

Principles of Communication- Part I
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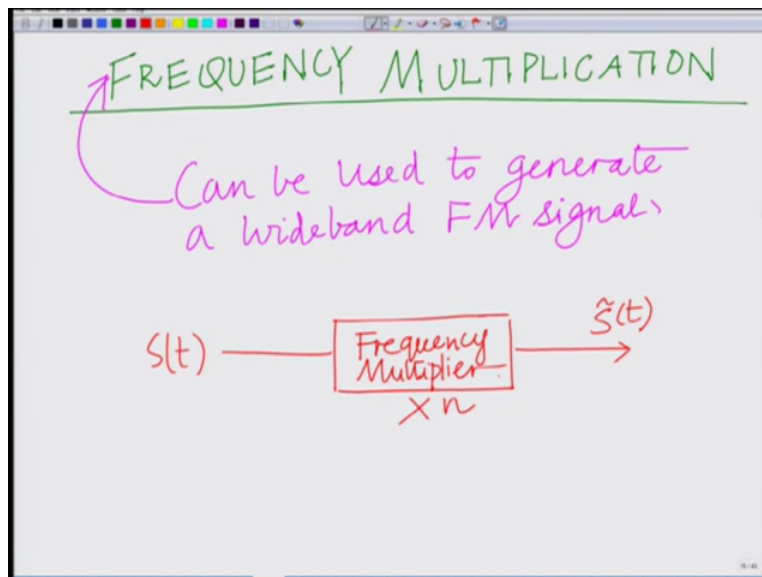
Module No 6

Lecture 31

Indirect Method for Generation of FM Signals - Generation of Wideband FM Signal through Frequency Multiplication

Hello and welcome to another module in this massive open online course. So we are looking at a generation of a narrowband frequency modulated signal we have said that a narrowband frequency modulated signal is one in which the modulation index beta is very (so) small significantly smaller than one. Let us now look at how to generate a wideband FM signal that is using the indirect method, correct? So first we generate the narrowband frequency modulated signal and then we convert it into a wideband FM signal by a frequency multiplier. So I can use frequency multiplication generate a wideband FM signal.

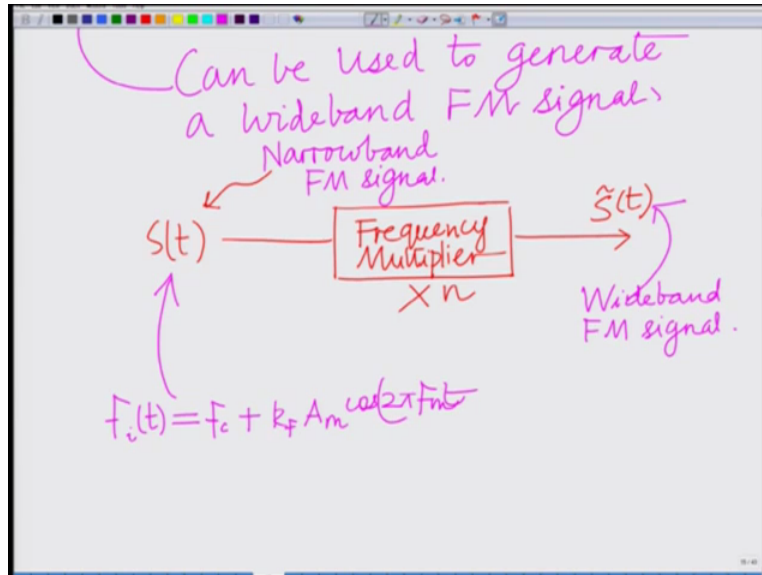
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So what we can do to generate a wideband wideband frequency modulated signal I can use a frequency multiplication, okay. So frequency multiplication this can be used to generate can be used this can be used can be used to generate a wideband signal, okay. So what do we, so what do we mean by frequency we pass our narrowband FM signal through a frequency multiplier,

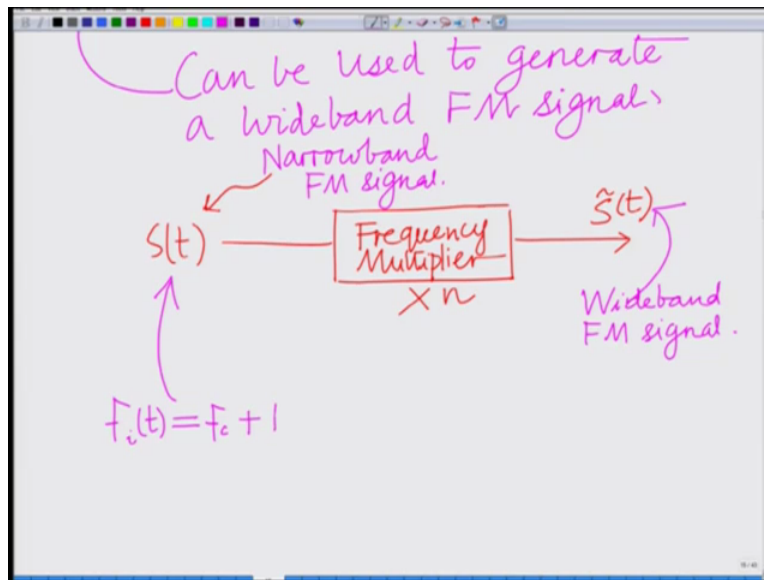
okay. So let us say this is a frequency multiplier and the output is $\tilde{S}(t)$, so this multiplies the frequency by a factor of n .

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So we input our narrowband, so this is let us say a narrowband FM signal this is our narrowband FM signal and this is our output as a wideband FM signal and this arises because let us see the input of the FM signal that is your narrowband modulated signal $S(t)$ has let us say an instantaneous frequency given by $f_i(t)$, so this has let us say input frequency $f_i(t)$ or instantaneous frequency that is given as $f_c + k_f A_m \cos(2\pi f_m t)$ this is my input frequency this is the input frequency, correct?

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Frequency or let us say frequency of the input narrowband FM signal or this is your instantaneous frequency of input narrowband FM and the output frequency you can see is $f_o(t)$ the output frequency will be since you are multiplying the frequency this will simply be n times $f_i(t)$ equal to n times f_c plus n times $k_f A_m \cos(2\pi f_m t)$ and this is the output frequency and now you can see the carrier frequency of the output signal the carrier frequency of this output signal the signal that is the output of the frequency multiplier is n times f_c . So the carrier frequency resulting carrier frequency equals n times f_c and further you can see the frequency deviation, now let us look at the frequency deviation, okay.

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The image shows a handwritten derivation on a digital whiteboard. At the top, an arrow points to the carrier frequency $n f_c$. Below it, the frequency deviation Δf is calculated as the maximum difference between the instantaneous frequency $f_o(t)$ and the carrier frequency $n f_c$. This is shown to be equal to the maximum value of $n k_f A_m \cos(2\pi f_m t)$, which simplifies to $n k_f A_m$, labeled as 'Max Freq. Deviation'. Finally, the modulation index β is derived as the ratio of the maximum frequency deviation to the message frequency f_m , resulting in $\beta = n \beta$.

$$\begin{aligned} \Delta f &= \max |f_o(t) - n f_c| \\ &= \max |n k_f A_m \cos(2\pi f_m t)| \\ \Delta f &= n k_f A_m \leftarrow \text{Max Freq. Deviation} \\ \beta &= \frac{n k_f A_m}{f_m} = n \beta = n \frac{k_f A_m}{f_m} \end{aligned}$$

So carrier frequency is n times f_c Δf that is frequency deviation is Max your $f_o(t)$ minus n times f_c which is equal to $\max n$ times $k_f A_m \cos(2\pi f_m t)$ and this is equal to n times $k_f A_m$, so Δf the maximum frequency deviation, okay. So this is basically your maximum frequency deviation or peak frequency deviation.

The maximum frequency deviation and therefore beta or beta output is this n times $k_f A_m$ divided by f_m which is equal to n times beta which is n times $k_f A_m$ divided by f_m this is your beta, alright. so basically what is happening using this frequency modulation using this frequency multiplication is that your the input as a narrowband frequency modulated signal the output is the signal with frequency that is a frequency of the instantaneous frequency of this narrowband frequency modulated signal multiplied by n therefore the resulting carrier frequency is multiplied by n times f_c , alright.

So carrier frequency becomes n times f_c and also the peak frequency deviation which was previously k_f times A_m is now n times $k_f A_m$ and the peak frequency deviation is increasing therefore the peak frequency deviation to the frequency of the message that is f_m becomes n times k_f n times $k_f A_m$ divided by f_m which gives us a modulation index beta output of n times beta, alright. So we are able to increase the modulation index, alright. Achieve a multiplication of n in the modulation index by using appropriate frequency multiplication.

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The image shows a digital whiteboard with a handwritten equation in purple ink: $\beta = \frac{\Delta f_m}{f_m} = \frac{1}{\beta} = \frac{1}{\beta} \frac{\Delta f_m}{f_m}$. Below the equation, there is a note in orange ink: "Enables us to increase β for wideband FM". Two curved arrows point from this note to the β terms in the equation.

So that basically gives us so this basically enables us to increase the or enables one to increase the beta for wideband for wideband FM. So frequency modulation helps us increase the beta and therefore we will see later when we will talk about the bandwidth of the frequency modulated signal as the beta increases in the bandwidth occupied by the frequency modulated signal increases that implies that the frequency modulated narrowband as the bandwidth increases it progressively becomes a wideband frequency modulated signal, okay. Now let us look at one of the techniques to achieve frequency modulation one of the techniques to achieve frequency modulation is to use of a non-linear device, so frequency modulation can be achieved frequency multiplication your frequency multiplication.

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Handwritten notes on a whiteboard:

$$\beta_s = \frac{11k_{FM}}{f_m} = 11\beta = 11 \frac{k_{FM}}{f_m}$$

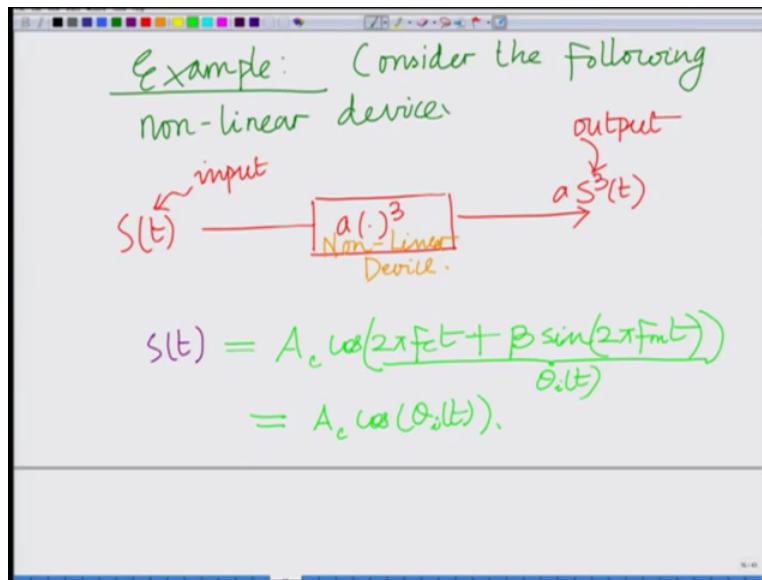
Enables us to increase β for wideband FM

FREQUENCY MULTIPLICATION

This can be achieved by passing $s(t)$ through a non-linear device.

Frequency multiplication this can be achieved through pass by passing through this can be achieved by passing is through a nonlinear ((8:24)) device. This can be achieved by passing the narrowband signal your $S(t)$ narrowband frequency modulated signal $S(t)$ through a non-linear device that is the key that is considering a non-linear (obvi) obviously the device can be cannot be used to frequency multiplication because for a linear (transi) linear time invariant system you can see that a Sinusoidal signals input with a certain frequency than the the frequency of the output Sinusoidal signal will also be the same frequency, alright. So a linear time invariant signal cannot be used for frequency multiplication however a non-linear device can be used for the same purpose and we are going to illustrate that, okay.

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So consider so that is the key we would like to use a non-linear device for frequency (multi) for example, okay for example consider the following non-linear device consider the following non-linear device where our input is $S(t)$ this is your non-linear device which performs a cube Law operation, so the output is a s cube of t, so input this is your input and this is your this is your output which is a times S cube t and you can clearly see this is a cubing operation.

So this is a non-linear device, alright. It is (())(10:41) for you to see that this is a non-linear operation, so this is straightforward to see this device is a non-linear device, okay. this device is a non-linear device, okay. So let us say the input signal to this is your narrowband Fm signal that is $S(t)$ equals your $A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$ that is the input to this let us call this as this is basically denoting this by $\theta_i(t)$ I can say this is $A_c \cos(\theta_i(t))$ this is my $S(t)$.

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$$\begin{aligned}\text{output} &= \tilde{S}(t) \\ &= a \cdot (A_c \cos(\theta_i(t)))^3 \\ &\quad \boxed{\cos^3 \theta = \frac{\cos 3\theta + 3 \cos \theta}{4}} \\ &\rightarrow = a A_c^3 \cos^3(\theta_i(t)) \\ &= a A_c^3 \frac{\cos 3\theta_i(t) + 3 \cos(\theta_i(t))}{4}\end{aligned}$$

So the output signal equals, so when $S(t)$ is the input the output let us call that as $\tilde{s}(t)$ that is equal to a times A_c times $s^3(t)$ which is $A_c \cos^3 \theta_i(t)$ which is basically now we will use a property of cosine cube of θ . Cosine cube of θ we will use the trigonometric property for cosines cube of θ . Cosine cube of θ is cosine θ or cosine 3θ rather cosine 3θ plus $3 \cos \theta$ divided by 4 . So this is the property of cosine cube θ that one can use.

So cosine cube θ is cosine 3θ plus $3 \cos \theta$ divided by 4 . I am going to use this property to simplify the output, so the output is $a A_c^3 \cos^3 \theta_i(t)$ which is $a A_c^3$ times cosine cube $\theta_i(t)$. I can write as cosine three $\theta_i(t)$ plus $3 \cos \theta_i(t)$ divided by 4 where here I have used this property cosine cube θ equals cosine 3θ plus $3 \cos \theta$ divided by 4 to simplify.

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The whiteboard shows the following steps:

$$\begin{aligned} &= a A_c^3 \cos^3(\theta_i(t)) \\ &= a A_c^3 \frac{\cos 3\theta_i(t) + 3 \cos(\theta_i(t))}{4} \end{aligned}$$

Substituting expression for $\theta_i(t)$

$$\begin{aligned} &= \frac{a A_c^3}{4} \cos(6\pi F_c t + 3\beta \sin(2\pi F_m t)) \\ &\quad + \frac{3a A_c^3}{4} \cos(2\pi F_c t + \beta \sin(2\pi F_m t)) \end{aligned}$$

Annotations on the whiteboard:

- Green text: "Freq. Multiplication by factor of 3" with an arrow pointing to the $6\pi F_c t$ term.
- Red text: "Freq. $3F_c$ " with an arrow pointing to the $6\pi F_c t$ term.
- Red text: "Frequency" with an arrow pointing to the $2\pi F_c t$ term.
- Red text: "output signal of non-linear Device." with an arrow pointing to the entire expression.

Now let us substitute this expression for $\theta_i(t)$ here and therefore you can see substituting expression substituting the expression for $\theta_i(t)$ we have this will be $A_c a$ times A_c cube well, cosine 3 $\theta_i(t)$ I have to write the long expression, so $a A_c$ cube divided by 4 cosine 3 $\theta_i(t)$ that is cosine 3 well, into $2\pi F_c t$, so that gives me cosine of $6\pi F_c t$ plus $3\beta \sin 2\pi F_m t$ plus well, $3a A_c$ cube divided by 4 cosine $\theta_i(t)$ that is again cosine $2\pi F_c t$ plus $\beta \sin 2\pi F_m t$. So this is the next expression for the, this is the net expression for the output of the non-linear device. Output signal of the night signal for the output of the non-linear device and now if you can look at this component this is at frequency F_c .

And this component is at frequency $3 F_c$ or $n F_c$, so this is achieving a frequency multiplication by a factor of 3 you can see this is basically frequency multiplication by factor of 3. So this is achieving frequency multiplication by a factor of 3 so I can retain this component filter the other component frequency F_c , alright. So if I have a filter that is centered at $3 F_c$ within appropriate bandwidth I can filter out I can block the other components at frequency F_c as a result I will get the component.

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Handwritten slide content:

$$+ \frac{3\alpha A_c^3}{4} \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

↑ Frequency

output signal of non-linear Device.

Pass through filter centered at $3f_c$
Block component at f_c

$$\text{output} = \frac{\alpha A_c^3}{4} \cos(6\pi f_c t + 3\beta \sin(2\pi f_m t))$$

↑ Frequency multiplication by a factor of 3

So pass this through filter so pass through filter centered at $3f_c$ and block and therefore block component signal component at f_c and therefore the output net output will be αA_c^3 divided by 4 cosine $6\pi f_c t$ plus $3\beta \sin 2\pi f_m t$, okay. So basically now we have achieved frequency multiplication by a factor of 3, so this has basically achieved frequency multiplication by a factor of 3, so this has achieved frequency multiplication by a factor of 3 therefore the net beta that is output beta output equals 3 times beta, okay.

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Pass through filter centered at $3f_c$
Block component at f_c

$$\text{output} = \frac{\alpha A_c^3}{4} \cos(6\pi f_c t + 3\beta \sin(2\pi f_m t))$$

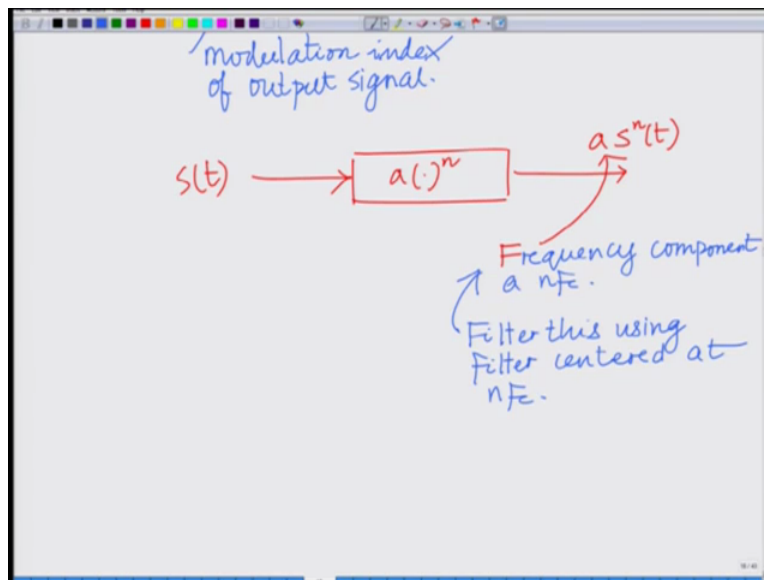
↑ Frequency multiplication by a factor of 3

$$\boxed{\beta_o = 3\beta}$$

modulation index of output signal.

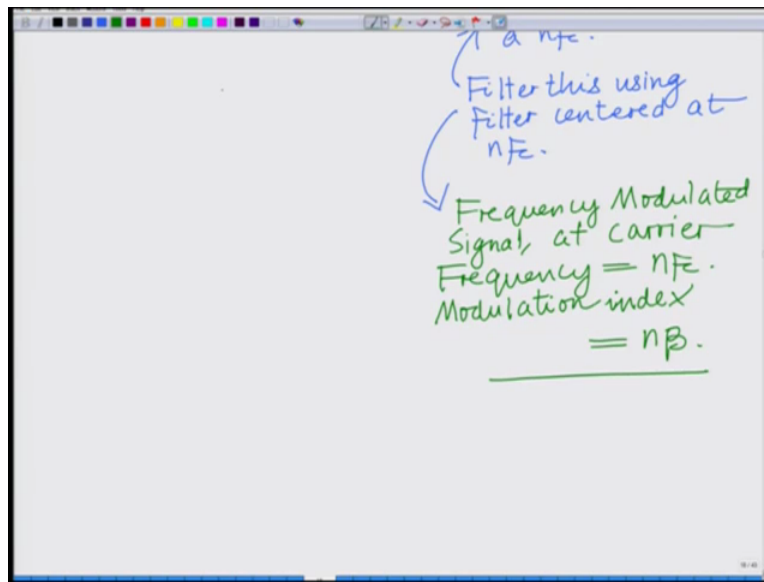
So the output beta so the modulation index beta output is the, this is the output modulation index or modulation index of the output signal. Modulation index of the modulation index of output signal is 3 times beta where beta is the modulation index of the input signal. Similarly we can consider other non-linear devices which are of the form raised to the which other forms signal raised to the power of n and that results in a frequency component output frequency component at nF_c .

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okay so if we consider to multiply it by a factor of n in general I can consider a non-linear device which is a raised to the power of n and therefore output will be a S_n to the power of t this will contain frequency component at nF_c as we have seen raised to the power of 3 contains frequency component $3 F_c$, in general signal raised to the power of n will contain frequency component at nF_c .

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Now filter this using filter centered at $n f_c$, using or bandpass filter, filter this using filter centered at $n f_c$ and that will result in a frequency modulated signal results in a frequency modulated signal gives your frequency modulated signal at carrier frequency equals naturally n beta a carrier frequency equals $n f_c$ and modulation index equals n beta. Modulation index equals n times beta, so that is the importance so that is the, that is how we generate a wideband frequency modulated signal using the indirect method first.

Basically as we have seen in the previous module first we passed the signal we modulate we use an approximation of the narrowband frequency modulated signal using the property of low beta, correct? We generate and now that is we use an indirect method to generate the narrowband frequency modulated signal, alright. using an approximation now once you have generated the narrowband frequency modulated signal I can scale I can increase make it make it a wideband frequency modulated signal through frequency multiplication that is if I pass it through a frequency multiplier than the carrier frequency that the incident in this frequency is multiplied by a factor of n therefore carrier frequency is multiplied by factor of n . The beta modulation index of the input signal frequency modulated signal is also multiplied by a factor of f .

And one way to achieve frequency modulation is bypassing the signal through a non-linear device the non-linear device for instance which raises the signal to the power of n , right? Results in a component at n times f_c , okay. Now filtering days at n times f_c now filtering this with a

filter centered at nF_c yields other FM signal with centre frequency carrier frequency nF_c and a modulation index n times β , alright. So this is the indirect method for generation of a frequency modulated signal we will stop here, thank you.