Principles of Communication- Part I
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Module No 6
Lecture 33
Bandwidth of Frequency Modulated (FM) Signals - Carson's Rule

Hello welcome to another module in this massive open online course so in the previous module we have seen the spectrum of a frequency modulated signal or an FM signal in this module let us look at the band with an FM signal, alright.

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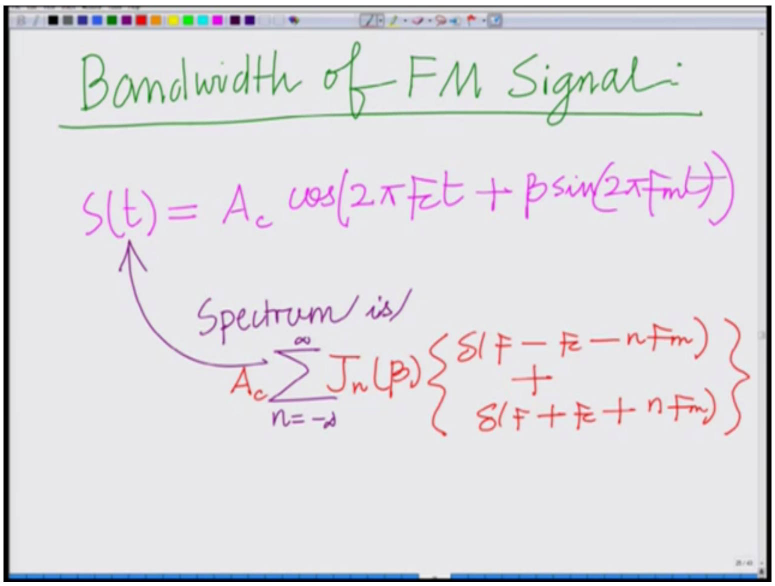

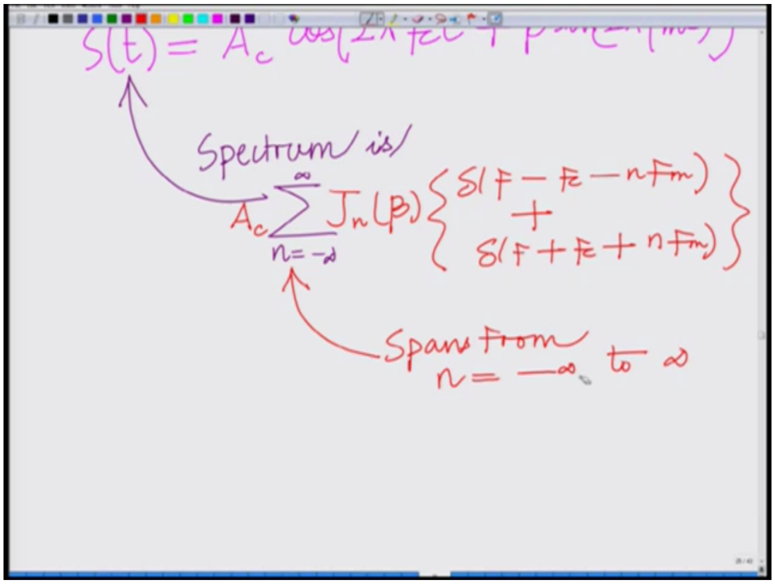

So what we want to look at is the, we want to look at the bandwidth of an SM FM signal, okay. Let the signal the S(t) equals Ac cosine for instance for an FM signal which is given as S(t) equals Ac cosine 2piFct plus beta sin 2 piFmt. Now the bandwidth of this signal or the spectrum of this signal we have seen the spectrum of the signal is given as the spectrum of this FM signal is given as summation n equal to minus infinity to infinity, well Ac times Jn beta into Delta F minus Fc minus nFm plus delta F plus Fc plus nFm, okay. So theoretically if you look at this this goes from n it spans from n equal to infinity to minus this spans from n equal to minus infinity to infinity.

So it spans from n equal to minus infinity to infinity that is theoretically it spans it has a spectrum that is span that spans the entire frequency band however that is not practical since when we have an Fm signal we would like to restrict to a certain transmission bandwidth, alright. Since the signals are only allocated in certain bandwidth be transmitted over the, alright. So we would like to find what is the practically relevant measure bandwidth?

That is although this has spectrum that spans minus infinity to infinity, alright? Typically the frequency decays very fast with increasing frequency, so we (woul) we would like to find what is a bandwidth which contains a significant portion of the FM signal? That is a significant portion of the energy of this frequency modulated signal, alright. So we want to evolve a bandwidth measure or a rule to compute the bandwidth of the FM signal.

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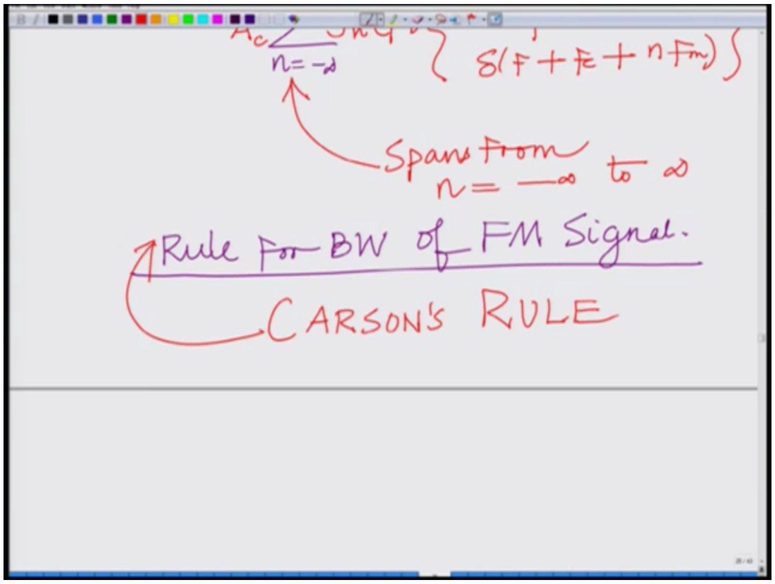

So what we would like to find is a rule, we would like to find the rule for a bandwidth of FM signal and this rule which gives us the bandwidth of an FM signal this is termed as the Carson's rule this is the Carson's rule, okay. So this is the rule which gives us the bandwidth, alright which helps us evaluate the bandwidth of the FM signal.

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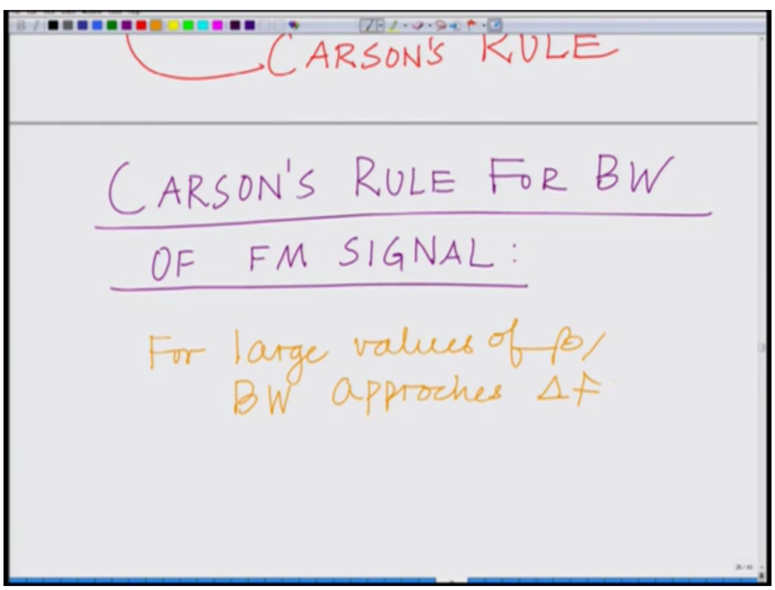

So now let us look at the Carson's rule to compute the bandwidth of the FM signal. Carson's rule for bandwidth of an Carson's rule for the bandwidth of an FM signal, alright. Now this Carson's rule is an a intuitive rules on how to be put it? It is a it is a rule of thumb or in a (prox) or in approximation which is valid to to a very large degree, alright which which is valid with a very high level of accuracy, alright. And this is based on the following observation that the bandwidth for large values of beta that is if you look at large values of the modulation index beta for large values of beta the bandwidth approaches Delta, okay. The bandwidth approaches Delta F where Delta F remember Delta F is the peak frequency deviation, okay.

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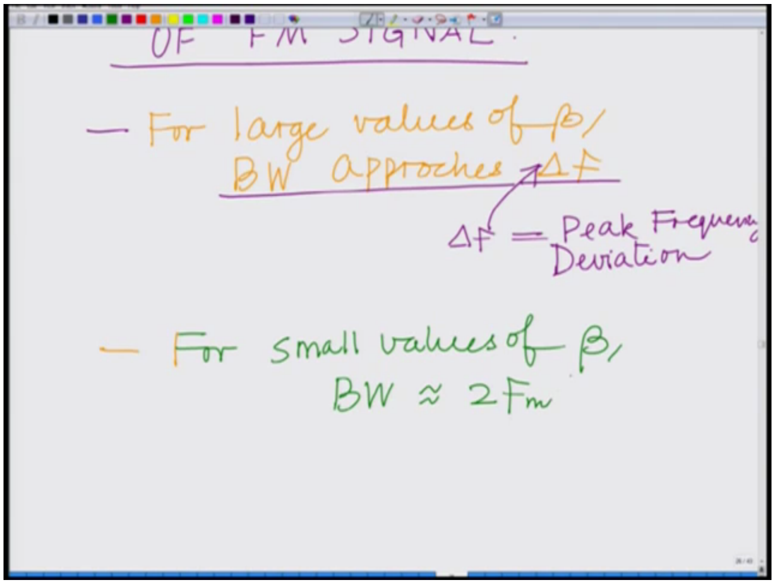

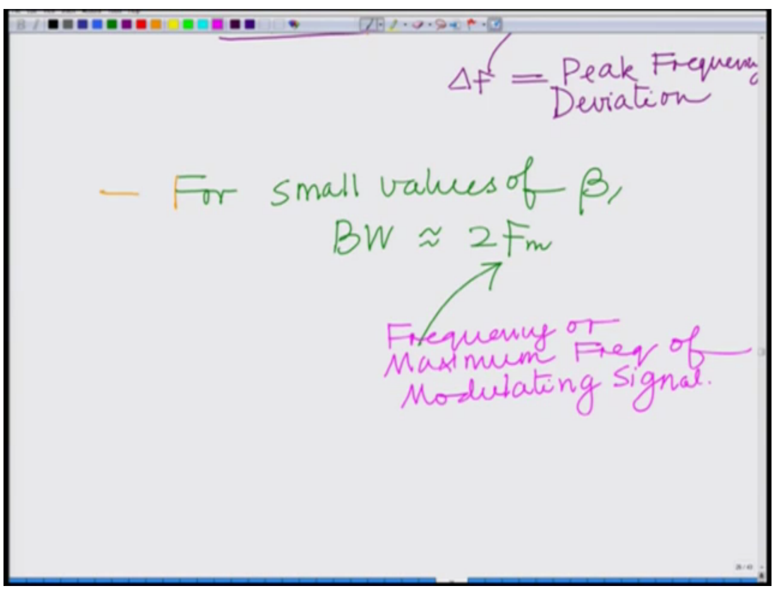

So for large values of data the bandwidth approaches Delta F however for small values of the modulation index beta for small values of for small values of beta the bandwidth is approximately 2Fm so 2Fm remember 2Fm is the frequency or you can think of it as a maximum frequency of modulating signal is either frequency or maximum frequency or the maximum frequency of maximum frequency of the modulating signal, okay.

So what we have seen is that it depends on the bandwidth occupied by the FM signal, significantly depends or majorly depends on the value of this modulation constant or the modulation index beta, for large values of beta the modulation index basically it is twice the twice Delta F where Delta F is the peak frequency division, okay. For small values of beta it is twice Delta F twice Fm where Fm is either the F frequency for a Sinusoidal modulating signal it can be simply simply the frequency of the Sinusoidal modulating signal, for a general modulating signal FM is the maximum frequency can be thought of as the maximum frequency of the modulating signal, okay.

And therefore to combining these 2 principles or combining these 2 insights the Carson's Carson's rule to calculate the bandwidth of the FM signal is given as follows. So combining these 2 principles so we have these 2 principles, okay.

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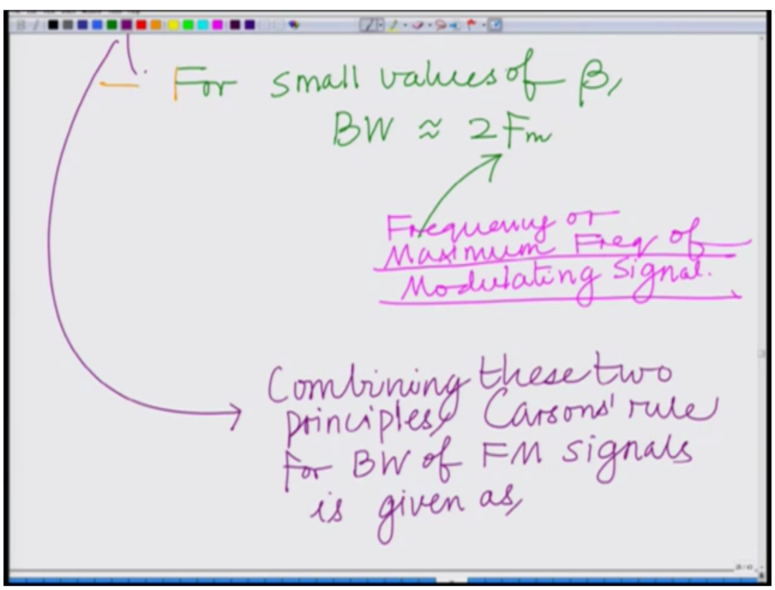

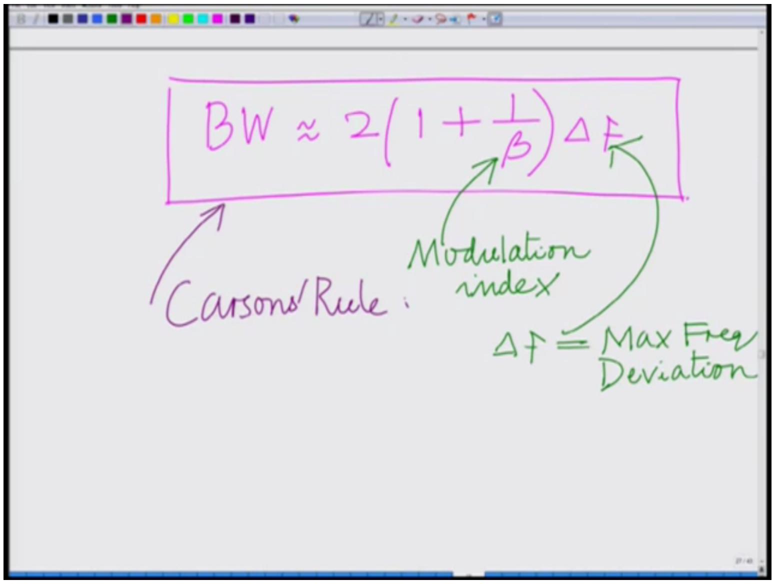

Now we combine these 2 principles so we combine combining these 2 principles Carson's rule (())(8:27) of FM signals the Carson's rule for bandwidth of FM signals is given as well the bandwidth of an FM signal is approximately equal to twice one plus one over beta into delta F where beta remember beta is the modulation index and Delta F equals the peak frequency deviation or peak or maximum frequency deviation, okay.

Simply this is the maximum frequency deviation of basically the instantaneous frequency from the carrier frequency the instantaneous frequency Fit from the carrier frequency, okay. This is the maximum frequency deviation, okay. So this is your Carson's rule this is basically the Carson's rule for FM band width, okay. Let me just write this here this is your Carson's rule, okay.

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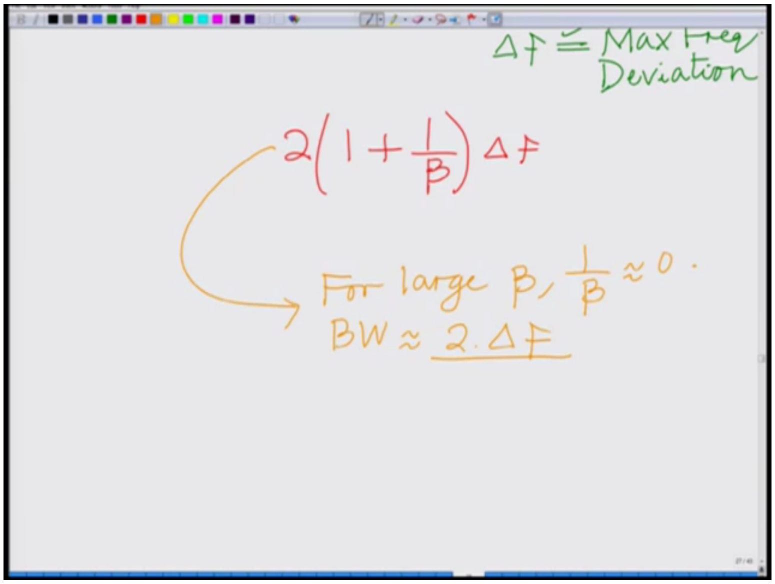

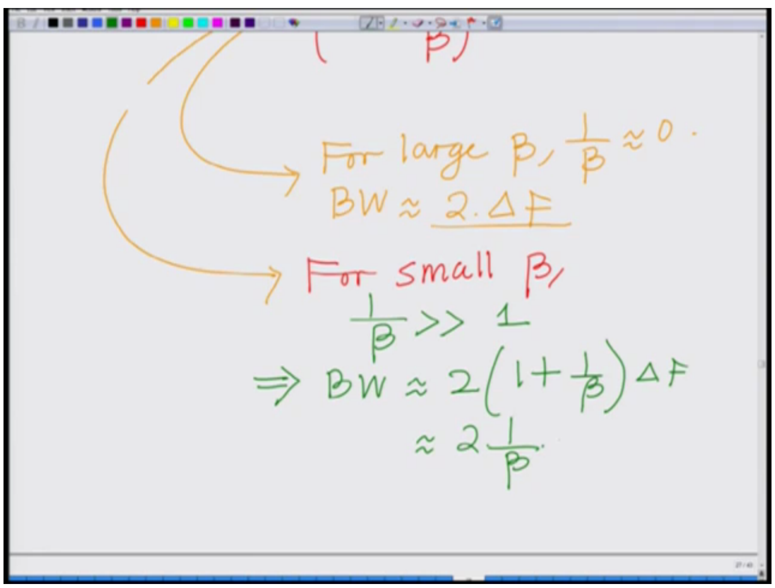

Now you can clearly see for instance if you take a look at this expression twice one plus one over beta Delta F, now for large beta we have for instance if you look at this for large beta we have one over beta approximately equal to 0 which means the bandwidth is approximately equal to twice into Delta F where Delta F is the well, Delta F is the maximum frequency deviation, alright.

So large values of beta 1 over beta is approximately equal to 0, so against this (expree) expression reduces to twice Delta similar to what we have seen in the principles earlier. Now for small values of beta for small beta now what happens is when beta is very small approximately close to 0 than 1 by beta is much greater than 1 implies the bandwidth is approximately equal to twice 1 plus 1 over beta Delta F which is approximately equal to since 1 over beta is much greater than 1, so this is approximately equal to twice 1 over beta into Delta F.

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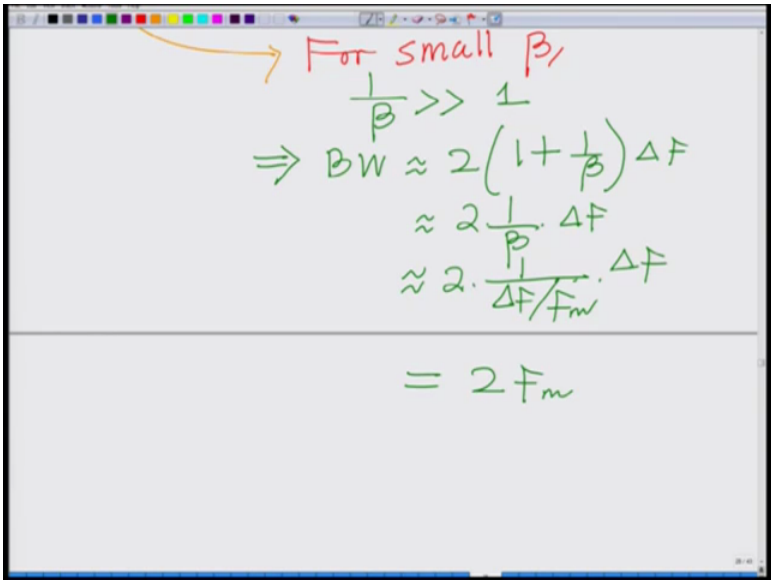

But remember beta is Delta F by FM so this is approximately twice 1over Delta F by Fm into delta F which is equal to Fm which is equal to basically FM or let me just write this a little bit clearly. So this is 1 over delta F divided by FM into Delta F which is equal to twice which is equal to twice Fm, okay.

So this again reduces to with the principles that we have seen earlier that is when beta is very high that is been for large beta this is approximately twice delta F for small beta there is approximately twice FM, alright. So that is basically rule of thumb or a very good approximation to the bandwidth occupied by an FM signal which can be used for to practically that is which can use to a practical communication systems, alright based on frequency modulation, okay.

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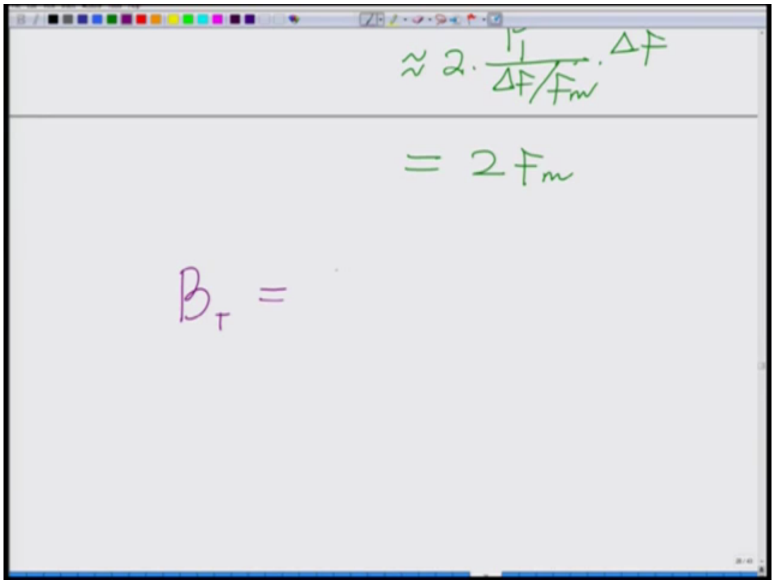

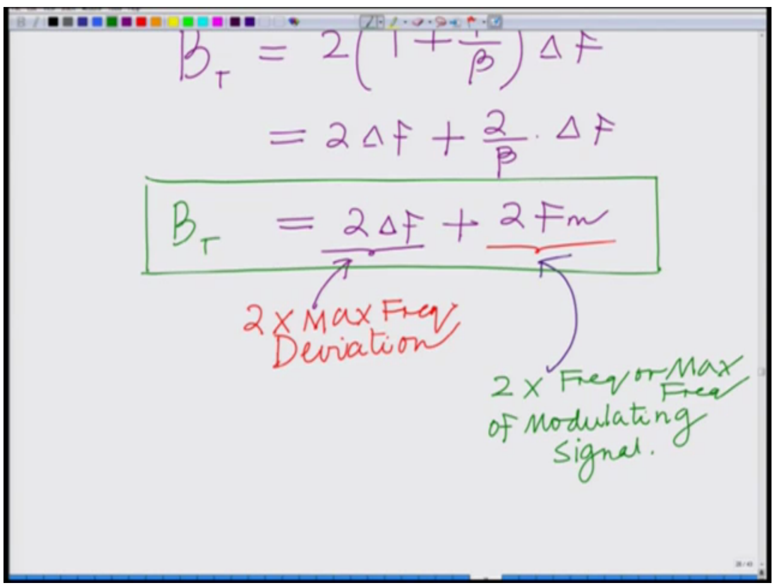

And therefore now if I can write it also I can realize that I can also write this as BT the bandwidth is twice 1 plus 1 over beta into Delta F which is twice Delta F plus twice over beta Delta F. Now Delta F over beta is Fm, so this is twice Delta F plus twice FM, so remember this is basically 2 into that is basically twice the maximum frequency deviation and this is basically your twice into frequency or a frequency or maximum frequency of or max frequency of the maximum frequency of the modulating signal, alright.

So I also express this as twice Delta F plus twice Fm it is an alternative way, right? Alternative way to write the Carson's rule for the bandwidth of an FM signal, alright. Now to understand this let us look at a simple example to compute to calculate the bandwidth of an FM signal and see how the Carson's rule can be applied, okay.

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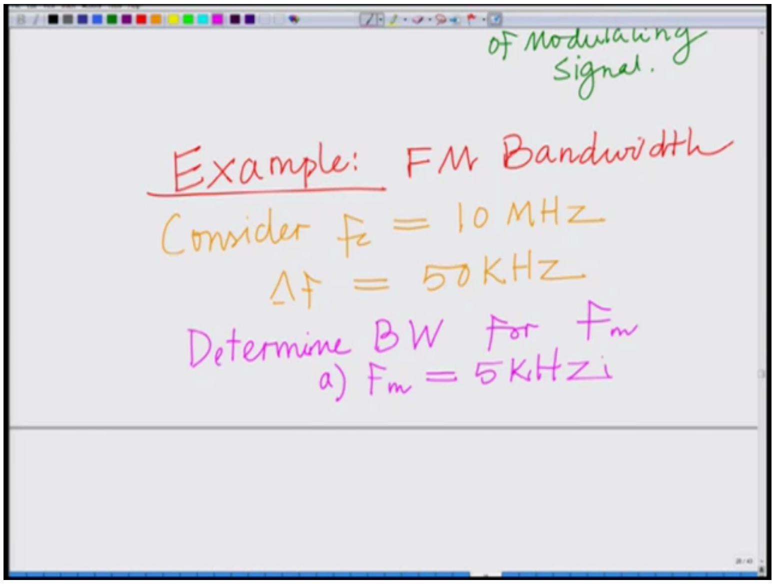

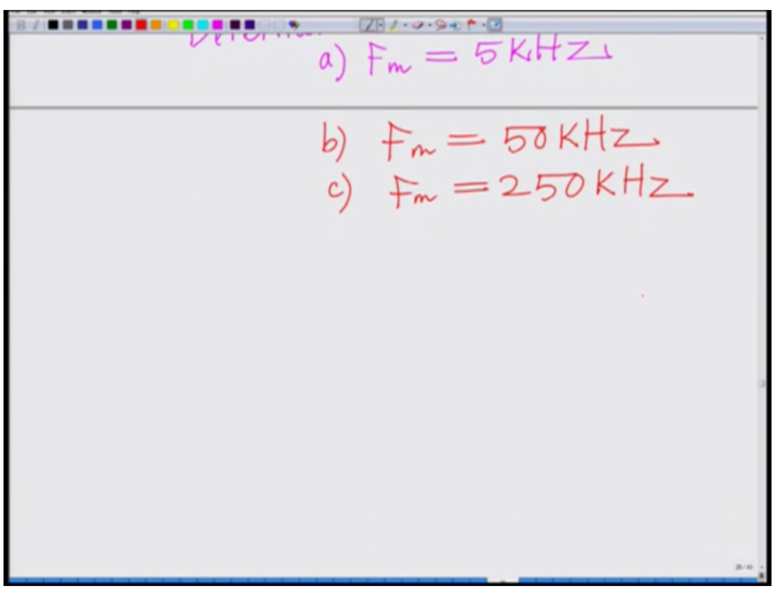

So let us look at a simple example for instance let us consider a carrier frequency equals 10 megahertz and Delta equals 50 kilohertz, now we would like to determine the bandwidth the bandwidth for FM corresponding to a) for FM that is values of Fm the maximum frequency FM frequency given below FM equals, well 5 kilohertz, b) for FM equals 50 kilohertz c) FM equals 250 kilohertz.

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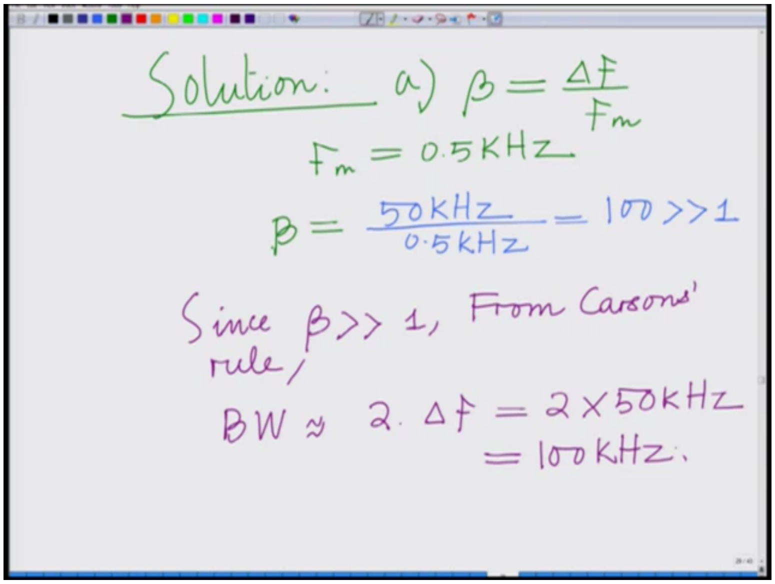

Now that is our problem and the solution can be found as follows the various the solutions now if you look at part a, now realize that beta let us start by calculating beta beta equals the Delta F by delta M, so what we have is we have given a carrier frequency, alright. And we are also given Delta F that is a peak frequency deviation and based on different values of FM that is a frequency of the modulating signal we have to calculate the bandwidth of the FM signal obvious using Carson's rule, okay.

So the first case let us start with beta equals delta f by Fm we have which is equal to well, we are given Fm equals 0.5 kilohertz and Delta F equals 50 kilohertz so beta equals well that is your Delta F which is 50 kilohertz divided by FM which is point 5 kilohertz so beta is equal to hundred which is significantly greater than 1 and therefore using Carson's rule since beta is significantly greater to then Carson's rule from Carson's rule from Carson's rule the bandwidth is approximately equal to twice Delta F which is equal to twice Delta F is 50 kilohertz twice 50 kilohertz which is equal to 100kilohertz, okay.

Therefore from Carson's rule we have the bandwidth, alright we have the bandwidth of the FM signal since beta is very large beta is 100 which is significantly greater than one the bandwidth is approximately twice Delta, okay which is hundred kilohertz, okay which is hundred kilohertz, okay.

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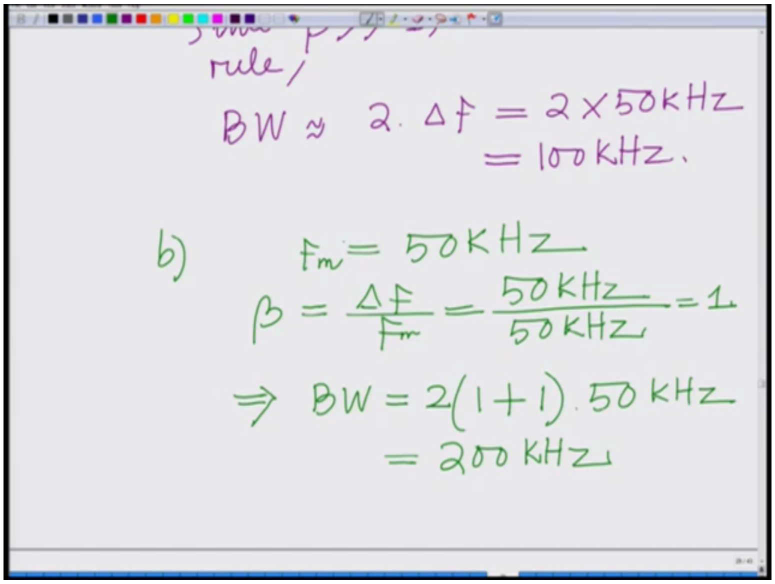

Now let us look at case B in case b) we have Delta F equals 50 kilohertz again beta equals Delta F by Fm delta f equals to 50 kilohertz Fm is also 50 kilohertz, okay. Delta F by Fm is equal to 50 kilohertz divided by 50 kilohertz which is equal to one, so in this case beta is x(t) exactly one, so we cannot use the approximation which means the bandwidth is twice 1 plus 1 over beta is 1 into Delta F which is 50 kilohertz which is basically twice into 2 that is 4times 50 kilohertz that is equal to your 200 kilohertz, okay. So the bandwidth is basically 200 kilohertz, okay.

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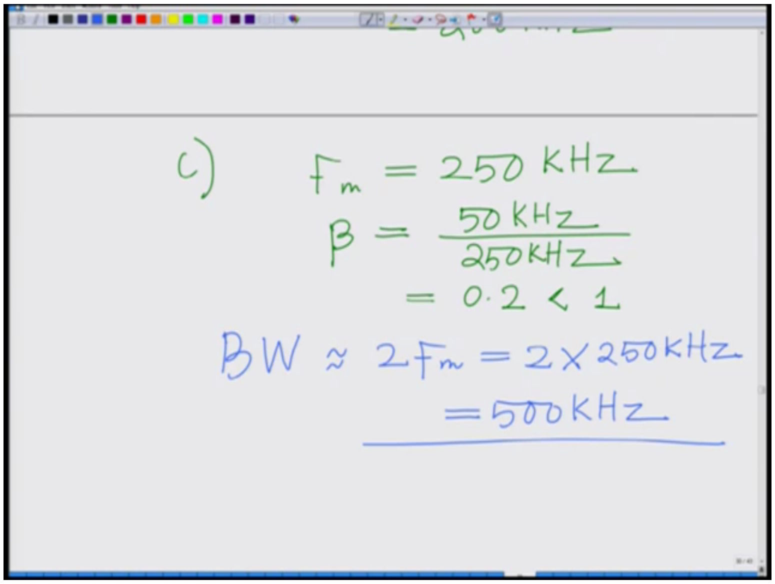

Now let us look at the last case in which k in which Fm is equal to 250 kilohertz, okay. The message frequency so therefore Delta F is equal to Delta F is basically 50 kilohertz divided by 250 kilohertz that is equal to that is the so your beta or basically your modulation index beta is equal to 50 kilohertz by 250 kilohertz and that is equal to 0.2 which is less than one. So I can use approximation here that were beta is less than one the bandwidth (approxi) is approximately equal to twice Fm which is equal to twice into 250 kilohertz it is equal to 500 kilohertz that is the bandwidth of the net bandwidth of the signal using Carson's rule, okay.

Of course here because beta is not very small in comparison to one I can also use the exact expression from the Carson's rule, however since beta is fairly small one can choose also to use the simple (approxo) approximation that that the bandwidth is simply twice FM, alright. But FM is a frequency of the modulating signal, alright. So basically this illustrates how the Carson's rule how Carson's rule can be used to compute the bandwidth of a typical FM signal depending of course on the peak frequency deviation, right?

The maximum frequency deviation delta Fm which is the frequency of the modulating message signal and also beta which is the modulating modulation index and infect beta plays a key role as the level of modulation or the modulation index plays a key role in the net bandwidth that is occupied or the net bandwidth that can be practically assumed to be occupied this this by this Fm signal in a practical scenario, okay.

So basically in this module we have seen a practical rule that is the Carson's rule to compute the bandwidth of a typical FM signal which can be used to determine the bandwidth that has to be allocated for this FM signal in a practical communication system. So we will stop here and continue with other aspects in the subsequent modules, thank you.