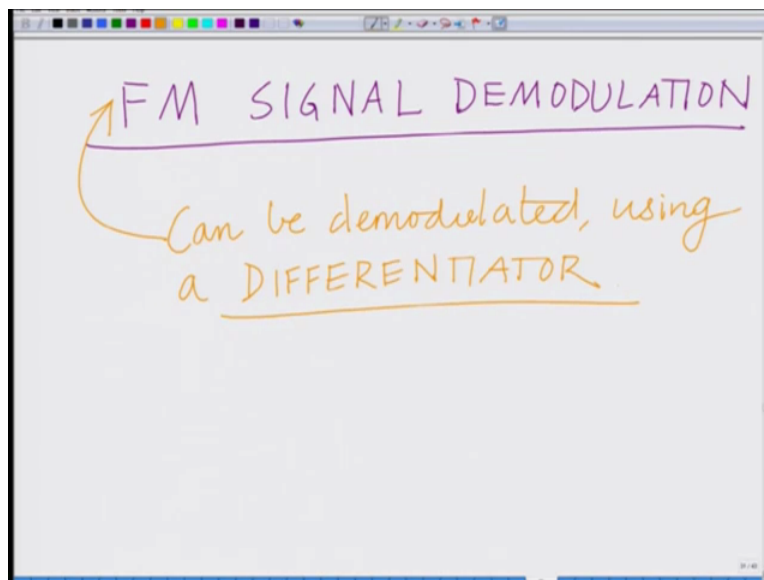


Principles of Communication- Part I
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Module No 6
Lecture 34

Demodulation of Frequency Modulated (FM) Signals, Condition of Envelope Detection

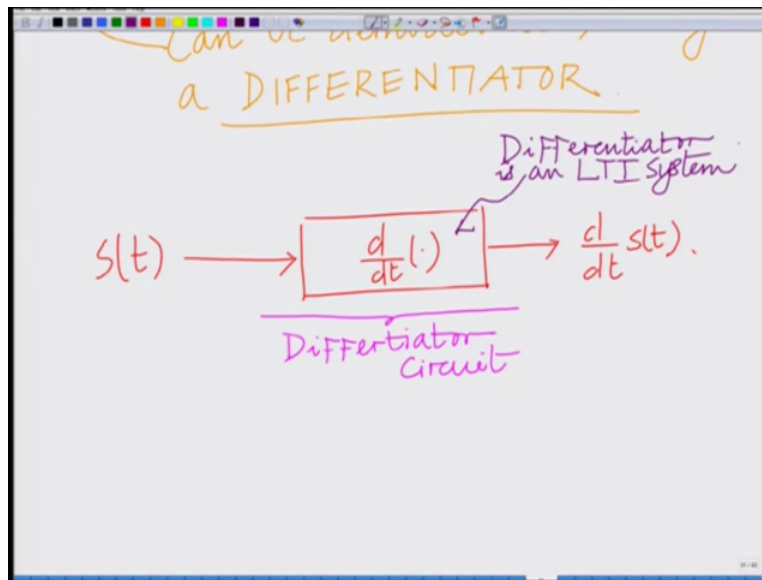
Hello welcome to the module in this massive open online so in the previous module we have looked at the bandwidth of an FM signal the approximate bandwidth occupied by an FM signal which is given by the Carson's rule, okay. So in this module let us start looking at the demodulation of an FM signal.

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So we want to start looking at basically we want to start looking start looking at FM signal demodulation that is a demodulation of an FM that is a demodulation of an FM signal. And FM signal can be demodulated the technique for demodulation the FM signal is through the use of a differentiator circuit, okay. So we are going to illustrate how to demodulate the FM signal using a differentiator. So FM signal can be demodulated this can be demodulated using a differentiator the idea is basically we have this FM signal, okay.

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So I have an FM signal $s(t)$ I can give it as input to a differentiator circuit which performs the operation which differentiates the input and the output is $\frac{d}{dt} s(t)$ this is your differentiator this is your differentiator circuit. By the way a differentiator you can realize you will realize is a linear time invariant system, okay. So it is this is a very important property of a differentiator, correct? So this is a linear time invariant system, okay. So you can also note that the differentiator the differentiator is an LTI system, okay.

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$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right)$

FM Signal

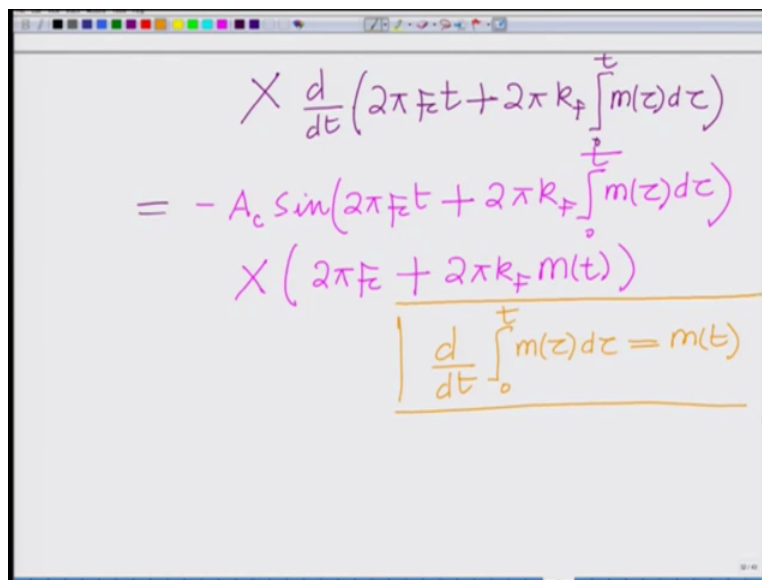
Pass this through a differentiator circuit

$\frac{d s(t)}{dt} = -A_c \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right)$

$\times \frac{d}{dt} \left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right)$

And now let us consider our FM signal our standard FM signal which is given as $s(t)$ equals $A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau)$, okay. This is my FM signal, okay. We have seen this several times before this is an example of a canonical or a general this is an example of a general FM signal. Now if we pass this through a differentiator now when we pass this through a differentiator so when we pass this through a when we pass this through a differentiator circuit the output is well, we have seen that it differentiates the input that is ds/dt that is the derivative of $s(t)$ which is $A_c \cos$ the signal is $A_c \cos$ the signal is $A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau)$ we can see the derivative of this is minus A_c derivative of cosine is minus sin, so minus sin $2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$ times the derivative of well, the argument that is $2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$ that is argument of the cosine function, okay.

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$$\begin{aligned}
 & \times \frac{d}{dt} \left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right) \\
 &= -A_c \sin \left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right) \\
 & \times \left(2\pi f_c + 2\pi k_f m(t) \right)
 \end{aligned}$$

$$\left| \frac{d}{dt} \int_0^t m(\tau) d\tau = m(t) \right|$$

So we are employing the chain rule for differentiation which is basically equals which basically equals minus A_c well, $\sin(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau)$ times now you can see the derivative of $2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$ is nothing but $2\pi f_c$ plus derivative of $2\pi k_f \int_0^t m(\tau) d\tau$ is nothing but $2\pi k_f m(t)$, okay. Here we have use the property the derivative we are using the property that the derivative which is respect to t of $\int_0^t m(\tau) d\tau$ is nothing but $m(t)$, okay. This is the property that this is the property

that we are that we are basically using, okay. It is a very simple property that if you differentiate the integral of a function you get the function itself, okay.

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$$\left| \frac{d}{dt} \int_0^t m(\tau) d\tau = m(t) \right|$$

$$\frac{ds(t)}{dt} = -A_c(2\pi F_c + 2\pi k_f m(t)) \times \sin\left(2\pi F_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right)$$

output of Differentiator

Envelope of output of Differentiator
 $= A_c(2\pi F_c + 2\pi k_f m(t))$

And therefore now I can rewrite this as minus A_c bringing the argument to the front $2\pi F_c$ plus $2\pi k_f m(t)$ times well, sin of $2\pi F_c t$ plus $2\pi k_f \int_0^t m(\tau) d\tau$ and now if you look at this signal the resultant signal, so this is basically your output of the differentiator, correct? That is this is basically your $ds(t)/dt$ and now if you can see look at the output of this differentiator you can see that the envelope of this is basically envelope of the output the output is a Sinusoidal signal the envelope the output is basically the envelope of output of differentiator equals $A_c 2\pi F_c$ plus $2\pi k_f m(t)$, so what you can see is that the envelope of the output of this differentiator is proportional to $m(t)$, okay.

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Differentiator
 $= A_c(2\pi f_c + 2\pi k_f m(t))$

Pass output of Differentiator through an Envelope Detector
 $= A_c(2\pi f_c + 2\pi k_f m(t))$

$$= 2\pi A_c(f_c + k_f m(t))$$

And therefore we pass this the output of the differentiator to an envelope detector, okay. So pass this through an envelope detector we have already seen the envelope detector so pass output of differentiator through envelope detector. So pass output of differentiator through the output of differentiator through an envelope detector, correct? And the resultant output is and the net output is that will give us the envelope which is basically $A_c 2\pi f_c$ plus well, $2\pi k_f m(t)$ which is basically now consolidating the terms this is basically A_c into 2π , correct?

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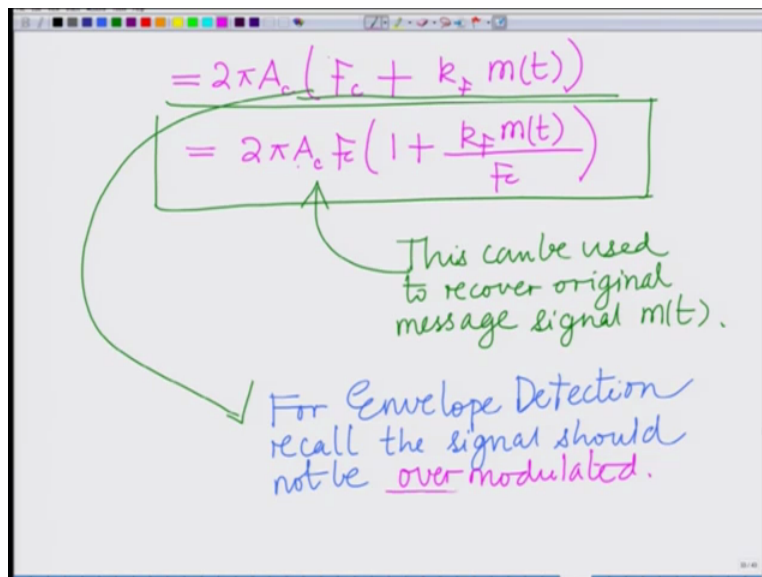
$$= A_c(2\pi f_c + 2\pi k_f m(t))$$
$$= 2\pi A_c(f_c + k_f m(t))$$
$$= 2\pi A_c f_c \left(1 + \frac{k_f m(t)}{f_c}\right)$$

This can be used to recover original message signal $m(t)$.

f_c plus $k_f m(t)$ and which is equal to basically now if you can look at this which is equal to $2\pi A_c f_c$ into 1 plus 1 plus $k_f m(t)$ divided by f_c , okay. And now we know, correct? So now we have the output of the envelope detector this is given by this is your output of the envelope detector which can be used to recover, so this is the output of envelope detector and this can be used to recover the original message signal $m(t)$. This can be used to recover the original message signal $m(t)$.

So this is the output of the envelope detector which can be used to recover the original message signal $m(t)$, however remember for no phase reversals, alright we need a condition, correct? That is the the the amplitude (modu) remember when we looked at an amplitude modulated signal for envelope detection the condition (wi) no phase reversal is that the signal should not be over modulated, correct?

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The image shows a digital whiteboard with handwritten mathematical equations and notes. At the top, the equation $= 2\pi A_c (f_c + k_f m(t))$ is written in pink. Below it, the equation $= 2\pi A_c f_c \left(1 + \frac{k_f m(t)}{f_c}\right)$ is written in pink and enclosed in a green rectangular box. A green arrow points from the boxed equation to the text "This can be used to recover original message signal $m(t)$." written in green. Another green arrow points from the boxed equation to the text "For Envelope Detection recall the signal should not be over modulated." written in blue, with the words "over modulated" underlined in pink.

$$= 2\pi A_c (f_c + k_f m(t))$$
$$= 2\pi A_c f_c \left(1 + \frac{k_f m(t)}{f_c}\right)$$

This can be used to recover original message signal $m(t)$.

For Envelope Detection recall the signal should not be over modulated.

Which means that is if you can look at this quantity here signal should not be over modulated which means for envelope detection recall that the signal should not be, for envelope detection recall the signal should not be over modulated, okay.

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not be overmodulated.

Since over modulation results in phase reversal NOT arising from the carrier.

$$2\pi A_c (F_c + k_f m(t))$$

Condition for No Envelope Distortion is

$$\Rightarrow F_c \geq k_f \max |m(t)|$$

Since over modulation results in phase reversals not arising from the carrier, remember since over modulation not arising from the carrier and the condition for and now if you look at this signal look at this signal $2\pi A_c$ times F_c plus $k_f m(t)$, now if you look at this signal if you look at this signal, okay. The condition for no envelope condition for no envelope distortion is that this has to be greater than 0 which implies basically F_c has to be greater than or equal to k_f times maximum of magnitude $m(t)$, okay.

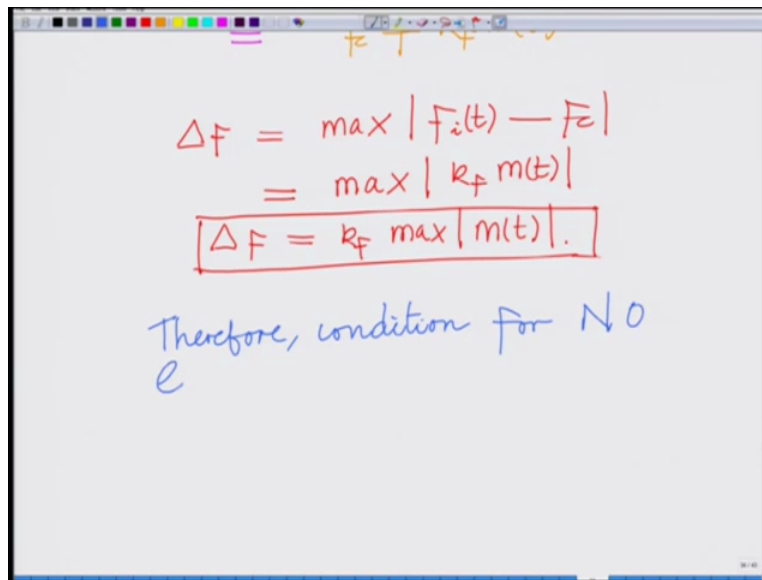
So basically your k_f has to be greater than equal to maximum of magnitude of, so this is the condition that we require this is the condition that we require remember this is a condition that we require for no envelope distortion that is F_c the carrier frequency has to be greater than equal to k_f times maximum magnitude $m(t)$ that is the maximum of the magnitude of the message signal. And now if you look at our frequency, correct?

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$$\begin{aligned}\theta_i(t) &= 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \\ f_i(t) &= \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} \\ &= f_c + k_f m(t) \\ \Delta f &= \max |f_i(t) - f_c| \\ &= \max |k_f m(t)| \\ \Delta f &= k_f \max |m(t)|\end{aligned}$$

Now let us go back to our frequency the frequency the instantaneous frequency $f_i(t)$ is basically if you look at that that is basically your well, the phase let us look at the phase 2π well, $f_c t$ plus $2\pi k_f \int_0^t m(\tau) d\tau$, now the instantaneous frequency is 1 over 2π derivative of the phase which is equal to well, $2\pi f_c$ plus $2\pi k_f m(t)$ which is equal to again $2\pi f_c$ $2\pi f_c$ plus plus $2\pi k_f$ $2\pi k_f m$ I am sorry f_c plus $k_f m(t)$ because there is a factor of 1 over 2π , so this is simply f_c plus k_f times $m(t)$ and therefore the frequency deviation, if remember our frequency deviation Δf equals maximum of magnitude $f_i(t)$ minus f_c this is how we derived the frequency deviation which is maximum of magnitude $m(t)$ which is k_f times maximum of magnitude maximum of magnitude, so our Δf our frequency deviation Δf is nothing but k_f times maximum of the magnitude of $m(t)$.

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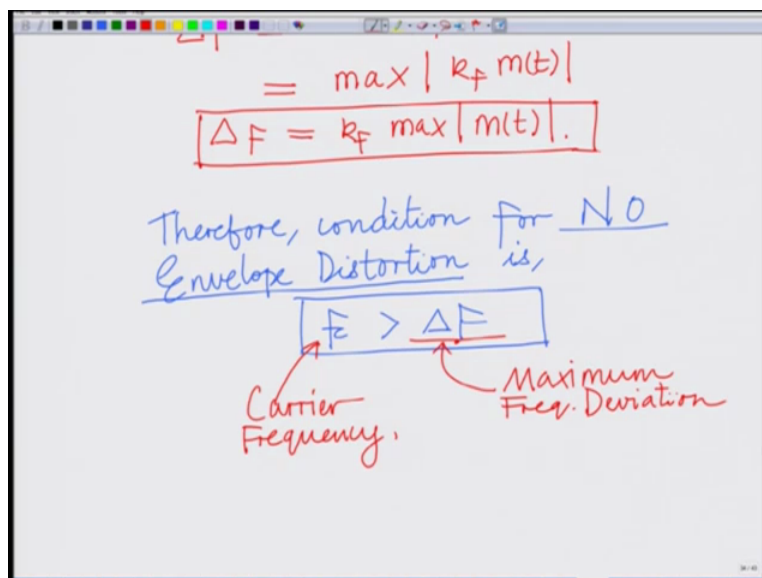
Handwritten derivation on a whiteboard:

$$\Delta f = \max |f_c(t) - f_c|$$
$$= \max |k_f m(t)|$$
$$\boxed{\Delta f = k_f \max |m(t)|}$$

Therefore, condition for No
e

And now if you look at this you will realize that this quantity k_f times maximum of magnitude $|m(t)|$ is nothing but Δf therefore the condition for no envelope distortion what it reduces to basically the fact that the frequency carrier frequency must be greater than the maximum the peak frequency deviation of the FM signal.

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Handwritten derivation on a whiteboard:

$$= \max |k_f m(t)|$$
$$\boxed{\Delta f = k_f \max |m(t)|}$$

Therefore, condition for No
Envelope Distortion is,

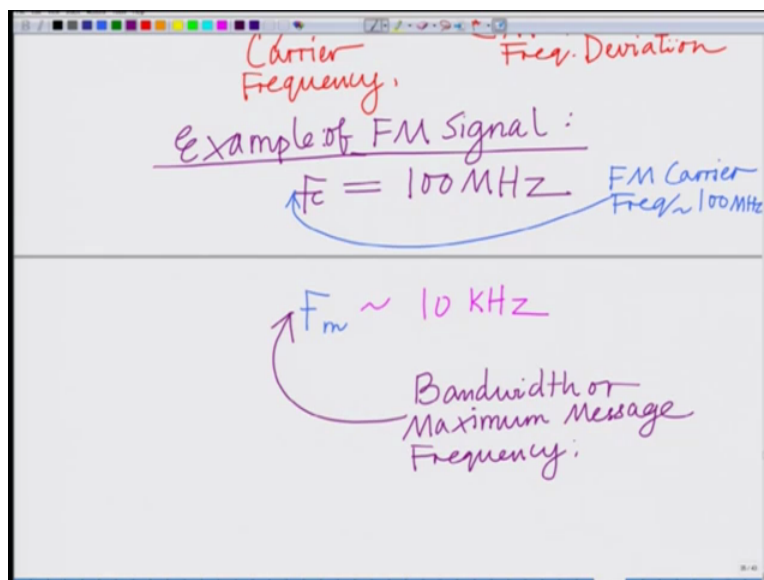
$$\boxed{f_c > \Delta f}$$

Carrier Frequency, Maximum Freq. Deviation

Therefore condition for no envelope distortion very interestingly the condition for no envelope distortion is basically f_c greater than Δf , okay. So f_c so if remember this is your maximum

frequency deviation, okay. Let me also mention that this is your this is your maximum frequency deviation and this is your carrier frequency this is the maximum frequency deviation and this is the carrier frequency and the condition for no envelope distortion is that F_c is greater than or equal to ΔF and therefore if this condition is satisfied the reason carrier frequency of the FM signal is greater than the maximum frequency deviation there will be no envelope distortion. However you can see that this is naturally true for an FM signal since the carrier frequency is a very high quantity compared to the frequency deviation for instance let us look at a typical example, okay.

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Let us take a typical example of an FM signal, okay. You can see the carrier frequency F_c equals F_c equals well, for FM signal that carrier frequencies are typically in the range of 100 megahertz these are the FM carrier (seque) frequencies, okay. Its 90 megahertz to about 105-110 megahertz, okay. FM carrier frequencies approximately or they are of the order of basically 100 megahertz and the bandwidth or the maximum message frequency you can say F_m , right. F_m is approximately of a order of let us say 10 kilohertz you can say this is the bandwidth or the maximum message frequency, okay.

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Handwritten calculations on a digital whiteboard:

$$\beta = 10 \quad \text{Modulation index}$$
$$\beta = \frac{\Delta F}{f_m} \Rightarrow \Delta F = \beta F_m$$
$$= 10 \times 10 \text{ kHz}$$
$$= 100 \text{ kHz}$$
$$\boxed{\Delta F = 0.1 \text{ MHz}}$$
$$\underset{0.1 \text{ MHz}}{\Delta F} \ll \underset{\approx 100 \text{ MHz}}{f_c}$$

It is a bandwidth or maximum message frequency and therefore now if we have a modulation index beta equals 10 let us say this is our modulation index and we know that Delta beta is basically Delta F divided by Fm the maximum frequency component which implies Fm which implies Delta F which implies the frequency deviation Delta F is beta Times Fm which is 10 times well, our bandwidth is well, its 10 times 10 kilohertz which is equal to 100 kilohertz which is equal to 0.1 megahertz.

So your Delta F is approximately 0.1 your carrier frequency Fc is basically of the order of hundred megahertz, so you can clearly see that Delta F is much smaller than Fc this is approximately 0.1 megahertz this is approximately 100 it is approximately 100 megahertz, okay. Fc so this is your Delta F this is Fc, so Fc is approximately 100 megahertz your Delta F is approximately 0.1 megahertz, so what you can see is that this Delta F this quantity Delta F is typically that is a frequency deviation it is typically much smaller than Fc carrier frequency for a frequency modulated signal, okay.

So this quantity since the frequency deviation is much less than carrier frequency, tracing back our steps it means that basically there is going to be no envelope distortion, alright. There is going to be no envelope distortion because envelope because this quantity Fc plus kF times m(t) is always positive there therefore there is going to be no envelope distortion and that in turn

means that differentiation followed by envelope detection can be used for that can be used as 1 of the techniques for demodulation of an Fm signal, okay.

And which is rather simple because it has a low complexity technique which requires a differentiator followed by a envelope detector again as we have seen envelope detector is a simple circuit that can be implemented with relatively low complexity, so we will stop this module here and look at other aspects in the subsequent modules, thank you.