Principles of Communication- Part I
Professor Aditya K. Jagannathan
Department of Electrical Engineering
Indian Institute of Technology Kanpur
Module No 5
Lecture 27
Properties of Vestigial Side Band Filter of Reconstruction of Message Signal without Distortion

Hello welcome to another module in this massive open online course. So we are looking at vestigial sideband modulation which is basically a form of upper sideband or single sideband modulation and which the filter is non-ideal and let us look at the properties of this non-ideal VSB filters.

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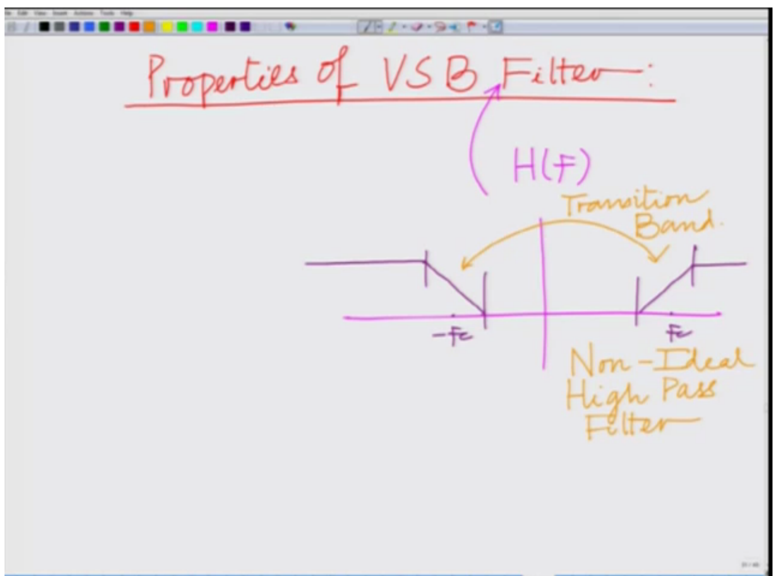

So we are looking at the property of this VSB filter and let us say this has impulse this has spectrum HF, correct? This non-ideal VSB filter has spectrum HF which as we already said it has it is a high pass filter but it is not an ideal high pass filter, so it has a finite transition band, correct? So this is minus Fc and this band these 2 bands these are your transition bands, correct? Where it is rising from where the impulse where the responses lies between 0 and 1 that it is rising from 0 to 1 that is known as the transition band and this is a non-ideal high pass filter, correct?

Non-ideal high pass filter at with cut-off frequency Fc this is a let me just write it again clearly this is a non-ideal high pass filter. As we already said we prefer this non-ideal high pass filter we prefer this because designing I Ideal high pass filters or implementing ideal high pass filters as a very high complexity, okay. Require a large number of stages to implement the ideal high pass filter that's one where is this non-ideal high pass filter can be implemented with relatively lower complexity, alright. So this is preferred in a practical implementation, okay. So let us look at a schematic for vestigial sideband modulation, okay.

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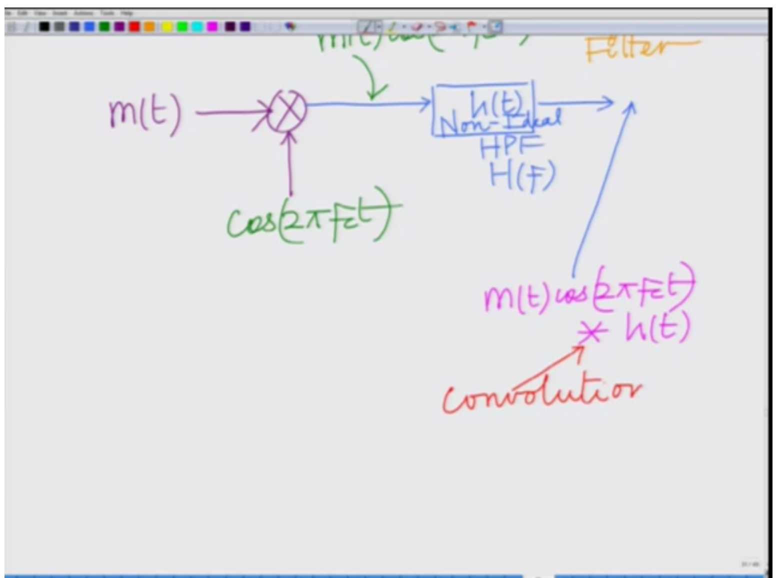

So we have our signal m(t) which is as usual modulated with cosine 2pi Fct first because remember we are using the frequency discrimination, okay. Frequency discrimination technique except that we are using non-ideal so it is modulated cosine 2pi Fct let us write this over here that is cosine 2pi Fct to generate m(t) cosine 2 pi Fct and that is pass through your linear time invariant system with impulse response ht or that is your non-ideal high pass filter impulse response ht and spectrum HF and this is basically the output. So naturally the output here will be this is basically your m(t) cosine 2pi Fct convolved with ht because modulated signal m(t) cosine 2pi Fct is passing through the non-ideal high pass filter with impulse response ht therefore the output is m(t) cosine 2 pi Fct convolved with the impulse response, okay. So this is the convolution, okay.

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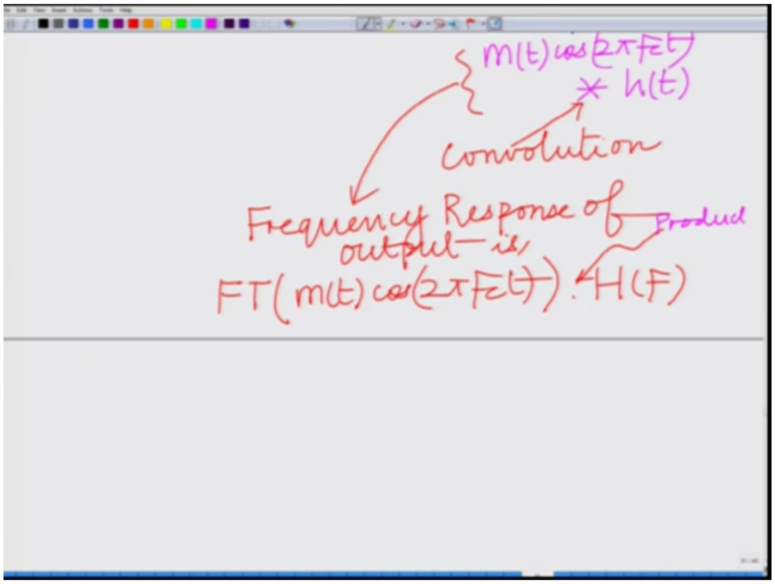

So that should be clear this is your convolution operation, okay which in the frequency domain means and then for the frequency response of this of the output frequency response of the output is that is the Fourier transform in the frequency domain it is the Fourier transform of m(t) cosine 2 pi Fct with multiplication because convolution in time domain this becomes your multiplication, okay. So that is also something that your must be very familiar with at this point. So this is this becomes a, your simple product. Let us write product instead of multiplication, okay.

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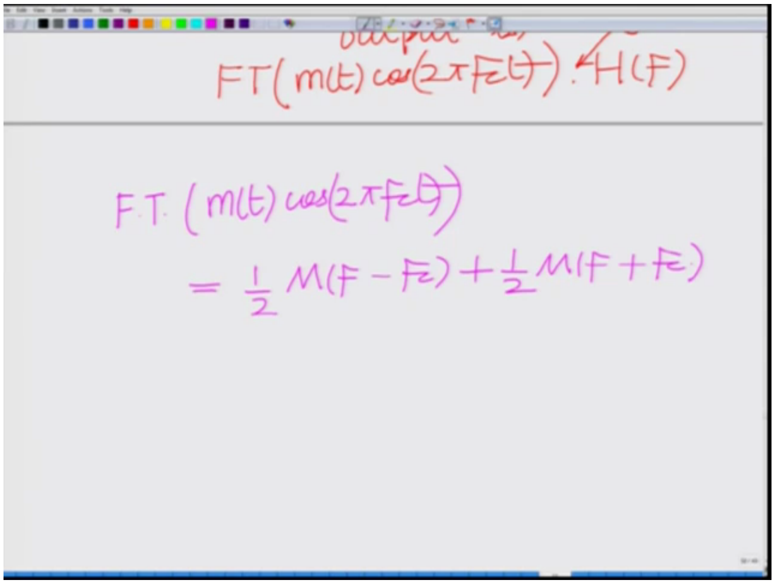

And now the Fourier transform of m(t) cosine 2pi Fct that is also something that we are very familiar with that is half MF minus there is Fourier transform MF shifted to Fc and MF shifted to minus Fc. So the Fourier transform of m(t) cosine 2pi Fct this is equal to half MF minus Fc plus half MF plus Fc.

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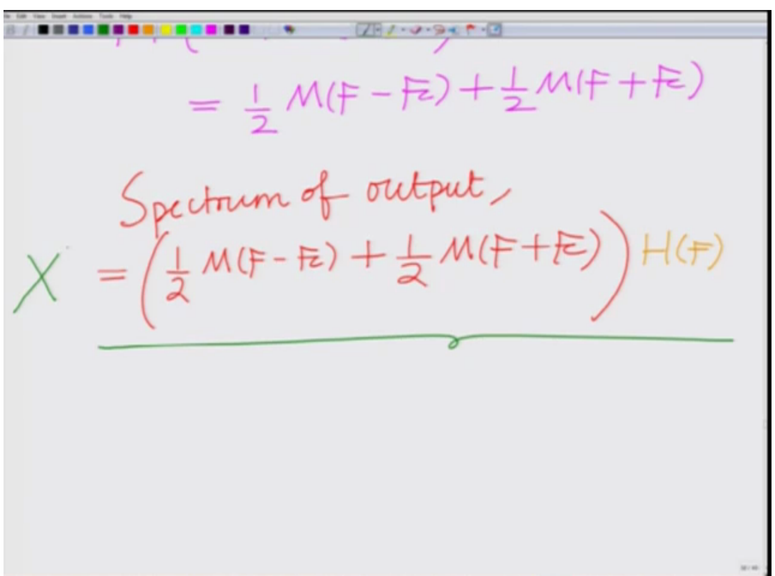

So the net Fourier spectrum of the output, so spectrum of the output of the VSB modulated signal that is spectrum of the VSB modulated signal is half MF minus Fc plus half MF plus Fc times HF, okay. So this is basically the spectrum let us call this as X(F).

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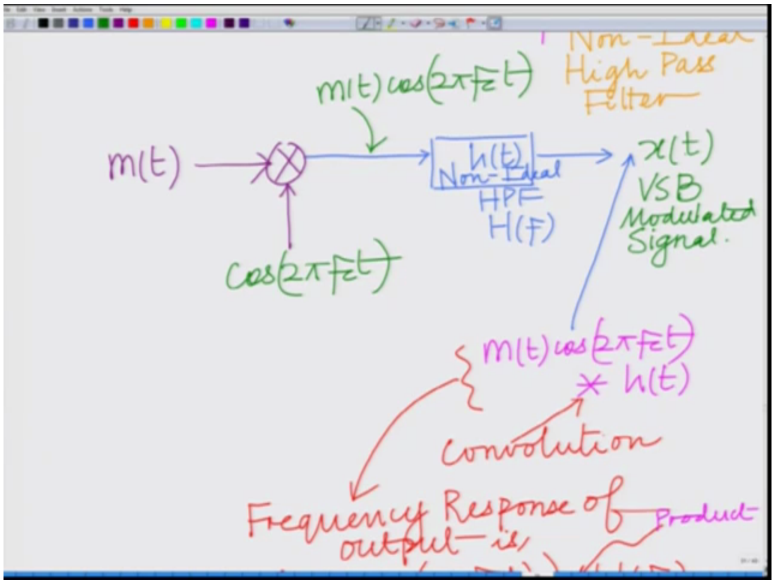

If we call the signal as x(t) here the VSB modulated signal as x(t) this is our VSB vestigial sideband modulated if x(t) is the vestigial sideband modulated signal let say it has spectrum X(F) that is this x(t) has spectrum X(F).

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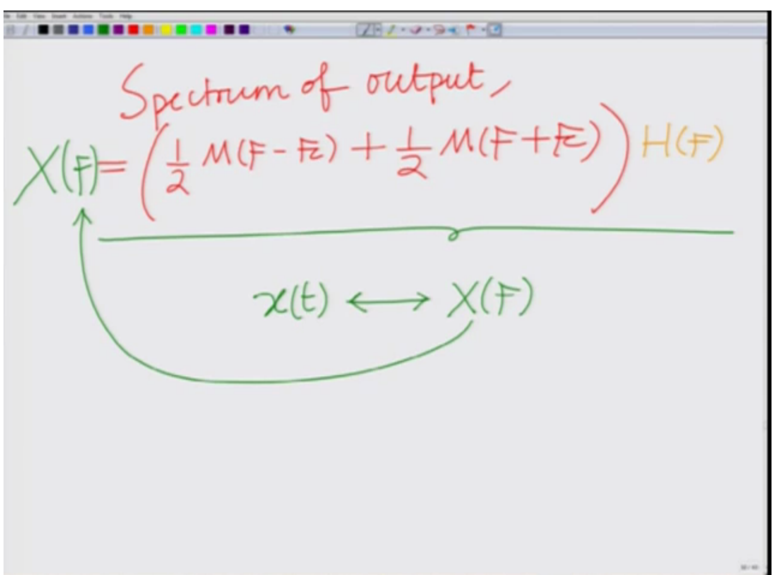

So output signal x(t) VSB modulated signal x(t) where X(F) x(t) has spectrum X(F) and that X(F) is given by this that X(F) is given by this that is half MF minus Fc plus half MF plus Fc that is the spectrum of m(t) cosine 2pi Fct multiplied by HF which is the spectrum of the non-ideal high pass filter, okay which has a transition band around Fc, okay.

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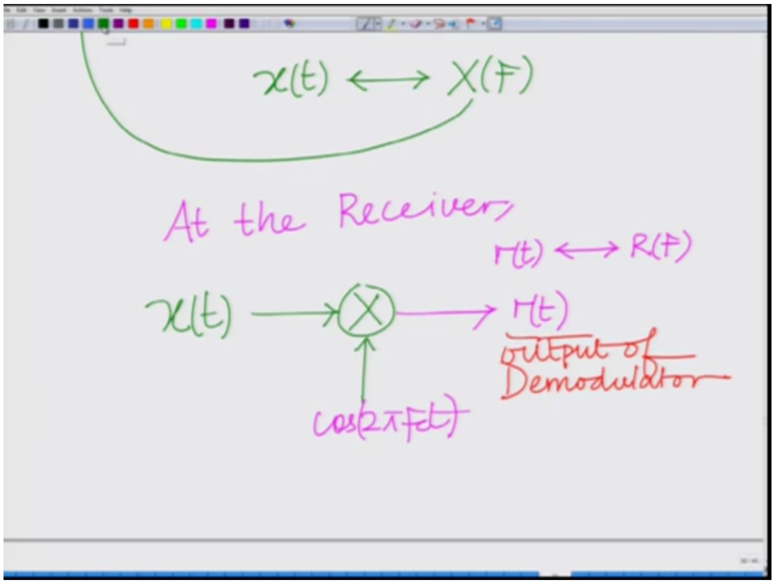

Now at the receiver consider demodulation at the receiver we have your x(t) which is the received signal this is again demodulated by passing through by demodulating with your cosine 2 pi Fct and that gives rise to the output signal, okay. So let us call this output signal as rt with spectrum let us say rt is the output of the demodulator that has spectrum RF, okay. So rt this is the output of this is the output of the demodulator, okay.

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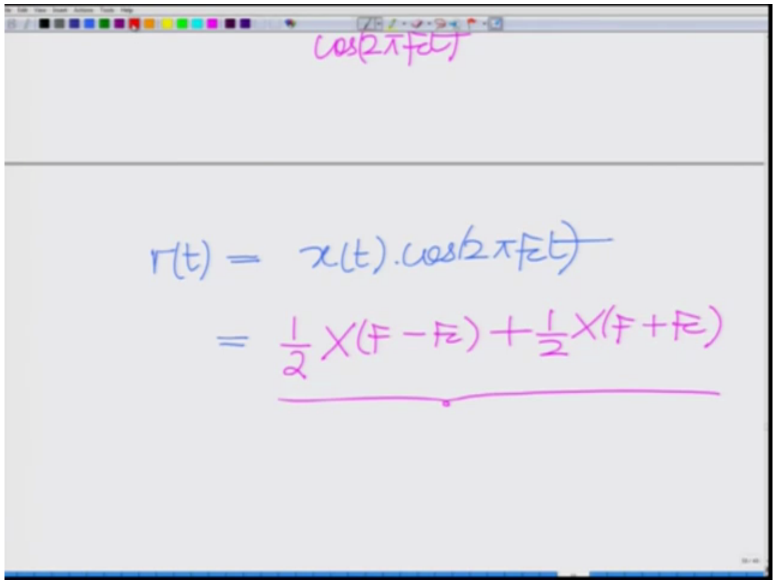

So this is basically rt and see rt is basically your x(t) times cosine 2 pi Fct and therefore rt is basically demodulated x(t) times cosine 2pi Fct therefore its spectrum is naturally half X(F) minus Fc plus half X(F) plus Fc, okay. So this so x(t) cosine 2pi Fct it's spectrum is half X(F) minus Fc plus half X(F) plus Fc, now substitute for X(F).

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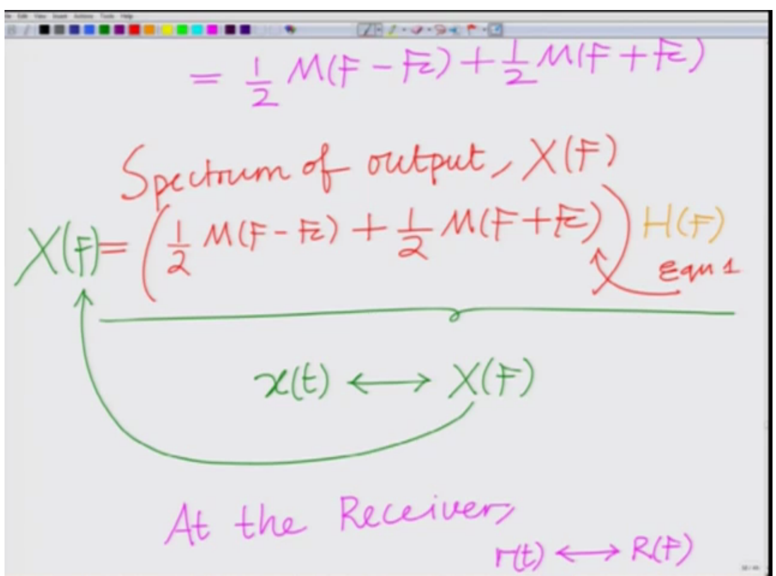

Now in this what we are going to do is substitute, so if we call this is a spectrum for X(F), okay spectrum of output that is X(F). So if you call this as equation 1, now what we are going to do is substitute for that is equation 1 tells us that X(F) is half MF minus Fc plus half MF plus Fc into HF substitute that spectrum expression for the spectrum X(F) in this expression for the spectrum rt, okay.

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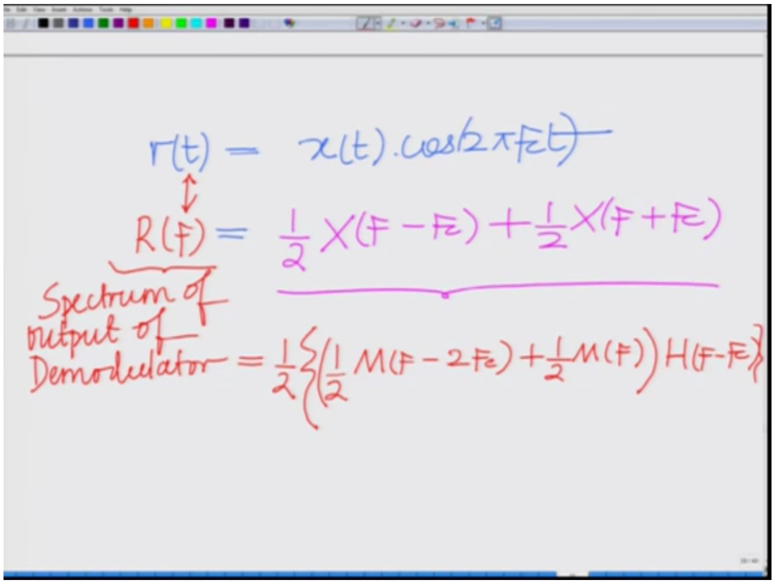

By the way, this is RF that is the spectrum of output, okay. This is a spectrum of output of spectrum of output of spectrum of the output of the demodulator that is equal to half, okay. And instead of X(F) minus Fc I am going to substitute that expression, so X(F) minus Fc so we have half X(F) minus Fc is half MF minus 2Fc plus half MF MF, so this is half so X(F) minus FC is half MF minus 2Fc MF plus Fc minus Fc that is half MF times HF minus FC, okay.

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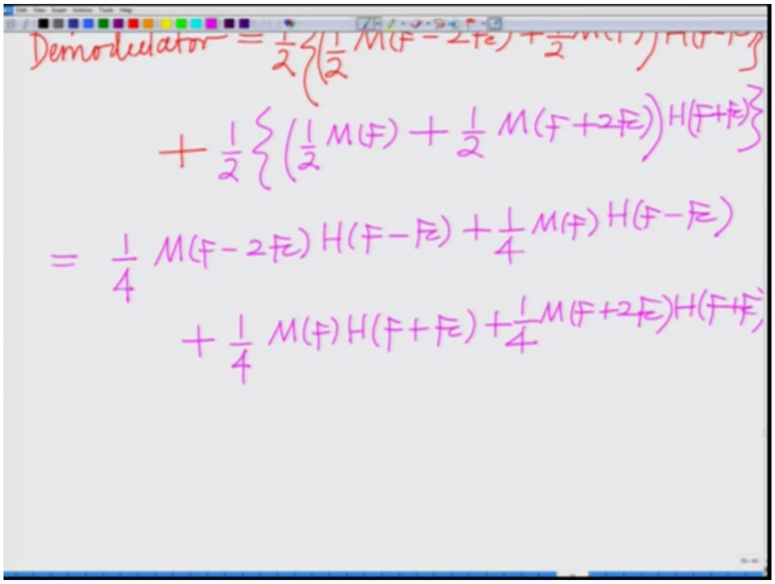

So this is your basically half X(F) minus Fc plus half now X(F) plus FC, so that will be half MF minus FC plus FC so half MF plus half MF plus FC plus Fc MF plus 2Fc into HF plus Fc, okay. So this is the net expression. Now if you look at this now let us write this term by term so this is 1 by 4 MF minus 2Fc into HF minus Fc plus 1 by 4 MF into HF minus Fc plus 1 by 4 MF into HF plus Fc plus 1 by 4 MF plus 2Fc into HF plus Fc, okay. So these are the 4 terms you get from the Expansion of the output of the demodulator. So this is still we are talking about the spectrum of the output signal of the demodulator that is we are talking about the spectrum RF.

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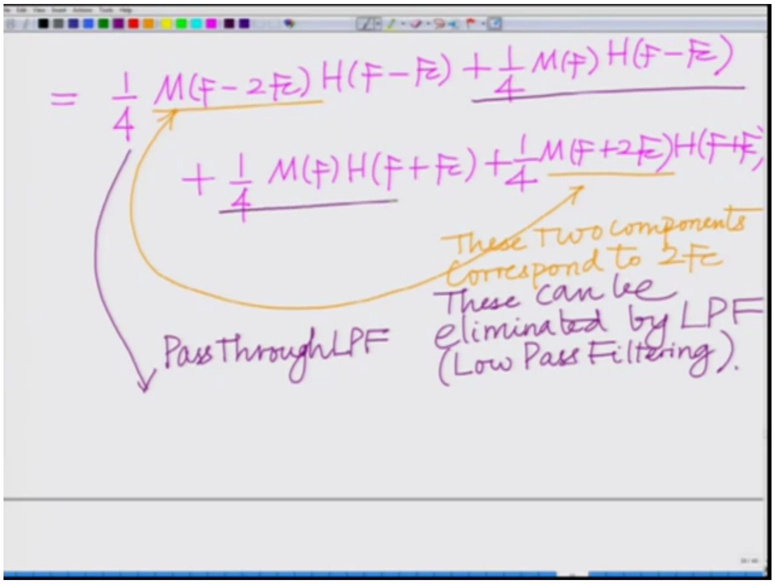

Now if you look at this spectrum which we have simplified by substituting the expression for the spectrum of X(F) that is the spectrum of the X(F) which is the spectrum of the VSB modulated signal x(t), now you can see that MF minus FC look at these 2 MF minus Fc and MF plus 2Fc, so these 2 components correspond to 2Fc are centered at 2Fc MF minus Fc MF minus 2 Fc is MF shifted to 2Fc and MF plus 2Fc is MF shifted to minus 2Fc, so these 2 are centered at 2Fc. So these can be eliminated by low pass filtering, hence these can be eliminated by low pass filter.

So when we eliminate these by low pass filtering only the 2 other components are going to remain, so that is the central idea. So now we are going to low pass filter it, now these can be eliminated by LPF, okay. That is your, hence now if you pass this through a low pass filter pass that is pass RT through LPF’s only these components are going to remain that is 1 by 4 MF into HF minus Fc into 1 by 4 into HF plus Fc.

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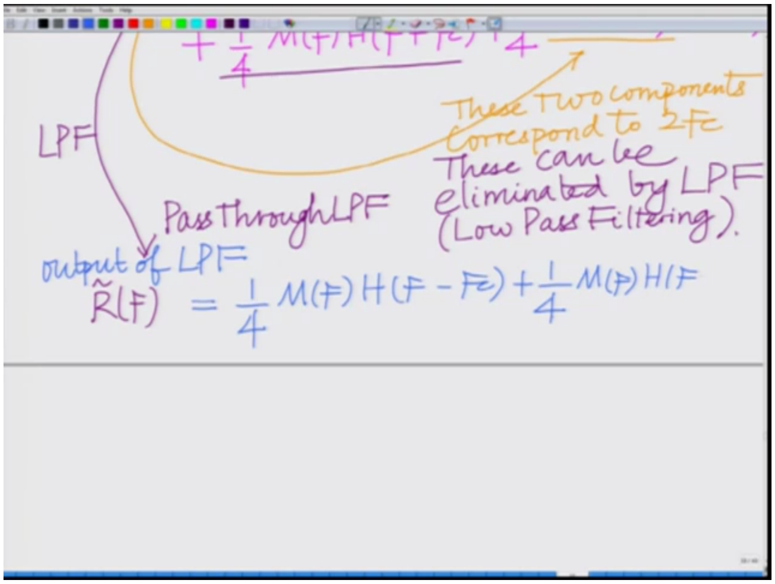

So once you pass this you LPF and let us say you get R tilde F that is output of LPF. So you are R tilde t is output of LPF which is 1 by 4 MF into HF minus Fc plus 1by 4 into MF into HF plus Fc, okay. So 1 by 4 MF into HF plus Fc.

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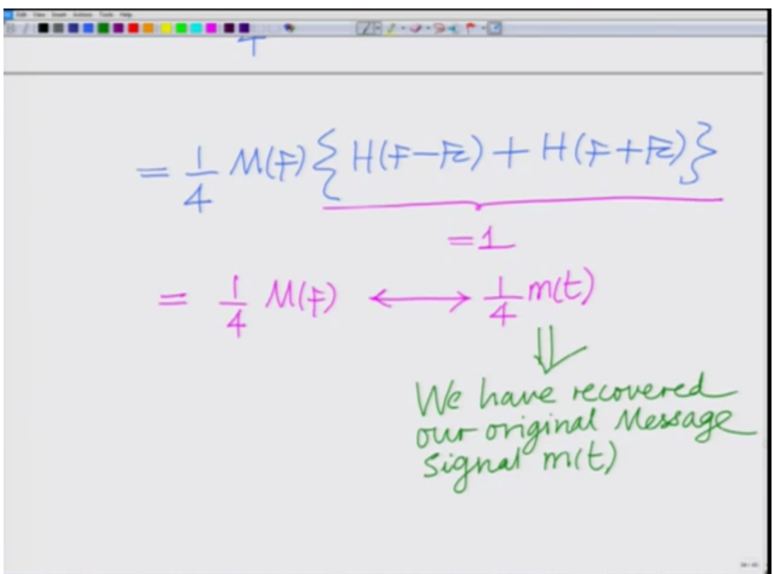

And now if you take MF common you can see this is 1 by 4 MF into HF minus Fc plus HF plus Fc. Now therefore if we can set these 2 a constant HF minus Fc, so you can see the net output spectrum after low pass filtering is 1 by 4 MF into HF minus Fc plus HF plus Fc. Now if you can set this HF minus Fc plus HF plus Fc equal to sum constant in particular let us say this is one than the output spectrum will be 1 by 4 MF which is basically proportional to the spectrum of m(t) that is in fact the output signal will be 1 by 4 m(t).

So if this is equal to one than this is simply 1 by 4 MF which in the time domain corresponds to 1 by 4 m(t) and therefore we would have recovered our which implies that we have recovered our implies that we have we have recovered our original we have recovered our original signal m(t). But note that the requirement is HF minus Fc plus HF plus Fc should be equal to one, so that is the property that the VSB filter that the non-ideal VSB filter must satisfy, okay. So let us look at that property, so we have HF minus Fc.

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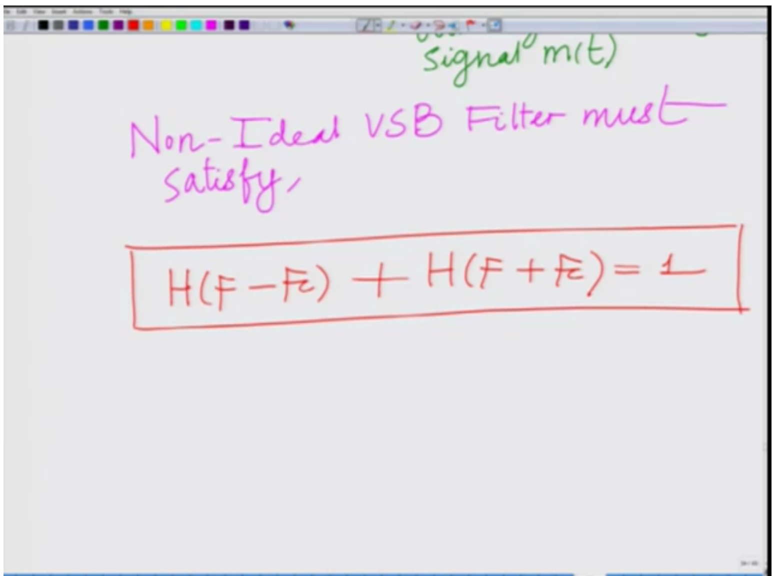

So VSB filter must satisfy so the non-ideal VSB filter must satisfy for recovery distortion-less recovery non-ideal VSB filter must satisfy the property HF minus Fc plus HF plus Fc equal to one non-ideal this is the property that your non-ideal VSB filter must satisfy, okay.

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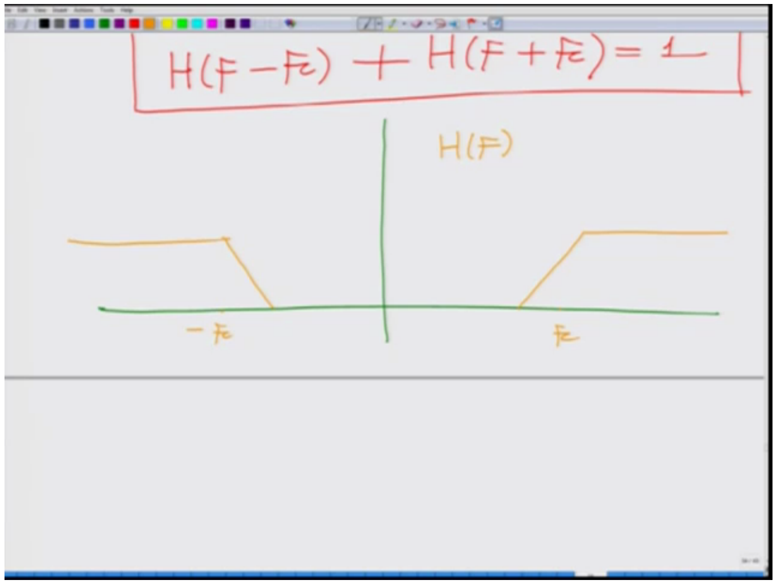

So let us look at this property that the non-ideal VSB filter must satisfy, so this is your HF, correct? And let us look at this; this is minus FC so this is your HF, okay.

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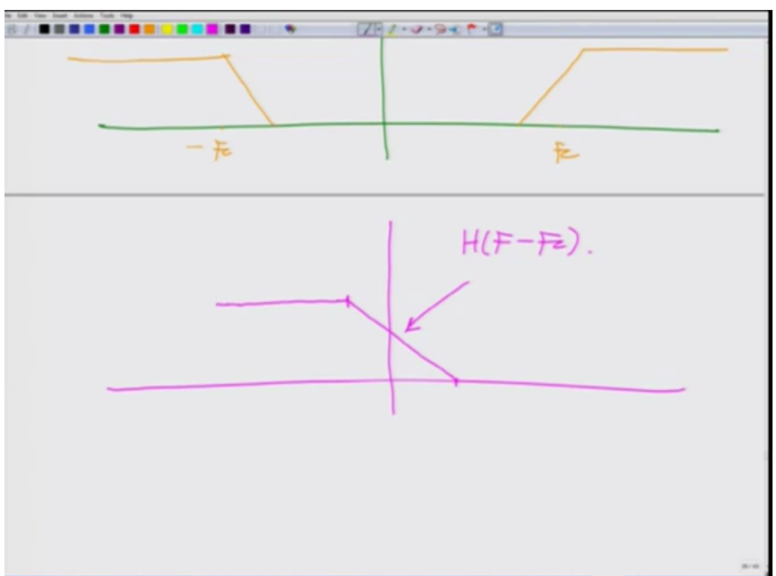

And now what we are going to do is HF minus Fc is basically HF shifted by Fc so HF minus Fc is (shi) HF shifted that is advance by Fc, so we have HF minus Fc so the band , okay. So this is basically your HF minus Fc and plus there will be some other components at 2 Fc we are not worried about that, alright. So we are only worried about this component which is in the baseband that is HF minus Fc.

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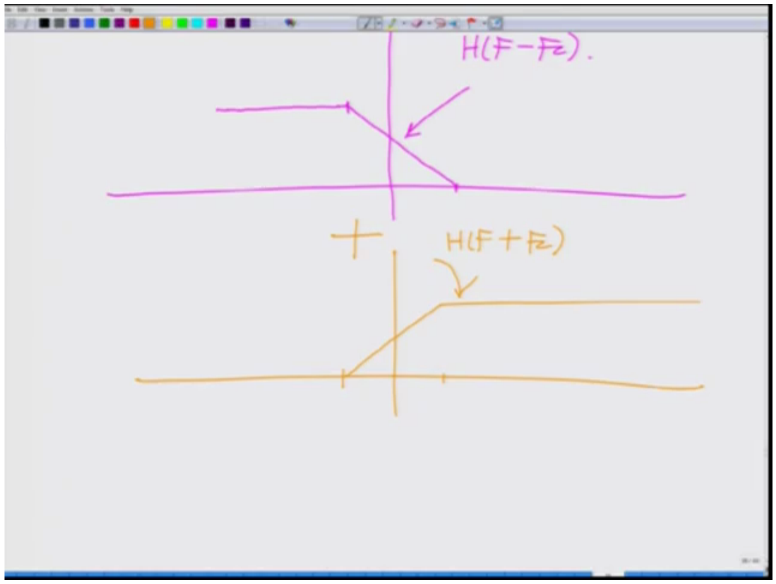

Now let us look at HF plus Fc and if you look at HF plus Fc that looks something like this. So this is your HF plus Fc that is basically your component that is at Fc the shifted backward to the 0 frequency that is HF plus Fc that is shifted by minus Fc that is shifted by Fc to the left.

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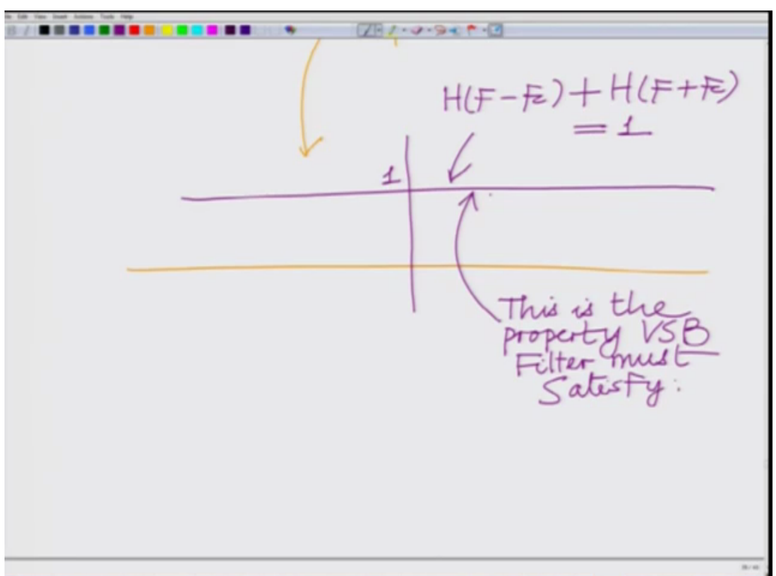

Now when you add these 2 that is HF minus FC plus HF plus FC the VSB (propert) the VSB filter should obey the property that VSB filter must satisfy the property that when you add these 2 you get you get unity that is you should get unity that is HF minus FC plus HF plus FC equal to one. So this is the property that the VSB filter must satisfy. So this is equal to one this is the property that the VSB filter must satisfy and you can see this property is satisfied if the VSB filter is symmetric about Fc.

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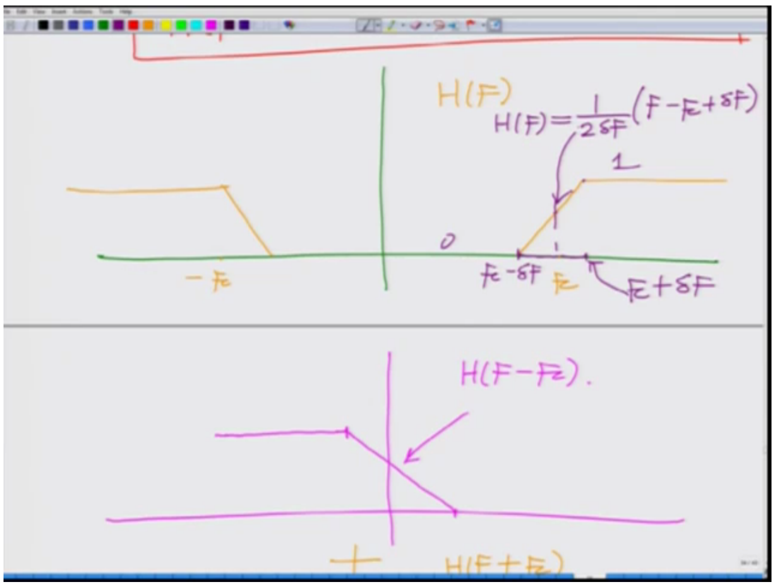

That is if you can see if these VSB filter is symmetric about FC that is basically if this is symmetric about Fc that is this is let us say Fc minus delta F this point is let us say Fc plus Delta F and then if its symmetric about Fc then you can see that symmetric about Fc in the sense that you can see that this is linear linearly increasing between FC minus Delta F to FC plus Delta F the midpoint is at FC. So if this is symmetric about Fc then this property is satisfied or in other words between FC minus Delta F to FC plus Delta F this has to be HF equals 1 over you can see this linear part corresponds to 1 over the slope is 1 over that is it is rising.

So this is one this is 0 so slope is 1 over it is rising to 1 in interval of 2 Delta 1 over 2 Delta F times F minus Fc plus Delta F. So if this property satisfied that is if this filter HF is symmetric about Fc that is starts from Fc minus Delta F to Fc plus delta F and rises from 0 to 1 linearly then you can see that when you shift H Fc to so when you consider HF plus Fc and HF minus FC and superimpose them or add them in the baseband they will sum up to 1.

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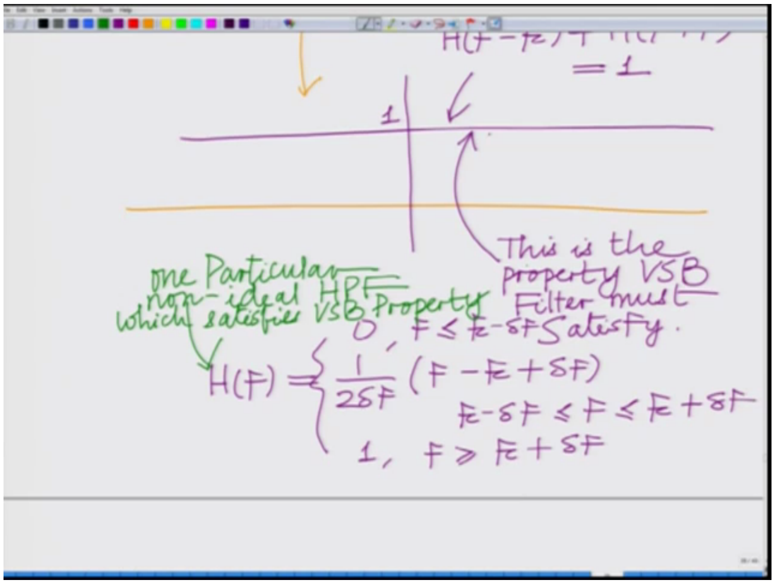

So the property that you can see the this is the property that the VSB (silter) filter must satisfy HF minus FC plus HF plus FC equal to one, one such filter has the response HF equals 1 over twice Delta F in F minus FC plus Delta F for FC minus Delta F less than or equal to F less than or equal to FC plus Delta F and HF, so HF equal to this equal to 0. So for F less than or equal to Fc minus delta equal to 1 for F greater than or equal to FC plus Delta F. So this is one particular filter one particular VSB filter.

So this is one particular filter non-ideal one particular ideal filter non-ideal HPF non-ideal HPF which satisfies VSB property, okay. This is one particular non-ideal high pass filter which satisfies the VSB property, alright. So what we have seen in this module what we have seen is basically we require that is one can implement a non-ideal version of single sideband modulation, alright. In this particular place we have chosen upper side band modulation and non-ideal version of upper sideband modulation which is termed as vestigial sideband modulation with a non-ideal high pass filter, alright.

And we have shown if the high pass filter satisfies certain property that is HF minus FC plus HF plus FC is equal to unity then you can reconstruct, correct? You can reconstruct this VSB you can reconstruct the transmitted message signal again m(t) without any distortion by again passing through the demodulator that is demodulating with cosine 2 pi Fct at the receiver. However the disadvantage of the VSB modulation still remains that it requires a slightly larger bandwidth in this particular place you can see that the transition bandwidth is FC minus F Delta F to Fc plus Delta F. So requires a (addi) additional bandwidth of Delta F, alright which the previous upper side modulation upper sideband modulation scheme did not require, alright.

So requires a slightly (addi) slightly larger bandwidth that is FM plus Delta F where FM is the maximum frequency, so previously upper sideband modulation upper sideband modulation required only FM, correct? Now this requires FM plus FC which is slightly larger than FM but of course it is a trade-off it allows for a lower complexity the design of the non-ideal high pass filter and if this non-ideal high pass filter satisfies this property HFc minus F HF minus Fc plus HF plus FC equal to unity then that allows the reconstruction of the message signal m(t) without any distortion by simply demodulating with cosine 2 pi Fct.

Now similar condition can also be derived for the vestigial sideband modulation with the lower sideband that is non-ideal low pass filter, alright. I will let you explore that, alright. So we will stop this module here and continue with other aspects in the subsequent modules, thank you.