Principles of Communication- Part I
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Module No 4
Lecture 23
Phase Shifting Method for Generation of Single Sideband (SSB) Modulated Signals based on Hilbert Transform

Hello! Welcome to this another module in this open online course, so we are looking we have current look at the Hilbert transform, correct? And also the impulse response of the Hilbert transformer and the spectrum that is a Fourier transform of the Hilbert transformer. Now let us look at the application that is let us look at the generation of the SSB signal that is a single sideband modulated signal using the Hilbert transform, alright.

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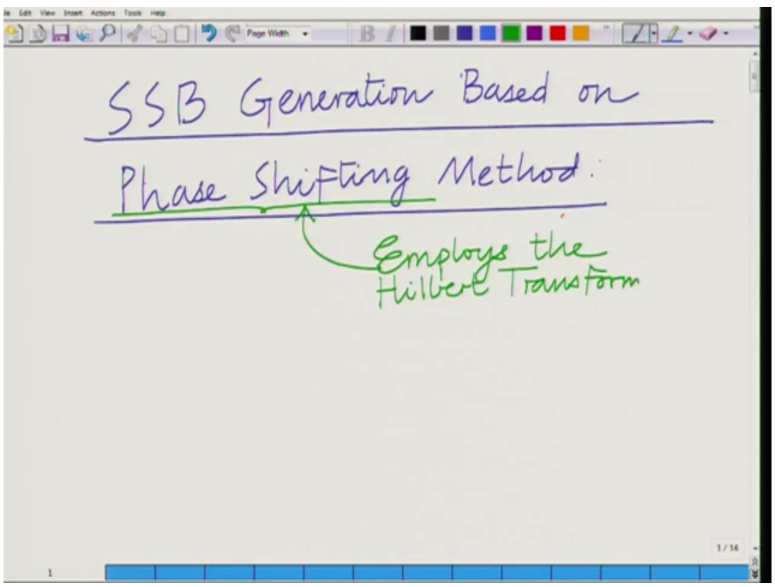

So today let us look at SSB generation using the Hilbert transformer or the phase shifting method which is based on the Hilbert transform is on the phase shifting method, okay. And as we have already said the phase shifting method this employs the Hilbert transform. Now for this purpose considered the message signal m(t), alright with spectrum M(F), okay.

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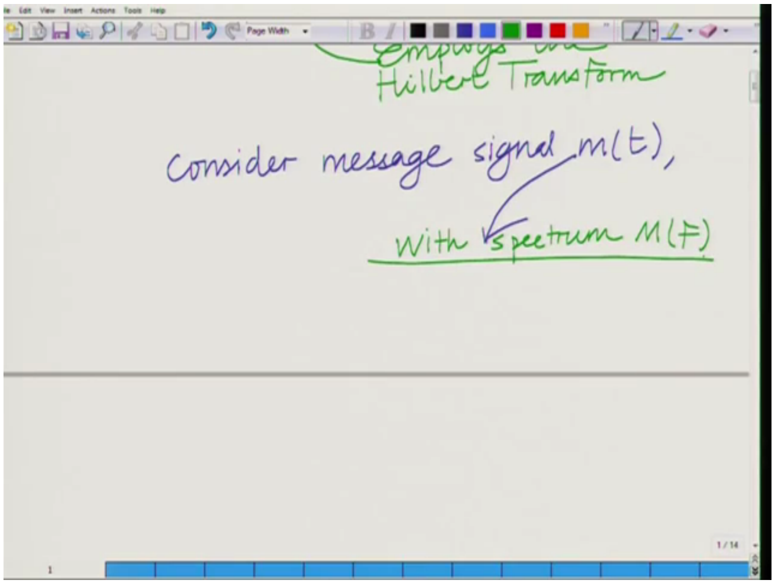

So to illustrate single sideband modulation consider m(t) with spectrum M(F), alright.

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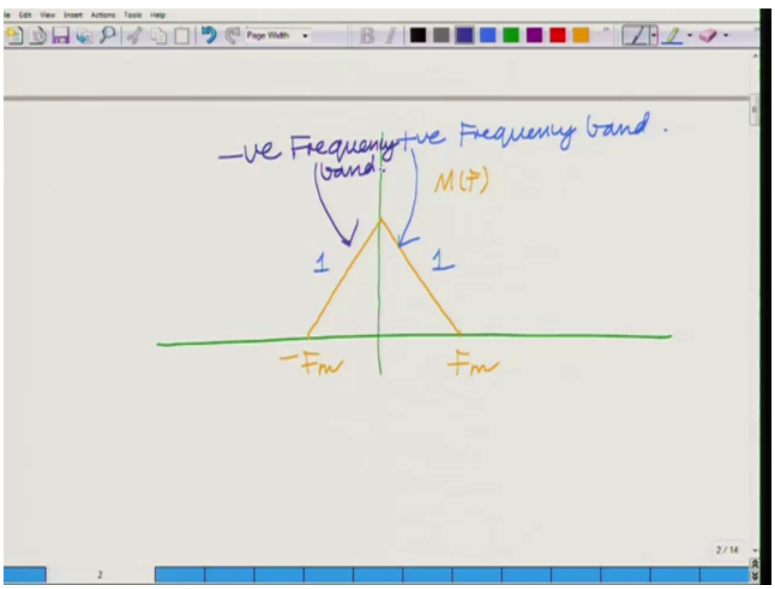

Let us draw this spectrum, let this spectrum be represented as follows this is your M(F), okay. Maximum frequency is Fm minus Fm. Now what we will denote is we will write these numbers 1 to indicate that this is the original baseband spectrum with the positive frequency band and the negative frequency bands scaled by unity, alright. So this is simply the original message spectrum just to keep track of the various scaling factors.

We are going to start with the original message spectrum denote that this message spectrum both the side bands, right. The positive side band and the the positive band that is comprising of the frequency components the positive frequencies and the negative frequencies are scaled by unity, alright. So this is your positive frequency band and this is similarly the negative frequency negative frequency band and both of these are scaled by unity. Scaled by unity means the original scaled by unity implies that there (ess) essentially (iden) they are essentially the original message spectrum, alright.

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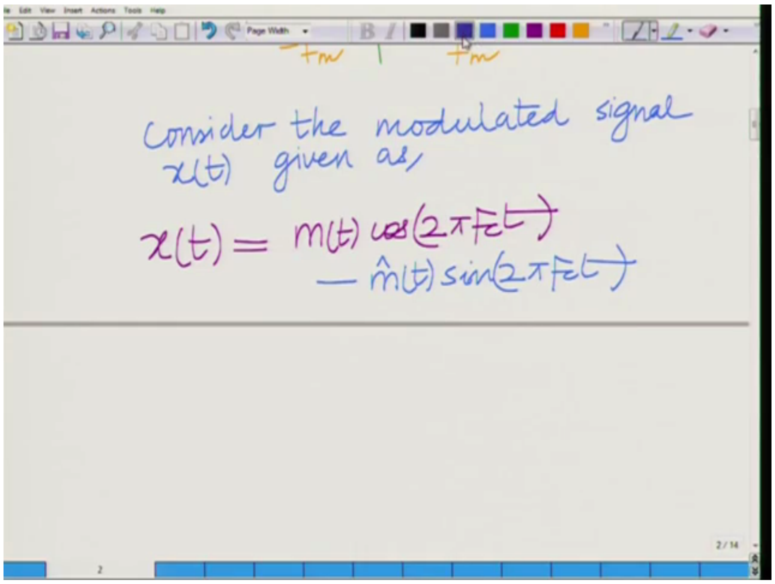

Now us consider the following signal x(t) given as, consider x(t) consider rather the modulated signal x(t) consider the modulated signal x(t) given as x(t) equals m(t) cosine 2pi Fct minus m hat t sin 2 pi Fct, alright where m hat of t, we already know this m hat of t is the Hilbert transform is the Hilbert transform.

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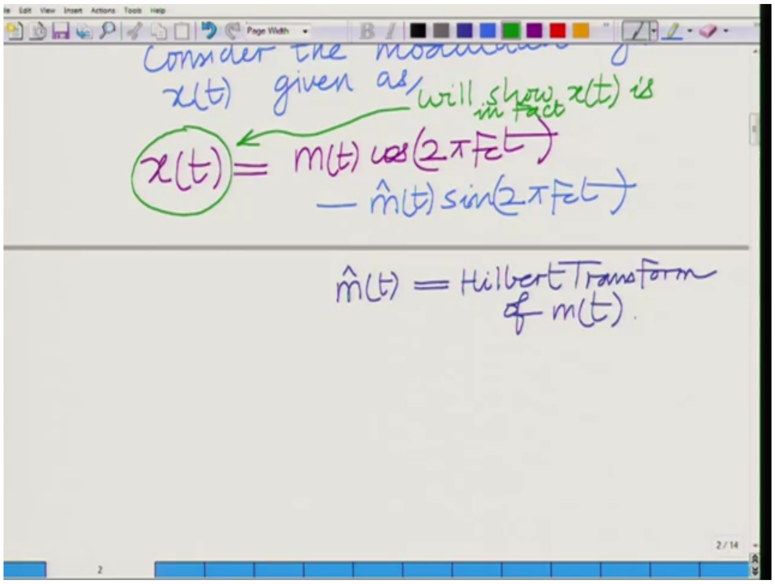

M hat of t is a Hilbert transform of the signal m(t) and x(t) is m(t) cosine 2 pi Fct minus m hat t sin 2 pi Fct, let Fc is the carrier frequency we already know that m hat t is the Hilbert transform of m(t) and we will show that this x(t) is in fact a single sideband modulated signal, so what we are going to show is that this x(t) will show that x(t) is in fact your SSB signal. So we will in fact show that xt is the SSB signal. So what we are doing given a message signal m(t) we are using m(t) and also using m hat t which is a Hilbert transform of m(t) or the phase shifted version of m(t) to generate the single sideband modulated signal therefore this is termed as the phase shifting method for the generation of single sideband modulated signal.

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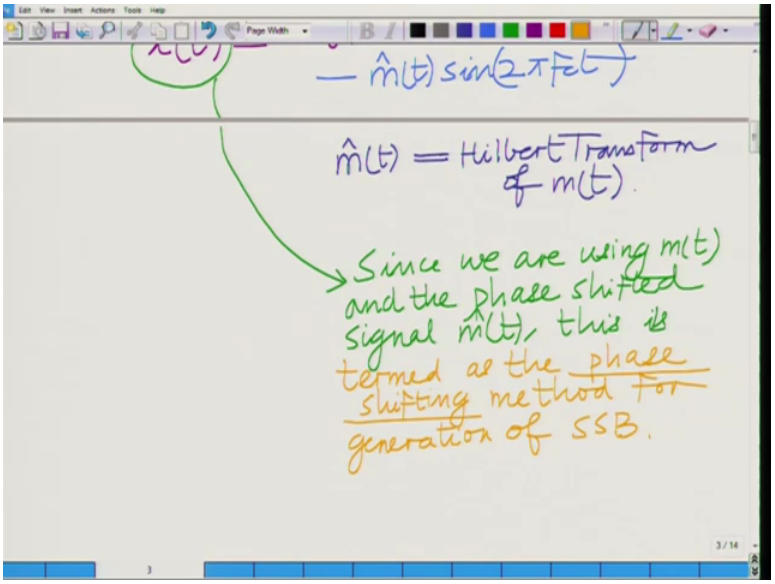

So basically since we are using m(t) and the phase shifted signal and the phase shifted signal m hat of t this is termed as the phase shifting method. A phase shifting method for generation of the phase shifting method for the generation of the SSB signal.

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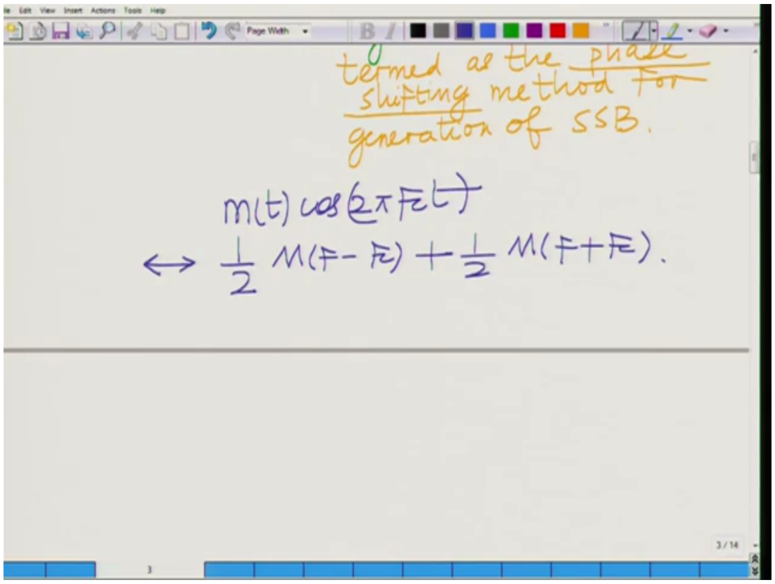

Now let us consider this component by component that is look at m(t) cosine 2 pi Fct, we have already seen this is simply your double band double sideband suppressed carrier signal which has spectrum which is given as the spectrum, so let us look at the spectrum of this that is equivalently we already know that is given as half MF minus Fc that is at plus half MF plus Fc that is spectrum MF shifted to Fc scaled by half, spectrum MF shifted to minus Fc scaled by half each of the sideband are scaled by half, okay.

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So if I draw this it is going to look like at Fc I am going to have minus Fm Fm and each of the sideband is scaled by is scaled by half, correct? So this is at minus Fc and what I have is from this is Fc, so this is Fc plus Fm, Fc minus Fc, minus Fc plus Fm, minus FC minus Fm and again each of these bands are scaled by half. This is your the right side is your MF minus FC and the left side is your MF plus FC, we have already seen this, correct? So what we have is that the baseband spectrum of of m(t) that is MF is shifted to Fc scaled by half shifted to minus Fc scaled by half. What is more interesting? Is a spectrum of m hat t times sin 2 pi Fct.

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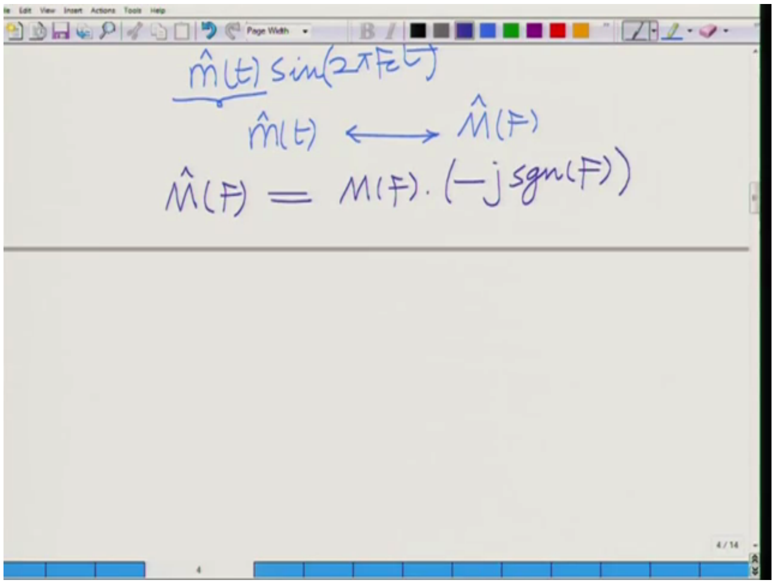

So let us look at that is your, now let us look at this m hat t sin 2 pi Fct, now first for this let us look at the spectrum of m hat of t that is let m hat of t have the spectrum, that is a Hilbert transform of m(t) have the spectrum m hat of f. Then we know that m hat of f is nothing but we have seen this in the properties of the Hilbert transform is the spectrum MF multiply by the Fourier transform of the Hilbert transformer which is minus j sgn F.

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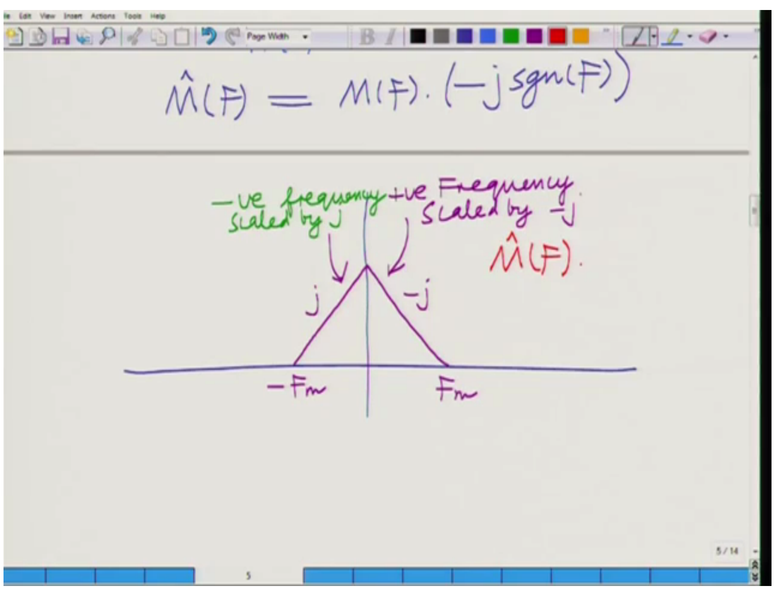

So therefore what you have this is MF original spectrum this is multiplied by minus j sgn F which means that the positive frequency side is multiplied by the scaling factor minus j, negative frequency side is multiplied by the scaling factor, this is remember positive frequency scaled by minus j and phase shifted by minus pi by 2.

This is your negative frequency band scaled by j, alright because the Hilbert transform remember the Fourier transform the spectrum of the Hilbert transform is (())(11:08) by minus j sgn F which means basically all the positive frequency components of the input signal are multiplied by minus J the negative frequency components are multiplied by j at 0 it is multiplied precisely by 0 but we are going to ignore that one point where it is multiplied by 0 because that it can be shown that that does not have a (sig) that does not lead to a so that does not lead to any significant any perceivable difference in the in the rest of the processing, right?

Because that single point at that single point 0 it is exactly 0 it is slightly cumbersome to draw that over here, so we will ignore that and we will simply say that the positive frequency band is scaled by minus j the negative frequency band scaled by j, so this is the spectrum m hat of F the spectrum of m hat of F.

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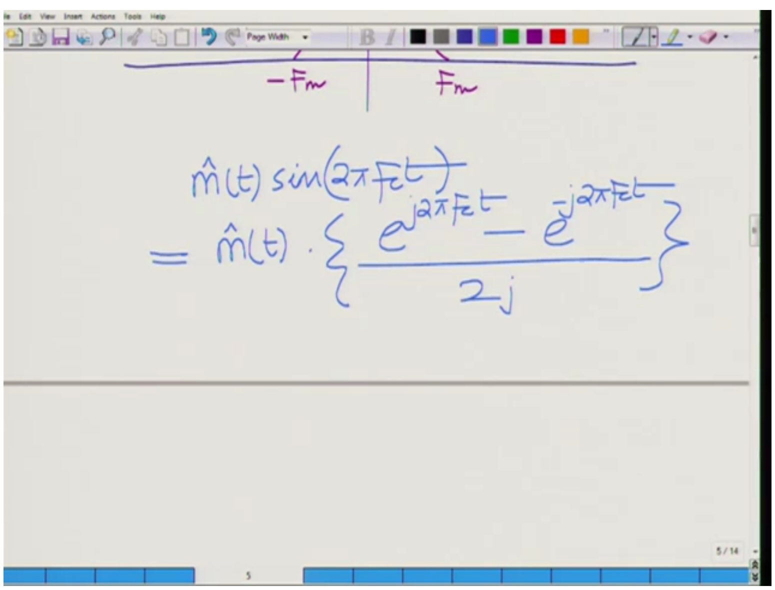

Now m hat of t sgn 2Pi Fct that is modulated by the sgn wave that is m hat of t sgn can be represented as a to the power of j 2pi Fct minus e to the power of minus j 2pi Fct divided by 2j, alright. I equivalent rewrite the sgn 2pi Fct as e to the power of j 2pi Fct minus e to the power of minus j 2pi Fct divided by 2j.

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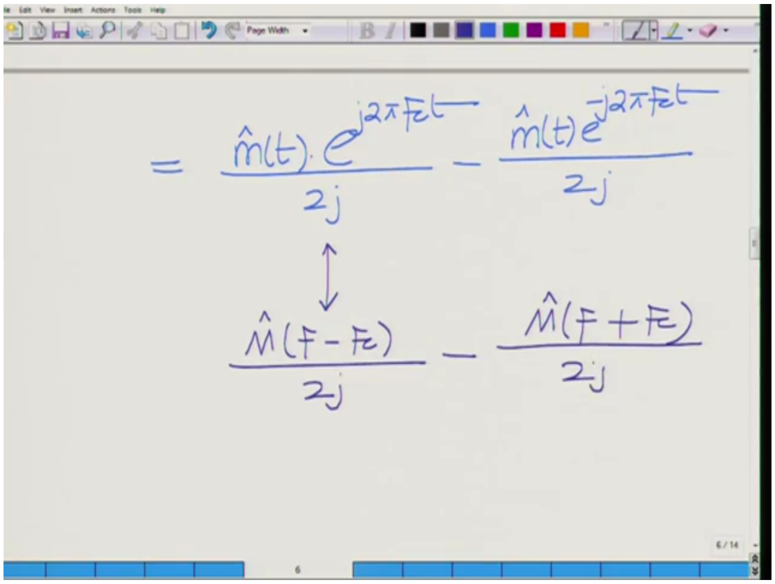

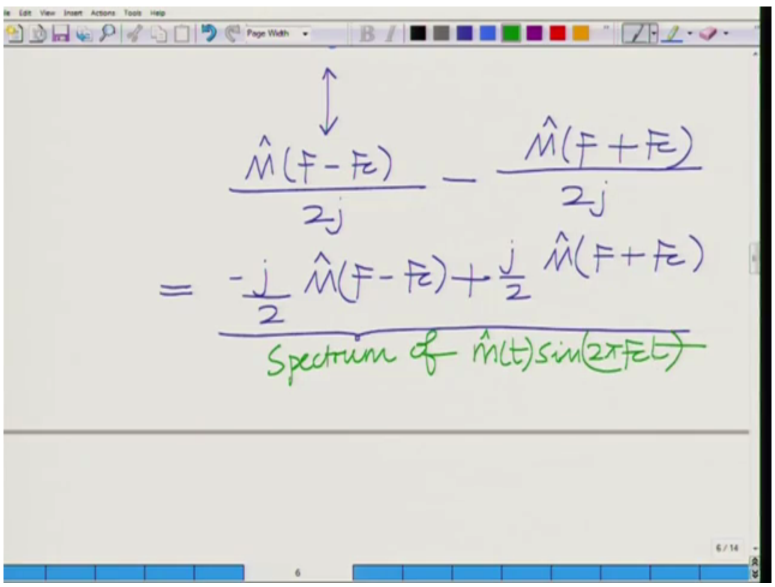

And therefore now this is basically separating this, this is m hat of the e to the power of j 2pi Fct by 2j minus m hat of t e to the power of minus j 2pi Fct divided by 2j and now if I take the spectrum of this spectrum of this is well, m hat of e to the power multiplying by e to the power of j 2pi Fct using the modulation property that is m hat F minus Fc divided by 2j minus multiplying by e to the power of minus j 2pi Fct that is modulation property shifting to minus Fc m hat of F plus Fc divided by 2j which is basically nothing but minus j over 2 m hat F minus Fc plus j over 2 m hat F plus Fc and what is this?

And this you can see basically this you can see basically is the spectrum this is the spectrum this is the spectrum of m hat of t sgn 2pi Fct. So it has m hat shifted to Fc that is m hat F minus Fc multiplied by minus j by 2 and m hat F plus Fc that is shifted to minus Fc and multiplied by j by 2.

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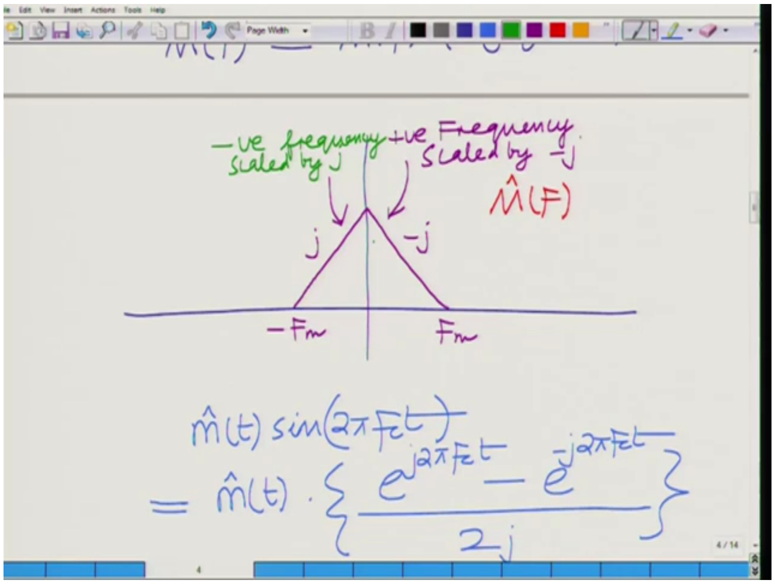

And therefore now we will have if you look at this, this is the spectrum of m hat of F that is positive band scaled by minus j negative band scaled by j.

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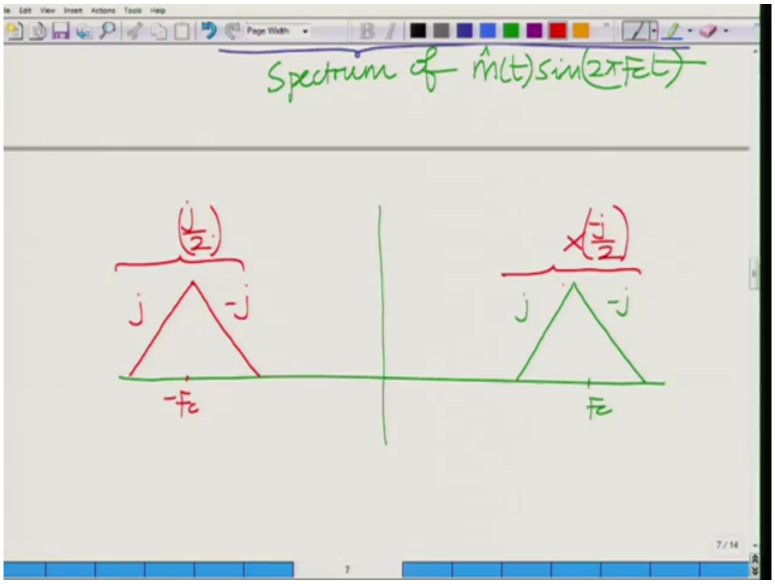

So therefore what we are going to do is, now remember shifted this to Fc we already have positive band scaled by j negative band quality band scaled by minus j negative band scaled by j shifted this to minus Fc and we are going to have again j minus j minus Fc. Now this has to be multiplied by j over 2 or rather this has to be multiplied by minus j over two and this has to be multiplied by j over two remember you have to shift it to Fc multiply by minus j over 2 shift it to minus Fc multiply by j over 2.

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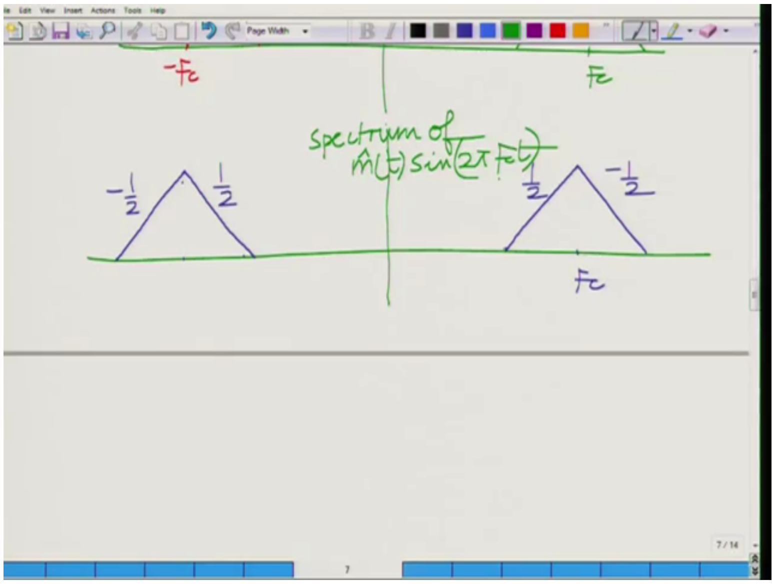

Therefore the net resulting spectrum is given as something like this the net resulting spectrum the net resulting spectrum will be well, at Fc you will have minus j multiplied by minus j by 2. So that gives positive band multiplied by minus F, j multiplied by minus j by 2 multiplied by this scaling factor is half this is at Fc. At minus Fc we will have something interesting j minus j multiplied by j by two, so that will give us the lower band this is multiplied by half j multiplied by j by 2 upper band multiplied by minus half and therefore this is now the spectrum now this is basically your spectrum of m hat of t sgn 2pi Fct and now you can see the various scaling factors some of the scaling factors are half some of the scaling factors are minus half this is a spectrum of m hat t sgn 2pi Fct.

For this, first we have derived the spectrum of m hat t which is m hat F which is MF minus j sgn F plus the modulation effect with respect to sgn 2pi Fct that is shifting to Fc multiplying by minus j by 2 shifting to minus Fc multiplying by j by 2 and that gives us this final spectrum. Now the spectrum of m hat m(t) cosine 2pi Fct minus m hat t sgn 2pi Fct is the subtraction of the 2 spectra that are derived about that is the spectra for remember this is the spectra of of m(t).

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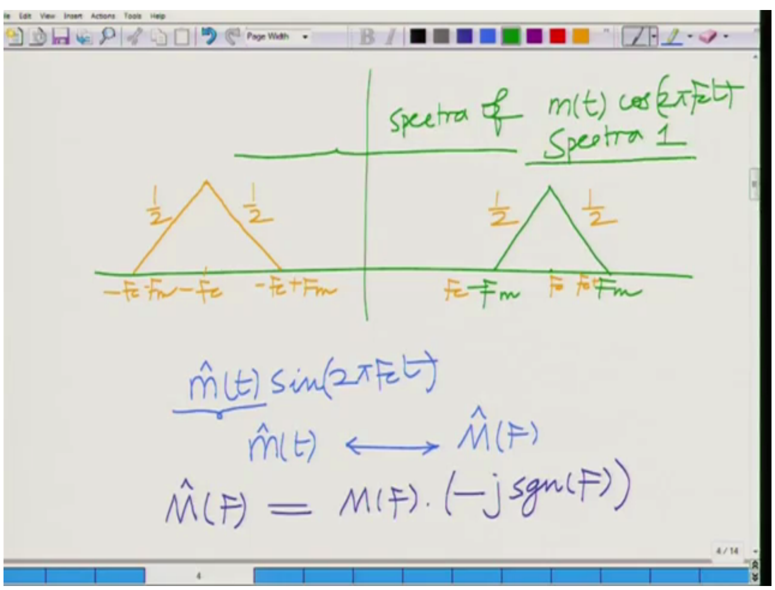

This is the spectra of m(t) of m(t) cosine 2pi Fct let us call this (spect) spectra 1, okay.

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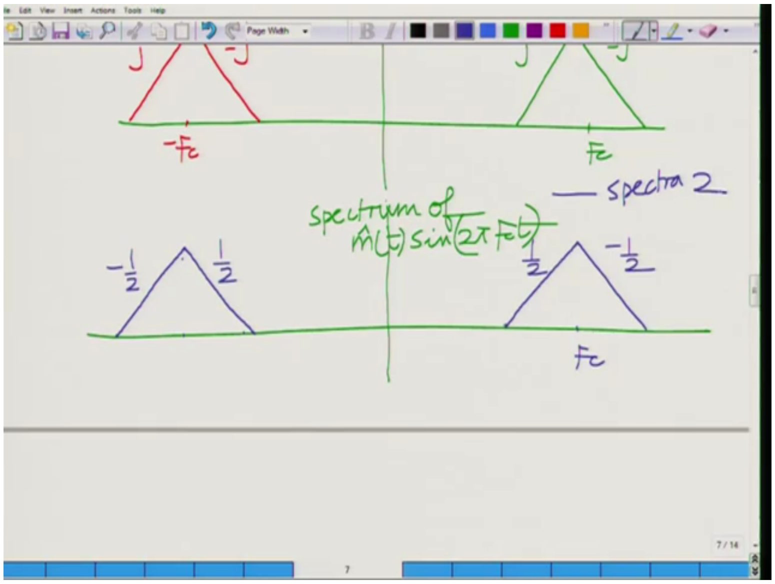

This is the spectra of m(t) m hat t sgn 2pi Fct, let us call this spectra 2 or the second spectrum.

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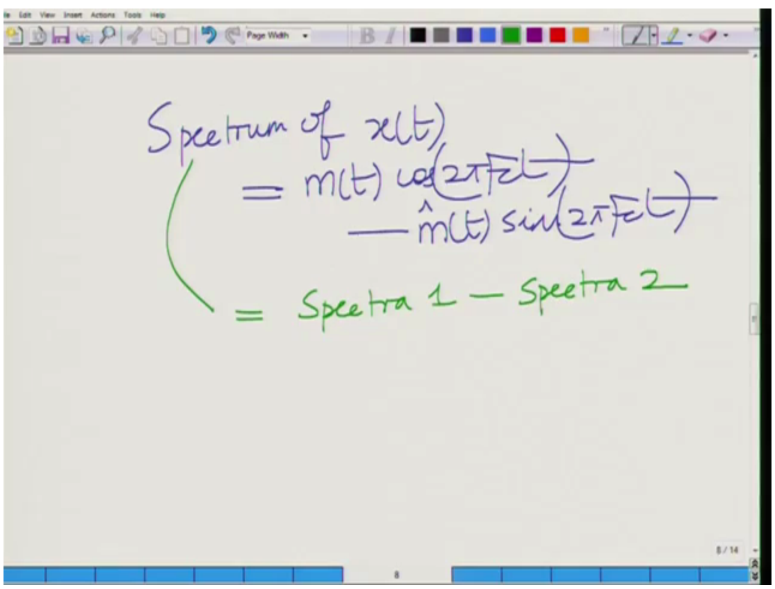

Now the spectrum of x(t) equals m(t) cosine 2pi Fct minus m hat t sin 2pi Fct this spectrum is equal to spectra of 1 that is m(t) cosine minus 2pi Fct minus spectra 2 that is spectrum of and that now if you look at…

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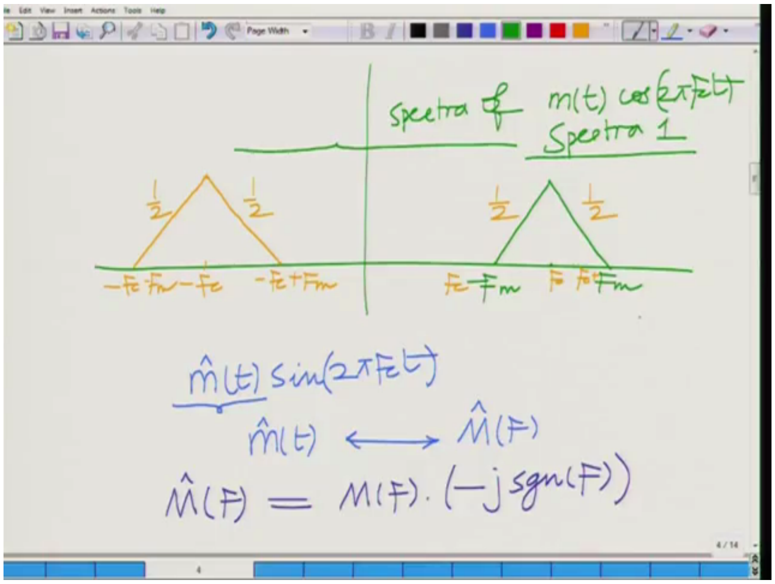

So now look at this if look at this in this the upper sideband if you look at this spectrum the upper side band scaled by half inner lower sideband is scaled by half.

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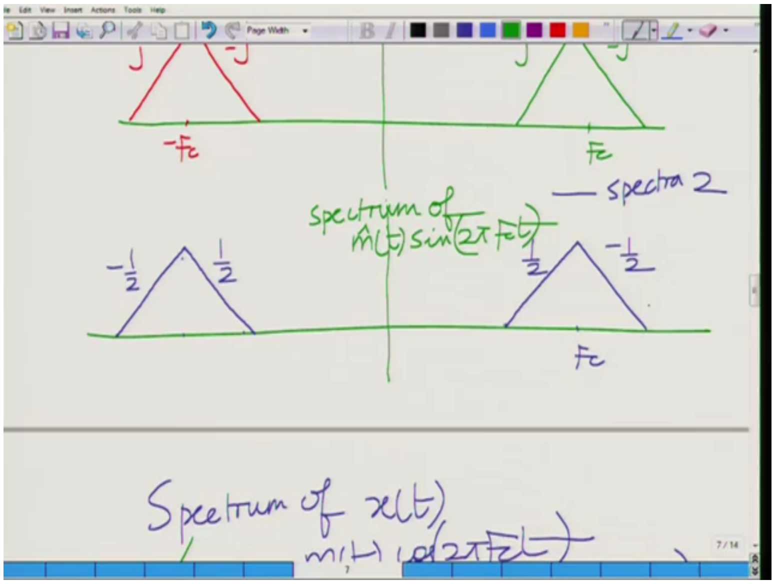

Here the upper sideband in m(t) m hat t sin 2pi Fct upper sideband is scaled by minus half lower sideband is scaled by half therefore you (sub) if you subtract spectra 2 from spectra 1, the upper sideband s will add half plus half the lower side bands will cancel half minus half therefore what is remain?

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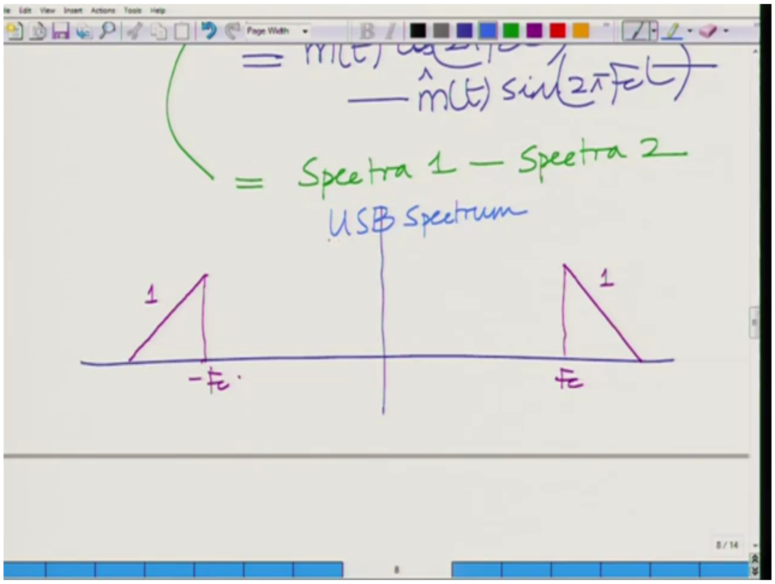

What remains is the upper sideband modulated signal and that will be given something like this. The upper sideband spectra 1minus spectra 2 that will be given as the upper side bands the factors of half and half will add that will give a scaling factor of 1 for the upper sideband scaling factor of 0 for the lower sideband and therefore what we will have is basically now if you look at this the net signal that you will get will be something like, okay. So this is Fc this is minus Fc this is the USB spectrum or spectrum of the USB signal.

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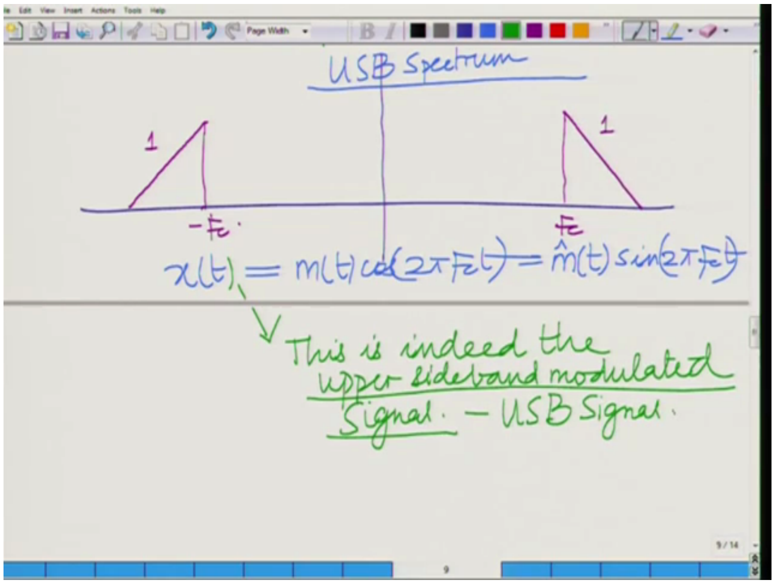

So x(t) equals m(t) cosine 2pi Fct minus m hat t sin 2pi Fct, what we have shown that this is indeed the upper sideband modulated signal. This is indeed the upper sideband modulated signal or basically the USB signal, okay. This is indeed the upper sideband. Similarly now it is not very difficult show that the other signal that is m(t) cosine 2pi Fct plus m hat t sin 2pi Fct will be the lower sideband modulated signal because when you add spectra 1 and spectra 2 the upper sideband which are scaling factors of half and minus half will cancel at 0, the inner the lower sideband which will have which has a scaling factor of half and half will add to give the scaling factor 1 therefore that will be the lower sideband modulated signal.

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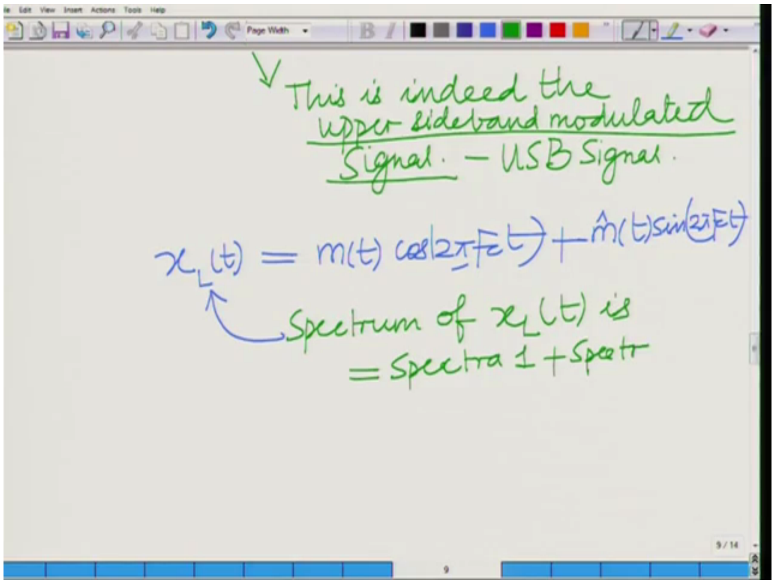

So other the other signal xLt let us call this or let us call the let us call this xLt equals m(t) cosine 2pi Fct plus m hat t and spectrum of xLt is equal to spectra 1 plus spectra 2.

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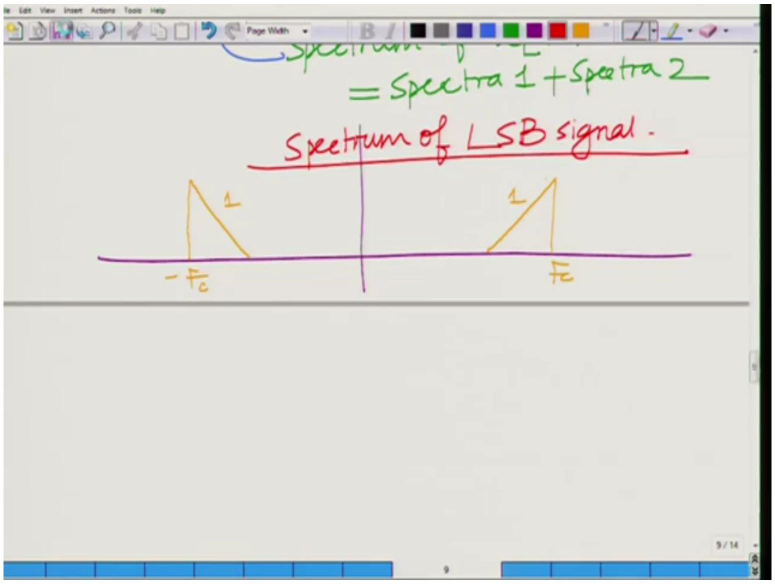

And if you do that you will notice that only the lower side bands only the lower sideband this is Fc the lower sideband only the lower sideband will survive and what you have is, this is basically spectrum of the LSB signal. So this is the spectrum of the LSB signal.

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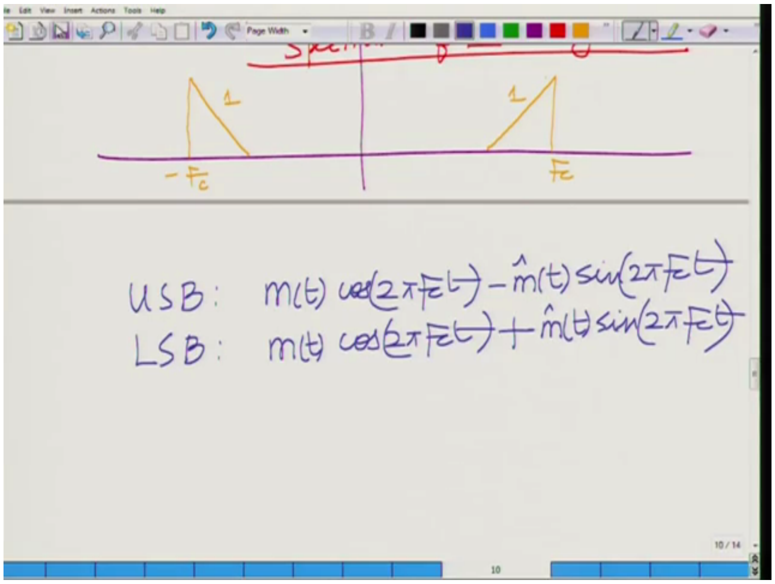

So basically to summarize what we have shown is that the USB signal upper sideband modulated signal is m(t) cosine 2pi Fct minus m hat t sin 2pi Fct and the lower sideband modulated signal is m(t) cosine 2pi Fct plus m hat of t sin 2pi Fc of t and since this employs the phase shifting the phase shifter that is the Hilbert transformer generate m hat of t this is basically based on the phase shift this is basically the phase shifting method to generate a single sideband modulated signal.

And therefore we have seen now without the use of any low pass high pass filter remember that was the disadvantage of the frequency discrimination technique because we needed low pass and high pass filters with very sharp cut-offs, alright. In the sense that the pass band and the stop band but no transition band if there is a transition band that needs that leads to basically picking up of some vestiges, right which are unwanted portions of the signal.

Therefore and it is very complex design such filters with very sharp cut-off and therefore this is an alternative technique does not use such filters with such sharp cut-offs but is based on the phase shifting based on the phase shifter or rather the Hilbert transformer to generate single sideband modulated signal can be used to generate both the upper sideband and the lower sideband modulated signals. By modulating m(t) and cosine 2pi Fct and the Hilbert transform of (tim) of m(t) that is m hat of t on sin 2pi Fct. So we will stop we will stop here and continue with other aspects in the subsequent modules, thank you.