

**Principles of Communication- Part I**  
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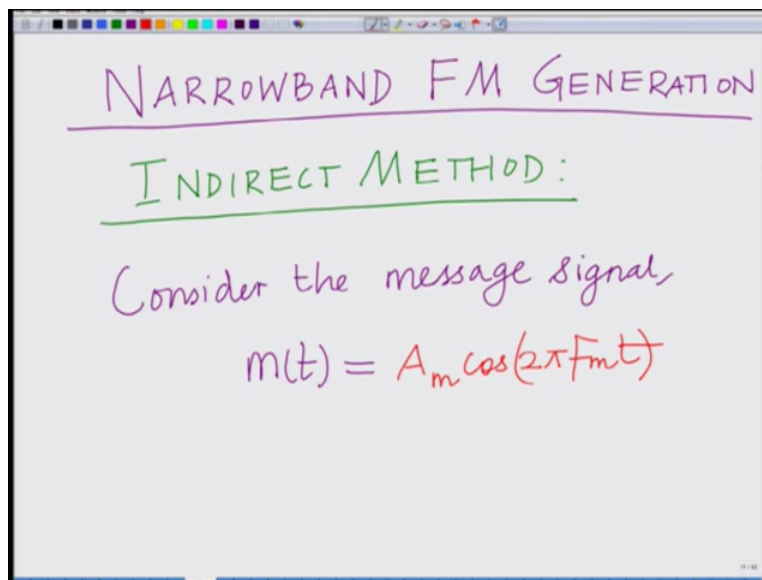
**Module No 5**

**Lecture 30**

**Indirect Method for Generation of FM Signals - Generation of Narrowband FM Signal**

Hello welcome to this another module in this massive open online course, so we are looking at angle modulation, so we are looking at angle modulation which comprises of basically two kinds of techniques either phase modulation and frequency modulation we have looked at the different kinds of we have looked at the properties of frequency modulated signals, alright and also representations of frequency modulated signals. In this module let us start a looking at the generation of frequency modulated signals, alright. In particular let us start with the generation of narrowband frequency modulated signals, okay.

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So what we would like to do is we would like to start with a discussion of generation of frequency modulated signal in particular let us start with a simplistic scenario where we have narrowband where we start consider narrowband narrowband Fm signals. Narrowband, so we want to look at narrowband Fm signals and we want to look at narrowband Fm generation in particular, okay. So basically we want to look at the generation of narrowband FM signals and in

this we will start with a technique known as the indirect the indirect method for narrowband Fm signal generation, okay.

So consider the message signal, so let us say consider the message let us say we have the message signal.  $M(t)$  equals  $A_m \cos(2\pi F_m t)$  this is our message signal, okay. So we are considering the message signal  $A_m \cos(2\pi F_m t)$  towards illustrating this process of generating a narrowband Fm signal, alright.

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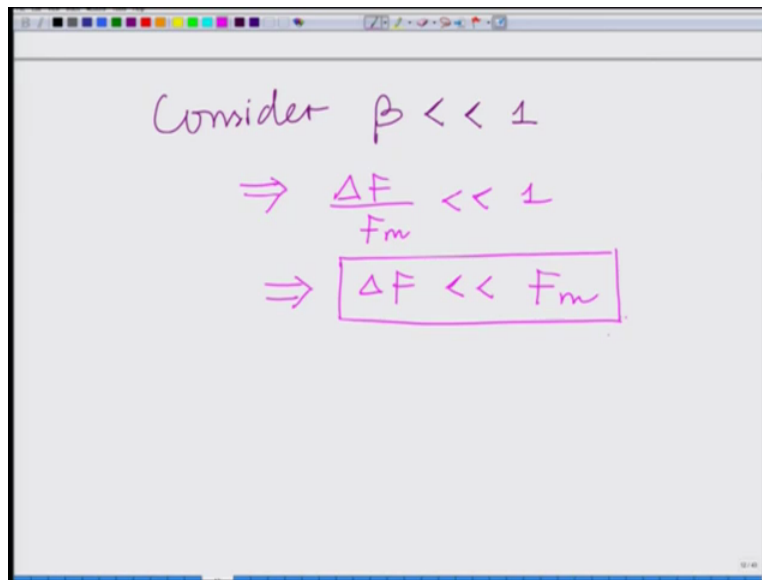
The handwritten notes on the whiteboard are as follows:

- At the top, the message signal is written as  $m(t) = A_m \cos(2\pi f_m t)$  in red.
- Below that, it says "The modulated FM signal is" in green.
- The modulated signal is given as  $S(t) = A \cos(2\pi F_c t + \beta \sin(2\pi F_m t))$  in orange.
- An arrow points from the text "Modulated Signal." in pink to the  $S(t)$  equation.
- The modulation index  $\beta$  is defined as  $\beta = \frac{k_f A_m}{F_m} = \frac{\Delta F}{F_m}$  in blue.
- An arrow points from the text "modulation index." in blue to the  $\beta$  in the equation.

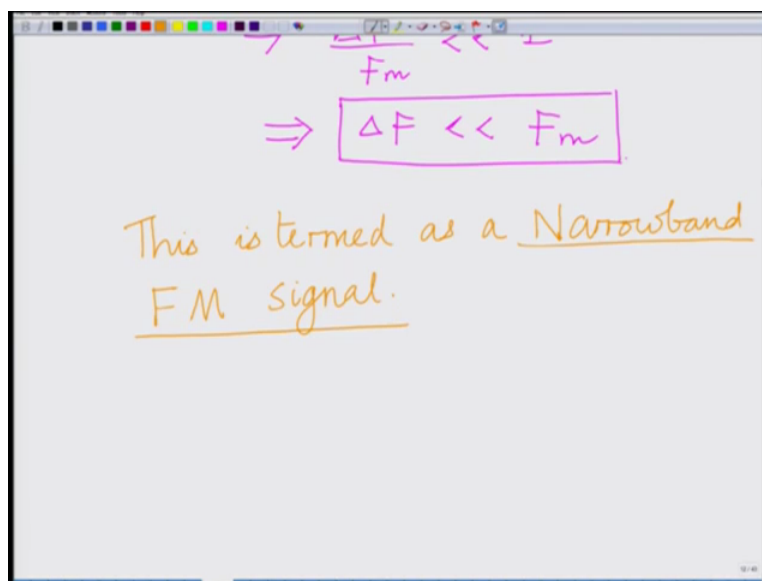
Okay, now let the modulated Fm signal be we have already seen that the modulated Fm signal is  $S(t)$  equals since this is a Sinusoidal modulating signal we have already seen the corresponding modulated signal  $A \cos(2\pi F_c t + \beta \sin(2\pi F_m t))$  this is your modulated signal, correct? So this is basically the this is basically the modulated signal and what we have seen is this value beta the value of this modulation index beta for this scenario is  $k_f A_m$  divided by  $F_m$  which is basically your  $\Delta F$  divided by  $F_m$ .

So this is your modulation index beta, okay. That also we know, so this is your this is the modulation index beta, okay. So now we would like to generate a narrowband Fm signal let us consider a signal in which this modulation index beta is very small, alright. Later we are going to (illus) later we are going to describe how to generate and Fm signal where beta is where beta is not very small or beta is beta has some nominal value, okay.

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Consider  $\beta \ll 1$   
 $\Rightarrow \frac{\Delta F}{F_m} \ll 1$   
 $\Rightarrow \boxed{\Delta F \ll F_m}$

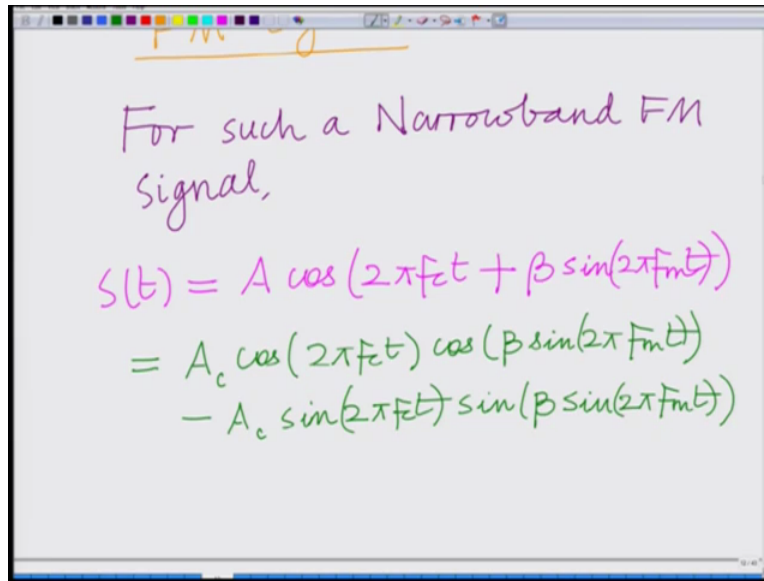


$\Rightarrow \boxed{\Delta F \ll F_m}$   
This is termed as a Narrowband FM signal.

So to begin with let us consider a scenario with very small beta that is a modulation index beta, alright. So consider to begin with consider beta significantly less than one this implies that basically your Delta F by Fm beta is Delta F by Fm is significantly less than one. Which implies Delta F is significantly less than M Fm, now this is termed as a narrowband FM signal. This is termed as a narrowband FM signal that is a signal for which beta is significantly less than one that is beta is very small this can be considered as a narrowband Fm signal for the reason that the spread the bandwidth occupied by this signal is going to be very small.

As we are going to see when we discuss the bandwidth of an FM signal, alright. So when beta this modulation index is very small the bandwidth of (6:29) by the FM signal is indeed very small, okay. So we call such an FM signal a narrowband FM signal, okay. And now this narrowband FM signal for this narrowband FM signal for such a narrowband FM signal we have.

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For such a Narrowband FM signal,

$$\begin{aligned}
 s(t) &= A \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \\
 &= A_c \cos(2\pi f_c t) \cos(\beta \sin(2\pi f_m t)) \\
 &\quad - A_c \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t))
 \end{aligned}$$

Now for such a narrowband FM signal now for such a narrowband FM signal we have  $S(t)$  equals  $A \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$ . Now this can be simplified for using the properties of the cosine function as using the trigonometric properties that is  $A \cos(a + b) = A \cos a \cos b - A \sin a \sin b$ , so this is  $A \cos(2\pi f_c t) \cos(\beta \sin(2\pi f_m t)) - A \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t))$ , alright. So we have simplified this modulated signal  $A \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$ , okay.

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The image shows a digital whiteboard with handwritten mathematical expressions and a note. At the top, the expression  $\cos(\beta \sin(2\pi f_m t)) \approx 1$  is written in purple. Below it, the expression  $\sin(\beta \sin(2\pi f_m t)) \approx \beta \sin(2\pi f_m t)$  is written in green. Two green curved arrows originate from these two expressions and point towards a red handwritten note below them. The note reads: "Substituting these expressions in the relation for the modulated signal  $s(t)$ , we have,".

$$\cos(\beta \sin(2\pi f_m t)) \approx 1$$
$$\sin(\beta \sin(2\pi f_m t)) \approx \beta \sin(2\pi f_m t)$$

Substituting these expressions in the relation for the modulated signal  $s(t)$ , we have,

Now we are going to employ the property that is for small beta. That is for small beta for small beta we have, well for small beta cosine beta sin two pi Fmt this is approximately equal to one because beta small, so cosine theta is small for small Theta cosine Theta is approximately one and for small Theta sin theta is approximately equal to theta so employing this approximation sin of beta sin two pi Fmt is approximately equal to beta sin two pi Fmt. For small Theta sin theta is approximately equal to theta.

So therefore sin beta sin two pi Fmt is simply beta approximately beta sin two pi Fmt, now when we substitute this in the expression so substituting these so now substituting these in the expression for the modulated signal  $S(t)$ , so what we are going to do is we are going to substituting these expressions substituting these expressions in the relation or in the expression for the modulated signal  $s(t)$ .

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Handwritten derivation of the modulated signal  $S(t)$  using trigonometric identities and small angle approximations.

$$S(t) = A \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$
$$= A_c \cos(2\pi f_c t) \underbrace{\cos(\beta \sin(2\pi f_m t))}_{\approx 1} - A_c \sin(2\pi f_c t) \underbrace{\sin(\beta \sin(2\pi f_m t))}_{\approx \beta \sin(2\pi f_m t)}$$

For small  $\beta$

$$\begin{cases} \cos(\beta \sin(2\pi f_m t)) \approx 1 \\ \sin(\beta \sin(2\pi f_m t)) \approx \beta \sin(2\pi f_m t) \end{cases}$$

We have  $S(t)$  equals well,  $A \cos 2\pi f_c t$   $A \cos 2\pi f_c t$  into  $\cos \beta \sin 2\pi f_m t$  that will be one. Since, so what we have here is this  $\cos \beta \sin 2\pi f_m t$  this is approximately one and  $\sin \beta \sin 2\pi f_m t$  this is approximately equal to  $\beta \sin 2\pi f_m t$ .

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Handwritten substitution of approximations into the signal equation and simplification.

Substituting these expressions in the relation for the modulated signal  $s(t)$ , we have,

$$S(t) \approx A \cos(2\pi f_c t) \cdot 1 - A \sin(2\pi f_c t) \cdot \beta \sin(2\pi f_m t)$$
$$= \boxed{A \cos(2\pi f_c t) - \beta \sin(2\pi f_m t) \times \sin(2\pi f_c t)}$$

The image shows a digital whiteboard with handwritten mathematical equations. At the top, the signal  $s(t)$  is approximated as the sum of two terms:  $A \cos(2\pi f_c t)$  and  $-A \sin(2\pi f_c t) \cdot \beta \sin(2\pi f_m t)$ . Below this, the expression is simplified into a single product term enclosed in a yellow box:  $A \cos(2\pi f_c t) - \beta \sin(2\pi f_m t) \times \sin(2\pi f_c t)$ . An arrow points from the text 'Approximation For narrowband modulated FM signal.' to the boxed equation.

$$s(t) \approx A \cos(2\pi f_c t) - A \sin(2\pi f_c t) \cdot \beta \sin(2\pi f_m t)$$

$$= A \cos(2\pi f_c t) - \beta \sin(2\pi f_m t) \times \sin(2\pi f_c t)$$

Approximation For narrowband modulated FM signal.

So substituting these we have  $A_c \cos(2\pi f_c t)$  this is approximately equal to  $A_c \cos(2\pi f_c t) - A \sin(2\pi f_c t) \cdot \beta \sin(2\pi f_m t)$  which is equal to  $A \cos(2\pi f_c t) - \beta \sin(2\pi f_m t) \sin(2\pi f_c t)$  and this is the approximation for your narrow band modulated, so this is basically we are going to employ this approximation for the narrowband modulated FM signal.

So this is an approximation this is the approximation for this is the approximation for the narrowband since the approximation for our narrowband modulated FM signal and we are going to employ this approximation for the generation of the FM signal and you can see this is reasonable as simple with this approximation because now the modulating signal is out of the phase component and it is in the (fmo) form of a product of the carrier and the message signal, alright.

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The image shows a handwritten derivation on a digital whiteboard. At the top, the signal  $s(t)$  is approximated as  $A \cos(2\pi f_c t) \cdot [1 - \beta \sin(2\pi f_m t)]$ . Below this, the expression is simplified to  $A \cos(2\pi f_c t) - \beta \frac{\sin(2\pi f_m t)}{\sin(2\pi f_c t)}$ , which is enclosed in a yellow box. An arrow points from this boxed expression to the text "Approximation For narrowband modulated FM signal."

$$s(t) \approx A \cos(2\pi f_c t) \cdot [1 - \beta \sin(2\pi f_m t)]$$
$$= A \cos(2\pi f_c t) - \beta \frac{\sin(2\pi f_m t)}{\sin(2\pi f_c t)}$$

Approximation For narrowband modulated FM signal.

Say you can see this is basically in the form of a product that is sin two pi Fmt into sin two pi Fct.

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The image shows a handwritten note on a digital whiteboard. It features the expression  $A \cos(2\pi f_c t) \times \sin(2\pi f_m t)$  enclosed in a yellow box. Two arrows point from this box to the text "Approximation For narrowband modulated FM signal." and "Can be employed for Nr".

$$A \cos(2\pi f_c t) \times \sin(2\pi f_m t)$$

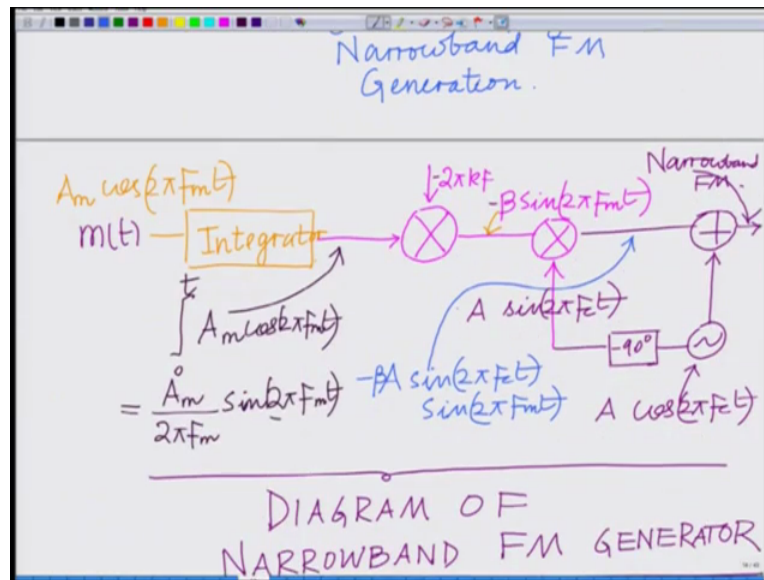
Approximation For narrowband modulated FM signal.

Can be employed for Nr

So now basically we can generate this narrowband, so now you can generate this narrowband modulated FM signal so this can be employed to generate the, so and this approximation can be employed so this approximation can be employed for this approximation can be employed for narrowband FM generation.



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So let us look at how to generate that? So I have  $m(t)$  which I can basically this is your message signal I can basically pass it through an integrator, right? Correct? That is your cosine two pi  $F_c t$  that integrate gives sin two pi, so I can pass this pass it through the integrator, correct? So this is let us say your  $A_m$  this is your  $A_m \cos(2\pi F_m t)$  you pass it through the integrator at this point you will have so you will have integral zero to  $t$   $A_m \cos(2\pi F_m t)$  at this point you will have  $A_m \cos(2\pi F_m t)$  integral zero to  $t$   $A_m \cos(2\pi F_m t)$  which is basically equal to, well this is  $A_m$  divided by two pi  $F_m$  sin two pi  $F_m$  sin two pi  $F_m t$ .

So this you multiply at this point you multiply this by when you multiply this by two pi into  $k_f$  so you multiply this by two pi into  $k_f$ . So you take the output of the integrator multiplied by 4 two pi into  $k_f$ , first multiplied by this factor two pi into  $k_f$  and also multiplied by this multiplied now this is basically multiplied by this if so that gives you beta so this gives you basically your beta sin two pi  $F_m t$ .

Which at this point you multiply by you multiply by you modulated with the cosine signal that is  $A_c \cos$  or  $A_c \sin$  two pi  $F_c t$   $A_c \sin$  two pi  $F_c t$  and so you have this  $A_c \sin$  two pi  $F_c t$  which can in turn be generated by a ninety degree phase shift by a ninety degree phase shift from a cosine modulator. So I have a cosine modulator which is generating your oscillator rather, so this is your  $A_c \cos$   $A_c \cos$  two pi  $F_c t$  and this signal.

Let us say and this signal you add this over here so you have  $A_c \cos(2\pi F_c t)$  minus or you have  $A \cos(2\pi F_c t)$  minus and let us say I have this minus  $2\pi k_f$ , so this will be so this will be minus  $\beta \sin(2\pi F_m t)$  times  $A \sin(2\pi F_c t)$ , so what we have here will be the narrowband FM so this will be narrowband FM the output of this will be your narrowband FM, okay so this is basically your schematic diagram or diagram of narrowband FM generator, okay.

So this is basically your diagram for a this is the diagram of your narrowband FM, diagram of a simple narrowband FM generator, so I am taking  $A_m \cos(2\pi F_m t)$  passing it through an integrator that gives me  $A_m$  by two by  $F_m \sin(2\pi F_m t)$ , so multiplying that by  $2\pi k_f$  or minus  $2\pi k_f$  that gives me, so minus  $\beta \sin(2\pi F_m t)$  that will be the output here, so this is modulated through this  $A \sin(2\pi F_c t)$ , alright.

And the same thing is also again to the same thing we add  $A_c \cos$  so you have at this point or at this point you have at this point you have  $A \sin(2\pi F_c t)$  or minus  $\beta A \sin(2\pi F_c t)$  into  $\sin(2\pi F_m t)$  that is what you have at this point, so minus  $\beta \sin(2\pi F_m t) \sin(2\pi F_c t)$  that is what you have at this point to that you add  $A \cos(2\pi F_c t)$  and that gives you your narrowband FM signal. So this is basically the narrowband FM generator, alright.

So this is a simple schematic diagram for the narrowband FM generator, alright. So in this module what we have seen is we have technique to generate the narrowband FM signal we have seen that the narrowband FM signal is the signal for which the  $\beta$  that is the modulation index  $\beta$  is significantly lower than one that so using an approximation for that, right. A reasonable approximation for this scenario where  $\beta$  is much less than one.

We have simplified this expression for the modulated narrowband FM signal and based on these approximation for the narrowband modulated FM signal we illustrate a simple scheme to generate this narrowband FM modulated signal, alright. By modulating the signal appropriately with a integrated version of the signal appropriately with  $\sin(2\pi F_c t)$  and adding the cosine carrier that is  $\cos(2\pi F_c t)$ , alright so we will stop here and continue with other aspects in the subsequent modules, thank you very much.