Principles of Communication- Part I

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Module No 4
Lecture 22

Time Domain Description of Hilbert Transform - Impulse Response of the Hilbert Transform

Welcome to another module in this massive open online course. So we are looking at the Hilbert transform for the generation of (oyo) single sideband modulated signals and we also looked at a frequency domain description of the Hilbert transform, alright.

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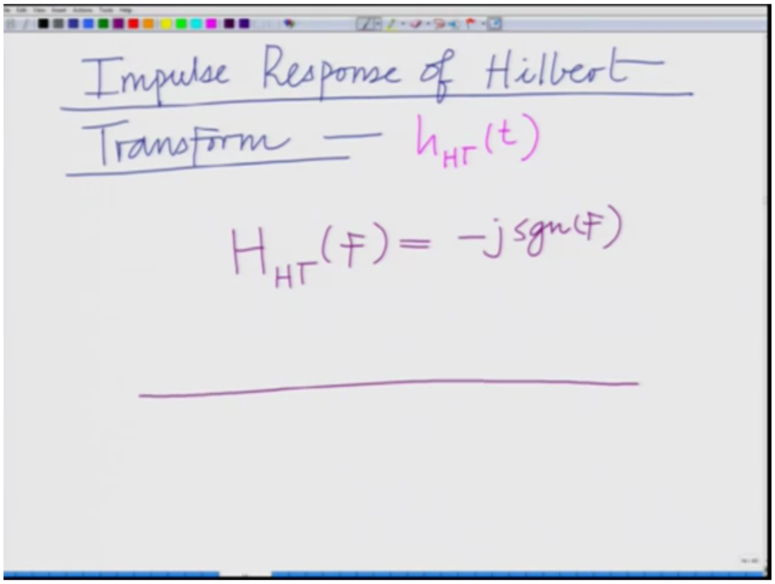

So now let us look at the time domain description that is the impulse response of the Hilbert transform. So we are looking at the impulse response of the (Hemb) Hilbert transform the impulse response of the Hilbert transform and we have denoted this by hHT of t this is the impulse response of the Hilbert transform, alright.

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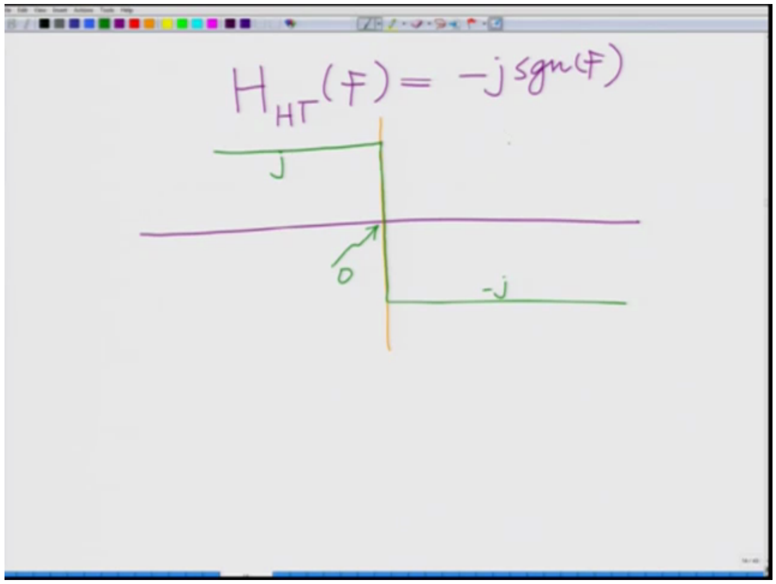

We are denoting it by hHT of t to start with let us recall that the frequency response of the Hilbert transform is minus J sgn F which looks something like this remember for convenience I can always denote it by this it's j for f less than 0 minus j for F greater than 0 and F equal to 0 this is 0. So this is your minus j sgn F to derive the impulse response of the Hilbert transform we already said that we are going to use that derivative property of the Fourier transform, alright. And I am going to talk about that shortly.

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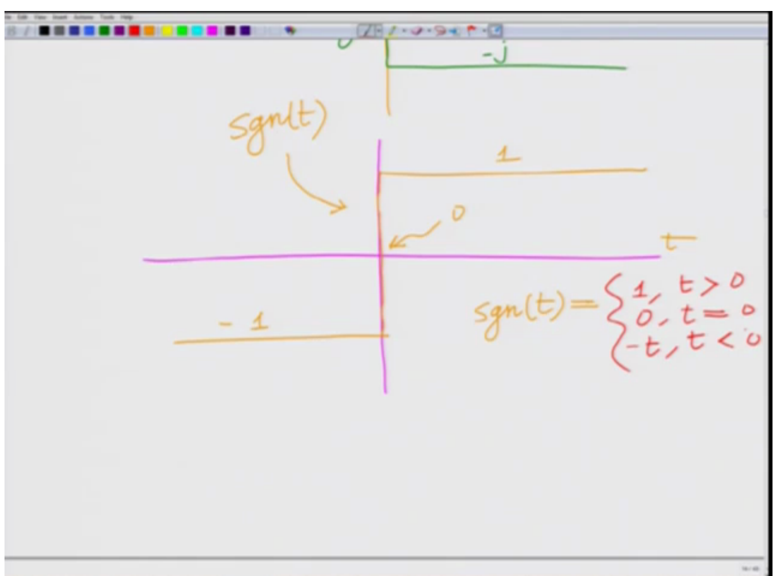

But first let us look at the derivative of the sgn function, let us go back looking at sgn of t, okay. So this is your sgn of t it is one for the greater than 0 minus one for t less than 0 and t equal to 0 it is 0.So sgn of t we already know sgn of t equals one for t greater than 0, 0 t equal to 0, minus t for t less than 0.

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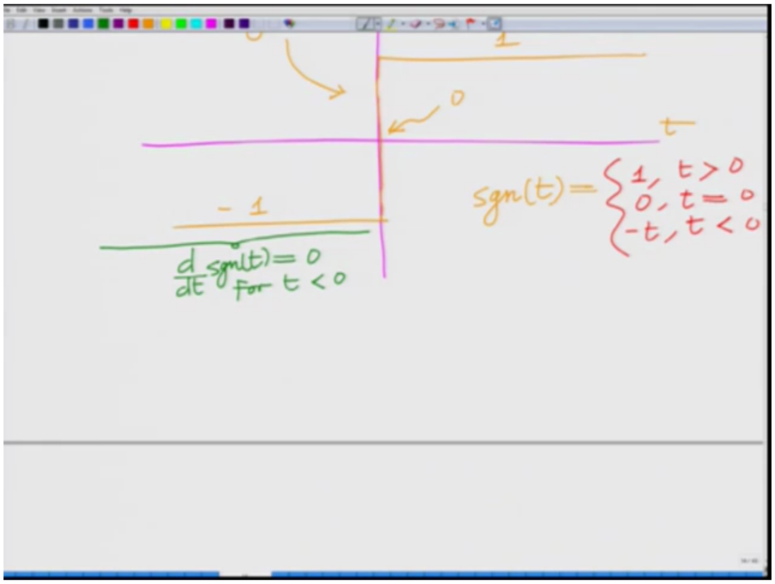

And now you can see the derivative of sgn of t, now here in this region sgn of t that is for t less than 0, sgn of t is constant. So in this region d by dt of sgn of t equal to 0 for t less than 0, right? You can see it is flat. For t less than 0 sgn of t is minus 1. So d by dt the derivative is 0 because it is a constant for t less than zero.

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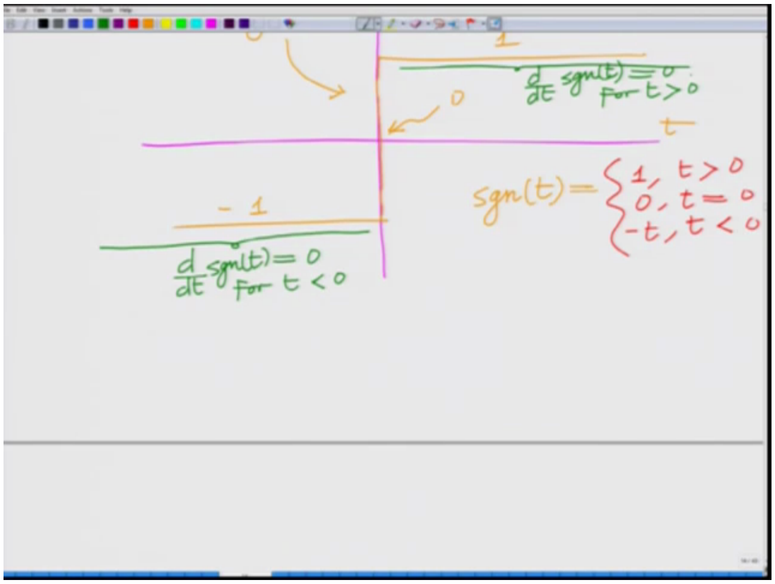

Similarly for t greater than 0 sgn of t is again a constant it is equal to one, so therefore the derivative is again 0, so in this region also d by dt sgn of t equal to 0 for t strictly greater than 0, alright. So for t both t strictly less than 0 and t strictly greater than 0 the derivative or sgn of t is 0 because sgn of t is constant in both these regions. The only place where the derivative of the sgn sgn function that is sgn of t is nonzero is at the point t equal to 0 where it is transitioning from minus 1 to minus it takes a step jump from minus 1 to 1 and there the derivative you can see, since the magnitude is changing from minus 1 to 1 that is magnitude is changing by 2 the derivative is 2 delta t, alright.

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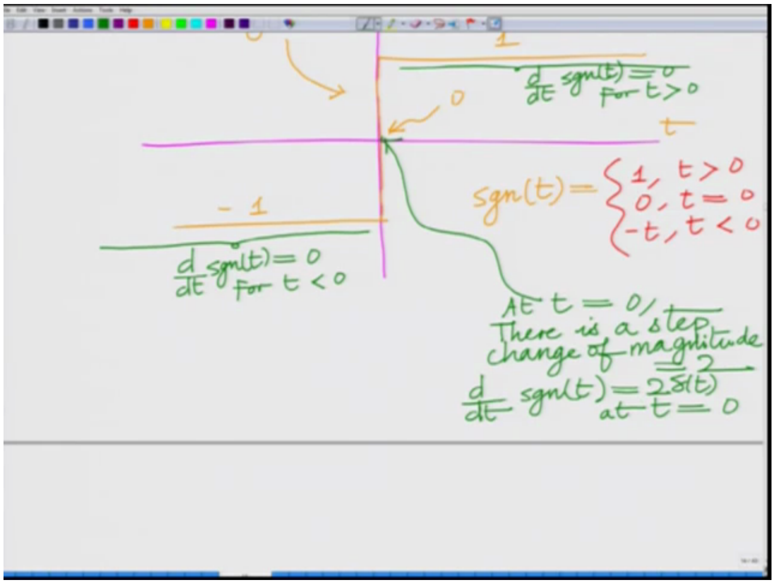

So in this position at t equal to 0 at t equal to 0 there is a step change of magnitude equal to 2 therefore d by dt of sgn t equal 2delta t at t equal to at t equal to 0. Therefore the derivative of this function, so you can see it is constant step change at t equal to 0 and again constant for t greater than or equal to 0, for t greater than 0 therefore the derivative is basically twice delta t, alright. Since the change occurs only at so t equal to 0.

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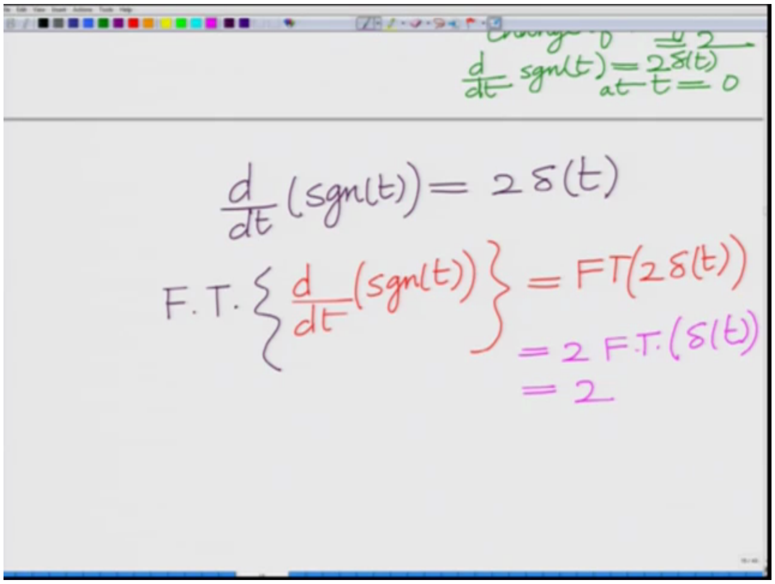

So therefore d by dt of sgn t equals two delta t which is nonzero only for t equal to zero, okay which implies, now let us start with the Fourier transform of the derivative I will show why this is convenient to find the Fourier transform the original function. Fourier transform of d by dt of Fourier transform of d by dt of sgn t d by dt of sgn t is 2delta t therefore the Fourier transform of d by dt sgn t equals the Fourier transform of 2delta t but the Fourier transform of 2delta t is simply 2 because of Fourier transform or we can write one more step that is twice the Fourier transform of delta t which is equal to two, right? Fourier transform of Delta t is simply one over the entire frequency domain.

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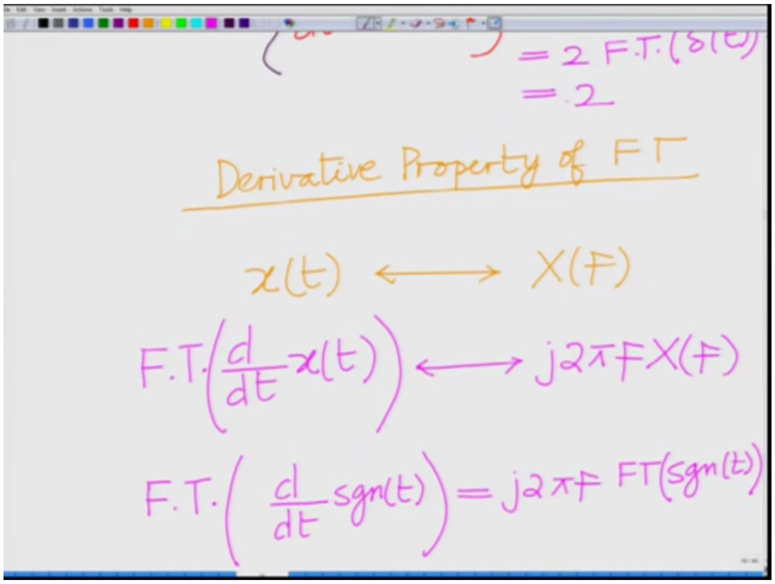

However we also have a result which states that the derivative of that is the Fourier transform of the derivative of a function that is if consider this is the derivative property of the Fourier transform states that if a signal x(t) has Fourier transform X(F) than the derivative of x(t) that is if I look at the derivative of x(t), the Fourier transform of the derivative of x(t) is j 2pi F times X(F). The Fourier transform of the derivative of a signal x(t) is j 2pi F X(F) where X(F) is a Fourier transform of x(t).

And using this property the Fourier transform of the derivative of sgn t therefore the Fourier transform of the derivative that is the FT of the derivative of sgn t that is j 2pi Fourier transform of sgn t however we have already seen that the derivative of the Fourier transform of sgn t from here. We have seen that the derivative of the Fourier transform of sgn t is 2 alright and therefore this is also equal to from above this is also equal to this result we have from above implies that these 2 must be equal.

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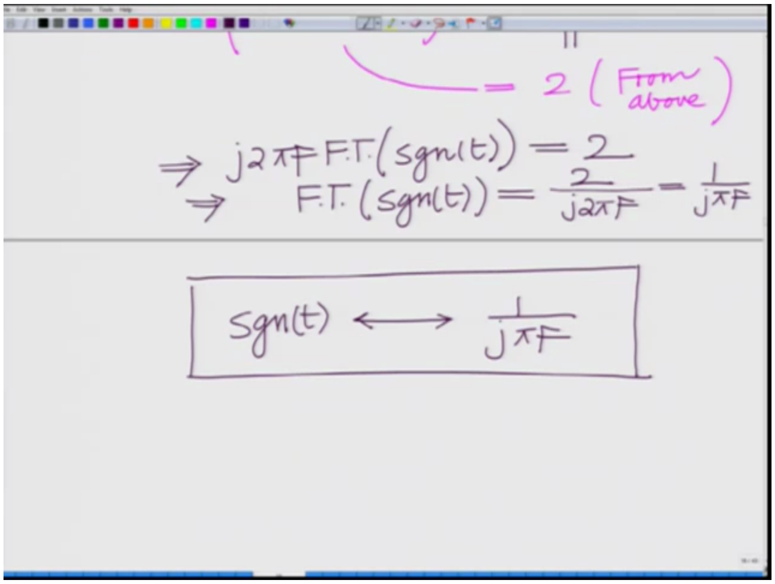

So this implies we must have j2piF Fourier transform of sgn t equals 2, alright. We have already shown that the derivative of the Fourier (trans) that that the derivative that is the derivative of sgn t is 2delta t therefore the Fourier transform of the derivative of sgn t is 2 and we also it is all from the derivative property of the Fourier transform we are saying that the derivative of sgn t has a Fourier transform that is j 2pif times the (fou) the Fourier transform of sgn t therefore the issue quantities must equal be equal which implies j 2piF Fourier transform of sgn t equals 2 which implies that Fourier transform of sgn t equals 2 by j 2piF which is basically equal to 1 by jpiF, therefore sgn t Fourier transform sgn t has the Fourier transform 1 by j 1 over j pi F this is the property that we have been able to (())(10:39) sgn t has the (pou) Fourier transform 1 over jpiF, alright.

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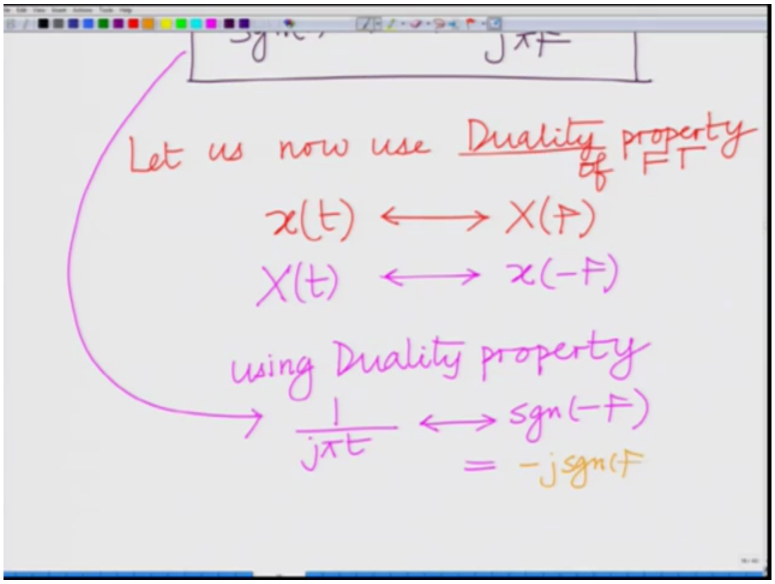

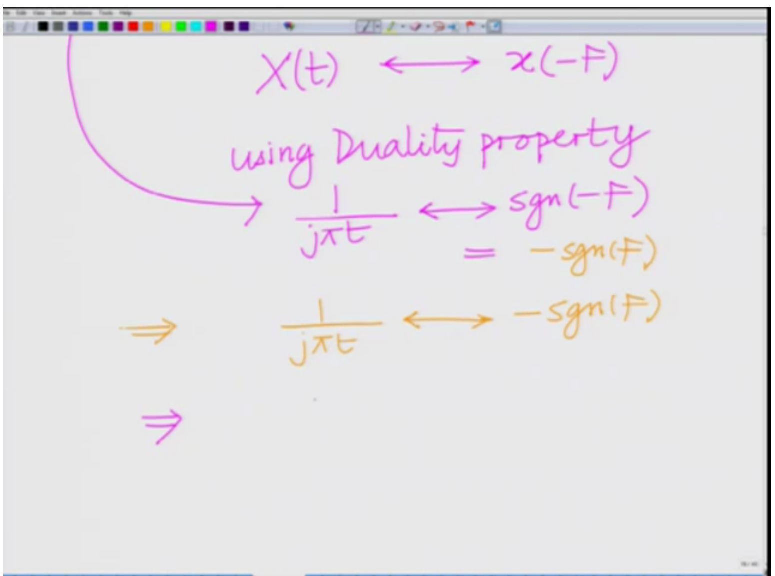

And now let us use (pru) duality let us now use duality and using duality remember duality states that if x(t) has duality property of Fourier transform which states that if x(t) has Fourier transform X(F) than x(t) has Fourier transform x of minus F using the duality property we have sgn t has Fourier transform 1 by j phi. So using the duality property of sgn t has Fourier transform 1 by jpiF 1 by jpit has Fourier transform x of minus F that is sgn of minus F.

Now remember sgn function is an odd function, right? Because it is 1 for t (grea) sgn t is 1 for t greater than 0 minus 1 for t less than 0.0 at equal to 0 this is an odd function. So sgn of minus F is minus j sgn F. This is sgn in the frequency domain by the way, so this is minus which is exactly what we want for the Hilbert transform this is minus j sgn F this implies that 1 by j pit has Fourier transform minus sgn. I am sorry not minus sgn of minus F is minus minus sgn F, so 1 by jpit has Fourier transform minus sgn F which implies.

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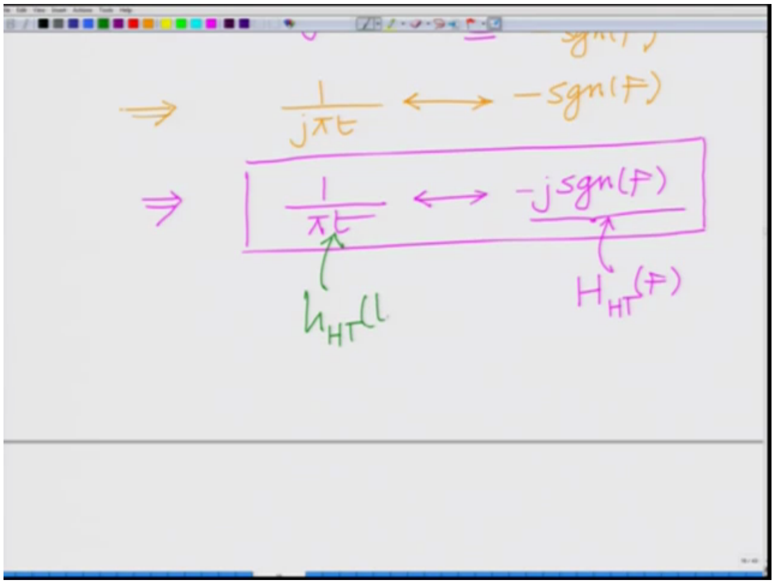

Now taking the j to the other side 1 by pit has the Fourier transform minus j sgn F, 1 by pit has the Fourier transform of minus j sgn F, now minus j sgn F is nothing but recall that minus j sgn F is nothing but the impulse response of the Hilbert transform HHt of F which implies that 1 by pit which is the inverse Fourier transform of this must be nothing but the impulse response of the Hilbert transform therefore the impulse response of the Hilbert transformer is 1 by pi.

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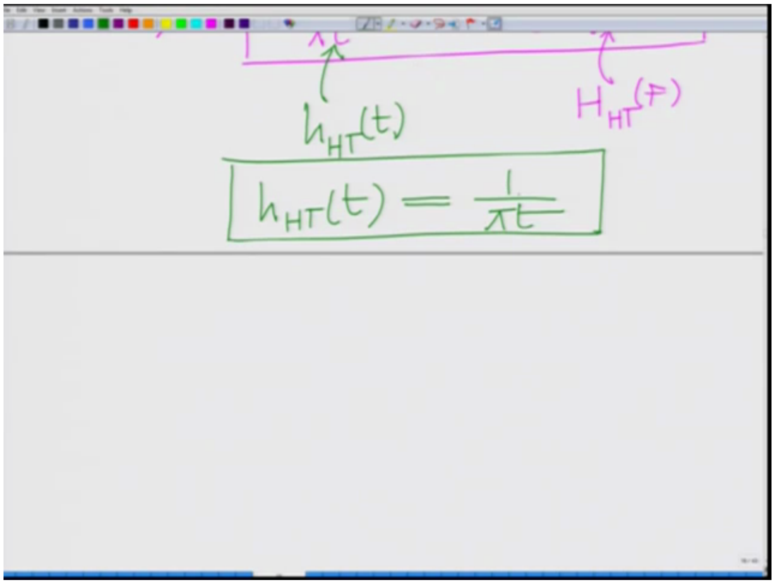

And that is what we have derived H of HT of t equals 1 by pi 1 by equals 1 by pi t, okay. So that is a property that we have been able to show that the impulse response of the, that is if remember the Hilbert transform is a linear time invariant system whose Fourier transform is given by minus j sgn F. In the time domain its impulse response is 1 by pi t therefore now if you look at the Hilbert transform remember the Hilbert transform is an LTI system, correct?

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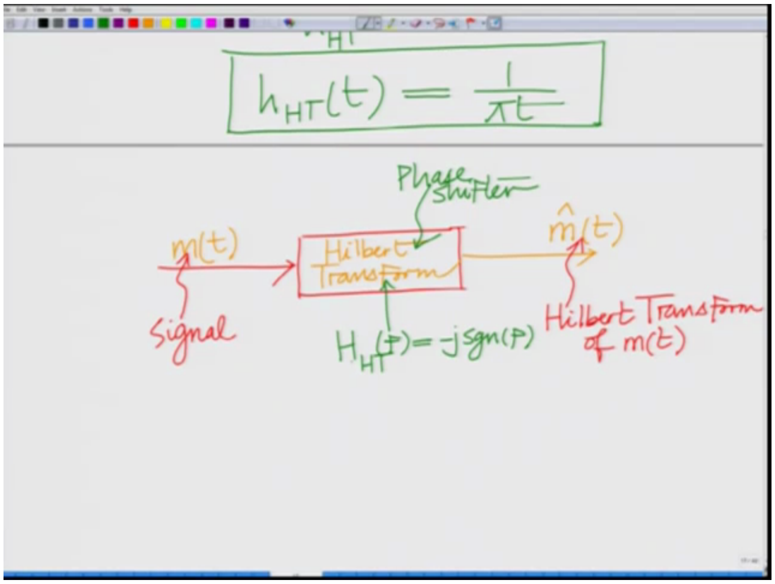

This is your Hilbert transform is an LTi system if you pass a signal m(t) outcomes the Hilbert transform m hat of t this is your signal this is its Hilbert transform of m(t) of m(t) and now this system has (impu) Fourier transform we have already seen this equals minus j sgn F in fact this is a phase shifter, correct? Remember this is a phase shifter that is it shifts the phase of all the positive frequency components by minus pi by two all the negative frequency components by pi by two. This is a phase shifter it has (impu) frequency spectrum the Fourier transform is minus j sgn F and the frequency.

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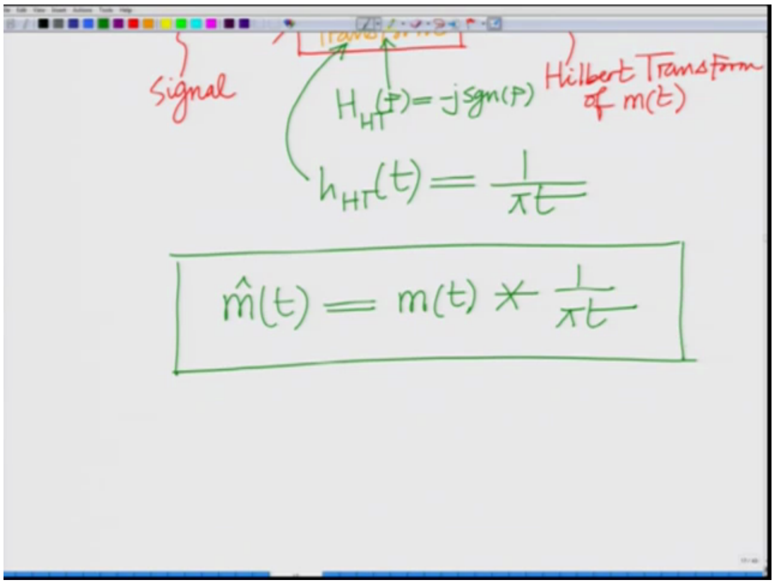

And now the impulse response we have derived the impulse response hHT of t equals one by pi t which implies in the time domain the Hilbert transform can be represented equivalently as m hat of t equals the input signal remember when you pass input signal through LTi system signal gets convolved with the impulse response, so m(t) convolved with 1 by pit.

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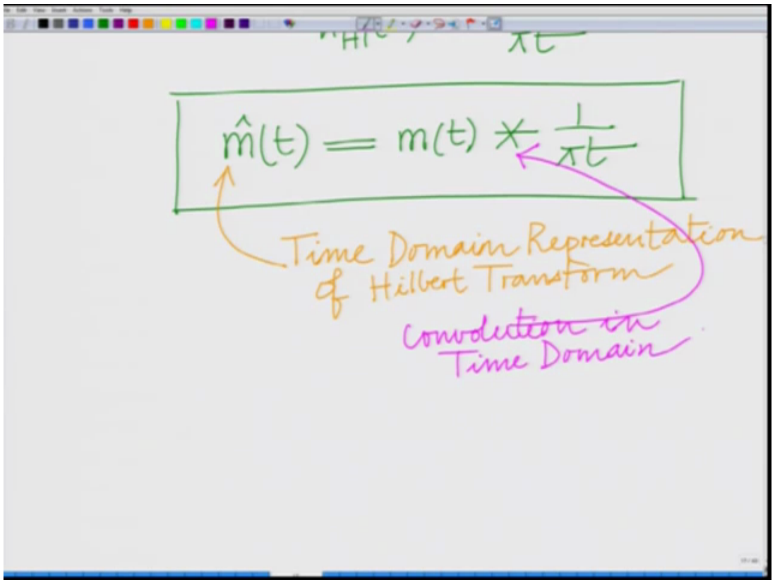

This is the time domain representation of the Hilbert transform this is the time domain representation of the Hilbert transform and this is remember convolution this is a time domain representation therefore it is a convolution in time domain. That is convolution of the input signal with the impulse response.

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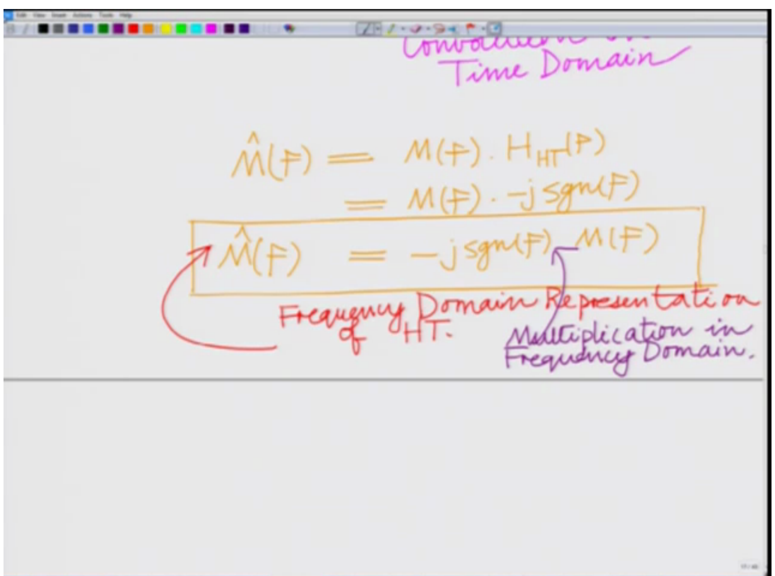

Now frequency domain representation that is the output Fourier transform remember convolution in time domain is multiplication in the frequency domain, so this is MF into H HT of F which is basically your MF times minus j a product, correct? This is a product or you can write this as minus j minus j sgn F times, okay. So this is the product in the frequency domain. So this is the frequency domain representation of HT Hilbert transform and this is a multiplication in the frequency domain alright.

So in this module we have developed the time domain representation of the impulse of the Hilbert transform that is we have calculated the impulse response of the Hilbert transformer using 2 properties one is the derivative property of the Fourier transform and the duality property of the Fourier transform to demonstrate that the impulse response of the Hilbert transform is 1 over pi t which corresponds to the spectrum of the Fourier transform of the Hilbert (tress) transform which is given by or the spectrum of the Hilbert transformer which is given by minus j sgn F, alright.

And therefore Hilbert transform can be represented in the time domain also equivalently by the convolution m hat of t which is a Hilbert transform of m(t) is given by the convolution of m(t) with 1 over pi t. However as you can see since 1 over pi t is a messy signal, correct? 1 over pi t is a quite a messy signal therefore it is much more easily and readily expressed in the it is much more this signal is much more readily expressed in the in the the Hilbert transform the operation of the Hilbert transform is much more readily expressed in the frequency domain rather than the time domain.

So the frequency domain specification of the Hilbert transform which is in the phase shifting property that is minus j sgn F, alright where it's which shifts the phase of all the positive frequency components by minus pi by 2 and the negative frequency components by pi by 2 is much more simpler compared to the times domain description of the Hilbert transform, alright. So we will stop this point over here and continue with the other aspects in the subsequent modules, thank you.