And I already know the Fourier transform of this the Fourier transform of the pulse which unit height between minus T by 2 to T by 2 this is basically my T sinc FT this is basically this pulse if you remember this is basically my P T by 2 of t that is where PT by 2 of t is if mod t less than or equal to T by 2 and 0 and 0 otherwise. So this is Pt by 2T by T by 2 so therefore Fourier transform of this using the modulating property that is x(t) minus t not Fourier transform will be a Fourier transform of x(t) is XF this will be X(F) e to the power of minus j 2 pi F t not.

Therefore t sinc, we have Pt our pulse Pt equals basically your Pt by 2 delayed by T by 2 therefore this will be PF the Fourier transform of this pulse will be well, it is going to be T sinc FT e to the power of minus j 2pi F t not which is equal to T by 2. This is e to the power of minus j e to the power of minus j pi Ft so the Fourier transform or the spectrum of the pulse is T sinc FT e to the power of minus j pi T this is your spectrum of pulse.

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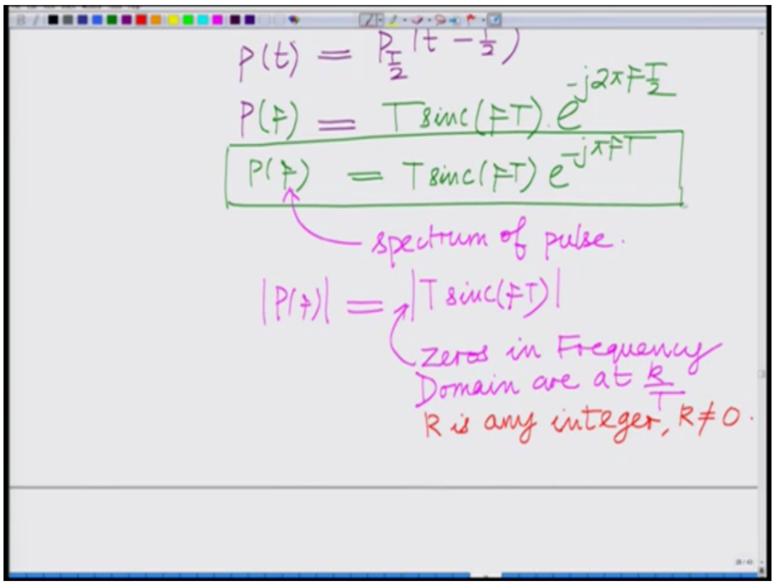

And interestingly if you look at the amplitude spectrum e to the power of minus j pi FT is simply a phase factor of so therefore amplitude spectrum is still the same that is magnitude T sinc FT and zeros of this in the frequency domain zeros in frequency domain are at k by T that is integer multiples of 1 over T where k is any integer but k not equal to 0. Of course, at k equal to 0 this is equal to T so if you look at the (magne) so the magnitude spectrum of P of F the pulse filter is again magnitude T sinc FT in the frequency domain with zeros at 1 over T. Now remember T this duration of the pulse is much less than Ts therefore 1 over T is much less than 1 over T is much greater than 1 over Ts or Fs, okay. So if you plot this noise look at this this is going to look something like this.

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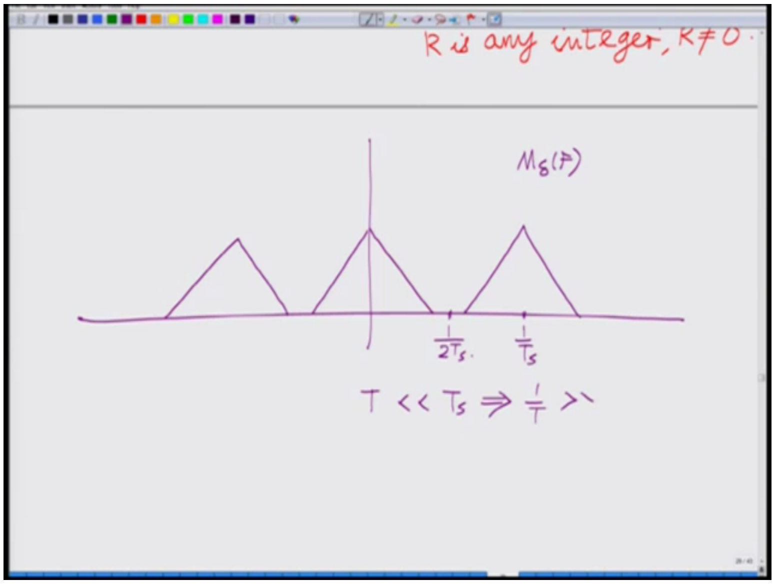

So we have let us plot this, so we have over m delta F which looks something like this let us say it is not aliased it looks something like this so this is your M delta F this is your this is your Fs which is 1 over Ts this is your Fs 1 over 2Ts, okay. And now we have T much less than Ts which implies 1 over T that is 1 over T is much greater than 1 over Time scale.

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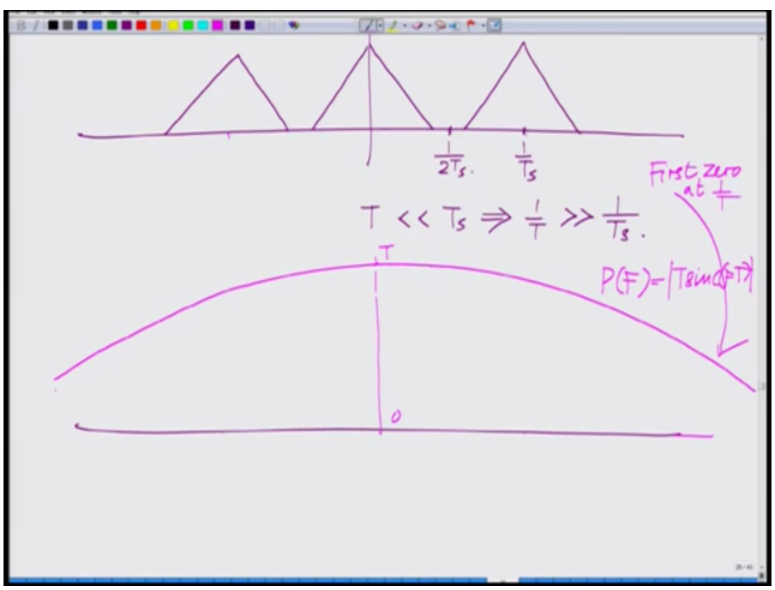

And therefore the pulse filter the 0 of the first 0 of the pulse filter will lie much beyond 1 over Ts so it will be something like so this is my minus TS and this is my 1 over TS, so the zeros will lie something over so it looks something like this, your sinc filter will look so this is your this is your P of F, correct? This is your P of F which is magnitude T sinc Ft of course at 0 this value is T and zeros of us to 0 let us put it that way first 0 lies much farther there.

So if you look at the first 0 first 0 is at 1 over T first 0 on the positive axis as at 1 over T at 1 over T on the positive axis and of course on the negative axis first 0 is that minus 1 over T and now what we have is this whole m delta F that is the impulse sampled signal is multiply spectrum of m delta spectrum m delta F of the impulse sampled signal is multiplied by P of F which is the spectrum of the pulse shaping filter therefore what you are going to have is the net spectrum of MPF will be you will again have several copies which will be multiplied by the sinc pulse shaping.

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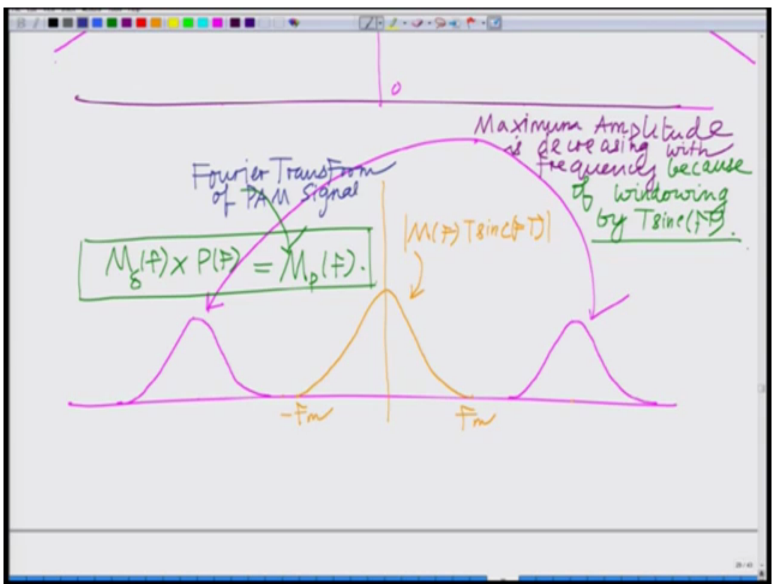

So this will be so if let us say this is Fm this is let us say minus Fm to Fm this is your M of F times the magnitude spectrum let us say we are plotting simply the magnitude spectrum magnitude M of F T sinc FT, okay. And this will be from Fs minus Fm to Fs plus Fm this will be something like this and this will be something like the amplitude the maximum amplitude is decreasing because of the sinc because of the windowing by the because of the windowing by the sinc.

So you can see the maximum amplitude is decreasing progressively as we go to the right which was not in the case in the impulse response frequency of the impulse sampled impulse sampled maximum, so first of all its distorted and maximum amplitude or peak amplitude I mean peak amplitude in the frequency domain is decreasing P is amplitude or which copy is decreasing with frequency because of windowing by the sinc, correct?

Because of windowing by the T sinc FT and therefore now reconstruction is again simple this is this is your basically this is your MP of F which is equal to remember M Delta of F times the pulse shaping filter P of F this is the Fourier transform MP of F or basically Fourier transform of pulse amplitude modulated signal, let us write that also Fourier transform of your PAM signal. Now obviously to reconstruct the original signal we have to isolate the the baseband component or the component around 0 frequency, alright. The the copy of the the spectrum around 0 frequency but now we also have to invert the distortion, so we have to also invert the distortion caused by the windowing by the sinc.

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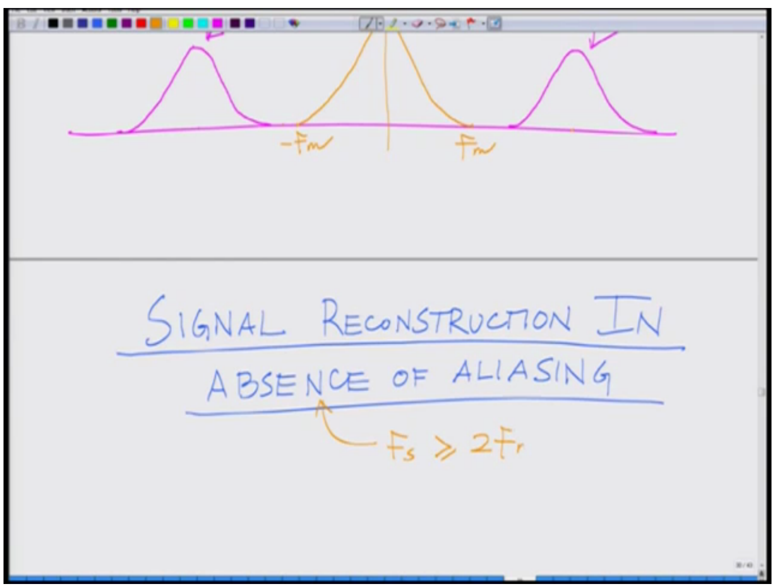

So first for reconstruction so let us now talk about reconstruction if there is no aliasing, alright. Obviously the condition for aliasing still is there so signal reconstruction in absence of aliasing which means absence of aliasing means that really it means that your Fs is greater than or equal to twice Fm.

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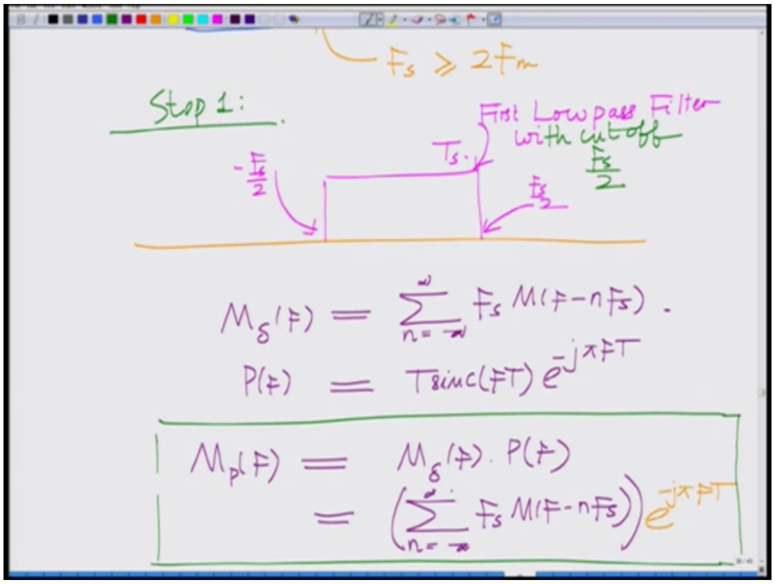

Now in that scenario first what we do is we low pass as usual low pass filter with cut-off as you are Fs by 2 and 1 over T which will basically and low pass filter where with so first low pass filter and this is basically your 1 over this has to be. So you have to adjust for the scaling for the scaling which has to be that scaling is basically Fs multiplied by Fs, so 1 over Fs or basically you have to multiply this by this has to be multiplied by Ts so you have to multiply by Ts and by the way the expression for this is basically your M delta F into PF.

Now M delta F remember, we have M delta F equals we know the expression for this, so let us write it at the appropriate place we know the expression for this well, we know the expression for m delta F so we know the expression for m delta F this is equal to summation or let us just write it down we know the expression for M delta F M delta F equals that is the response of the impulse sampled signal that is summation n equal to minus infinity to infinity Fs M F minus nFs, PF we know that is basically T sinc FT e to the power of minus j pi FT and therefore MP of F that is the response of the pulse amplitude modulated signal that is M delta F times PF which is basically your summation n equal to minus infinity to infinity n equal to minus infinity to infinity Fs MF minus nFs times e to the power of minus j pi Ft.

Okay times e to the power of minus j pi Ft, okay this is the response. Now to reconstruct it first low pass filter with cut-off again Fs by 2 that is a first step this is your first step, now in step 2 you also have to cause the invert the distortion of this amplitude spectrum of the copy around 0 frequency which is caused by windowing with the sinc function.

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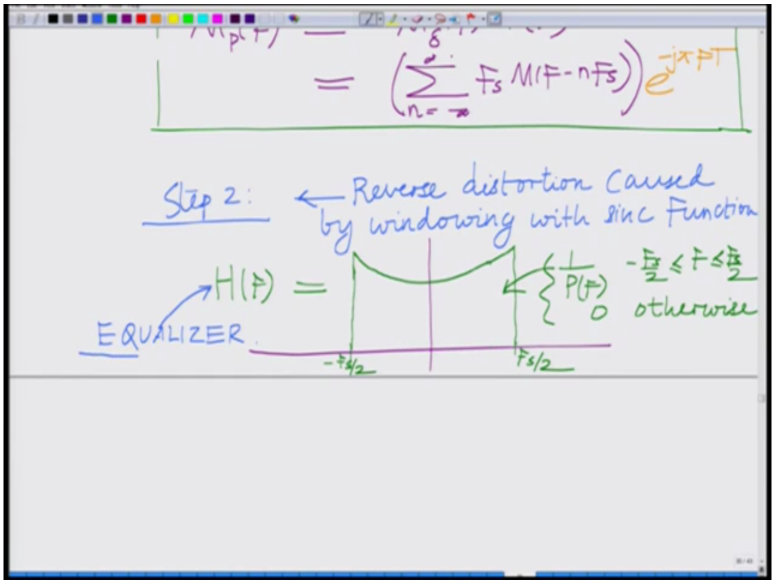

So in step 2 you have to in that frequency band step 2 reverse basically distortion caused by windowing with the caused by windowing with sinc function. So basically you have to multiply by the response in Fs by 2 to Fs by 2 which is 1 over pF, correct? So that will be the inverse of the sinc which will be something like this, correct? Something like this that is 1 over your PF in but this is necessary to be done only in your Fs by 2 to Fs by 2 minus Fs by 2, so it is necessary to do this only in Fs by 2 to minus Fs by 2, correct? So this is your 1 over PF in minus Fs by 2 less than or equal to F less than or equal to Fs by 2 and 0 otherwise because we do not care about the reconstruction in the outside band, alright.

We only want to reconstruct the Central band or the component around the, so this is your reconstruction filter if you call this H of F this is basically your reconstruction filter or we can call it the equalizer and the reason for this is as follows, let me just write it and explain this, this is also known as an equalizer the reason for this is that the original sinc windowing is multiplying by the sinc, alright. Which has which has basically a decreasing amplitude.

Now what you are doing you are trying to invert that so as to make the network the net response that is the multiplication of this this filter with the sinc to be unity, alright. So that way you are distorted so whatever distortion is caused by the sinc windowing you are reversing that by inverting that, so the net response if you look at it will be flat that is you are equalizing it across the frequency domain. So this is also known as the equalizer, alright.

So first you pass it through a low pass filter similar to reconstruction from the impulse ideal impulse sampled signal followed by an equalizer to invert this distortion caused by windowing due to the sinc function between minus Fs by 2 to Fs by 2 therefore the reconstruction will be as follows the reconstruction will be.

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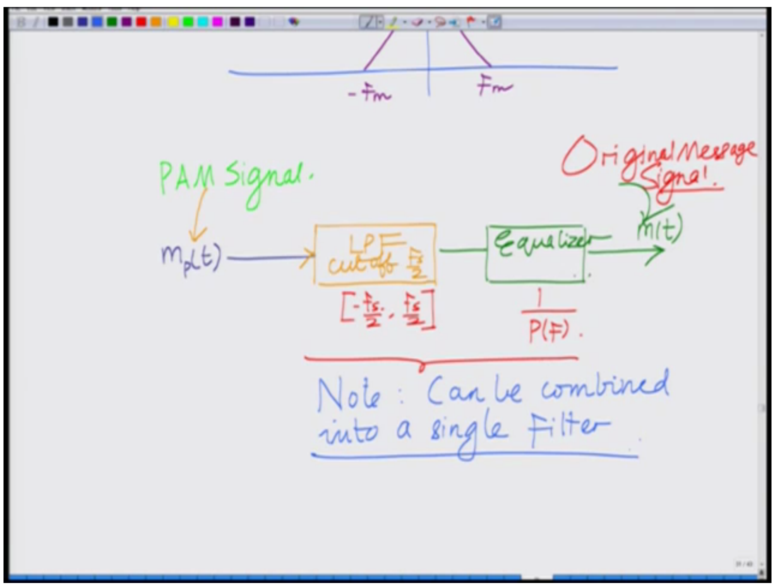

And then once you do that what you will have is, you will have your original message spectrum which is well, minus Fm to Fm this is your M of F, so therefore the reconstruction procedure would be I have my PAM signal or my MPt I pass this through first LPF cut-off is Fs by 2, okay this is mine PAM signal followed this by equalizer that is 1 over PF and then you get back m(t) which is the original message signal.

Of course noting that if there is no aliasing this is your original message signal this is the original message signal, okay. So you get back so you pass this is the take the PAM signal you have your low pass filter that is low pass filter between minus Fs by 2 to Fs by 2, okay. That is the pass band of this low pass filter that is pass band until Fs by 2 cut-off frequency at Fs by 2, okay. And then you pass it through an equalizer what the equalizer is doing is simply it is inverting the distortion caused by the by the sinc windowing function the frequency domain and then you get back the original signal m(t), alright.

So basically the pulse amplitude modulated signal has to go through a 2step reconstruction first, first low pass filtering to extract the component at 0 filtering and of course subsequently the equalizer and of course you can combine both these things into a single filter that is the low pass filter itself if you design the low pass filter response such that it is 1 over PF in that frequency band then naturally you can reconstruct, alright.

So but it is easier to think it think of it in terms of first low pass filtering operation followed by an equalizer operation although both there can be a combined filter, so these can be (de) these can be put together because both these things are linear operations these can be we can also note that note that these can be combined these can be combined into a single can be combined into a filter, alright and then 1 can reconstruct the original signal m(t) from the pulse amplitude modulated signal m(t) of t, alright. So we will stop here and we will look at other aspects in the subsequent modules, thank you.