Principles of CommunicationminusPart 1
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Module 1
Lecture No 3
Discrete Fourier series Example and Parseval's Theorem for Periodic Signals

Hello, welcome to another module in this massive open online course. So we are looking at the Courier series representation of periodic signals that is also we said apart of the spectral representation for computing the spectrum of a signal, which is basically representation of the signal in the frequency domain. So what we are looking at is the Fourier series representation or also the discrete Courier series for a periodic signal, correct.

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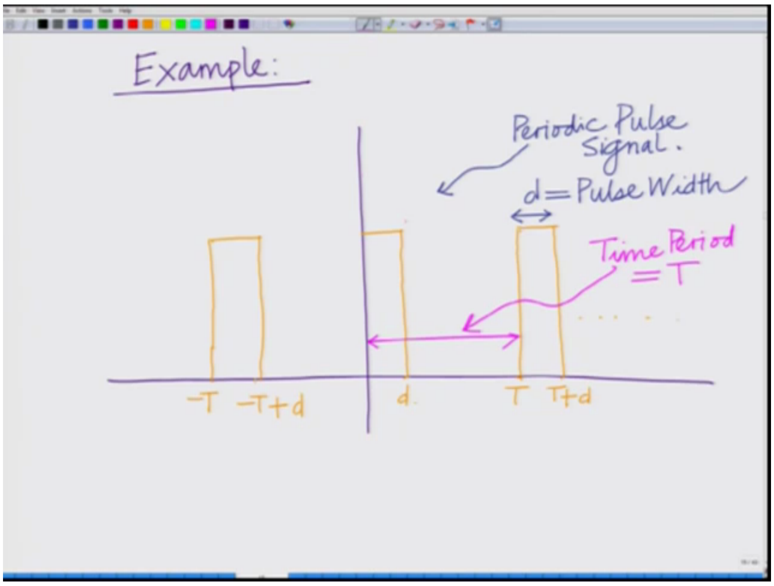

The discrete Fourier series for a periodic signal; let us look at an example to illustrate how the Fourier series can be computed and let us look at a simple example of a periodic signal. Consider a periodic stream of pulses for instance, we have each pulse of width d and let us say that this is a periodic signal of period T. So this is a periodic signal, so this is minus T this is minus T plus d, so this is a periodic pulse signal. The pulse width is the, let us say this is the width of the pulse and the period of the signal, the time period of this signal is T. This is the notation that we have used previously also, the time period of the signal is T this is a particular pulse, alright. Let us set d equals to T by 4 for the purpose of this example. So we are setting d equals to T by 4, where the equals to the pulse width the width of the pulse in time.

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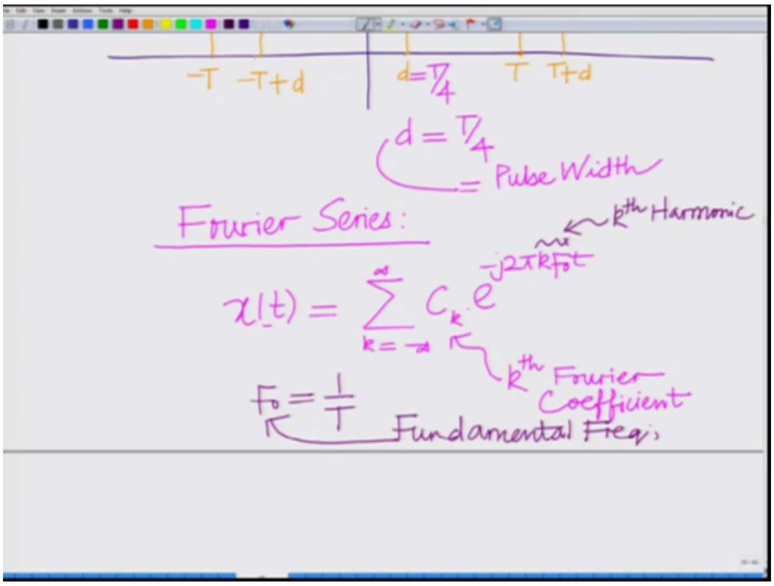

Now let us compute, now this is a periodic signal so we can compute the Fourier series representation basically, which is nothing but the computation of the various Fourier coefficients of the periodic signal, alright. So let us compute the Courier series representation of this periodic signal. So want to compute the Courier series for this periodic signal and we have shown that the Lth Fourier coefficient, correct. We have said any periodic signal x(t) with period t it can be expressed as an in finite sum of complex Sinusoid that is CL e to the power of minus j 2 k 2 Pi k F0 t, correct? This Ck is the kth Fourier cominusefficient this frequency k F0, remember we said that it is the kth harmonic that is the kth multiple of this is the kth harmonic, where F0 equals to 1 over T this is your fundamental frequency.

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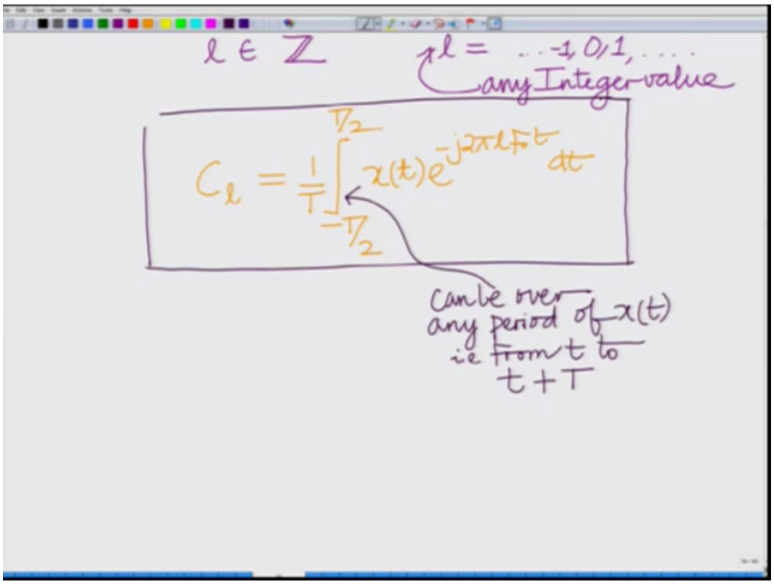

This is the fundamental frequency and we also illustrate develop the relation to compute this Fourier cominusefficient CL this the Lth Courier cominusefficient CL for L for any integer L that is L belongs to the set of integers, that is L equals to 0 1 so on minus 1 so on. So L equals to so L can basically take on any integer value and this cominusefficient CL can be integrated can be is given or by this integral minus T over 2 to T over 2 1 over T x(t) e to the power of minus j 2 Pi L F0 T dt, this is the relation we have derived for the Lth cominusefficient of the Fourier series, correct.

And what we have said and we have and in general this integral here need not be only from minus T by 2 to T by 2 it can be over any period, that is this integral need not strictly be from minus T by 2 to T by 2, it can be from any t that is small t to small t plus capital T where capital T is the period, so this integral can be over any period that is from t to t plus T. That is it is start at any small t and you can carry out the integration from any small t to small t plus capital T where capital T is the period because after all the signal x(t) is periodic, okay.

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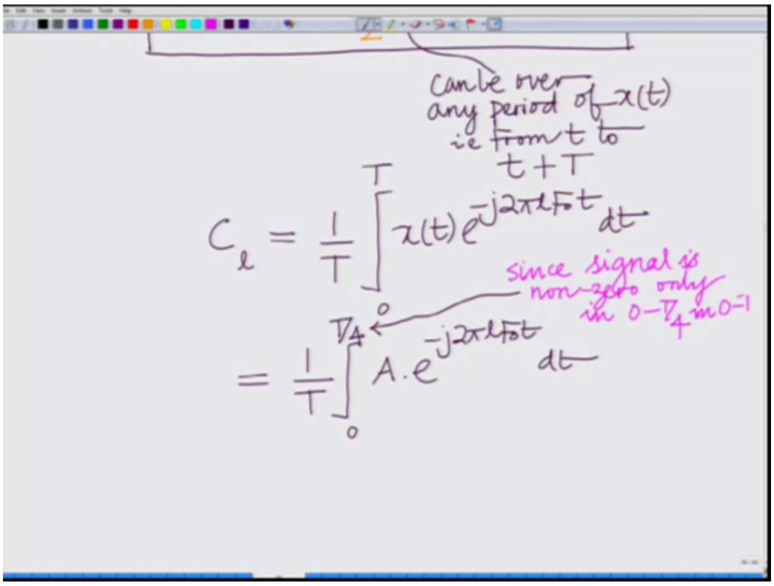

And now what we are going to do is, we are going to derive this for our particular signal. So we have CL, let us carry out the integral from 0 to capital T this is integral as I said it can be carried out over any period 0 to capital T of x(t) e to the power of minus j 2 Pi L F0 t and now this is equals to remember our signal is a pulse of width capital T by 4 and let us make this height as A that is the pulse height is A, okay. And therefore this is A 4 0 less than equals to t less than equals to T by 4, so I can replace this as 1 over T because now it is nonzero only in 0 to T by 4 A times e to the power of minus j 2 Pi L F0 t dt and we are now considering T by

4 since signal is nonzero since only in 0 to T by 4 that is 0 to T by 4 in the period 0 to T in the period 0 to T.

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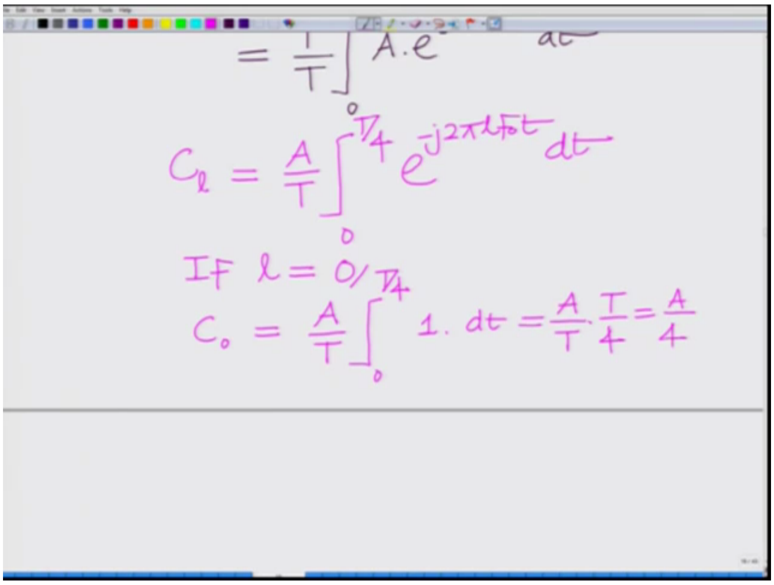

And now A is a constant, so I can bring it outside the integral so this is A over T integral 0 to T by 4 e to the power of minus j 2 Pi L F0 of t, now if L is equals to 0, let us integrate this, let us carry out this integration considering 2 separate cases, one if L is equals to 0 that is C 0 is A over T integral 0 to T by 4 since L equals to 0 this will simply be 1 integral 1 times dt, so this is A over T times T by 4, once you substitute the limits of integration, so this is A 0 A over 4. So this cominusefficient C0 corresponding to frequency 0, the Fourier cominusefficient C0 corresponding to that is your L equals to 0 that is CL corresponding to L equals to 0 that is C C0 is A over 4, this is also known as the DC cominusefficient that is the DC coefficient since it corresponds to the 0 frequency.

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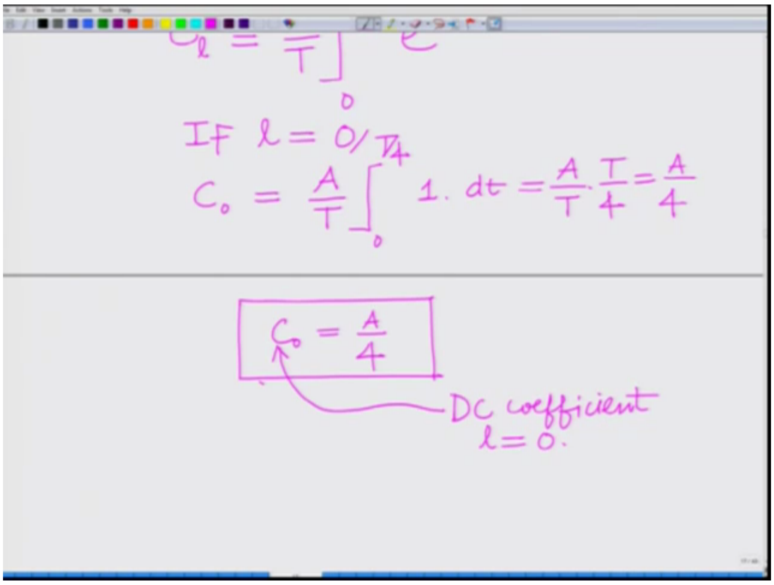

So this is basically your DC cominusefficient, the rest all which corresponds to nonminuszero frequencies, which are basically your k not equals to 0, so this corresponds to your L equals to to zero. This is known as the DC cominusefficient all the rest of them which corresponds to L not equals to 0, so those are known as the AC coefficients, AC is something which varies with time and DC is something DC because it is a 0 frequency it is constant with time. So this is the DC cominusefficient which we have evaluated as A over 4.

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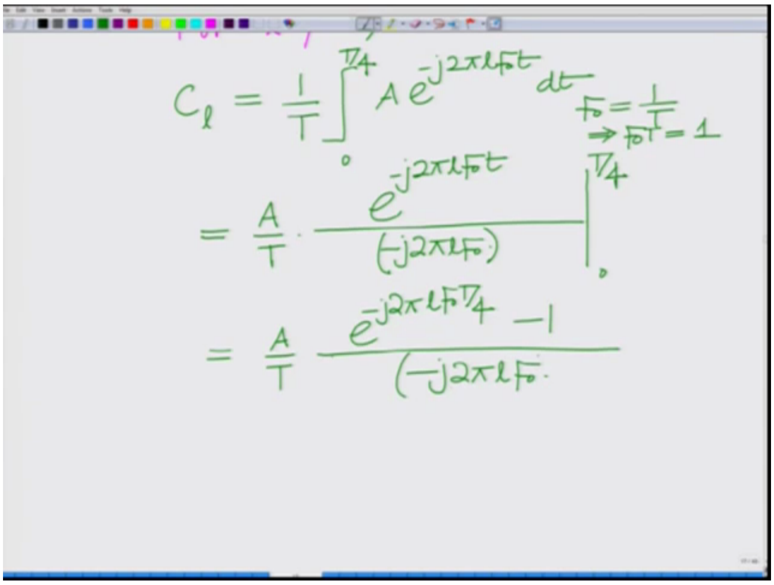

Now let us evaluate the rest of the cominusefficient the general CL for L not equals to 0 we have C L equals to 1 over T integral 0 to T by 4 A e to the power of minus j 2 Pi L F0 t dt, which is basically bringing A outside A over t integral e to the power of minus j L F0 t is e to the power of minus j 2 Pi L F0 t divided by minus j 2 Pi L F0 from the limits 0 to T by 4 and notice that F0 equals to 1 by T implies F0 into T equals to 1 and we are going to use that principle in the simplification here, so this is A over T substituting this it is e to the power of minus j 2 Pi L F0, let us substitute T by 4 e to the power of minus j 2 Pi L F0 T by 4 minus 1 minus j 2 Pi L F0.

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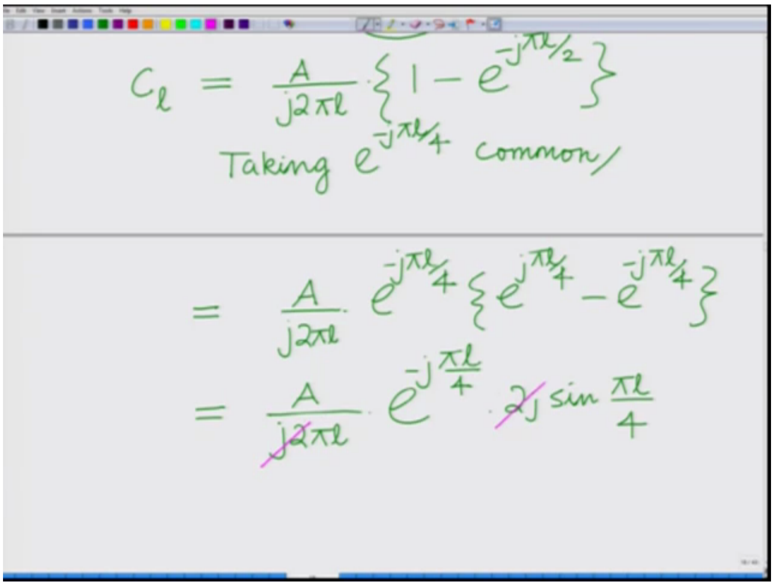

Now we know F0 T equals to 1. So therefore I will use that simple and also there is a negative sign, so I am going to simply write e to the power of invert multiply the numerator also by minus sign of minus 1 and what I get 1 minus e power minus j Pi by 2 L or 1 minus e by j Pi L 1 minus e power minus j Pi L minus Pi L by 2 divided by j 2 Pi L F0 and also in the denominator there is T and then F0 the product of these 2 is 1. So therefore, I can simply write this now also as A or rather A over j 2 Pi L into 1 minus e to the power of minus j Pi L by, 2 this is your Cl.

Now taking e to the power of minus j Pi L by 4 common from this that is or taking e to the power of minus j, what we have this can be further simplified as A divided by j 2 Pi L e to the power of minus j Pi L by 4, e to the power of j Pi L by 4, e to the power of minus j Pi L by 4 and you can see e to the power of j Pi L by 4 minus e to the power of minus j Pi L by 4 is nothing but 2 j sine Pi L by 4. That is you employ the substitution e to the power of j theta is cosine theta plus j sine theta. What we have is this is A over J 2 Pi l, e to the power of minus j Pi L by 4 into 2 j sine Pi L by 4.

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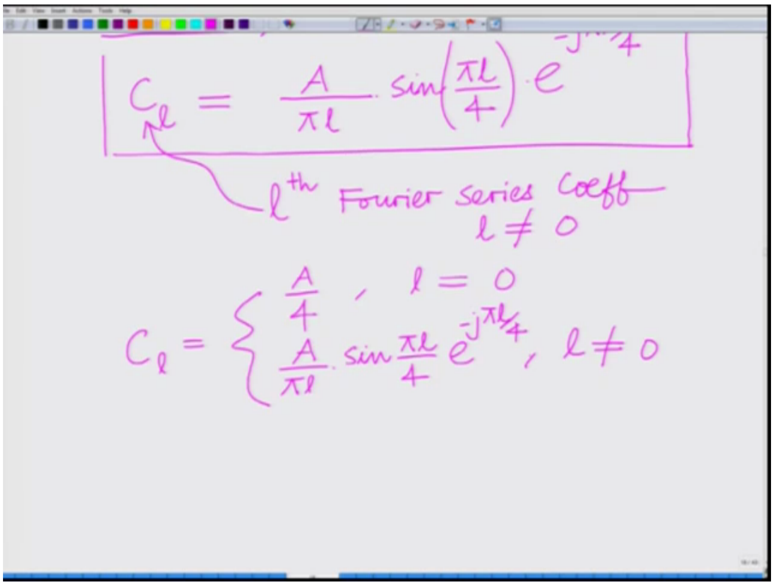

The 2 js canceled and therefore what we have is A over Pi l, sine Pi L by 4 times e to the power of minus j Pi L by 4 and this is our cominusefficient e to the power of and , correct, so this is your Lth Fourier series coefficient, where L not equals to 0. So now we can finally write CL equals to A by 4 for L equals to 0 CL equals to A by Pi L sine Pi L by 4 e to the power of minus j Pi L by 4 L not equals to 0 and further if you are only interested in the magnitude, therefore we have derived Lth Fourier series coefficient for this periodic pulse signal.

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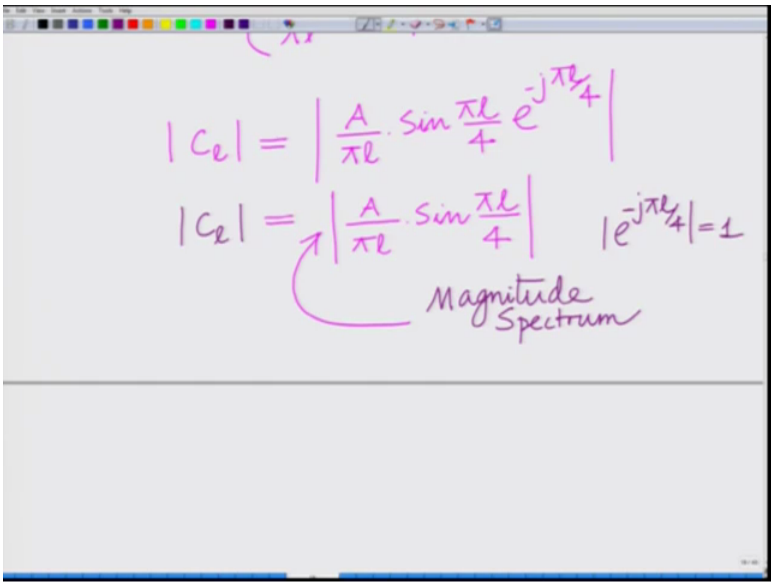

And further we can look at the magnitude, that is magnitude of this signal CL is magnitude Pi L sine Pi L by 4 e to the power of minus j Pi L by 4 and magnitude A to the power of minus j

Pi L by 4 is 1, so this is simply magnitude A over Pi L times sine Pi L by 4, this is known as the magnitude spectrum, okay. This is known in general the spectrum is a complex quantity there is a spectral cominusefficient, as the Lth the Lth frequency component that is at L times F0 is a complex quantity, so it has a magnitude and a phase, the phase constitutes the phase spectrum the magnitude of CL constitutes the magnitude corresponds to the magnitude spectrum, alright.

And also as I have said in this simplification we have employed the fact that magnitude e power minus j theta equals to 1 that is magnitude e power minus j Pi L by 4 is equals to 1. So this shows basically the computation of a so this is magnitude CL so this is basically a simple example for the computation of the discrete Fourier series that is the discrete Fourier series spectrum of this periodic pulse signal with period capital T and pulse width capital T over 4, alright. Now let us look at another important aspect of a periodic signal that is, let us look at the power of the periodic signal in terms of the Fourier series. We have already looked at the power; let us revisit this again that is, let us look at the power of the periodic signal. So let us revisit this power computation again, let us look at the, power of a periodic signal, correct.

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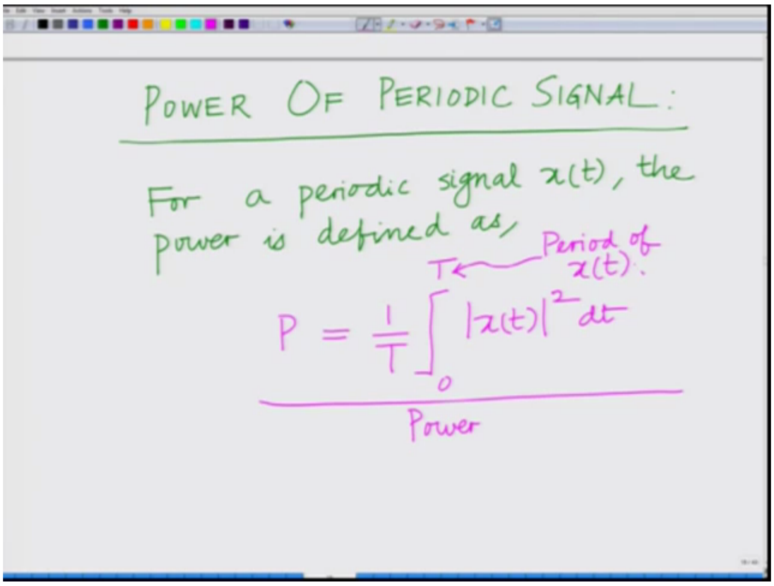

We have already seen that for a periodic signal, that is for a periodic signal x(t) the power is defined as P or the power is defined as well P equals to 1 over T integral 0 to T, again in fact, this integral can be carried out over any period, so 1 over T integral 0 to T magnitude x(t) square dt, this is the power of a periodic signal. This is the power and the expression for the power of the periodic signal x(t) that is 1 over T, 0 to capital T magnitude x(t) square dt where again of course it goes without saying capital T is the period, which is this very specific quantity which is the period fundamental period of this periodic signal x(t), so this is the period of x(t).

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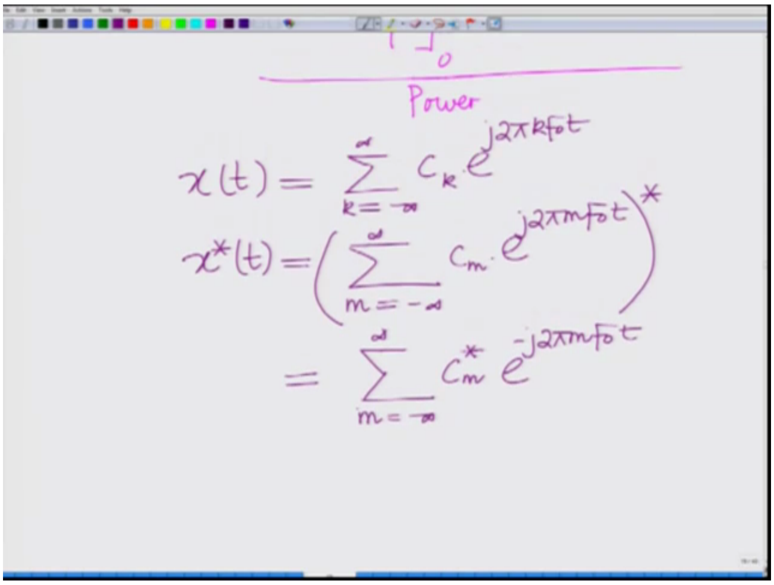

Now let us represent x(t) in terms of its Fourier series representation, we said any periodic signal x(t) can be represented in its courier series, so x(t) equals to summation k equals to minus infinity to infinity Ck e to the power of j 2 k F0 t, x conjugate, now to compute magnitude x(t) square we need X conjugate of t, so x conjugate of t can be given as the following. Let us employ different index, it is simply the index so I can change it so it is minus infinity to m equals to minus infinity to infinity Cm e to the power of that is conjugate of this j 2 Pi m F0 t, which is the integral m equals to minus infinity to infinity Cm conjugate e to the power of minus j 2 Pi F0 t, this is your x conjugate of t, correct?

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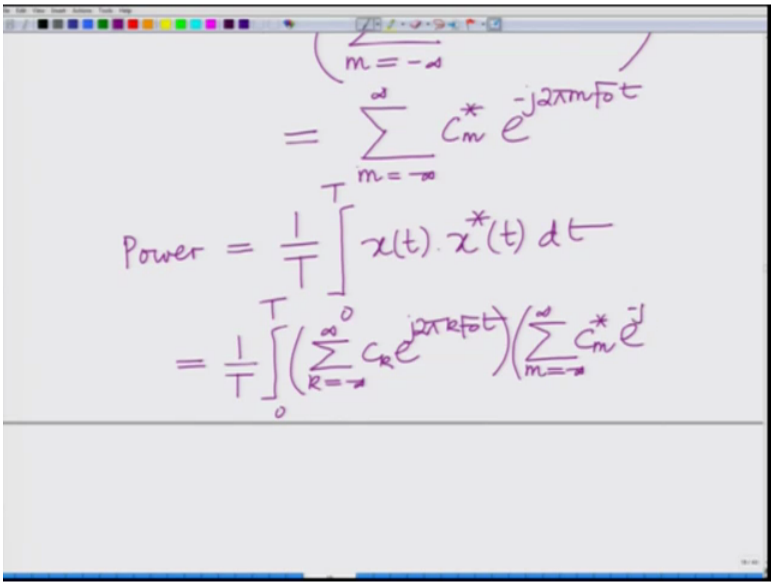

Now let us compute the power of this periodic signal, the power equals to 1 over T integral magnitude x(t) square, which is x(t) into x conjugate t dt which is 1 over T integral 0 to capital T x(t) we have already developed the expression x(t) is summation k equals to minus infinity to infinity e to the power of j 2 Pi k F0 t times x conjugate t that is m equals to minus infinity to infinity Cm conjugate e to the power of minus j 2 Pi m F0 t whole into dt.

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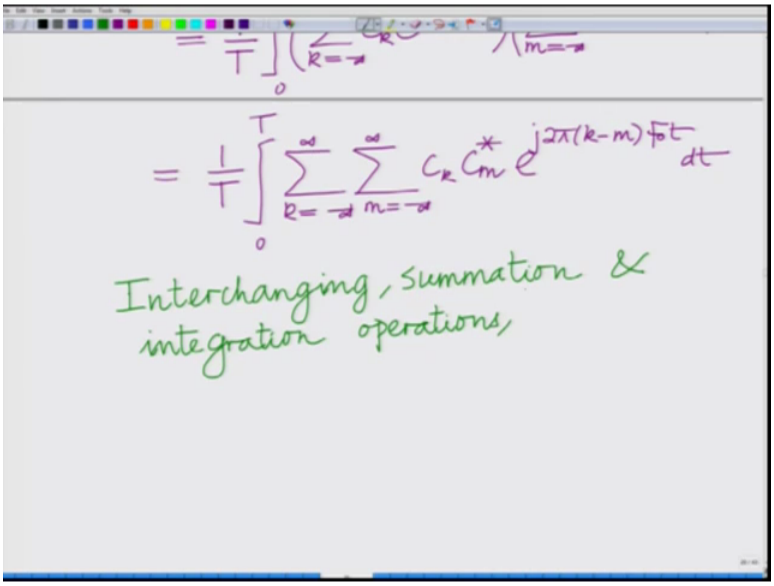

Now let us expand this product, remove the brackets and bring multiply these things term by term the product that is the sums in both the summations term by term and what they are going to get his basically 1 over T summation 0 to T, k equals to minus infinity m equals to minus infinity to infinity Ck Cm conjugate e to the power of j 2 Pi k minus m F0 dt. Now what we are going to do similar to our derivation of the courier series coefficient, what we are going to do is we are going to interchange the summation and the integration operations, alright. And what we are going to have then is basically interchanging summations and, or rather interchanging the summation and integration operations, what we are going to have is the power.

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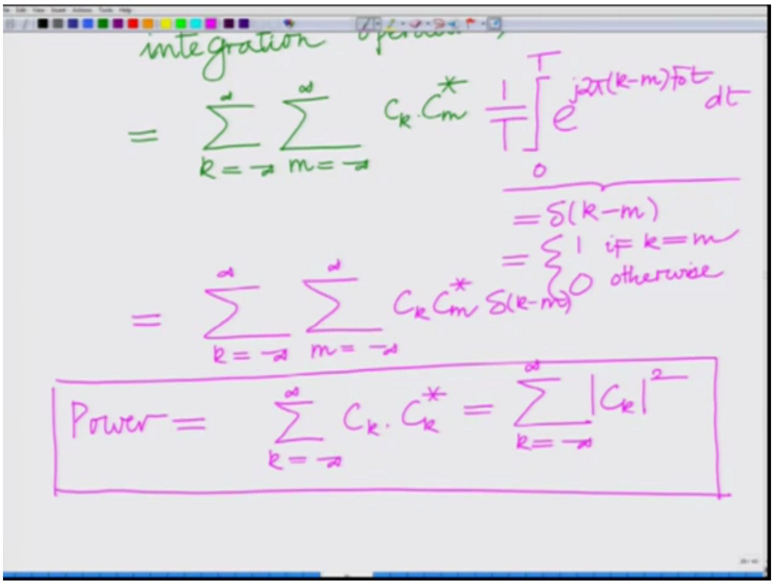

I am going to bring the summations outside summation k equals to minus infinity to infinity, summation m equals to minus infinity to infinity and Ck into Cm conjugate into 1 over T times integral 0 to T. If you look at this 1 over T integral 0 to T e to the power of j 2 Pi k minus m F0 t that is e to the power of j 2 Pi k minus m F0 t dt, and if you look at this we have already seen this that is for any k not equals to F that is 1 over T integral 0 to T e to the power of j 2 Pi k minus m F0 t dt that is if k not equals to m this integral is 0 if k equals to m this integral is 1, remember we said this integral is therefore delta k minus m. This integral we have already derived this, this is simply your delta k minus 1 that is equals to 1 if and only if k equals to m, 0 otherwise, so this is 1 if k equals to m, this is 0 otherwise.

So this is summation k equals to minus infinity to infinity summation m equals to to minus infinity to infinity Ck Cm conjugate into delta k minus m and therefore in these double summation the only term that will that only terms that survives are when because delta k minus m equals to to 0, if k is not equals to m the only times that terms that survive are the terms corresponding to k equals to m and therefore, when k equals to to m Ck into Cm conjugate will be Ck into Ck conjugate which is magnitude Ck square. And therefore in that scenario this will be k equals to minus infinity to infinity Ck Ck conjugate equals to summation k equals to minus infinity to infinity magnitude Ck square and this is the expression for the power and what you can see is you have a very nice expression for the power of the periodic signal in terms of now in terms of now the Fourier coefficients that is the power of the periodic signal is nothing but the magnitude.

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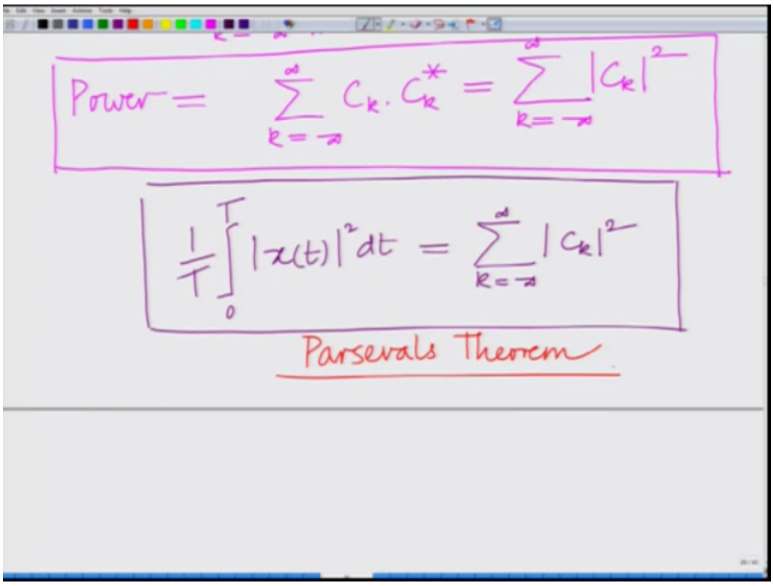

There is nothing but the sum squares of the magnitude of all the Fourier series coefficients from minus infinity to infinity. And and this is a very interesting and this is a very elegant expression for the power. So therefore what we have is the power has 2 expressions, 1 is we have the traditional time domain expression 1 over T 0 to T it is magnitude x(t) square dt equals to summation k equals to minus t minus infinity to infinity magnitude k Ck square, so now we have a frequency domain expression also expression for the power in the frequency domain that is in terms of it is spectrum and this is known as the Parsevals theorem, it is a very popular result this is known as the, this is known as the Parsevals Theorem, that is which give us the power of the signal and what this is really giving is the power of the signal in terms of the spectrum. This is giving the power in terms of the spectrum, alright.

So we have 1 over T integral 0 to T magnitude x(t) square dt which is the power, it is a time domain computation and now we also have a very interesting expression to compute the power from the spectrum that is in the frequency domain that is magnitude k equals to summation k equals to minus infinity to infinity magnitude Ck square, where Ck is the Kth Fourier series coefficient, alright. This is a very, this is a lot of applications and it is also a very interesting result, this is known as the Parsevals theorem, it gives an equivalent way to compute the power (()) (31:25) and it shows that these 2 things the power computed from the time domain and also from the using the coefficient that is spectral coefficient is the frequency domain are equal and helps us to characterize the power distribution amongst the various spectral components.

For instance, one can think of Ck or magnitude Ck square as a power in the kth spectral component that is power at the Harmonic k times F0 and that gives us a very interesting interpretation, of course we are going to keep looking at such this interpretation, alright okay. So remember this and we are going to keep looking that is once we look at courier transform of a continuous signal again we are going to relook at this interpretation of what is the distribution of the power? How is the various spectral components that is amongst the various frequency components and this Parsevals relation gives us a way to interpret this power of the periodic signal as being distributed amongst the various spectral components, which magnitude Ck square characterizing the power of the signal in the kth spectral component or at the kth harmonic k times F0, alright.

So let us stop this conclude this module here, what we have looked at in this module is an example of how to derive the discrete courier series representation of a simple periodic signal that is a periodic pulse tray and also the Parsevals Theorem for the power of a periodic signal, alright. So we will stop here and continue with other aspects in the subsequent modules, thank you.