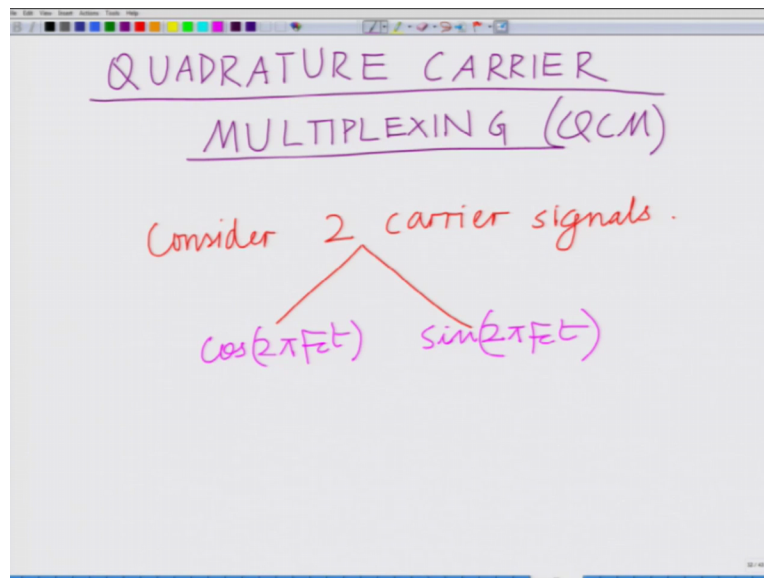


Principles of Communication- Part I
Professor Aditya K. Jagannathan
Department of Electrical Engineering
Indian Institute of Technology Kanpur
Module No 4
Lecture 18

Introduction to Quadrature Amplitude Modulation (QAM) and Demodulation of QAM

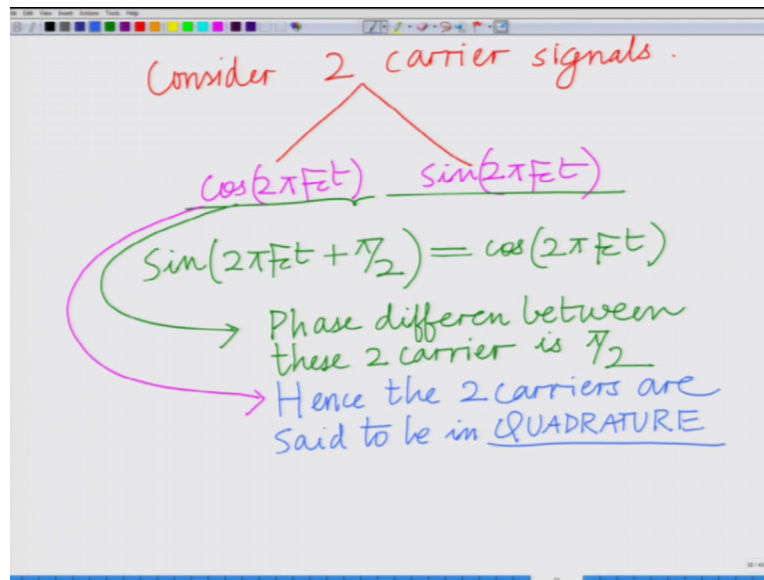
Hello welcome to another module in this massive open online course. Today let us take a look at a different modulation scheme that is quadrature carrier multiplexing, alright.

(Refer Slide Time: 0:30)



So we want to start looking at quadrature carrier multiplexing that is QCM. Now for this purpose consider there are 2 carriers or 2 carriers. Consider the carrier signals let us consider the following 2 carrier signals that is cosine $2\pi F_c t$ which is the standard carrier signal at carrier frequency F_c and also now $\sin 2\pi F_c t$, so these are our 2 carriers that is cosine $2\pi F_c t$ and $\sin 2\pi F_c t$ and F_c as usual is the carrier frequency.

(Refer Slide Time: 2:08)



Now observe that the phase difference between these 2 carriers is 90 degree that is if you look at $\sin(2\pi F_c t + \pi/2)$ that is equal to $\cos(2\pi F_c t)$ therefore these 2 carriers therefore phase difference between these 2 carriers these 2 carriers is 90 degrees or basically $\pi/2$ radian is $\pi/2$ hence these carriers to be in quadrature, quadrature denotes the phase quadrature basically denotes 90 degrees or a phase of $\pi/2$ and cities to carriers that is $\cos(2\pi F_c t)$ and the $\sin(2\pi F_c t)$ are said to be in quadrature, hence the 2 carriers that is $\cos(2\pi F_c t)$ and $\sin(2\pi F_c t)$ these are these are said to be in these are said to be in quadrature these 2 quadrature is $\cos(2\pi F_c t) \sin(2\pi F_c t)$, since the phase difference between them is $\pi/2$ these 2 carriers are said to be in quadrature.

(Refer Slide Time: 4:10)

Handwritten text: given as

$$x(t) = A_c m_I(t) \cos(2\pi f_c t) - A_c m_Q(t) \sin(2\pi f_c t)$$

Handwritten text: 2 Message signals: $m_I(t)$, $m_Q(t)$

Now let us consider a modulated signal $x(t)$ which is given as follows, alright. So we have $x(t)$ consider the modulated signal $x(t)$ given as consider the modulated signal $x(t)$ equals $x(t)$ is given as follows $x(t)$ is equal to uhh $A_c m_I(t)$ or we can just write this as $A_c m_I(t) \cos(2\pi f_c t)$ minus $A_c m_Q(t) \sin(2\pi f_c t)$ this is our modulated signal, now look at this there are 2 message signals in the above modulated signal previously we have considered only one message signal $m(t)$, now there are 2 message signals namely $m_I(t)$ and $m_Q(t)$, $m_I(t)$ is the message on the cosine.

(Refer Slide Time: 6:13)

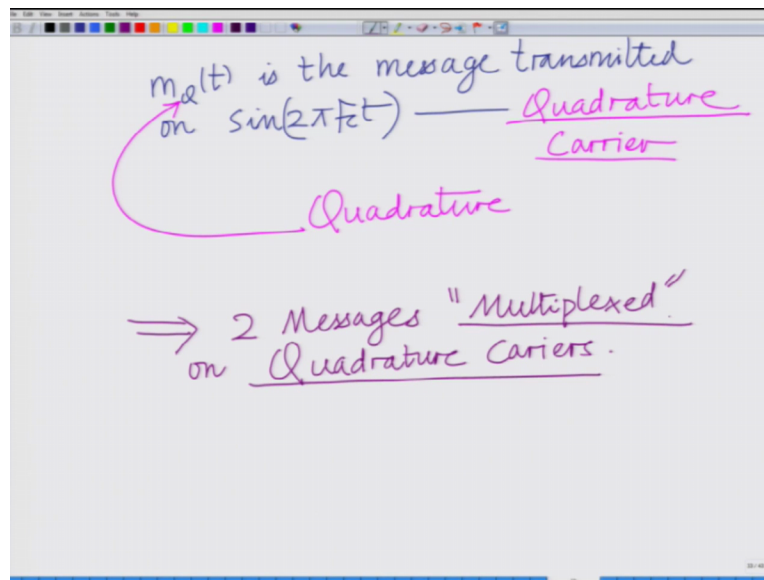
Handwritten text: 2 Message signals: $m_I(t)$, $m_Q(t)$

Handwritten text: $m_I(t)$ is the message modulated on $\cos(2\pi f_c t)$ — In-phase carrier

Handwritten text: In-phase

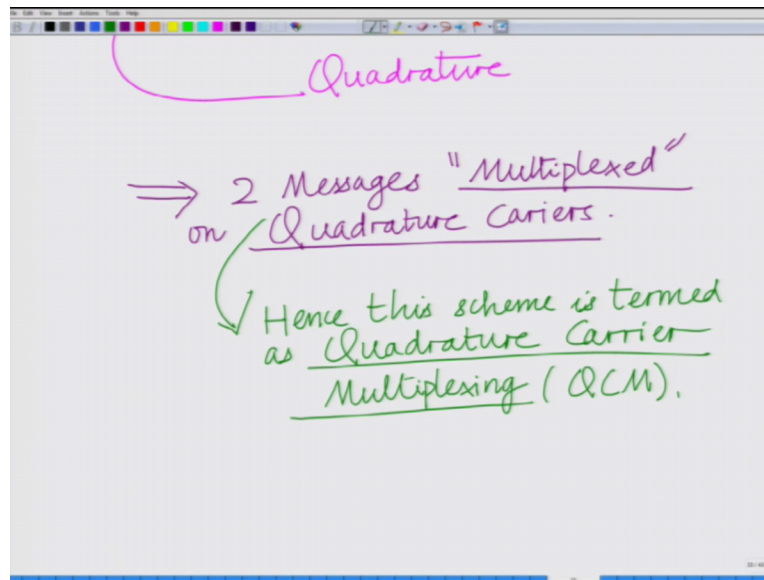
So we have $m_I(t)$ and $m_Q(t)$, observe $m_I(t)$ is the message which is modulated message which is modulated on the cosine carrier $\cos(2\pi f_c t)$, this is also termed as the in phase carrier, the cosine carrier is also termed as in phase carrier. So $m_I(t)$ with I subscript I stands for in phase, correct? The subscript I stands for in phase, correct? So $m_I(t)$ there are 2 messages $A_c m_I(t) \cos(2\pi f_c t)$ which is $A_c m_I(t) \cos(2\pi f_c t)$ minus $A_c m_Q(t) \sin(2\pi f_c t)$ there are 2 messages $m_I(t)$ and $m_Q(t)$ we are saying $m_I(t)$ is a message that is transmitted on the cosine carrier which is also termed as in phase carrier.

(Refer Slide Time: 7:28)



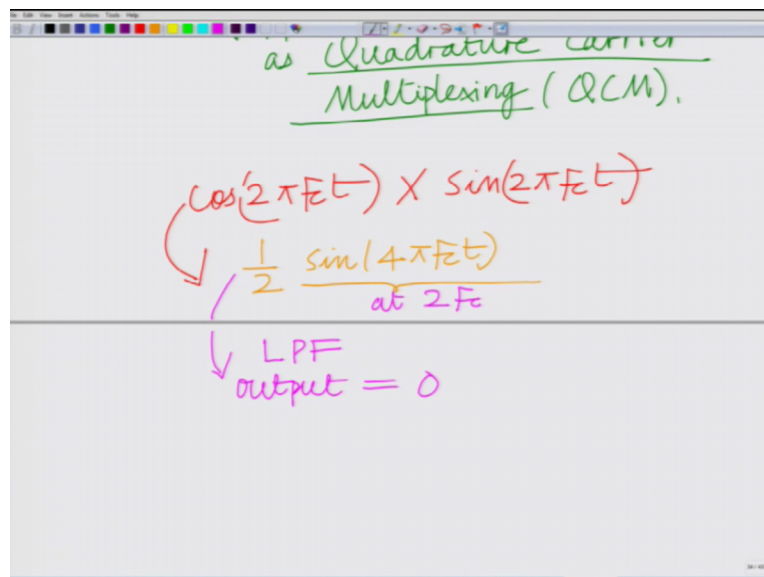
And $m_Q(t)$ (simul) similarly $m_Q(t)$ is the $m_Q(t)$ is the message transmitted on the quadrature on on $\sin(2\pi f_c t)$ which is also termed as a quadrature carrier, remember because $\sin(2\pi f_c t)$ is in quadrature with $\cos(2\pi f_c t)$ so this is also termed as a quadrature carrier. So this Q subscript Q stands for the term quadrature. So we have 2 message signals $m_I(t)$, $m_Q(t)$ $m_I(t)$ is a message signal there is an in phase message signal transmitted on the in phase carrier $\cos(2\pi f_c t)$, $m_Q(t)$ is a quadrature message signal transmitted on the (quadra) quadrature carrier $\sin(2\pi f_c t)$, alright. So we have 2 messages remember multiplexed on the on 2 quadrature carrier, so implies net we have 2 messages which are multiplex, correct? That is simultaneously transmitted on quadrature carriers.

(Refer Slide Time: 9:58)



We have 2 messages which are multiplexed on quadrature carriers hence this scheme is termed as quadrature carrier multiplex this scheme is termed as quadrature carrier multiplexing which is abbreviated as QCM scheme will be transmitting 2 message signal is different message signals $m_1(t)$ $m_2(t)$ on quadrature carriers is termed as quadrature carrier multiplexing further notice an interesting property of these quadrature carriers notice an interesting.

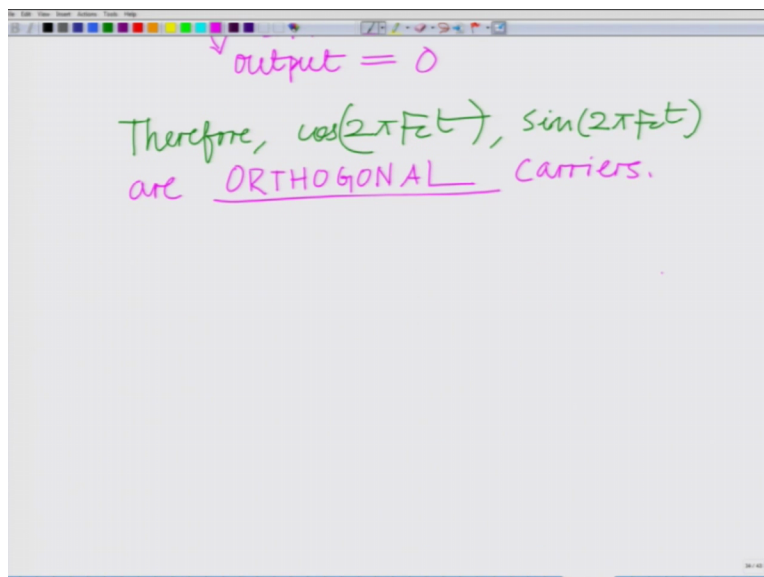
(Refer Slide Time: 10:28)

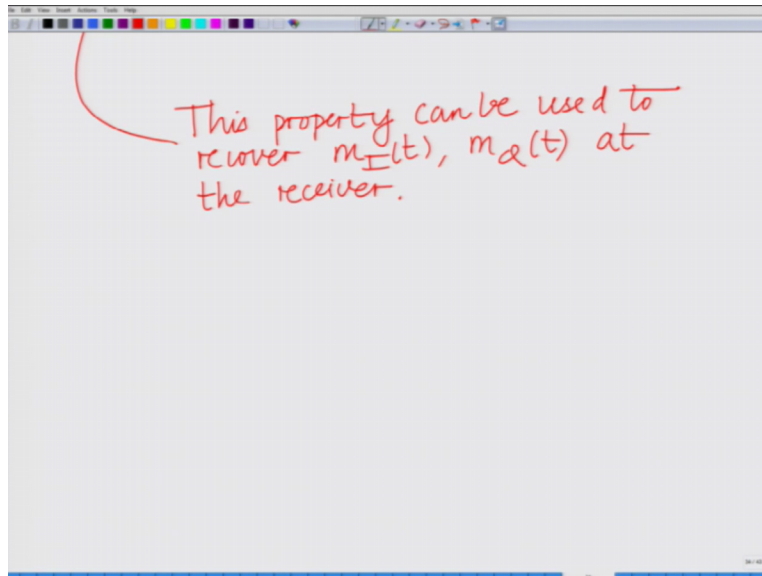


Consider now for instance let us say we multiply or demodulate cosine $2\pi F_c t$ by $\sin 2\pi F_c t$ that is we are multiplying at the receiver by $\sin 2\pi F_c t$ demodulating uhh cosine $2\pi F_c t$ by $\sin 2\pi F_c t$ that gives half $\sin 4\pi F_c t$ which is basically at 2 (freq) 2 times a carrier frequency $2F_c$ therefore if I low pass filter this LPF if we pass it through a low pass filter the resulting output equal to 0.

So if I demodulated cosine with the sin or sin with cosine, alright. You can see that they works either way, so cosine $2\pi F_c t$ into $\sin 2\pi F_c t$ is half $\sin 4\pi F_c t$, right? Which is the component which is basically at $2F_c$ which is at twice the carrier frequency? So if it I pass it through a low pass filter such as uhh which is basically which is cut off frequency in the baseband then obviously the output of the low pass filter is going to be 0. Therefore these 2 carriers the cosines cosine $2\pi F_c t$ and $\sin 2\pi F_c t$ are orthogonal because if you demodulated cosine with sin or sin with cosine and low pass filter it the output is 0.

(Refer Slide Time: 11:53)





Therefore the cosine & sin $2\pi f_c t$ therefore these 2 carriers cosine $2\pi f_c t$ these 2 carriers cosine $2\pi f_c t$, sin $2\pi f_c t$ these 2 carriers are orthogonal. These are this is an important property these are orthogonal carriers. Now this property can be used for demodulation of or this (mod) property can be used to demodulation or recover this property that is orthogonality can be used to recover can be used to recover $m_I(t)$, $m_Q(t)$ at the receiver, so we recover this the 2 message signals the in phase message signal $m_I(t)$ and the quadrature message signal $m_Q(t)$ at the receiver using this orthogonality property of the carriers. Now how is that done?

(Refer Slide Time: 13:29)

Demodulate with $\cos(2\pi f_c t)$

$$\begin{aligned}
 x(t) \times \cos(2\pi f_c t) &= (A_c m_I(t) \cos(2\pi f_c t) - A_c m_Q(t) \sin(2\pi f_c t)) \times \cos(2\pi f_c t) \\
 &= \frac{A_c m_I(t)}{2} (1 + \cos(4\pi f_c t))
 \end{aligned}$$

$$\begin{aligned}
 x(t) \times \cos(2\pi f_c t) &= (A_c m_I(t) \cos(2\pi f_c t) - A_c m_Q(t) \sin(2\pi f_c t)) \\
 &\quad \times \cos(2\pi f_c t) \\
 &= \frac{A_c m_I(t)}{2} (1 + \cos(4\pi f_c t)) \\
 &\quad - \frac{A_c m_Q(t)}{2} \sin(4\pi f_c t)
 \end{aligned}$$

Let us illustrate that at the receiver for instance consider demodulation with cosine $2\pi f_c t$ if you demodulated cosine $2\pi f_c t$, what we have? Your $x(t)$ into cosine $2\pi f_c t$ which is equal to A_c or let me write it $A_c m_I(t) \cos(2\pi f_c t) - A_c m_Q(t) \sin(2\pi f_c t)$ into cosine $2\pi f_c t$ which is equal to, now look at this $A_c m_I(t) \cos^2(2\pi f_c t)$. First-term is $A_c m_I(t) \cos^2(2\pi f_c t)$ which is which I write as $A_c m_I(t)$ divided by 2 into $1 + \cos(4\pi f_c t)$ minus $A_c m_Q(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$ which I can write as $A_c m_Q(t)$ divided by 2 into $2 \sin(2\pi f_c t) \cos(2\pi f_c t)$ which is $\sin(4\pi f_c t)$, okay.

(Refer Slide Time: 15:40)

$$\begin{aligned}
 &= \frac{A_c m_I(t)}{2} + \frac{A_c m_I(t)}{2} \cos(4\pi f_c t) \\
 &\quad - \frac{A_c m_Q(t)}{2} \sin(4\pi f_c t)
 \end{aligned}$$

Baseband component

And now what I am going to do simplify this a little bit this is basically $A_c m_I(t)$ divided by 2 plus $A_c m_Q(t)$ divided by 2 $\cos 4\pi F_c t$ minus $A_c m_Q(t)$ divided by 2 $\sin 4\pi F_c t$ now if you look at this you will realize that these 2 the last 2 components that is $A_c m_I(t)$ by 2 $\cos 4\pi F_c t$ and minus $A_c m_Q(t)$ divided by 2 $\sin 4\pi F_c t$ these 2 components are at $2F_c$, alright. So twice the carrier frequency, so this is at $2F_c$ this is at $2F_c$ of course, this is the baseband component $A_c m_I(t)$ by 2 is simply $m_I(t)$ is scaled by a constant, so this is your baseband component which is centered at 0 frequency, correct? And let me just this is your baseband.

(Refer Slide Time: 17:29)

$2F_c$

Baseband component

LPF - Low Pass Filter

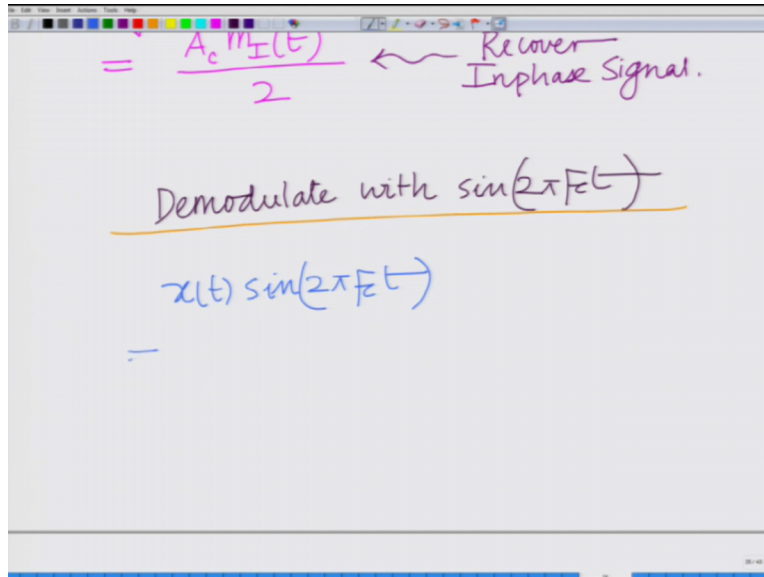
= $\frac{A_c m_I(t)}{2}$ ← Recover Inphase Signal.

So if I pass this through a low pass filter I pass this through a low pass filter of appropriate cut-off frequency low pass filter that is basically I low pass filter this is the components at $2F_c$ are filtered away and what I have is $A_c m_I(t)$ by 2 that is recover the in phase signal that is what I am able to do is I am able to recover the in phase signal and you can see that this is possible because of the orthogonality because once I demodulate with $\cos 2\pi F_c t$ at the receiver the sine carrier which is orthogonal, right?

The sin carrier is orthogonal to $\cos 2\pi F_c t$ the cosine carrier therefore uhh the component that is $m_Q(t) \sin 2\pi F_c t$ when it is demodulated with the cosine that vanishes, alright and therefore I am able to recover the in phase messages signal, now similarly to recover the quadrature message signal obviously I am going to demodulate with the $\sin 2\pi F_c t$, alright. And the in

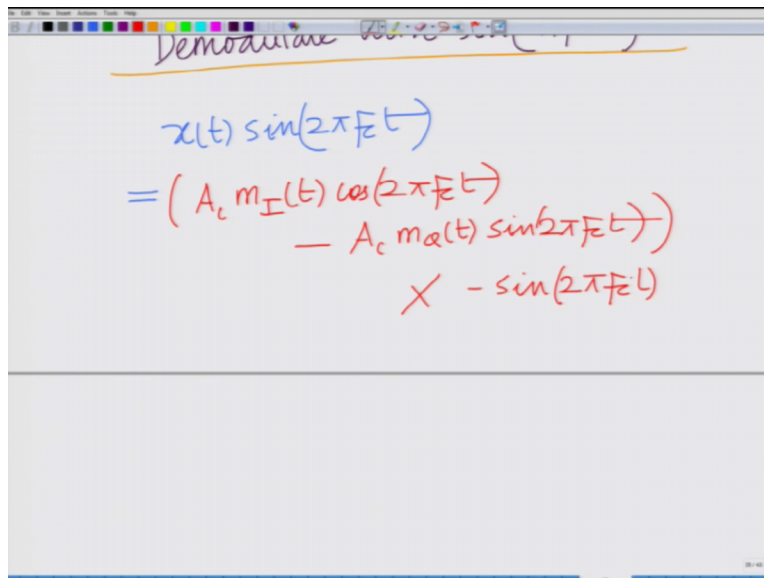
phase component which is on cosine $2\pi f_c t$ is going to vanish because this is orthogonal to the quadrature carrier that is $\sin 2\pi f_c t$.

(Refer Slide Time: 18:50)


$$= \frac{A_c m_I(t)}{2} \leftarrow \text{Recover Inphase Signal.}$$

Demodulate with $\sin(2\pi f_c t)$

$$x(t) \sin(2\pi f_c t)$$



Demodulate with $\sin(2\pi f_c t)$

$$x(t) \sin(2\pi f_c t)$$
$$= (A_c m_I(t) \cos(2\pi f_c t) - A_c m_Q(t) \sin(2\pi f_c t))$$
$$\quad \quad \quad \times - \sin(2\pi f_c t)$$

$$\begin{aligned}
 & + \frac{A_c m_a(t)}{2} (1 - \cos(4\pi F_c t)) \\
 = & \frac{-A_c m_I(t)}{2} \sin(4\pi F_c t) \\
 & + \frac{A_c m_a(t)}{2} - \frac{A_c m_a(t)}{2} \cos(4\pi F_c t) \\
 & \text{Baseband} \qquad \qquad \qquad 2F_c
 \end{aligned}$$

Therefore to recover $m_Q(t)$, let me illustrate that also briefly demodulate with $\sin 2\pi F_c t$ and when I demodulate with $\sin \pi F_c t$ I have $x(t) \sin 2\pi F_c t$ which is equal to $A_c m_I(t) \cos 2\pi F_c t$ minus $A_c m_Q(t) \sin 2\pi F_c t$ times $\sin 2\pi F_c t$ or let us demodulate with minus $\sin 2\pi F_c t$ is simply a phase changed version of $\sin 2\pi F_c t$, okay. Demodulating with minus $\sin 2\pi F_c t$, now the first term is minus $A_c m_I(t)$ by 2, cosine $2\pi F_c t$ into $\sin 2\pi F_c t$ or 2 minus $A_c m_Q(t)$ by 2 into 2 cosine $2\pi F_c t \sin 2\pi F_c t$ which is $\sin 4\pi F_c t$, correct?

Now minus or plus $A_c m_Q(t)$ by 2 into $A_c m_Q(t)$ by 2 uhh $A_c m_Q(t)$ into \sin^2 uhh $A_c m_Q(t)$ into $\sin^2 2\pi F_c t$ which is basically $1 - \cos 4\pi F_c t$ divided by 2 again simplifying this this is going to be minus $A_c m(t) m_I(t)$ divided by 2 uhh $\sin 4\pi F_c t$ $4\pi F_c t$ plus $A_c m_Q(t)$ divided by 2, correct? Minus $A_c m_Q(t)$ divided by 2 cosine $4\pi F_c t$ and now if you can look at this this component is at $2F_c$ this component is at $2F_c$ this is your baseband component, so once I low pass filter this again the components at $2F_c$ they go away and what I am left with is?

(Refer Slide Time: 22:40)

The diagram illustrates the demodulation process for a quadrature multiplexed signal. It shows the following steps:

- The input signal is a sum of two terms: $\frac{A_c m_a(t)}{2}$ (labeled "Baseband") and $\frac{A_c m_a(t)}{2} \cos(4\pi F_c t)$ (labeled "2F").
- An arrow labeled "LPF" (Low Pass Filter) points from the first term to the second term.
- The output of the LPF is $\frac{A_c m_a(t)}{2}$, which is labeled "Recover Quadrature signal".

Now I am left with this quadrature message signal $A_c m_Q(t)$ by 2. So basically able to recover the by recover the quadrature message signal. By so what we are doing is by demodulating with each of the 2 orthogonal sub carriers by the cosine carrier I am able to recover the in phase message signal by the quadrature carrier demodulating with the quadrature carrier that is $\sin 2\pi F_c t$ I am able to recover the quadrature message signal and the property that we are using essentially is that these 2 carriers the cosine $2\pi F_c t$ and $\sin 2\pi F_c t$ which are quadrature carriers are orthogonal, alright.

So that is the importance and therefore we are able to multiplex 2 signals in parallel, right $m_I(t)$ the in phase message signal and the quadrature message signal respectively on these 2 carriers quadrature carriers which are orthogonal and this is basically termed as quadrature carrier multiplexe, okay.

(Refer Slide Time: 24:26)

2

Therefore, using orthogonal carriers
 $\cos(2\pi f_c t)$, $\sin(2\pi f_c t)$, 2 parallel
message signals, $m_I(t)$, $m_Q(t)$ can
be transmitted on the same channel

message signals, $m_I(t)$, $m_Q(t)$ can
be transmitted on the same channel

Sharing same BW

This scheme is termed
as Quadrature Carrier
Multiplexing (QCM).

So therefore to summarize this therefore using orthogonal carriers to summarize the idea using orthogonal carriers that is cosine $2\pi f_c t$, sin $2\pi f_c t$ 2 parallel streams of information or 2 parallel message signals and these 2 parallel sig message signals are $m_I(t)$, $m_Q(t)$ can be transmitted on the same and that is important we are able to transmit on the same channel meaning the same sharing the same bandwidth. What we mean by this? We are transmitting them on the same channel is where basically they are sharing the same sharing the same bandwidth and this phenomenon this is termed as basically this principle or this scheme is termed as and we have

seen this before this scheme is termed as quadrature carrier multiplexing or basically quadrature carrier multiplexing or basically QCM.

So what we have demonstrated in this module is how to use these 2 quadrature carriers, $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$ which are orthogonal to multiplex 2 different message signals that is the in phase message signal and the quadrature message signal simultaneously on these 2 quadrature carriers which are orthogonal and uses orthogonality property at the receiver to recover both the in phase message signal and the quadrature message signal by alternatively demodulating first with $\cos 2\pi f_c t$ and later with $\sin 2\pi f_c t$, thank you. So we will stop here and continue with other aspects in the subsequent modules thank you.