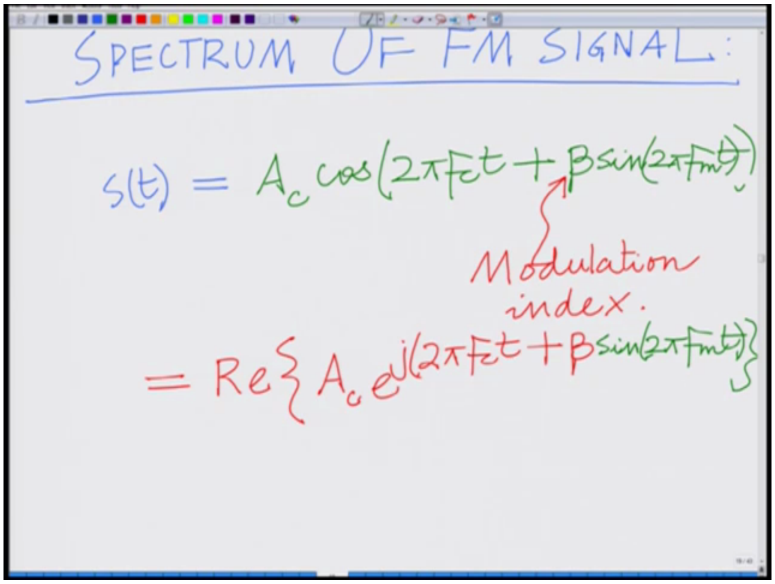
Principles of Communication- Part I
Professor Aditya K. Jagannathan
Department of Electrical Engineering
Indian Institute of Technology Kanpur
Module No 6
Lecture 32
Spectrum of Frequency Modulated (FM) Signals

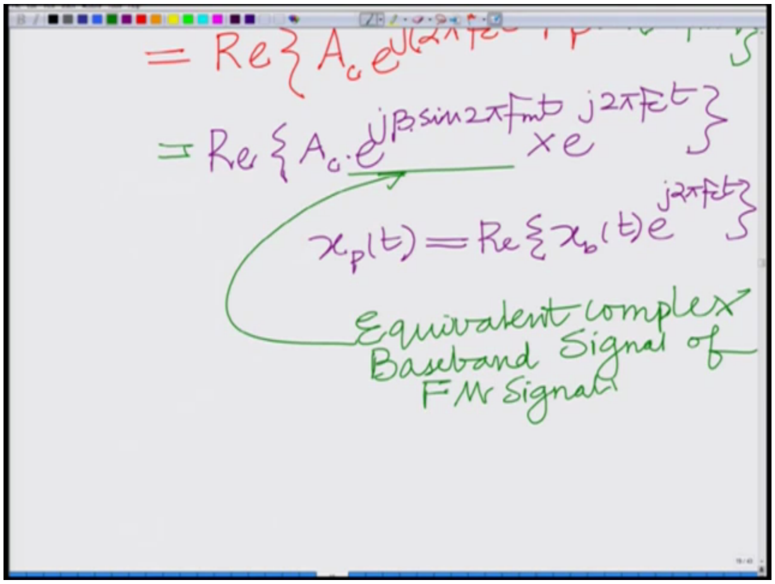
Hello welcome to another module in this massive open online course, so this module let us start looking at the spectrum of the FM signal, alright.

(Refer Slide Time: 0:25)



So we are looking at frequency modulation let us now start looking at the spectrum of a spectrum of an of an Am signal, so we want to look at the spectrum or spectrum of a FM signal. Spectrum of an FM signal, alright. Let us write our FM signal st equals Ac let us write this as Ac cosine 2 pi Fct plus beta sin 2pi Fmt as we have seen where beta is the where beta is our modulation index, okay. So beta is our this is our you have seen this before beta is the modulation index. Now what I am going to do I am going to write it as this, I can write this as the real part of Ac e to the power of j 2pi Fct times beta sin 2 pi Fmt which is equal to well.

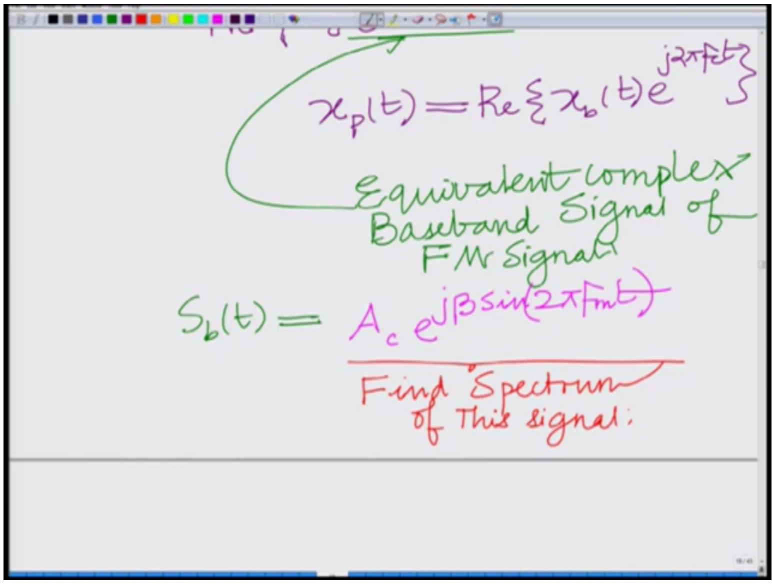
(Refer Slide Time: 2:39)



Now I can expand this as so first I am writing Ac cosine 2 pi Fct plus beta sin 2pi Fmt as the real part of Ac e to the power of j 2pi Fct plus beta Sinusoid 2pi Fmt. Now I can split this as Ac e to the power of j e to the power of j beta sin 2pi Fmt times e to the power of j 2pi Fct.

And we know that whenever I write a signal in this session that is real part of sum signal times e to the power of j 2pi Fct this becomes this is the complex baseband equivalent signal, okay. So we know that when we have a pass band signal xPt xPt is the real part of xbt which is a complex baseband equivalent signal times e to the power of j 2pi Fct, okay. So this is basically your equivalent complex baseband signal this is the equivalent complex baseband signal this is your equivalent complex baseband signal of the FM signal of the FM signal, okay.

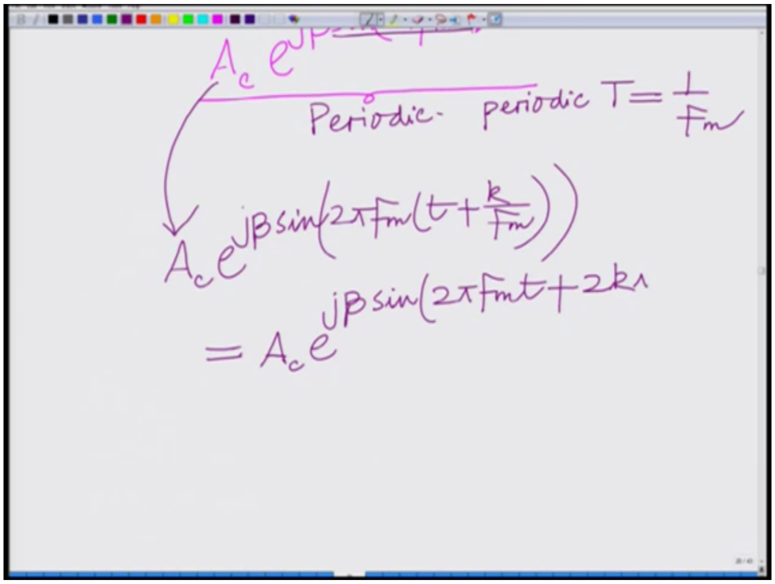
(Refer Slide Time: 4:34)

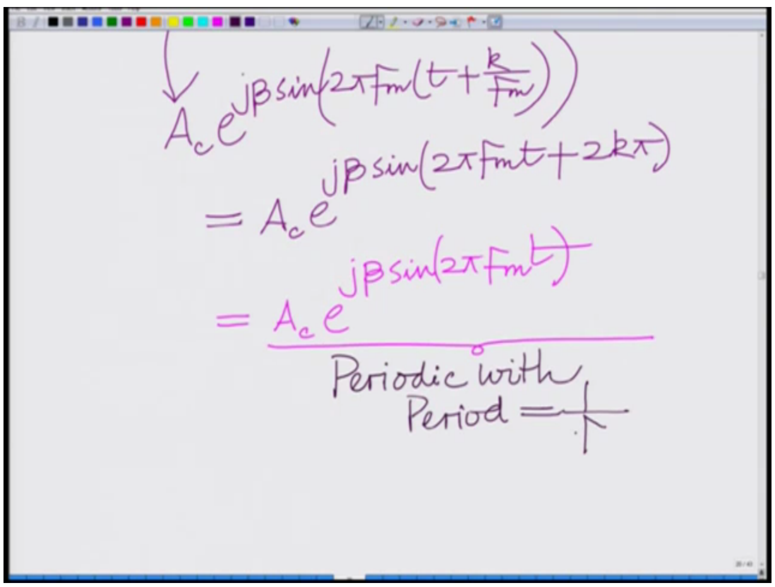


And therefore the complex baseband signal is well, that is your Sbt equals Ac e to the power of j beta sin 2pi Fmt, now what I can do is I can equivalently find the spectrum of the complex baseband so I can find the baseband spectrum then the pass band spectrum is nothing but simply a translation of the baseband spectrum, okay. So I am first first going to find the spectrum of this complex baseband signal and basically going to translate it appropriately because modulation is nothing but translation in the frequency domain, okay.

So first let us start by finding the spectrum of this equivalent complex equivalent baseband FM signal, so we will find the spectrum of this signal we will find the spectrum of this signal which is basically now if you look at this that will be, now first look at this signal Ac e to the power of j beta sin 2pi Fmt.

(Refer Slide Time: 6:16)

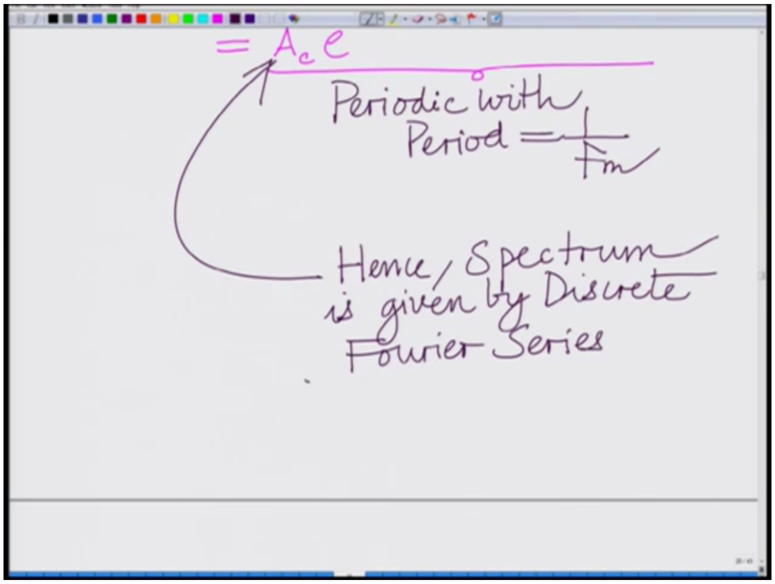




Now realize that this signal is periodic, okay. sin 2pi Fmt is periodic with period well, period is T equals 1 over Fm naturally sin 2pi Fmt is a periodic signal so e to the power of j beta sin 2pi Fmt is also a periodic signal, okay. And with the same period with period T equal to 1 over Fm, okay. In fact if I look at this I look at at kP leads later I have e to the power of j beta sin 2pi Fm t plus k by Fm that is going to be e to the (pow) Ac e to the power of e to the power of j beta well, sin 2pi Fmt plus 2kpi which is nothing but Ac e to the power of j we just j beta sin 2pi Fmt plus 2kpi is nothing but beta sin 2pi Fmt, so this is simply e to the power j 2pi e to the power j beta sin 2pi

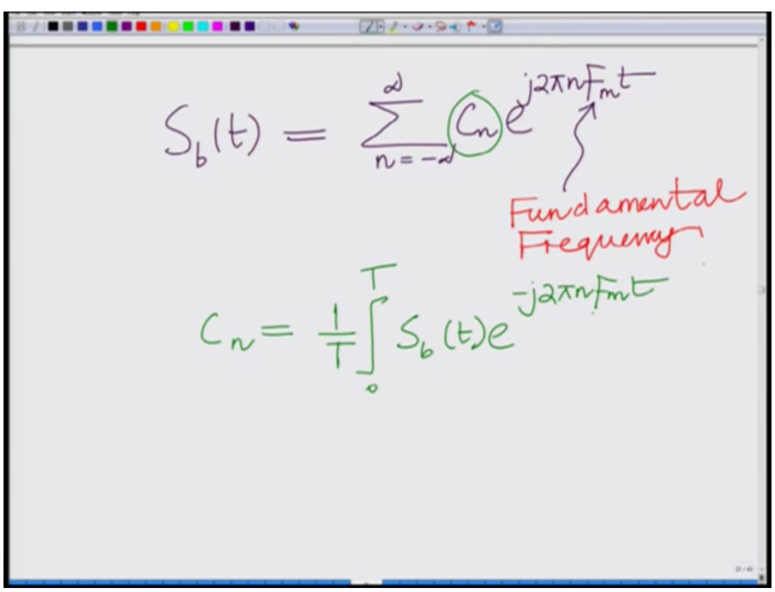
Fmt and therefore this is periodic with period equals with period this is periodic with period equal to 1.

(Refer Slide Time: 7:48)



Therefore now we can find the equivalent spectrum would be now we said that whenever remember that we have a periodic signal the spectrum is given by the Fourier series the discrete Fourier series therefore we have to find the discrete Fourier series hence spectrum will be given by hence spectrum is given by the discrete Fourier series.

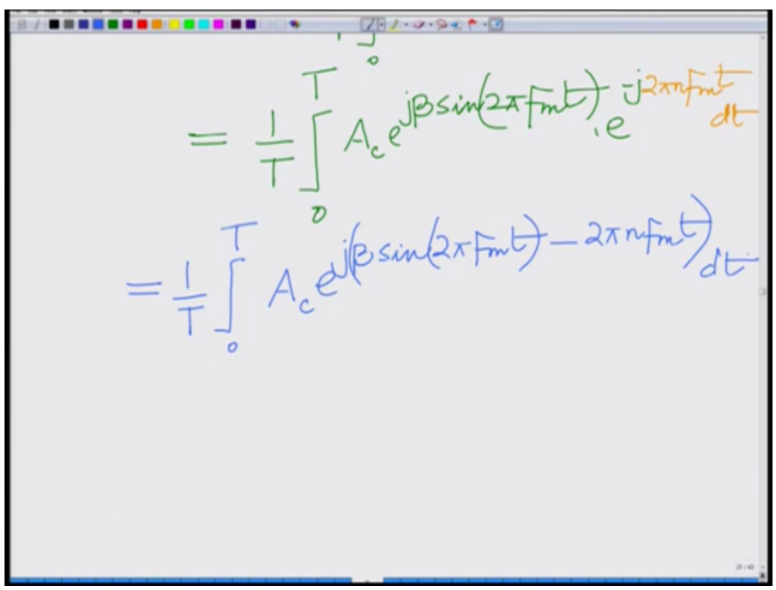
(Refer Slide Time: 8:32)



In other words I can write this signal your Sbt which is periodic as summation, remember the discrete Fourier series at the fundamental frequency and the harmonics that is Cn e to the power of j 2pi n Fmt this Fm remember we already said Fm is the fundamental frequency this the fundamental frequency, so I can write it as a as a linear combination in finite summation of harmonics at Fm and the at Fm that is frequency Fm and multiples of Fm, Fm is the fundamental frequency alright. This is the discrete Fourier series this is the representation, alright. (Differ) this is the representation that is the discrete Fourier series representation of your periodic signal, okay.

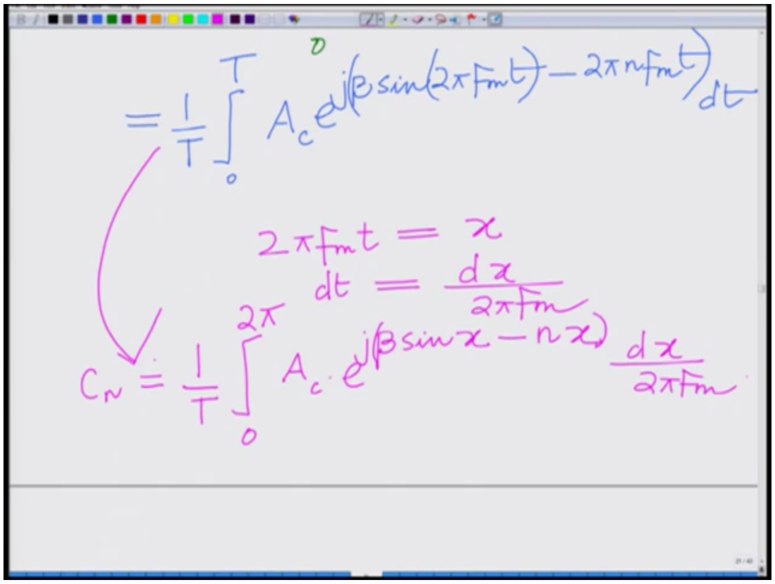
So now what I have to do is I have to find these coefficients Cn, these are the coefficients and we also know each Cn is basically 1 over t where t is the period times 0 to capital T Ac e to the power of j beta sin e to the or for a general baseband signal this is simply your Sbt e to the power of well, minus j 2pi n Fmt, okay.

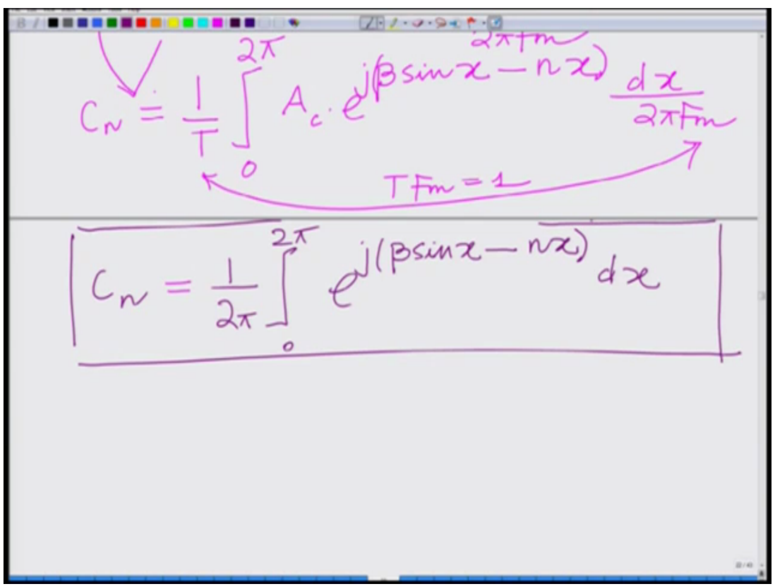
(Refer Slide Time: 10:28)



The nth frequency (coef) that is Cn nth coefficient of the discrete Fourier series which is 1 over T 0 to T Sbt Ac e to the power of j j beta sin 2pi Fmt times e to the power of minus j n Fmt dt is basically 1 over T 0 to T Ac e to the power of j beta sin 2pi e to the power of well, let us now use well, I can write this as j beta sin 2pi Fmt minus j beta minus Fmt into dt.

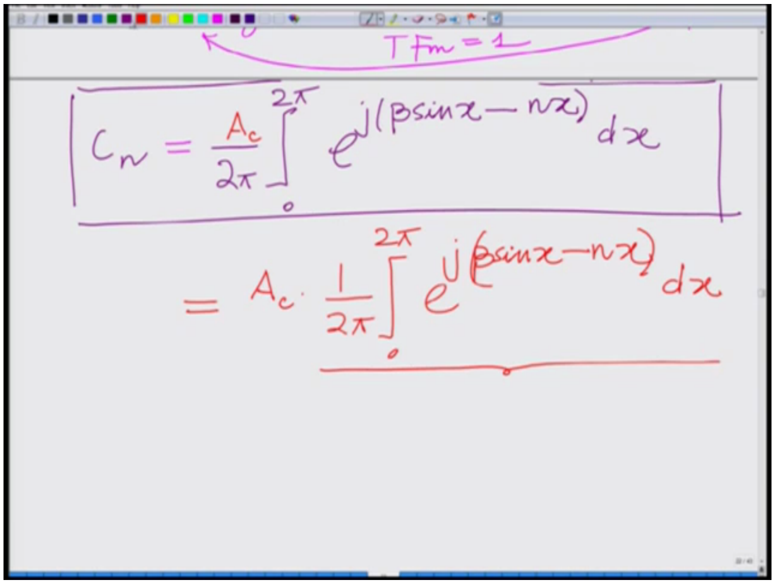
(Refer Slide Time: 11:54)





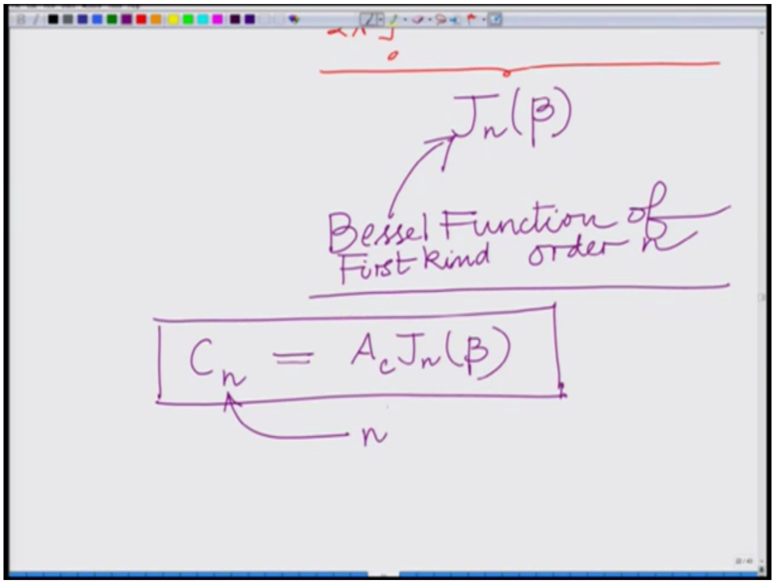
Now let us employ a substitution I can employ the substitution 2pi Fmt equal to x which means that dt becomes dx by 2pi Fm and therefore this integral for Cn will become Cn equals well, substitute 1 over T limit becomes 0 and well, x equals 2pi Fmt 2pi Fm so the upper limit will be 2pi Fm times capital T but Fm equals 1 by T, so this is simply 2 by 2pi F 2pi Ac e to the power of j beta sin 2pi Fmt will be 2pi Fm equals sin 2pi Fmt it's 2pi FFmt is x, so this is simply j sin beta x minus 2pi Fmt so is x so this is simply minus nx times dt is dx by 2pi Fm of course, T into Fm remember T is 1 over Fm so T into Fm equals we have T into Fm equals 1 and therefore this will be your well, 1 over T into Fm becomes 1 so this will be 1 or simply one over 2pi 0 to 2pi 0 to 2pi e to the power of j beta sin x minus nx dx that will be your Cn that will be your Cn.

(Refer Slide Time: 14:35)



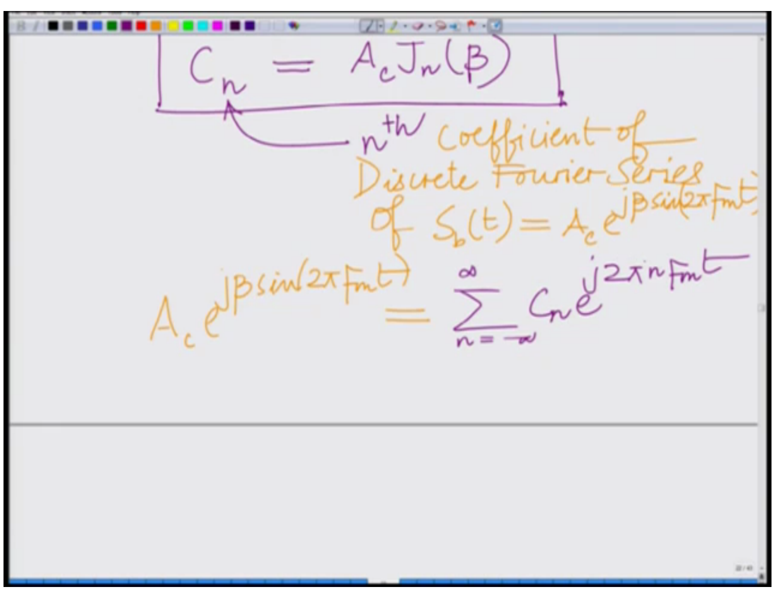
And you can see this is basically your Cn this is basically in fact I have an Ac also and this integral now you can see I can write this let me just write this as well, Ac into 1 over 2pi integral 0 to 2pi e to the power of j beta sin x minus nx now you can see this is a standard integral this is basically the special function this is basically Jn Jn of beta this is the nth order this is the Bessel function of the first this is the Bessel function of the first guide this is your Bessel function where your this is a standard mathematical function this is a Bessel function of first kind and order n.

(Refer Slide Time: 14:42)



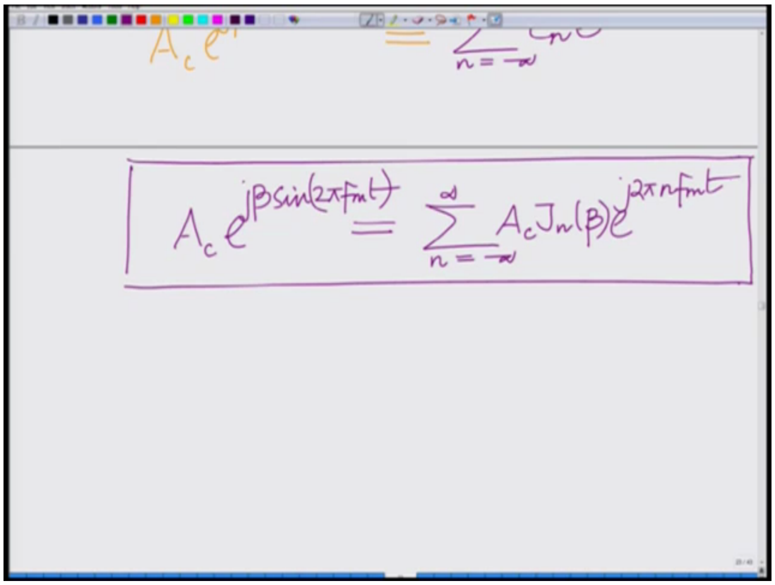
So this is basically your Bessel function of the first kind and order n which means your Cn basically what we have shown is that your Cn equals Ac times Jn beta. So this is what we are able to show that is this is the nth coefficient of the discrete Fourier series this is remember what is this? This is the nth coefficient in the discrete Fourier series but not of the FM signal the discrete Fourier series of the complex baseband equivalent signal of the pass band signal, okay.

(Refer Slide Time: 15:56)



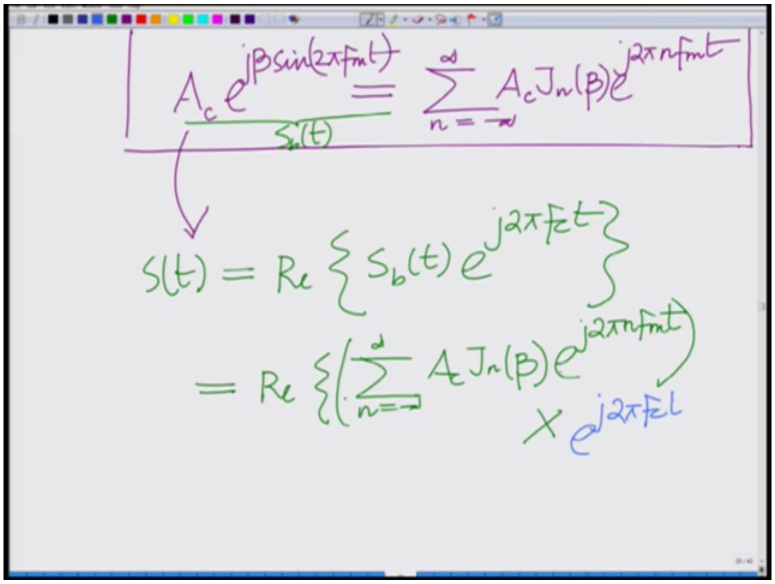
So this is the nth coefficient of the discrete Fourier series Sbt equals Ac e to the power of j beta sin 2pi Fmt, okay. So this is the nth coefficient of the discrete Fourier series. This is the nth coefficient of the discrete Fourier series, okay. Now therefore I have Cn equal to Ac jn beta now I have therefore I can write the expression or j beta sin 2pi Fmt this is going to be well, we have already seen this is your summation n equal to minus infinity to infinity Cn e raise to power of j 2pi n Fmt j 2pi n Fmt now substitute for the expression of Cn.

(Refer Slide Time: 17:21)



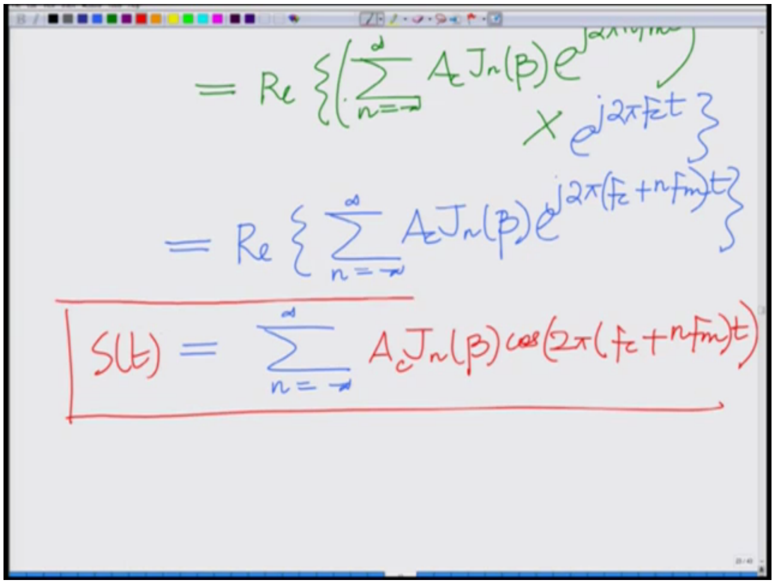
So therefore this becomes n equal to minus infinity to infinity well, that is Cn is Ac Jn beta e to the power of j 2pi n Fmt Ac j, this is basically the expression of the DFS of the discrete Fourier series of the complex baseband equivalent signal of the pass band FM signal, okay.

(Refer Slide Time: 18:22)



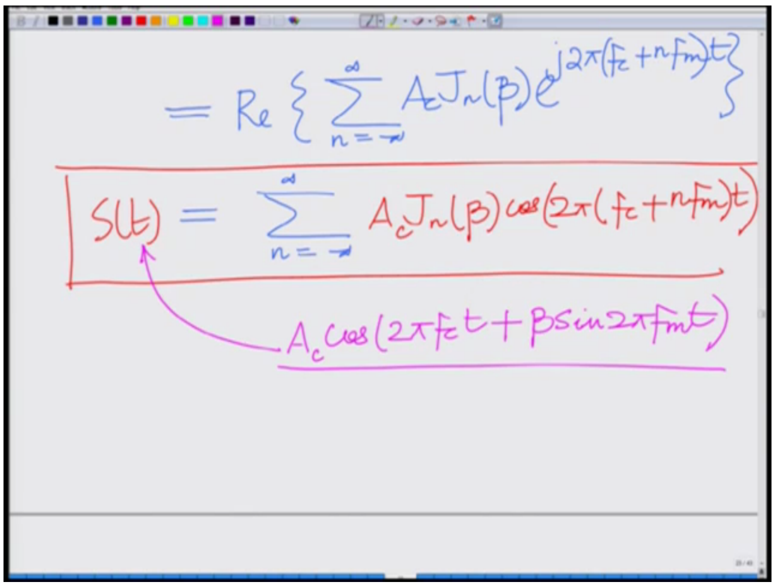
Now from this we can derive the pass band signal remember pass band signal St is nothing but the real part of this is your Sbt complex equivalent baseband signal, so the real part of Sbt e to the power of j 2pi Fct which is basically the real part of well, if I substitute this n equal to minus infinity to infinity Ac Jn beta e to the power of j 2pi n Fmt times this whole thing times e to the power of j e to the power of j 2pi Fct which is equal to…

(Refer Slide Time: 19:17)



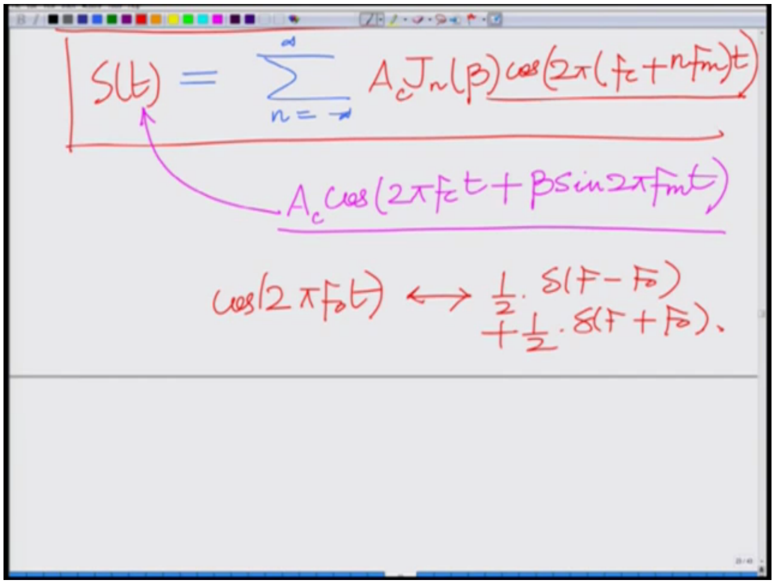
Now taking e to the power of the 2pi Fct inside this is simply real part of n equal to minus infinity to infinity Ac Jn beta e to the power of j 2pi well, e to the power of j 2pi Fc plus n Fmt which is now simply basically if you take the real part of this that is simply now you can see real part of e to the power of j 2pi Fc plus nFmt. Simply cosine 2pi Fc plus nFmt, so this is simply summation n equal to minus infinity to infinity Ac Jn beta cosine 2 pi Fc or 2pi into FC plus nFmt that is your St which is your pass band signal, so this is your pass band signal Start.

(Refer Slide Time: 20:40)



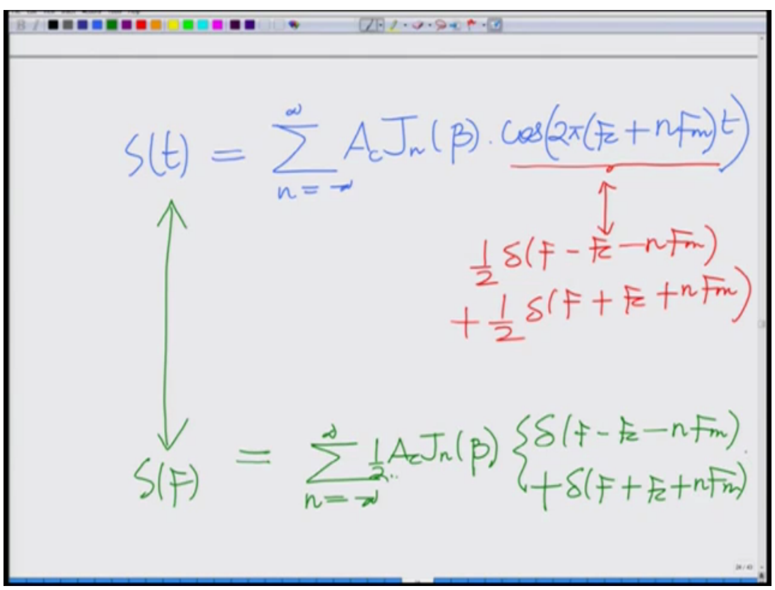
And remember the expression for St is basically St remember is simply Ac cosine 2 pi Fct plus beta sin 2pi Fmt, so this is basically summation n equal to minus infinity to infinity Ac Jn beta cosine 2 pi Fc plus nFmt.

(Refer Slide Time: 21:26)



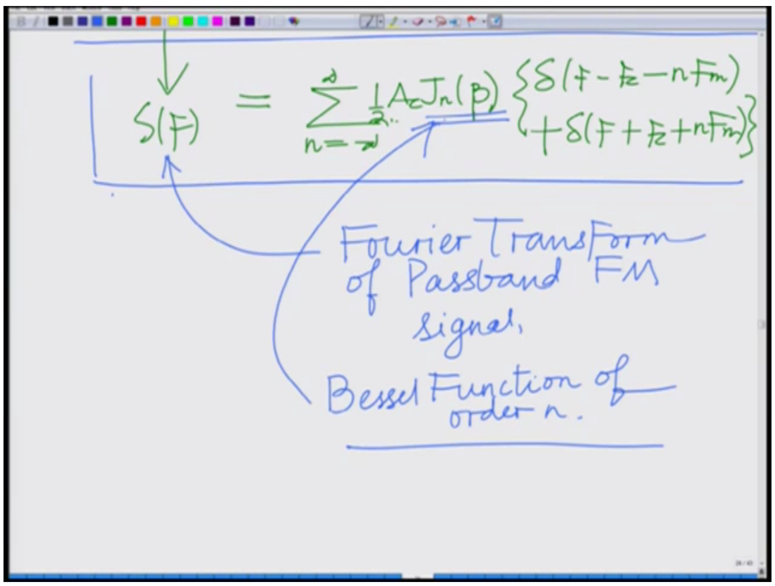
Now we know each cosine, cosine of 2 pi that is we know there is Fourier transform of cosine 2pi F not t has Fourier transform impulse of amplitude half at F minus Fnot plus half Delta F plus F not therefore each cosine 2pi Fc plus nFm basically is one impulse if you take the spectrum of that that is half impulse of my amplitude half at Fc plus nFm and impulse of amplitude half at minus Fc minus Fm therefore now if you look at Start.

(Refer Slide Time: 22:16)



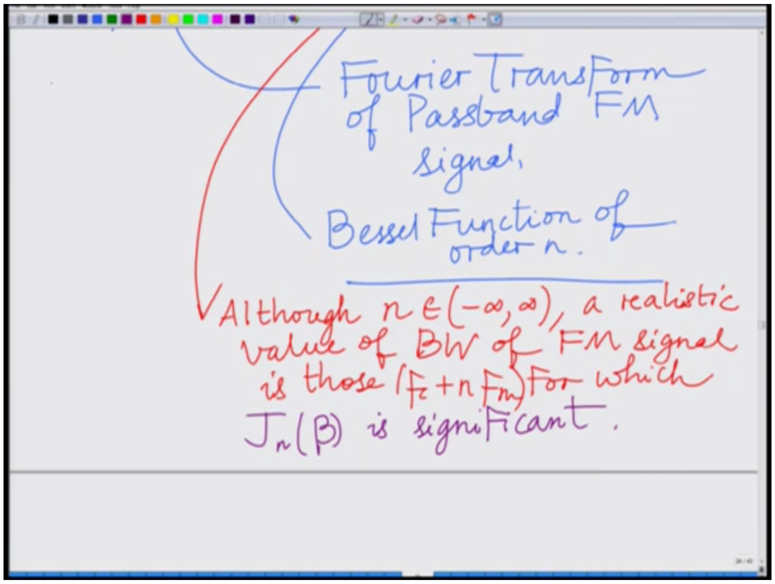
Let me rewrite St again St equals summation n equals minus infinity to infinity Ac Jn beta cosine 2pi Fc plus Fc plus nFm into t. Cosine 2pi Fc plus nFm into t and now each of this cosine term has Fourier transform half Delta F minus Fc minus Fm that is half Delta F minus F not plus half Delta F plus Fc plus Fm which implies that the Fourier transform of St therefore now I have simply I have to (trans) substitute the Fourier transform of the cosine that that is basically summation Ac Jn beta that is half or Ac Jn beta into well, Delta or half you can say half write here half Delta F minus Fc minus nFm plus half Delta F plus FC plus nFm, so this is the Fourier transform of the you are finally variable to derive the Fourier transform of the variable to derive the Fourier transform of the pass band FM signal.

(Refer Slide Time: 24:03)



What is this? This is the Fourier (trans) the Fourier transform of the of the pass band FM signal and you realize that it depends on this new function that we have seen this is the Bessel function Jnb Bessel function of order n this is the Bessel function of order n, okay. So this is your Bessel function of order n, okay. So this is the spectrum of this and naturally therefore it depends on Jn beta naturally of course, this is although is going from n equal to minus infinity n equal to infinity obviously since we have to trans in practice we have to limit the bandwidth or spectrum of this transmitted FM signal we have to choose a realistic value of bandwidth. So a realistic value of bandwidth for the signal is the range of frequencies for which this quantity Jn beta is significant, okay.

(Refer Slide Time: 25:26)



So that is also another thing that can be noted a realistic value of the bandwidth, so although Jn goes from minus infinity to infinity although n belongs to well, minus infinity to infinity a realistic value of bandwidth of FM signal is those Jn beta for which is those is those frequencies that is Fc plus nFm for which Jn beta is significant, right? So we have to choose we have to choose an appropriate range of n or appropriate range of these frequencies, right? For which Jn beta is significant that captures most of the signal power most of the pass band FM signal power, okay. So that gives us an estimate or a measure of a the bandwidth of this FM signal and then once we get the estimate of the bandwidth of this FM signal then we can calculate various other aspects, alright various other quantities, okay.

So that is what we have so what we have now is basically we have considered a pass band FM signal we have derived it's the expression for its spectrum now we have seen that this spectrum literally spans from I mean if you look at it if you look at it analytically or the precise mathematical expression it spans from over the entire frequency band, alright. So realistically we have to come up with a practical limit for the transmit bandwidth or a practical way to characterize a practical and a meaningful how do you put it? A measure to characterize the spectrum of this FM signal and that is given and that is something that we are going to look at in the subsequent modules that is out of this spectrum, how can we extract what is a practically feasible bandwidth? Or a practically good measure of the bandwidth of this FM system, alright. So we will stop here and look at other aspects in the subsequent modules, thank you.