Principles of CommunicationminusPart 1
Professor Aditya K. Jagannathan
Department of Electrical Engineering
Indian Institute of Technology Kanpur
Module 1
Lecture No 2
Frequency Domain Representation and Introduction to Discrete Fourier series

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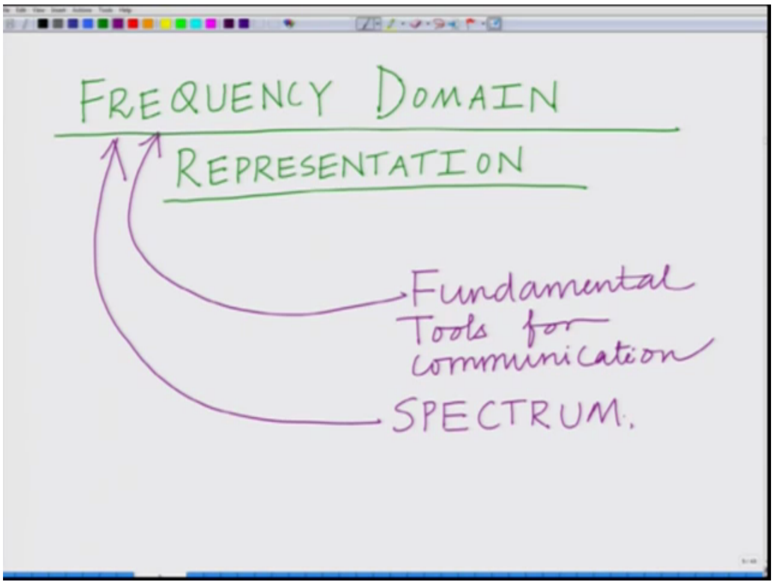

Hello, welcome to another module in this massive open online course, so today's let us start looking at the frequency domain representation of signals, alright. So today we will start looking at, what is known as and what you must be familiar with to some extent the frequency domain representation of signals. So we are going to look at the frequency domain representation, alright and this is 1 of the fundamental tools in communication, the frequency domain representation is 1 of the fundamental tools available. This is 1 of the fundamental tools required or 1 of the fundamental tools used in communication and this frequency domain representation of the signal is termed as the spectrum of the signal.

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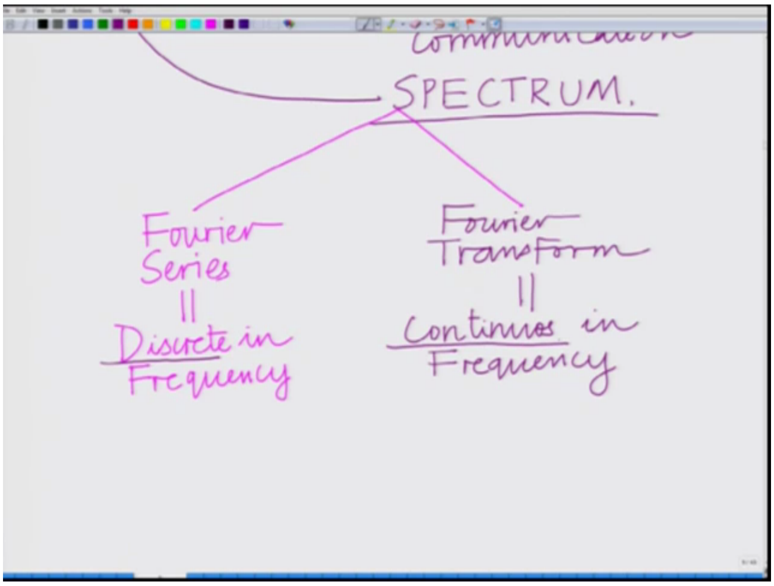

So this is termed as the frequency domain signal representation is termed as the spectrum right, the spectrum of the signal. And further this spectrum can be of 2 types, 1 is we can have a Fourier series which is discrete in frequency, this Fourier series is discrete in frequency and the other representation is the Fourier transform and this Fourier transform this is continuous so we have the Fourier series which is discrete the Fourier transform which is continuous in frequency.

So this way we have to different kinds of, of course we have more kinds of signal or spectrum or frequency domain representation, but for the purpose of this particular course, these 2 are going to be sufficient, 1 is the Fourier series which is discrete in frequency we are going to look at it in greater detail and the other is the Fourier transform which is continuous in the frequency domain. First let us start looking at the Fourier series representation, okay.

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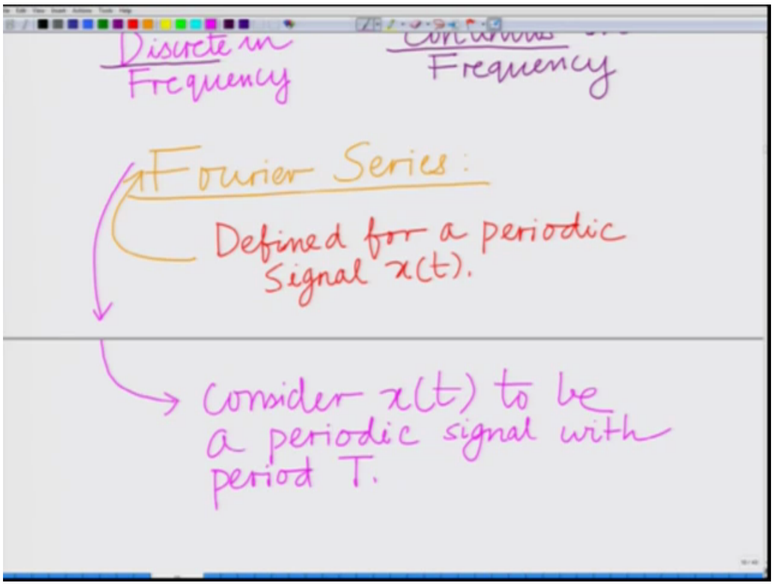

So we start by looking at the Fourier, we start by looking let us start by looking at the Fourier series and the Courier series is defined it is important to remember this is defined for a periodic signal. This is defined for a, periodic signal x(t), so let us consider x(t) to be a periodic signal, so consider x(t) to be a periodic signal with period capital T, alright. So x(t) is a periodic signal, alright we all are familiar with the notion of a periodic signal, so consider x(t) to be a periodic signal with period capital T, okay.

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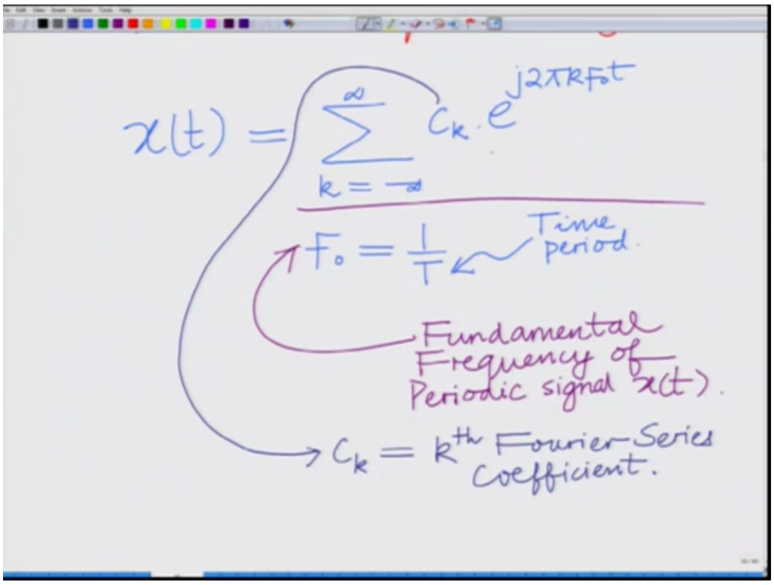

Now the Fourier series of x(t) is given as x(t) this is given as let us write this down x(t) equals to summation k equals to minus infinity to infinity, so let me just write it a bit more Clearly this is summation k equals to minus infinity to infinity Ck e to the power of j 2 Pi k F0 t, where F0 equals to 1 over the time period, remember capital T this is the time period of the periodic signal F0 equals to 1 over the T this is termed as the fundamental frequency. F0 is the fundamental frequency of periodic signal, x(t), okay.

And this is the Fourier series representation correct, Ck e to the power summation Ck need to the power of j 2 Pi k F0 t and this quantity Ck is the kth Courier series cominusefficient. This quantity Ck, alright the summation Ck e to the power of j 2 Pi k F0 t, F0 is the fundamental frequency which is 1 over T, where T is a time period of the periodic signal and F0 is the fundamental frequency, right? And Ck is the Kth discrete Courier series cominusefficient of this periodic signal x(k) okay.

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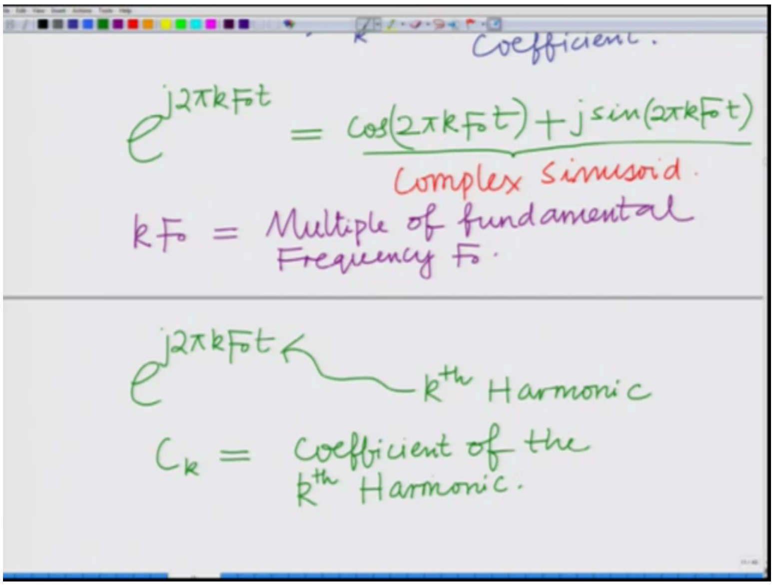

And now look at this, now let us try to understand this a little bit better if you look at this quantity e to the power of j 2 Pi k F0 t, look at this quantity e to the power of j 2 Pi k F0 t this is equals to cosine, e to the power of j theta is cosine theta + j sine theta. So this is cosine 2 Pi k F0 t + j sine 2 power k F0 and this is also, this is basically a complex Sinusoid, correct? Remember we said cosines and sines of Sinusoid, so cosine 2 Pi k F0 t + j sine 2 Pi k F0 T is a complex Sinusoid. So what we are saying is that we can look at this, the summation Ck e to the power of j 2 Pi k F0 t is basically nothing but a linear combination of Sinusoids at the fundamental frequency F0 and multiples k F0 of the fundamental frequency, alright.

So what we are saying is if you can look at it, each k F0 is equals to a multiple of fundamental, this is a multiple of the fundamental frequency F0 the kth multiple of the fundamental frequency F0, correct? So summation Ck e to the power of j 2 Pi k F0 t is basically nothing but a linear combination of these various complex Sinusoids, at either frequency F0 or k F0, k F0 is basically a multiple of the fundamental frequency F0.

So this is a linear combination, so what we are saying is every periodic signal x(t) with period with fundamental frequency F0 can be expressed as an infinite as a as a linear combination of an infinite number of Sinsusoids at either the fundamental of frequency of F0 or an integer multiple of the fundamental frequency F0 that is at k F0, alright. And that is the power of all that is the intuitive explanation of this discrete Fourier of the discrete Fourier series expansion of a periodic signal x(t), okay.

And this Sinusoid at k F0, that is this Sinusoid and k F0 that is e to the power of j 2 Pi k F0, correct? This Sinusoid that is Sinusoid at integer multiple of F0 t this is termed as the kth harmonic and Ck is basically the cominusefficient of the kth harmonic in the linear combination. So Ck is the cominusefficient of the Kth, so e to the power of j 2 Pi k F0 is k F0 t is the kth harmonic, correct, which is associated the kth multiple of the fundamental frequency F0 and Ck is the discrete Fourier series cominusefficient associated with the kth harmonic in this discreet Fourier series expansion, okay.

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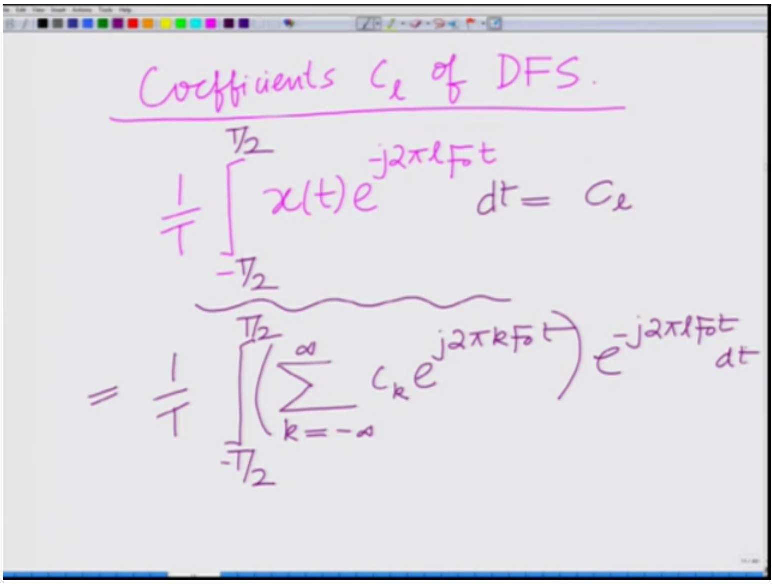

Now, let us look at how to find out the cominusefficient Ck in the discrete Fourier series, okay. so let us now look at cominusefficient , how to find the coefficients Ck of the discrete Fourier series? Let us try to make it CL since we are already using k as the index of the discrete Fourier series. Now we will show let us consider this quantity integral 1 over T minus infinity to T x(t) e to the power of j minus j 2 Pi L F0 t, we will show that this quantity this integral here boils down to yield the Lth cominusefficient CL of the discrete Courier series. For that what I am going to do is I am going to substitute the expansion minus infinity to infinity summation, I am sorry this integral is from is over 1 period, so that is from minus T over 2 to capital T over 2 minus T over 2 minus T over 2 to T over 2 summation k equals to minus infinity to infinity Ck e to the power of j 2 Pi k F0 t, correct, Multiplied by e to the power of minus j 2 Pi has to be dt e to the power of minus j 2 Pi L F0 t times dt.

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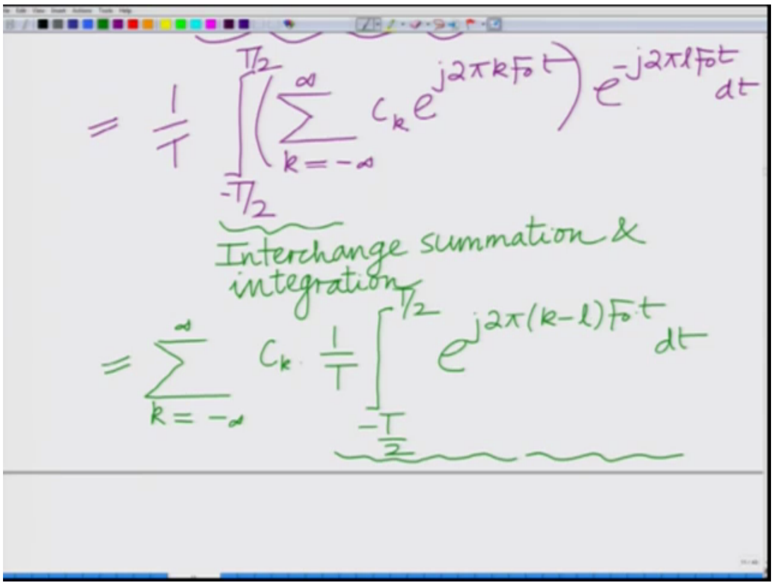

Now interchange the summation and integration that is interchange summation and or rather interchange order of summation and integration and that yields take the summation outside that gives k equals to minus infinity to infinity, correct? Ck times 1 over T integral minus T by 2 to T by 2 e to the power of j 2 Pi k minus L F0 t dt. Now let us look at this integral 1 over T integral minus T by 2 to T by 2 e the power of j 2 Pi k minus L F0 t let us take look at this integral, let us try to infer the properties or let us try to find out the properties of this integral and we will see that this is a very interesting property, that is let us look at this integral 1 over T integral minus T by 2 to T by 2 e to the power of j 2 Pi k minus L F0 t.

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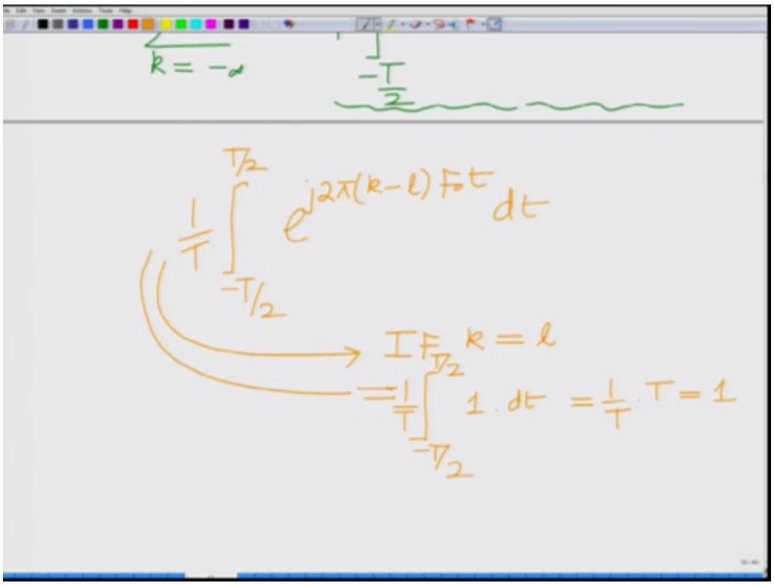

Now if k equals to L than e to the power of k minus L equals to 0, so e to the power of j 2 Pi k minus L F0 t that is e to the power 0 that is 1, so this integral becomes, if k equals to L this integral becomes integral 1 over T minus T by 2 to T by 2 1 times dt, which is 1 over T times T, so this is 1. So what we are saying is if k is equals to L than 1 over T integral minus T over 2 to T over 2 e to the power of j 2 Pi k minus L F0 dt is 1.

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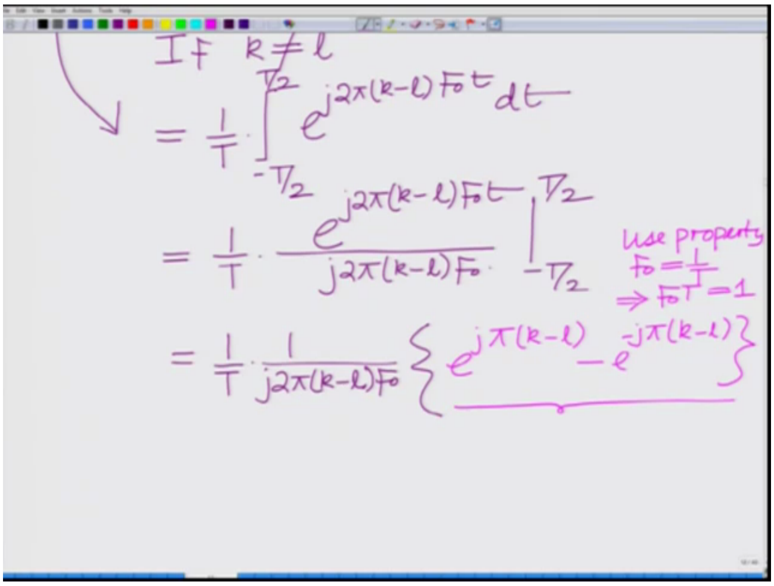

Now if k is not equals to l, that is k minus L is not equals to 0 than this integral becomes 1 over T, well let me just write it again minus T by 2 to T by 2 e to the power of j 2 Pi k minus L F0 t dt, so k minus L is not equals to 0 which means I can now evaluate this integral as 1 over T e to the power of j 2 Pi k minus L F0 t divided by j 2 Pi k minus L F0 between the limits minus T by 2 to T by 2, which is equals to 1 over T 1 over j 2 Pi k minus L F0.

Now e to the power of j 2 Pi k minus L F0 t evaluated with capital T by 2 using the property, use property remember we have to use the property F0 equals to 1 by T which implies F0 T equals to 1. So using this property this becomes e to the power of j 2 Pi k minus L F0 into T by 2 F0 T is 1, so this becomes Pi K minus L minus e to the power of minus j Pi at minus T by 2 it becomes e to the power of minus j Pi k minus l. Now look at this, this is a Sinusoid evaluated at phase Pi k minus L and phase minus Pi k minus l. The difference between the 2 phases is 2 Pi times k minus L that is integer multiple of 2 pi, therefore these 2 complex Sinusoids are equal hence the difference is 0.

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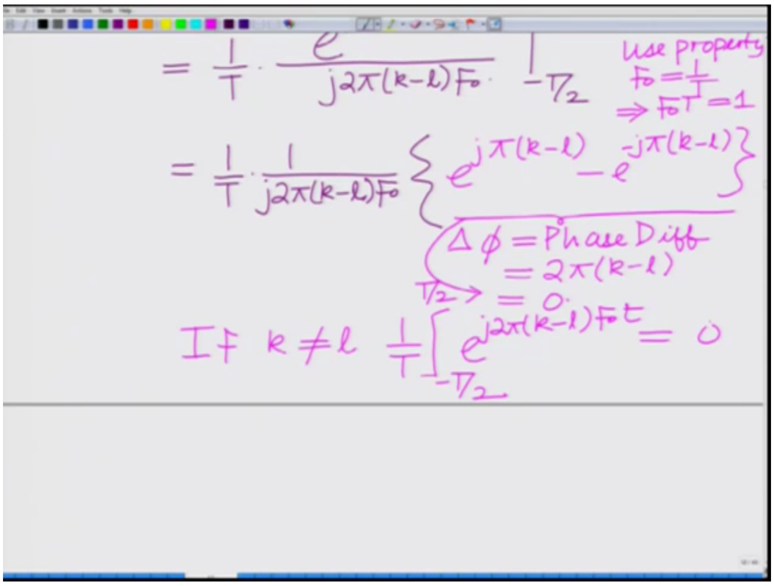

So phase difference equals to 2 Pi k minus L which means this quantity, here this is equals to 0 therefore if he k is not equals to L therefore to summarize if k is not equals to L than 1 over T integral minus T by 2 to T by 2 r to the power of j 2 Pi k minus L F0 t is equals to 0, okay.

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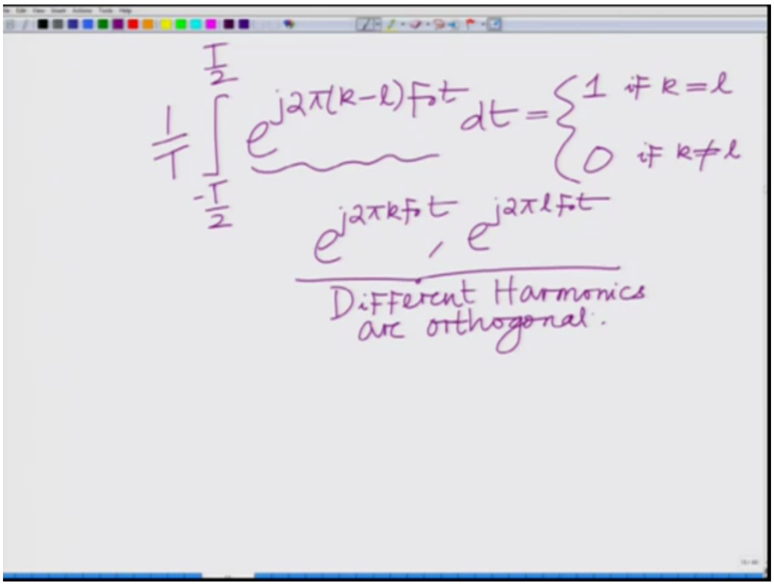

So what we have shown is that this quantity 1 over T integral minus T by 2 to T by 2 e to the power of j 2 Pi k minus L F0 t dt, this is equals to 1 if k is equals to 1 if k equals to l, 0 if k for all integers k not equals to L this is 0, okay. So and this is an incredible property that is if you can look at this, shows nothing but the fact that these different harmonics e to the power of j 2 Pi k F0 t, e to the power of j 2 Pi L F0 t, these are orthogonal the different harmonics the kth harmonic and Lth harmonic, these different harmonics that is e to the power of j 2 Pi k F0 t, e to the power of j 2 Pi L F0 t when k is not equal, these different harmonics there are different harmonics right, at different multiples of F0 k F0 and L F0 are orthogonal that is why you have integral 1 over T integral minus T over 2 to T over 2 e to the power of j 2 Pi k F0 t times e to the power of j 2 Pi L F0 t Conjugate,tThat is e to the power of minus j 2 Pi L F0 t dt that is equals to 0.

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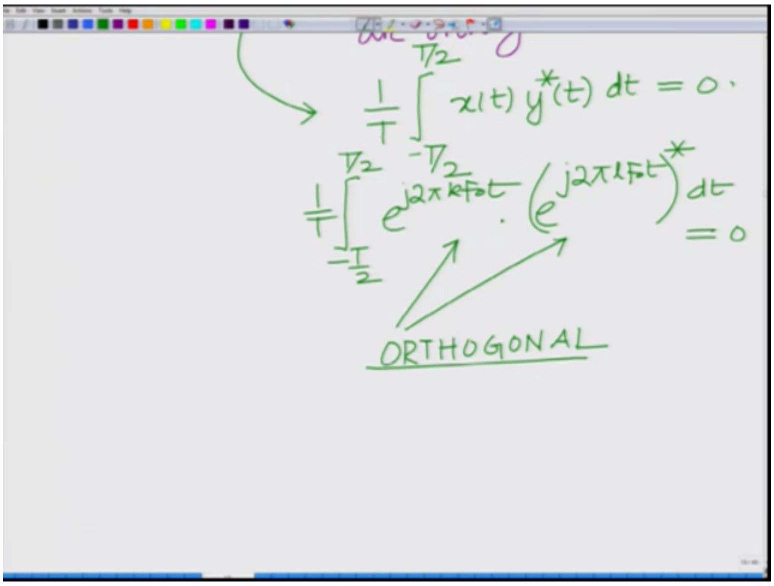

That is property, let me just write it again Clearly to illustrate the orthogonality. We say x(t) and y(t) are orthogonal if this dot product or the inner product minus 1 over T minus T by 2 x(t) into y conjugate (t) dt is equals to 0. Here we have integral minus T over 2 to T over 2 1 over T e to the power of j 2 Pi k F0 T times e to the power of minus j times e to the power of j 2 Pi L F0 t conjugate into dt equals to 0, hence these 2 are, in fact, there is a very important property, these 2 are that is the different harmonics at k F0 and harmonics at L F0, the kth harmonic and Lth harmonic are orthogonal.

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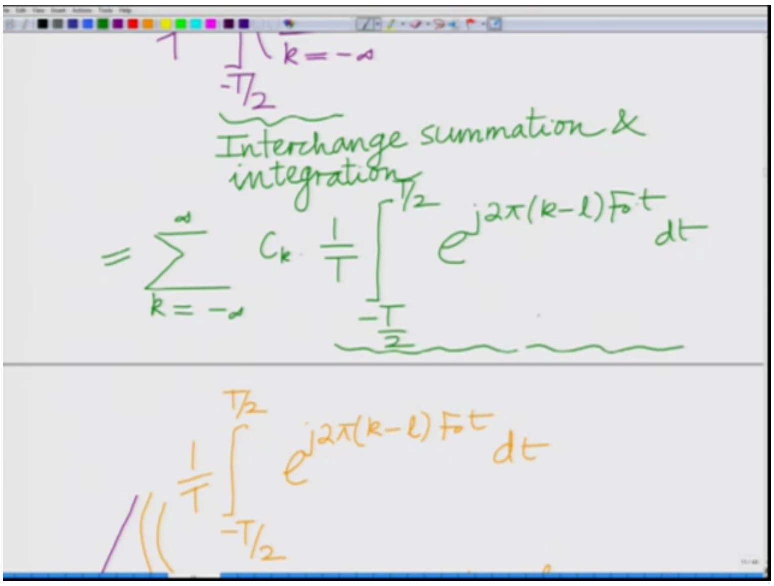

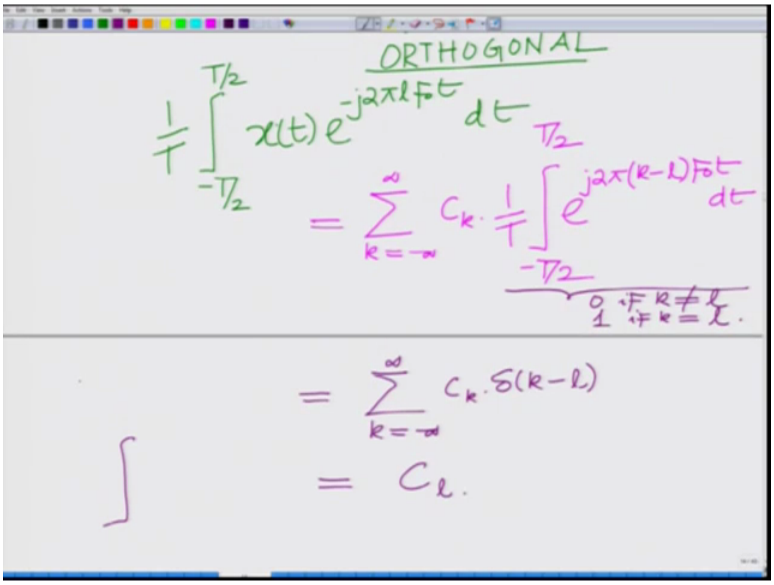

And we will now use this property to evaluate Ck CL and that can be simply shown as follows because look at this if you go back here, this integral alright. Let us go back to this integral that is summation we have shown that 1 over T integral minus T over 2 to T over 2 x(t) e to the power of minus j 2 Pi L F0 t dt, this is equals to summation k equals to minus infinity to infinity Ck times 1 over T integral minus T by 2 to T by 2 e to the power of j 2 Pi k minus L F0 t dt, this integral is nonzero this integral is 0. If this integral is 0 if k not equals to l, 1 if k equals to l. So it is going to survive only when k equals to l. So only the term corresponding to CL is going to remain and therefore this is simply equals to.

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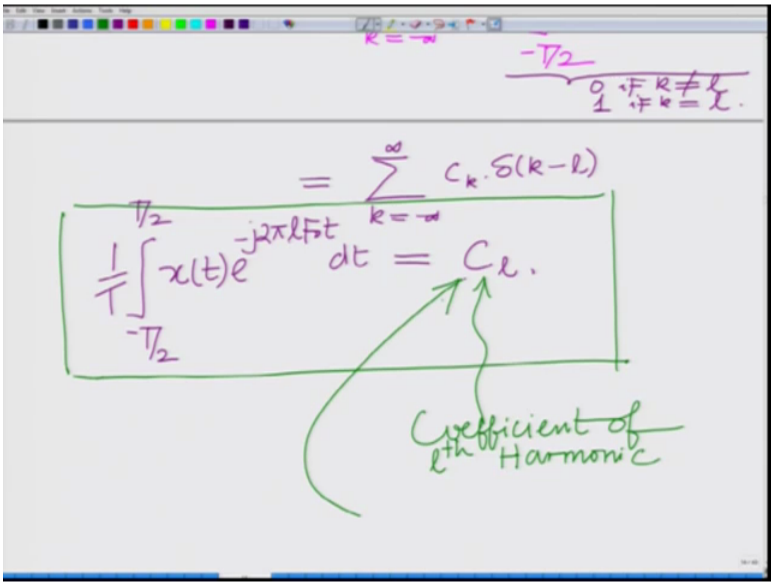

You can also write this as summation k equals to minus infinity to infinity Ck delta k minus L because this is a delta function with delta of 0 is 1 and delta of any other integer N is 0 if N is not equals to 0 that is basically therefore this is basically simply equals to your CL, okay. So fourth we have is basically that completing this we have integral x(t) e to the power of minus j 2 Pi L F0 equals to CL. That is the cominusefficient of the where CL is cominusofficiate of Lth harmonic or Lth cominusefficient in discrete Fourier series representation of x(t) or the Lth of coefficient in the discrete Fourier Lth cominusefficient in discrete Fourier series representation of x(t) or rather let us characterize this or let us qualify this as a periodic signal x of the representation of the periodic signal x(t), correct, Lth coefficient in the discrete Fourier series representation of the periodic signal x(t).

So what we have done so far is what we have done in this module is basically we have started to look at the Fourier domain or Frequency domain representation of the signal x(t), looked at the Fourier discrete Fourier series representation of a periodic signal x(t). We said that any periodic signal x(t) can be represented as with fundamental frequency F0, where F0 equals to 1 over T where capital T is the period of the signal can be represented as the summation or the linear combination of a infinite number of complex Sinusoids at the fundamental frequency F0 and multiples of the fundamental frequency that is k F0, that is the Sinusoid corresponding to the complex Sinusoid corresponding to the fundamental frequency F0 and its various harmonics, correct.

And we have also shown how to compute the discrete Fourier series cominusefficient Ck corresponding to kth harmonic in this discrete Fourier series representation. So we will stop this module here and continue with other aspects in the subsequent modules, thank you very much.