Principles of Communication- Part I
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Module No 5
Lecture 24

Hello welcome to another module in this massive open online course, so today let us start looking at the (con) concept of the complex envelope and complex pre-envelope of a signal, okay.

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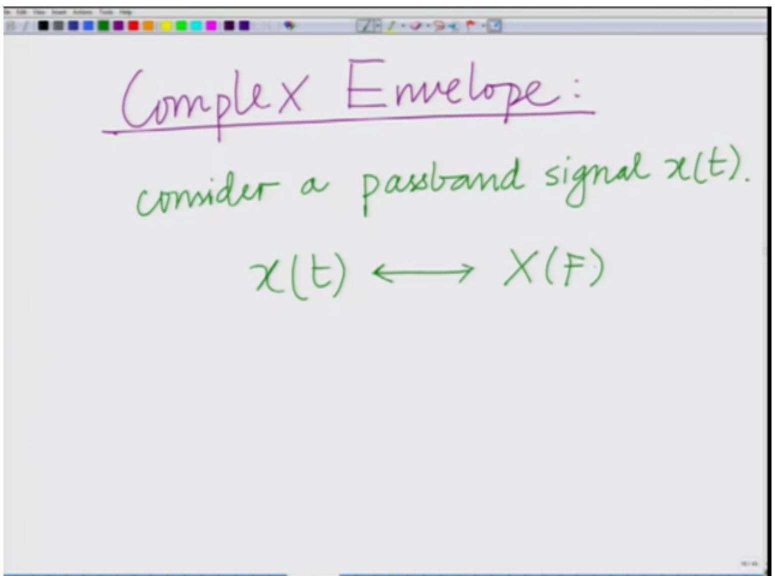

So you want to look at the complex complex envelope but to understand that first we have understand the concept of a complex pre-envelope, so towards this consider a pass band signal, consider a pass band signal so consider a pass band signal x(t). Consider a pass band signal x(t) give with Fourier transform X(F), so naturally a pass band signal has a Fourier transform that is centered around carrier frequency Fc.

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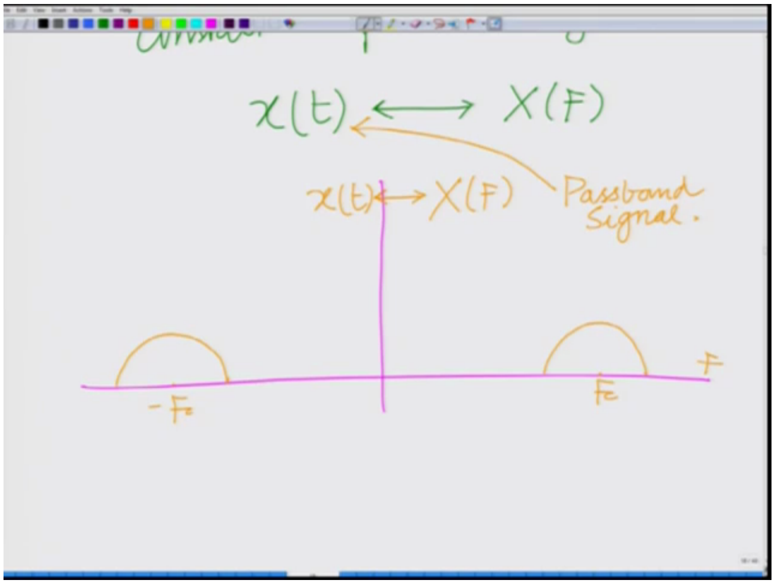

So let us start by considering a pass band signal X(F) pass band signal x(t) which has Fourier transform that is centered around the carrier frequency that is Fc, okay. So so I am doing the spectrum of this pass band signal which is centered at at carrier frequency Fc, so let us say this is the spectrum X(F), okay. We have x(t) is the pass band signal it has for spectrum X(F), so your x(t) this is basically the pass this is basically the pass band signal. Pass band signal in the sense that it has been modulated by a carrier therefore its spectrum is centered at the (carr) frequency carrier frequency that is that Fc and at minus Fc, since it is a real signal it is symmetric, correct? So it has spectral band it has basically it has 2 bands one at centered at Fc and one centered at minus Fc.

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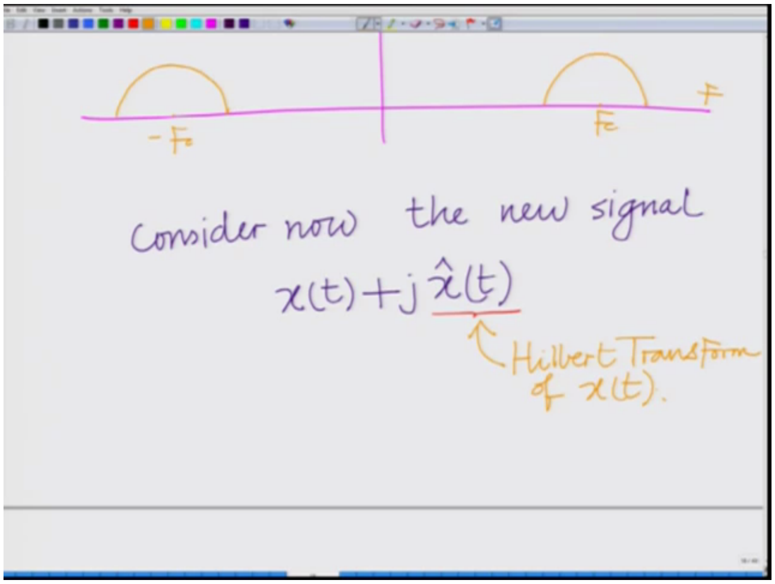

Now consider the signal consider now before my new signal, now the new signal x(t) plus j c hat of t where c hat of t we know this is the Hilbert transform of x(t) x hat of t is the Hilbert transform of x(t) is the Hilbert transform of x(t) , okay. So x we are now forming a new signal which is x(t) plus j x hat of t where x hat of t is the Hilbert transform of xt we have already seen the concept of Hilbert transform.

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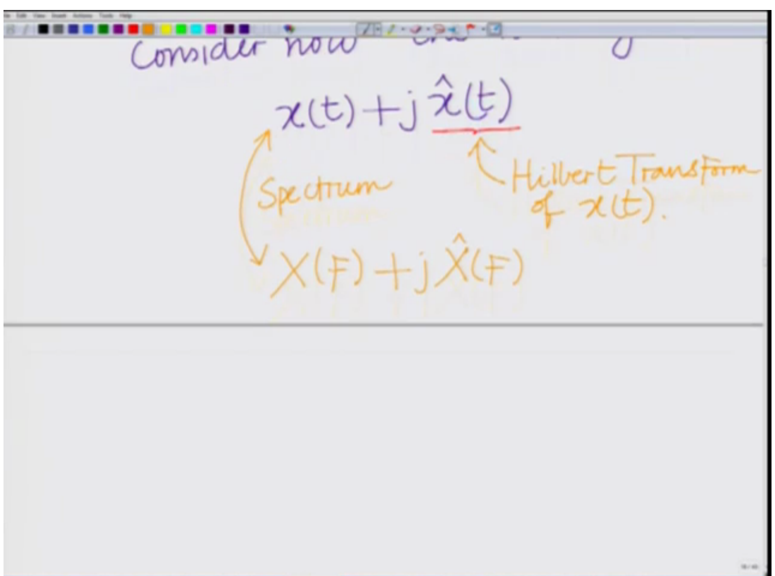

Therefore the spectrum of this signal X(F) will be j X(F) plus times spectrum of x hat of t which is X(F). So this is a spectrum of this. So it's spectrum or Fourier transform of this signal x(t) plus j x hat of t will be X(F) plus j x hat of F, we have already drawn X(F) that is a spectrum of x(t). Let us now see what is a spectrum x hat F going to? What is a spectrum of X(F) plus j x hat? To do that first letter start with x hat of F.

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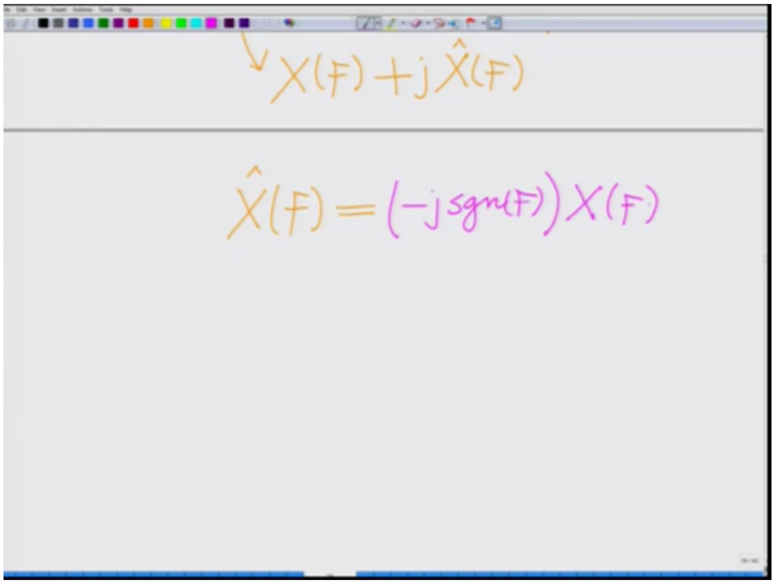

Now we know x hat of F is the Hilbert transform spectrum of the Hilbert transform this is equal to basically minus j sgn F minus j sgn F times X(F).

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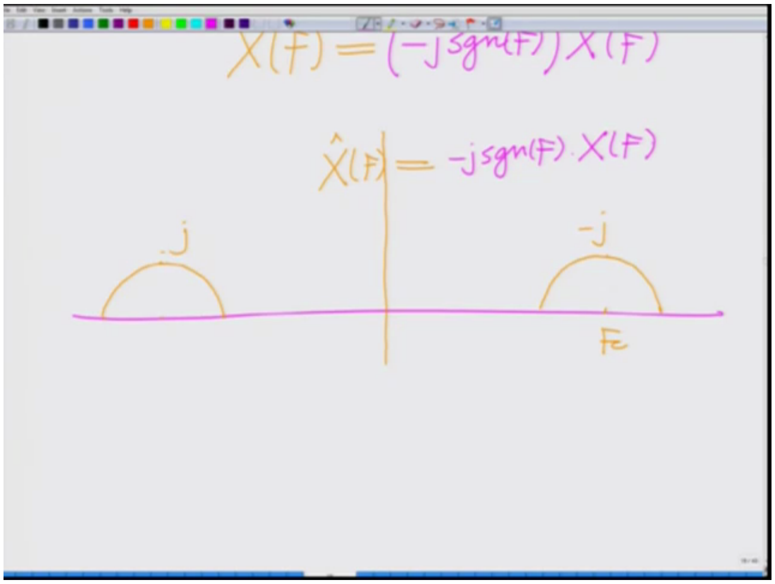

Now minus j sgn F is minus j if F is greater than 0 and plus j if F is less than 0, let us draw the spectrum x hat F which means it is minus j, so x hat of F is minus j sgn F times X(F), correct? Which means the positive band at Fc is multiplied by minus j this is minus j sgn F and the negative band that is if you look at this band which is at minus Fc this band this band will be multiplied by, this band will be multiplied by j, correct? So since this is minus j sgn F, so this band will be multiplied by j so this is x that from x hat of F spectrum of x hat of T which is minus j sgn F times x of F.

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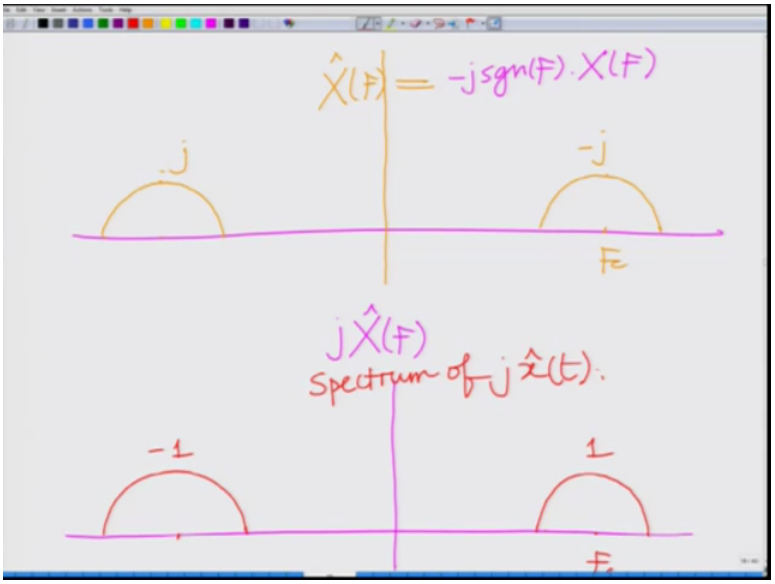

Now j times x hat of F, if you consider j times x hat of F that is going to be multiply the spectrum of x hat of F by j which means the positive side band which is already multiplied by minus j will be multiplied by that will be multiplied by j, so j minus j into j that will give us a scaling of basically 1. So this will give us scaling of 1, this is at your carrier frequency Fc and this is at carrier frequency minus c which multiplied by the negative side the negative band is multiplied by j multiply that by another j that is j x hat of F that will give j times j which is j times j which is minus 1. So this is the spectrum of spectrum of j, this is basically j x hat of F where x hat of F is a spectrum of x hat of t, okay. Or you can say this is a spectrum of j x hat of t, j x hat of F is a spectrum of j x hat of t.

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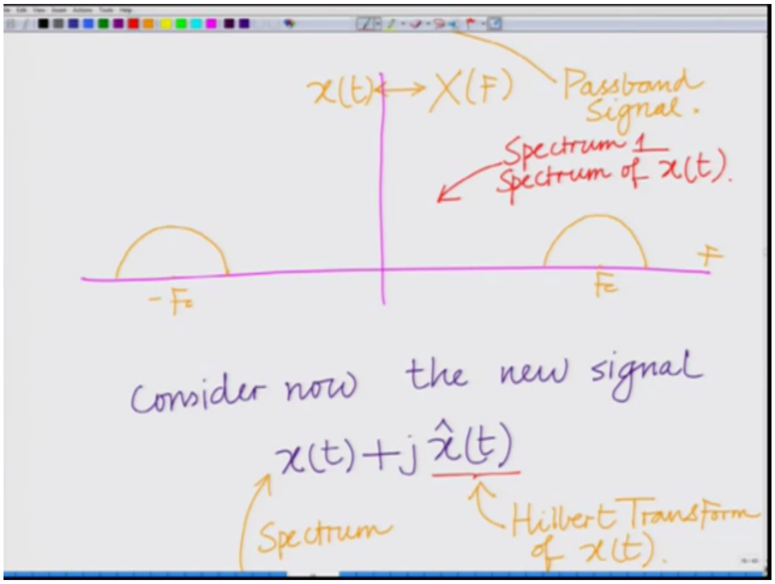

Now the spectrum of x(t) plus j x hat of t, so let us say because this spectrum one that is spectrum of x(t) spectrum 1 this is basically your spectrum of x(t) this is spectrum one.

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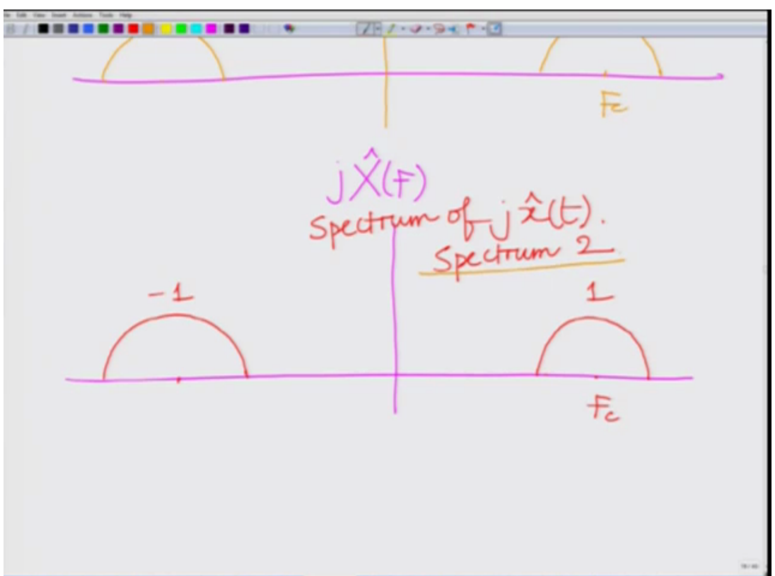

And this is let say spectrum 2, spectrum of j x hat of t, this is spectrum 2, okay. This is so we have spectrum one and we have spectrum 2, alright. And now we have to find the spectrum of x(t) plus j x hat of t, j x hat of t.

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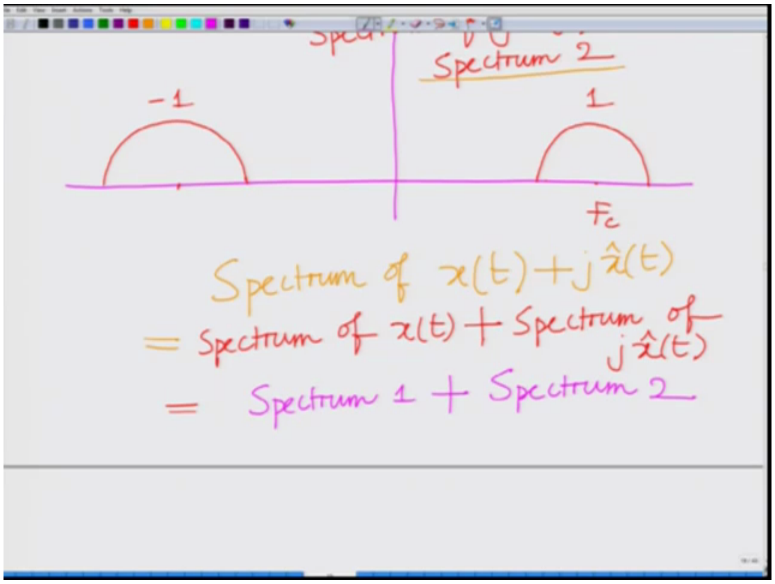

Naturally that will be spectrum of x(t) plus spectrum of j x hat of t, spectrum of x(t) plus spectrum of j x hat of t which is basically equal to spectrum 1 plus the spectrum 1 which is spectrum of x(t) plus spectrum of j x hat of t which is the spectrum 2. So what we have derived is we are trying to find the spectrum of x(t) plus j x hat of t. We derive the spectrum of x(t) we have derived the spectrum j x hat of t we are now trying to add these 2 spectra and when you add these you can see that here if you look at the spectrum of x(t).

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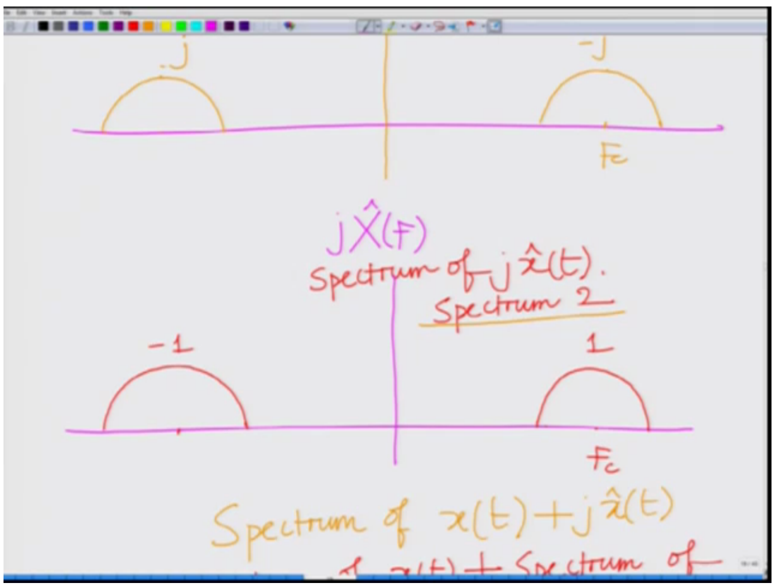

Now spectrum of x(t) is simply both the side bands scaled by one that is unity factor that is the default, now if you look at the spectrum of j x hat of t you can see that the positive band the band that is at Fc is scaled by 1 the band at minus Fc that is the negative band the negative frequency band is scaled by minus 1. So when you add these 2 the positive frequency band that is the frequency bands corresponding to the positive frequencies will add.

So you will have a net scaling factor of 2 and the frequency bands corresponding to (nev) negative frequency will cancel out each other that is 1 plus minus 1 0. So what you will have is net scaling factor 2 for the frequency band corresponding to the positive frequency and scaling factor 0 for the frequency band corresponding to the negative frequency and therefore what you will have the spectrum?

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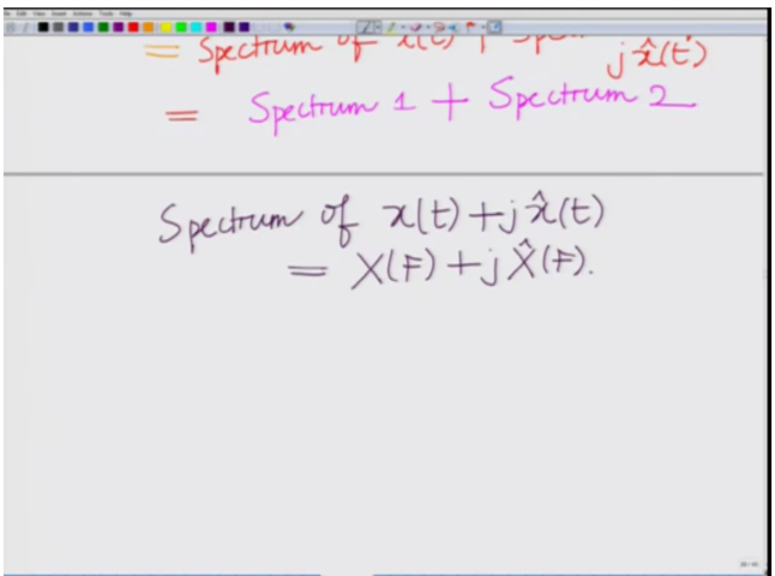

And now it is easy to (())(11:12) spectrum of x(t) plus j x hat of t equals X(F) plus j x hat of F.

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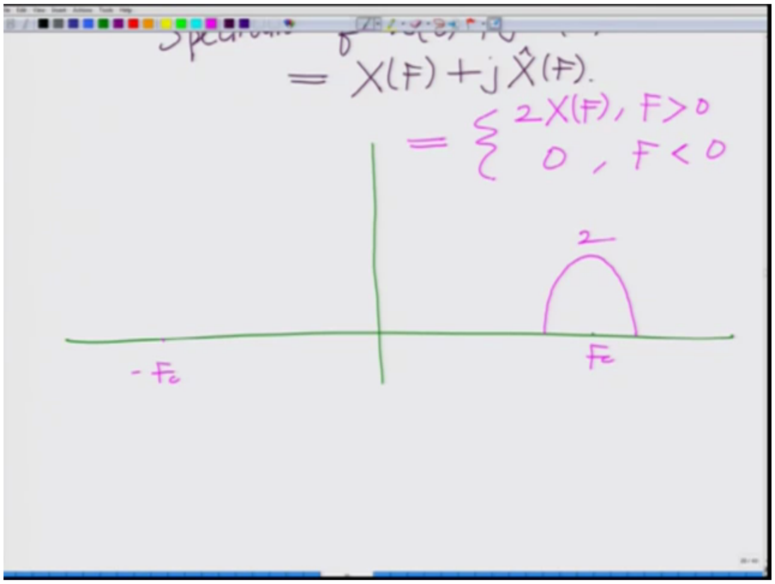

And we already found this. X(F) and j x hat of F, now when you add these in the positive frequency what you will have is you will have a scaling by a factor, so this is Fc the positive band is scaled by a factor of 2 the negative band is simply scaled by a factor of 0, so you basically you will have nothing so basically all you will have this is equal to, now you can see this is equal to twice X(F) for F greater than 0. 0 for F less than 0, so x(t) plus j times x hat of t has a spectrum X(F) plus j x hat of F which is basically the positive the the positive band.

That is the bands corresponding to frequencies greater than 0 these add up so you will have a scaling factor of 2 corresponding to the band of positive frequencies, correct? And the negative frequencies the bands the bands in the negative frequencies they cancel and therefore you have a scaling factor 0. So basically it is twice the original spectrum of X(F) twice the original spectrum X(F) for F greater than 0 that is for the positive frequencies and that is 0 for F less than 0 this is known as the complex pre-envelope of the pass band signal x(t).

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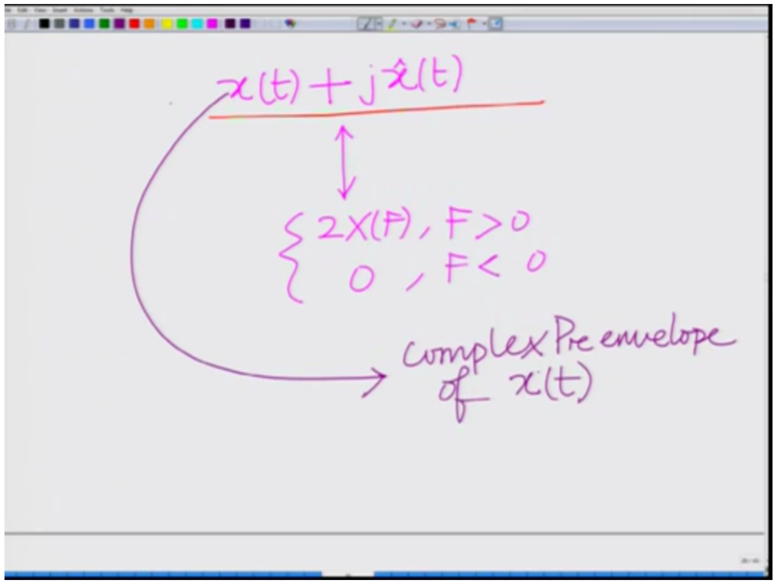

So this basically x(t) plus j x hat of t which basically has the spectrum which basically has the spectrum twice X(F) for F greater than 0, 0 for F less than 0 this signal is termed as the complex pre-envelope the complex pre-envelope of x(t) this signal is termed as the complex pre-envelope of x(t). That is we consider only the positive frequency band and scale it by a the negative frequency band is set to 0 consider the positive frequency band in x(t) and scale it by a factor of 2 negative frequency band is set to 0.

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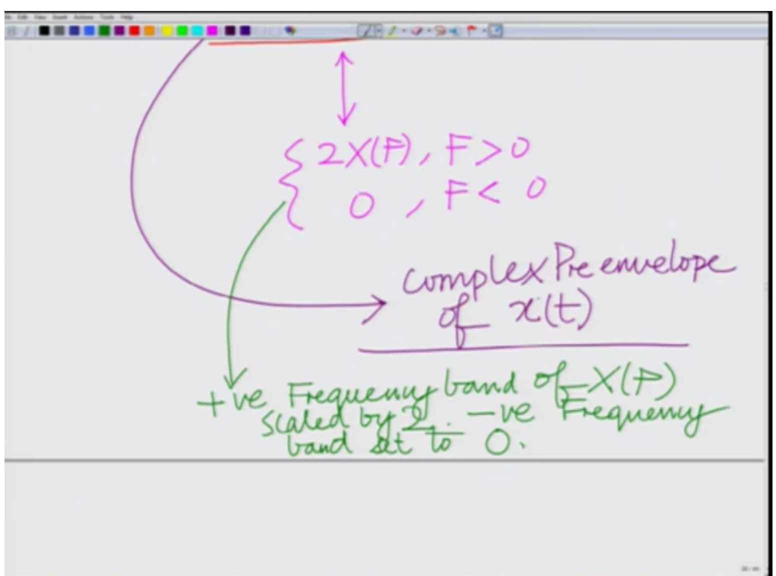

That is basically to describe it simply positive frequency band in x(t) x(t) or X(F) X(F) scaled by 2 and negative frequency band set to 0 this is known as the complex pre-envelope. Now you can see that this complex pre-envelope has only a band at Fc, if I shift it by minus Fc that is it is at Fc centered at Fc if I shift it by minus FC that is shift it to the left by Fc it will be centered at 0, so that will give me a (com) baseband signal but it will be a complex baseband signal, alright. So if I take this and let me draw it in a separate figure this is my complex pre-envelope sideband scaled by a factor of 2 if I take this and shift this by minus FC.

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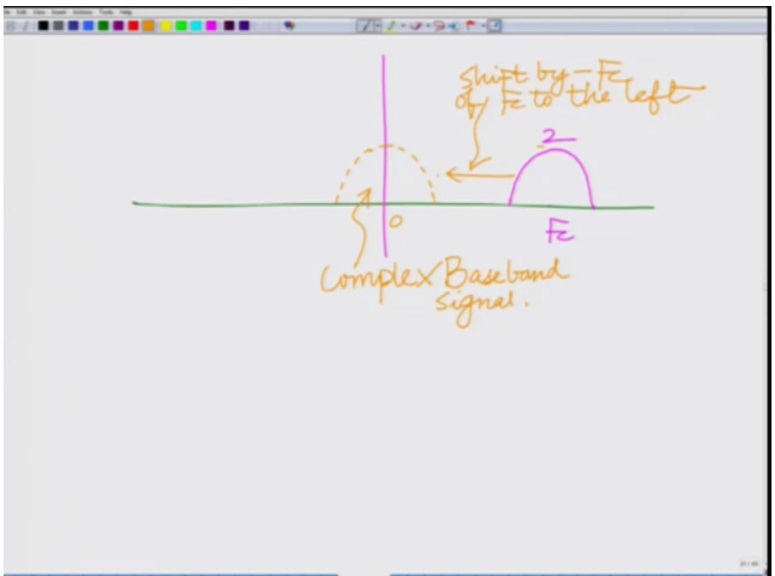

So let me draw this biased to shift it. So this is basically shift by minus Fc or Fc to the left this will be a complex baseband spectrum this will be spectrum of a complex baseband signal and remember shifting by minus Fc or shifting by Fc to the left is nothing but using the modulation property it is basically multiplying by e to the power of minus j 2pi Fct. So if I take this complex pre-envelope x(t) multiplied by e to the power of minus j 2pi Fct I will get complex I will get a baseband signal a complex baseband (sig) signal. This is known as the complex baseband equivalent of the pass band signal x(t).

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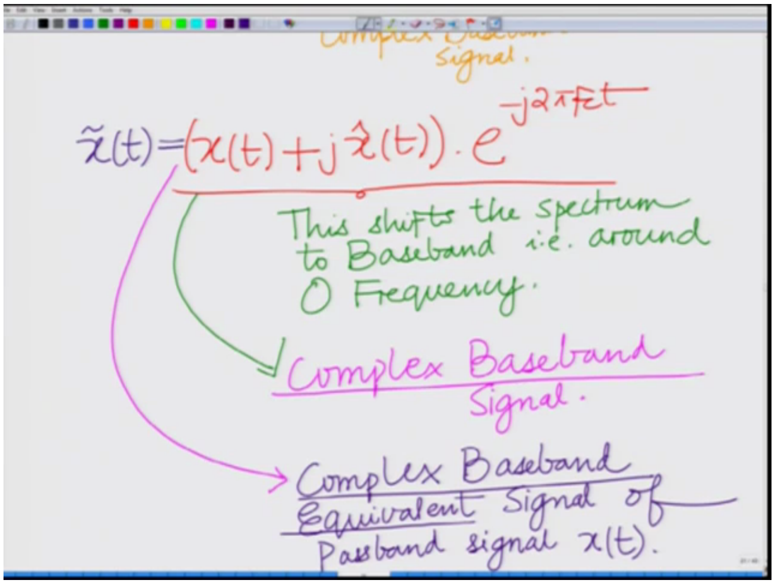

So basically now if I look at this complex pre-envelope and multiply it modulate it with e to the power of minus j 2pi Fct, this signal has a spectrum this shift the spectrum to the to the this shifts the spectrum to the baseband that is around 0 frequency, shifts the spectrum to around 0 frequency this is termed as the complex baseband equivalent of, so this is a complex baseband signal so this is a complex baseband signal this is a complex baseband signal this is termed as the complex baseband equivalent of the pass band signal x(t).

Complex baseband equivalent signal of complex baseband equivalent signal of the pass band complex baseband equivalent signal of the pass band signal x(t) that is and this is denoted by x tilde t. And you can see we have already shown this as a spectrum that is you take the positive frequency band of x(t) scaled by a factor of 2 shift it to the 0 frequency that is shifted by Fc to the left, okay.

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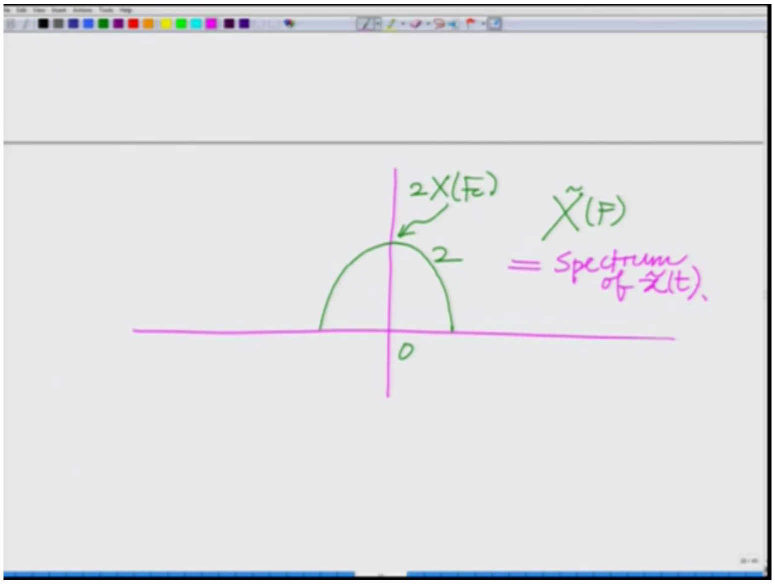

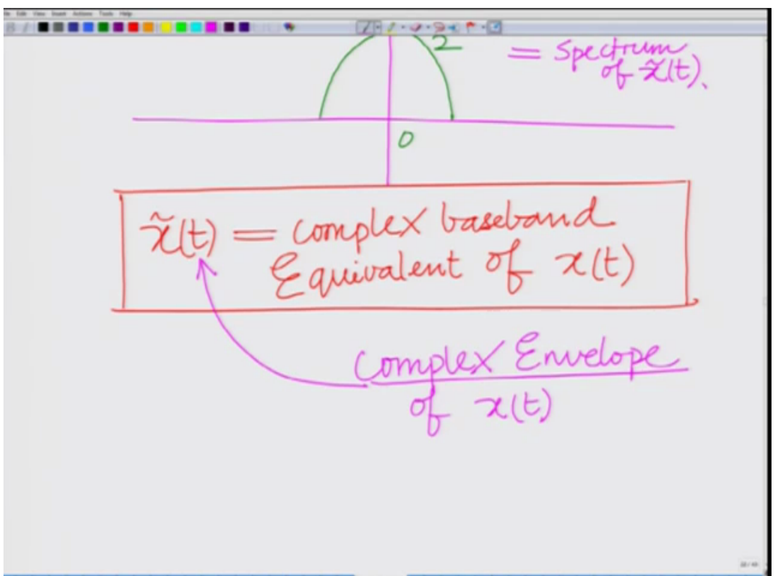

So the spectrum of the complex baseband equivalent signal is simply the positive frequency band of x scaled by a factor of scaled by a factor of 2. So therefore this point will be twice x of Fc that is the original spectrum x which is at Fc that scaled by a factor of x at point Fc scaled by a factor of 2. So this is basically this is basically at this point at the 0 frequency and this is basically what we are saying is this is your x tilde F this is X tilde F so this is X tilde F which is spectrum of X tilde t or the complex baseband equivalent signal of x(t).

So this is spectrum of x tilde t and it is worth noting again that x, so what is so we are taking this pass band signal x(t) and we have demonstrated that this passband signal any pass band signal x(t) can be reduced to an equivalent baseband signal a complex baseband signal this complex baseband equivalent (equiv) complex baseband equivalent signal is (rem) denoted by X tilde t which is given as x(t) plus j x hat of t times e to the power of minus j 2pi Fct.

So x tilde t is the complex baseband equivalent of the pass band signal x(t), okay. And it has a spectrum which is the (pos) spectrum the positive sideband the sideband correspond the bad corresponding to the positive frequencies shifted to the 0 frequency and scaled by a factor of 2. Further also note that this x tilde t is also known as the complex envelope of x(t) it is the complex baseband equivalent it is also known as the complex envelope of x(t) this is also another term this is also known as the, so we have seen the complex pre-envelope that is x(t) plus a times j times x hat of t. If you multiply that by e to the power of minus j 2pi Fct where Fc is a carrier frequency we will get the complex envelope that is shifted to 0 frequency given the complex envelope also the complex baseband equivalent of x(t), okay. So if the x tilde t is the complex envelope, okay. And why is this known as a complete envelope that is if you can look at this.

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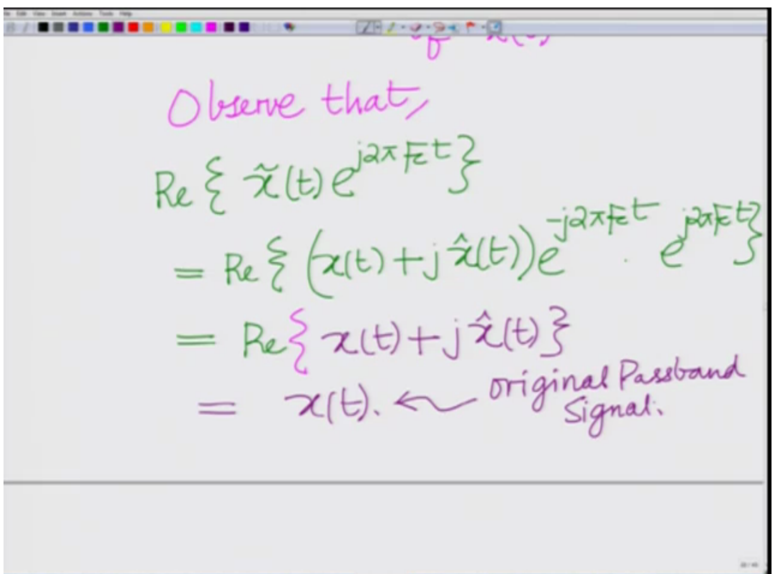

Let us look at this observe that observe observe that if you consider the real part of x tilde t e to the power of j 2pi Fct that will be the real part of, now x tilde t is x(t) plus j x hat of t into e to the power of j 2pi Fct into e to the power of minus j or into e to the power minus j 2pi Fct xet x(t) plus j x hat t to e to the power of minus j 2pi Fct into e to the power of j 2pi Fct. This e to the power of minus j 2pi Fct e to the power of j 2pi Fct these cancel, so I am left with the real part of x(t) plus j x hat of t and the real part of x(t) plus j x hat of t is simply x(t) which is your original pass band signal and this is the original this is the original pass band signal.

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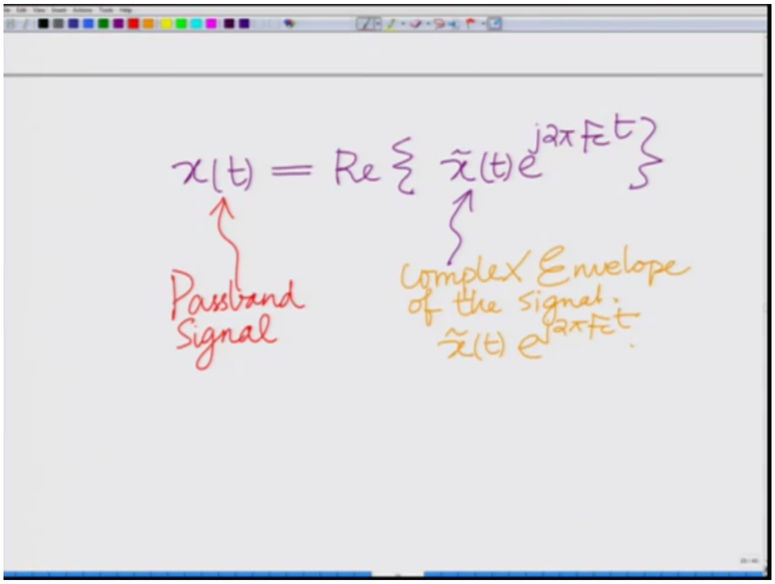

Therefore one can write the pass band signal x(t) as x(t) equals real part of x(t) plus real part of x tilde t e to the power of j 2 pi Fct. So this is x tilde t is the envelope of e to the power of because if you look at magnitude of this quantity x tilde t e to the power of j 2 pi Fct that is that is magnitude x tilde t, so this is acting as the amplitude of e to the power of j 2pi Fct or this is basically the complex envelope of the signal x tilde t e to the power of j 2pi Fct, okay. And real part of that is basically your pass band signal, so if you look at this the pass band signal x(t) is nothing but real part of x tilde t e to the power of into e to the power of j 2pi Fct, so you can see it's as if there is this (expo) there is a Sinusoid e to the power of j 2pi Fct with a complex envelope x tilde t therefore x tilde t is known as the complex envelope of this pass band signal x(t), okay.

So it’s as if, it is acting it is an envelope strictly speaking it is not an envelope because the envelope is simply the amplitude of the Sinusoid therefore this is a complex envelope x tilde t into e to the power of j 2pi Fct it is it can be seen it’s as if the amplitude of the Sinusoid complex Sinusoid is modulated by this complex envelope x tilde t therefore x(t) is the pass band signal x tilde t is it's complex envelope or also its complex baseband equivalent because it has a spectrum which is in the baseband, okay.

So this is the complex envelope of the signal x(t), x tilde t is a complete envelope and x(t) is your pass band signal x tilde x(t) is the pass band signal x tilde t is the complex envelope or the complex, so x(t) is the pass band signal and x tilde t is a complex envelope or the complex baseband equivalent of the pass band signal xt and therefore you can see that every (comp) this is an important principle of (comin) communication that is every pass band signal x(t) can be reduced to an equivalent complex baseband signal x tilde t and this is very useful for the analyses of the communication systems because now what this says is all the analyses of communication systems can be carried out in the baseband by establishing an equivalence to this complex baseband (si) signal.

So one remove the affect not remove the effect of carrier frequency or rather basically normalize it, so that all the analyses can be carried out in the baseband for all the signals irrespective of the carrier frequency Fc that is a unified framework can be developed for the analyses of all these communications in the baseband by reducing this pass band signal is to the equivalent complex baseband signals, alright.

So that is another so that is the use of Hilbert transform, so where we use the Hilbert transform x hat of t of x(t) to construct first the complex pre-envelope as x(t) plus j x hat of t and then shift the resulting spectrum to the baseband to derive the complex envelope of the complex baseband signal corresponding to x(t), alright. So this is the the principle of the complex envelope of a pass band signal x(t). We will stop here and continue in the subsequent modules, thank you.