

Principles of Communication- Part I
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Module No 4

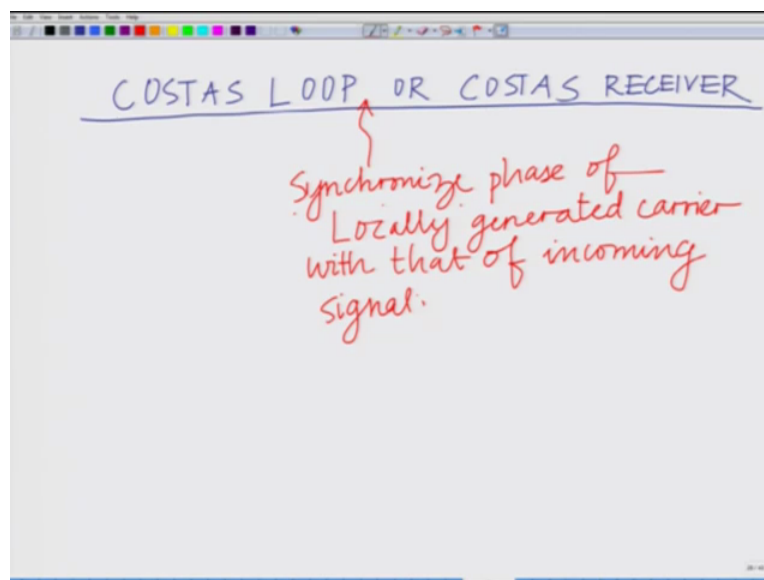
Lecture 17

Phase Synchronization using Costas Receiver for Double Sideband (DSB) Suppressed Carrier (SC) Demodulation

Hello welcome to another module in this massive open online course. So we are looking at the importance of phase synchronization between the carrier wave component in the incoming signal and the locally generated carrier at the oscillator and we can we have seen that first a loss of phase synchronization results in a loss of SNR, right? Decrease in the SNR output and the the (progre) SNR progressively decreases SNR this SNR progressively worsens as ϕ increases, correct?

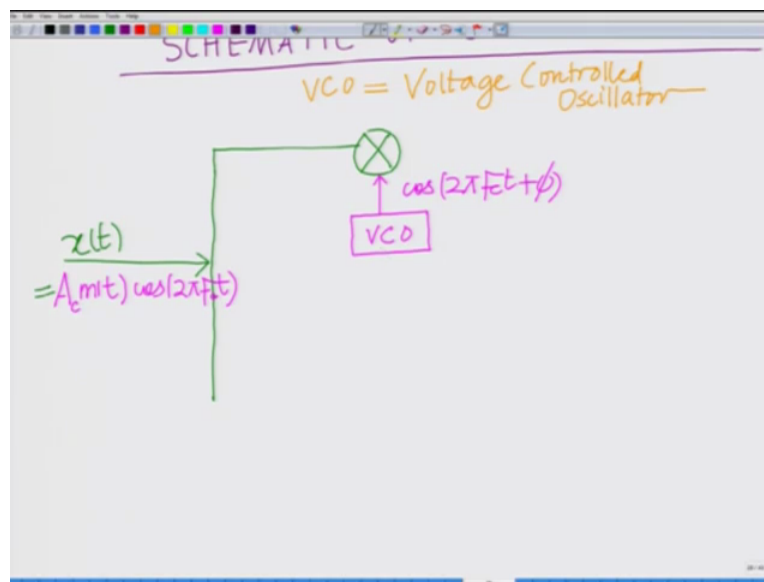
And also we have seen that this phase synchronization, right? Is frequently lost because of the mobility of the user typically in a wireless channel scenario where the distances are changing and because of that the delay and from the (de) due to the delay the phase offset of the carrier wave of the incoming signal is constantly varying and therefore this phase of the locally generated carrier needs to be constantly re-synchronized with that of the phase of the incoming carrier wave and this is done using a Costas loop.

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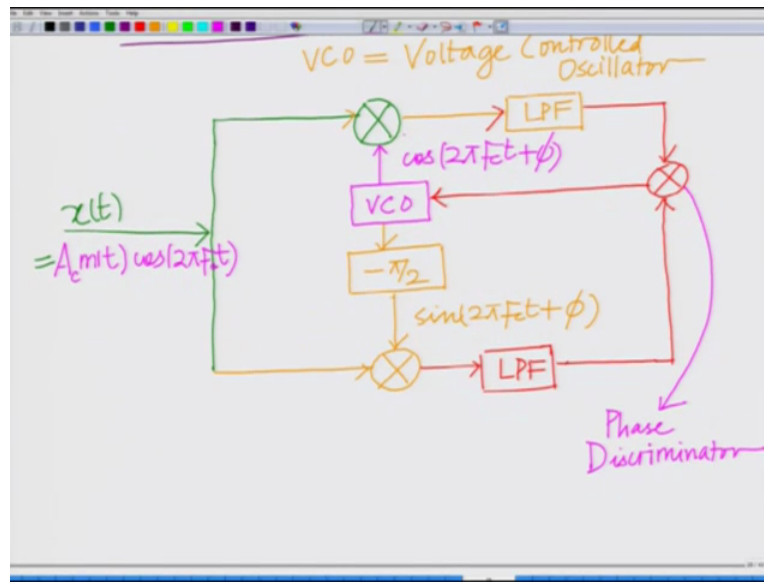
So today let us work look at the working of the Costas loop or the Costas receiver, okay. So in this module let us look at the working of the Costas loop or also termed as a Costas receiver and the purpose of the Costas loop is to synchronize, remember this we said is to synchronize purpose of Costas loop is to synchronize phase of incoming carrier wave phase of a incoming signal with that or put it the other way phase of synchronize phase of locally generated carrier, correct? Synchronize phase of locally generated synchronize phase of locally generated signal or locally generated carrier with that of the, with that of the incoming signal, alright.

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Okay and for that let us consider the structure of the Costas receiver, alright. The Costas receiver, I am going to draw the schematic of the Costas receiver the schematic diagram of the Costas receiver and that is given as follows, correct? I have the incoming signal which is, so let us draw the incoming signal. That is $x(t)$, now this is passed through two demodulators, okay. So $x(t)$ equals remember $A_c m(t) \cos(2\pi f_c t)$, okay this is the signal $x(t)$ it is the modulated with, alright. First it is demodulated pass to 2 demodulator s, first one is $\cos(2\pi f_c t + \phi)$ remember the locally generated carrier can have a phase offset with respect to the incoming signal, okay.

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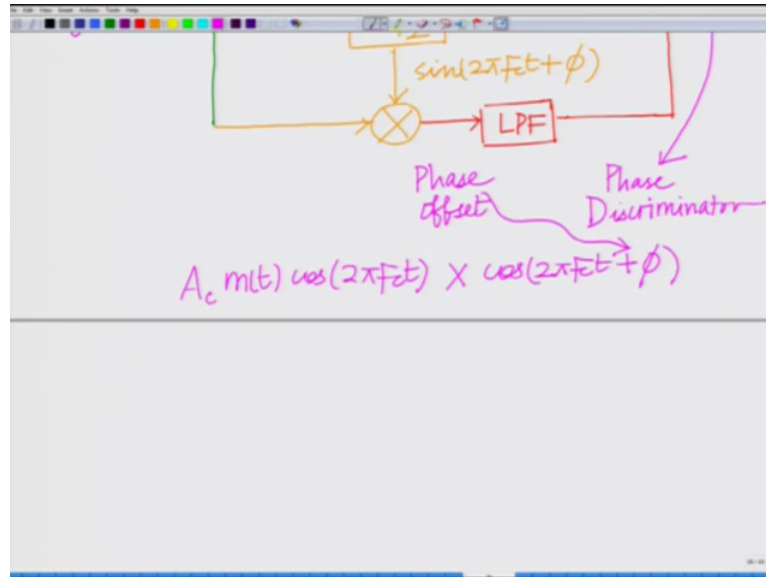
So I have the VCO this is termed as the VCO this unit here is termed as the VCO which basically stands for voltage control oscillator. So one can tune this oscillator by an in through an input signal. So this is VCO equals voltage or stands for the voltage control oscillator, alright. So we have the incoming signal which is passed simultaneously to 2 demodulators the input to one of the demodulators is cosine $2\pi f_c t$ plus ϕ , alright. Notice that this can possibly have a phase offset, alright. Because that is what that is the reason for which the Costas loop is used that is to recall that is unique in this phase synchronization.

That is to synchronize the phase of the local carrier with that of the incoming signal so there is a phase offset ϕ initially which we hope to drive down to 0 eventually, okay. And the input to the other demodulator is derived from the same VCO voltage control oscillator by shifting it by 90 degrees to generate a sine wave, correct? So cosine $2\pi f_c t$ plus ϕ minus $\pi/2$ gives rise to $\sin 2\pi f_c t$ this gives rise to $\sin 2\pi f_c t$ plus ϕ , okay. And this is passed through a low pass filter this is also passed through a low pass filter followed by both these are passed through what is known as a phase discriminator which is can be thought of as a simple multiplier this is passed through a phase discriminator, okay.

So this is basically a phase discriminator which can be thought of a simple multiplier, this is a phase discriminator, okay which in other words performs a simple can be thought of as equivalently it can be equivalently thought of as performing a simple multiplication operation,

alright. Now so what you are doing, alright and output of this phase discriminator that is given to the voltage control oscillator.

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Now if you can look at this so $A_c \cos(2\pi f_c t)$ demodulated with $A_c \cos(2\pi f_c t + \phi)$. We have already seen that, so $A_c m(t)$ let me describe that $A_c \cos(2\pi f_c t)$ or rather $A_c m(t) \cos(2\pi f_c t)$ that is the incoming message signal demodulated with $\cos(2\pi f_c t + \phi)$, this ϕ is the phase offset which is possibly nonzero but we would like to progressively drive this towards 0 there is we we would like to adjust the aim of the Costas loop is to adjust this phase of the voltage control oscillator that is the carrier that is generated by the local oscillator such that it is in phase with that of the incoming signal, alright. That is the aim of the Costas loop, okay.

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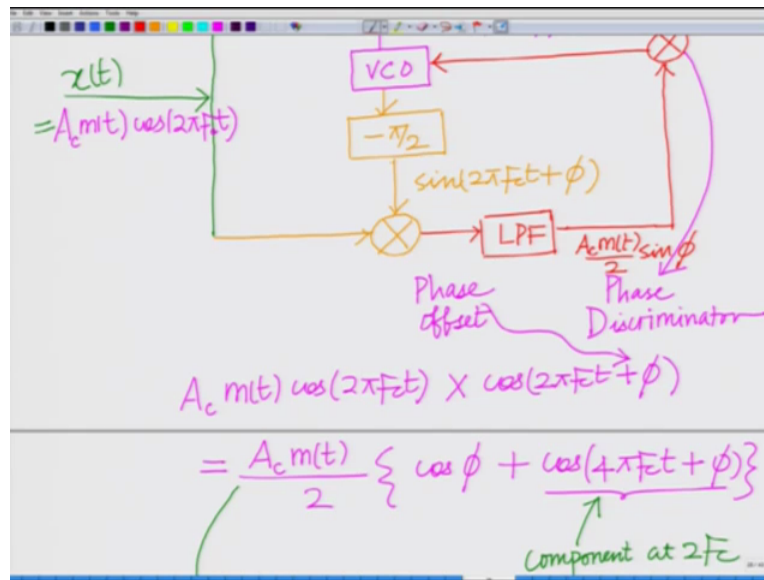
$$A_c m(t) \cos(2\pi f_c t) \times \cos(2\pi f_c t + \phi)$$
$$= \frac{A_c m(t)}{2} \{ \cos \phi + \underbrace{\cos(4\pi f_c t + \phi)}_{\text{Component at } 2f_c} \}$$

The whiteboard shows the derivation of the trigonometric identity for the product of two cosines. The first line is $A_c m(t) \cos(2\pi f_c t) \times \cos(2\pi f_c t + \phi)$. The second line shows the result: $= \frac{A_c m(t)}{2} \{ \cos \phi + \cos(4\pi f_c t + \phi) \}$. A green arrow points from the $\frac{A_c m(t)}{2}$ term to the first line. Another green arrow points from the $\cos(4\pi f_c t + \phi)$ term to the text "Component at $2f_c$ ".

So you (dim) but although initially there is going to be a phase offset once you start the synchronizing synchronization process and therefore now you can see the output of this we have already seen this this is $A_c m(t)$ by 2 cosine $2 f_c t$ plus ϕ cosine $2\pi f_c t$ minus ϕ this is cosine ϕ minus cosine this is cosine ϕ , correct? This is cosine ϕ plus cosine $4\pi f_c t$ cosine $4\pi f_c t$ plus ϕ this is the component at $2 f_c$. This is a component $2f_c$, so once I pass it through a low pass filter, correct?

Okay, pass it through a low pass filter LPF, once I pass it through the low pass filter the output is going to be $A_c m(t)$ by 2 cosine ϕ $m(t)$ by 2 cosine ϕ therefore the output of this low pass filter at the output of the cosine modulator is $A_c m(t)$ by 2 cosine ϕ .

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Now let us look at the output of the second demodulator remember the second demodulator is derived from the first one that is a carrier wave of the second demodulator which is cosine $2\pi f_c t$ plus phi minus pi by 2 is derived from the first is derived from the same voltage control oscillator but it's offset by pi by 2 therefore the output will be sin $2\pi f_c t$ plus phi. And therefore the output of the second demodulator will be, so second demodulator output or let us call this lower demodulator, so the above one was upper demodulator so this is lower demodulator output this is $A_c m(t) \cos(2\pi f_c t)$ times sin $2\pi f_c t$ plus phi which is equal to $A_c m(t)$ by 2 sin of sin of phi you are using the relation sin a cos b is half sin a plus half sin a plus b plus half sin half sin a minus b that is sin phi plus sin $4\pi f_c t$ plus phi again this is the component at $2F_c$.

This is a component at $2F_c$ that is twice the carrier frequency, so I low pass filter this I get $A_c m(t)$ by 2 sin phi, so this is the output of the low pass filter, okay. So the output of the low pass filter of the lower branch is $A_c m(t)$ by 2 sin phi, okay.

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$$A_c m(t) \cos(2\pi F_c t) \times \cos(2\pi F_c t + \phi)$$

$$= \frac{A_c m(t)}{2} \{ \cos \phi + \cos(4\pi F_c t + \phi) \}$$

LPF = Low Pass Filter

$$= \frac{A_c m(t)}{2} \cos \phi$$

Lower Demodulator output

So the output of the low pass filter in the upper branch is $A_c m(t) \cos \phi$ output of the low pass filter in the lower branch is $A_c m(t) \sin \phi$ because it is demodulated using $\sin 2\pi F_c t + \phi$. And the phase discriminator output is going to be the product which is therefore $A_c^2 m^2(t) \sin \phi \cos \phi$.

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$$\text{Output of Phase Discriminator}$$

$$= \frac{A_c m(t)}{2} \cos \phi \cdot \frac{A_c m(t)}{2} \sin \phi$$

$$= \frac{A_c^2 m^2(t)}{4} \sin \phi \cos \phi$$

So output of phase discriminator so let us write this down output of the phase discriminator equals $A_c m(t) \sin 2\phi$ that is product of the outputs of the low pass filter on the upper branch and

low pass filter on the lower branch this is $A_c m(t)$ by $2 \sin \phi$ which is $A_c m(t)$ by 4, A_c square m square t by $4 \sin \phi$ into $\cos \phi$. And now observe that A_c square m square t by 4 is always going to be greater than or equal to 0, however \sin and \cos ϕ for ϕ positive or negative is always is $\cos \phi$, now $\sin \phi$ into $\cos \phi$.

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$$= \frac{A_c m(t)}{4} \sin \phi \cos \phi.$$

Assume, ϕ small... $-\pi/2 \leq \phi \leq \pi/2$

If $\phi \geq 0$, we have

$$\text{PD output} = \frac{A_c^2 m^2(t)}{4} \cos \phi \sin \phi \geq 0$$

Now let us look at this for a small ϕ that is ϕ that is a small phase offset ϕ because we do not typically expect the phase offset to be very large, alright. So the phase offset let us look at a scenario where the phase offset is relatively small. That is the (received) incoming signal but phase (frequency) synchronization has been lost and so we are trying to resynchronize these 2 signals that is re-synchronize the locally generated carrier, so let us say ϕ is very small, alright. So ϕ is small ϕ is or let us assume that $0 < \phi$, so let us assume ϕ to be small phase offset ϕ to be small that is let us we can say that $0 < \phi \leq \pi/2$ that there is phase offset is small.

Now if ϕ is greater than 0 then the output observe now ϕ greater than 0 or if ϕ greater than 0 we have the phase discriminator output the phase discriminator, let us denote this by PD the phase discriminator can be denoted by PD the PD output equals A_c square, correct? M square t divided by 4 $\cos \phi \sin \phi$ now each of these components that's $\cos \phi$ is positive $\sin \phi$

is positive of course $A_c^2 m^2(t)$ by 4 is positive, so this is greater if ϕ is greater than or equal to 0 this is greater than or equal to 0.

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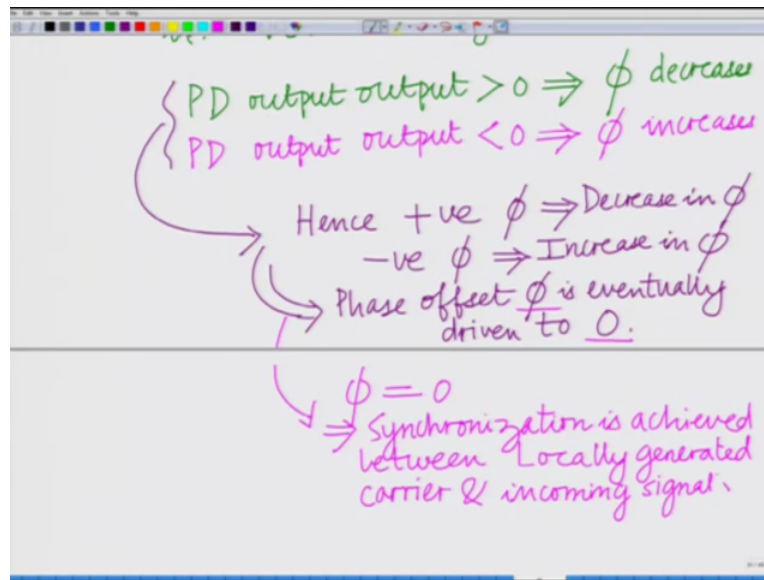
Similarly, if $\phi < 0$,

$$\text{PD output} = \frac{A_c^2 m^2(t)}{4} \underbrace{\cos \phi}_{\geq 0} \underbrace{\sin \phi}_{< 0} < 0$$

Similarly if ϕ is less than 0 the PD out phase discriminator output equals $A_c^2 m^2(t)$ by 4 cosine ϕ sin ϕ this is less than 0 because this is remember we are still considering ϕ small that is minus $\pi/2$ less than or equal to ϕ less than equal to $\pi/2$ this is greater than or equal to 0 in fact this is greater than 0 because this is the square of quantities unless $m(t)$ is equal to 0 this is greater than equal to 0 cosine ϕ is greater than equal to 0, however sin ϕ cosine ϕ is greater than 0 however sin ϕ is basically less than zero therefore this is less than 0.

So what we are seeing is that interestingly if ϕ is positive, correct? If ϕ is positive if ϕ is greater than 0 then the phase discriminate that is ϕ is greater than 0 but only slightly greater than 0 there is a phase discriminator is output is positive and if ϕ is less than 0 then the phase discriminator output is negative and the VCO is configured such that if the input to it is positive then the phase ϕ is driven down because remember we are considering a voltage tunable voltage tunable that is voltage (con) control oscillator whose phase can be tuned, so if the if the input to it is positive the phase is driven down. If the input to is negative the phase is driven up and therefore eventually the phase will settle at 0. So that the phase eventually our eventually it will convert the phase will tend towards 0.

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So the PD, so the VCO is configured such that the voltage control oscillator is configured such that PD output the (pha) the phase discriminator output greater than 0 greater than 0 implies phi decreases similarly PD output less than 0 implies phi increases, so the VCO is configured such that phi, if phi is greater than 0 than the PD output is positive the PD output is positive the phase (dis) the VCO is configured such that the phase Phi is driven down. Therefore phi is, if the phase Phi is greater than 0 it is driven down it is decreased basically driven down towards 0 similarly if the phase Phi is less than 0 than the PD output is negative, correct?

The PD output is negative the input to the VCO is negative when the input to the VCO is negative the phase Phi is driven up and therefore now this negative value of phi is driven up therefore the phase Phi basically again tends to 0. So therefore the phase we can see again here that the phase phi eventually tends to 0 or hence positive phi decreases positive phi implies decrease in phi. A Positive phi implies a decrease in phi negative value of phi implies increase in phi and therefore this implies phase offset phi eventually is eventually driven to is eventually is eventually driven to 0 and therefore phase synchronization is achieved.

So what is happening is you are looking at the phase discriminator output which is fed to the voltage control oscillator and if phi is greater than 0 than the input turns out that the input to the VCO is positive which drives down phi if Phi is less than 0 than the input to the VCO is negative, correct? Which drives up phi and therefore Phi is eventually driven to 0 which means if

Φ is 0 which means there is phase synchronization between the locally generated carrier and the incoming carrier, so $\Phi = 0$ implies carrier synchronization is achieved that is implies synchronization is achieved let us put it that way synchronization is achieved between locally generated between the locally generated carrier and the incoming signal.

So by this process what we have is basically we are eventually the Costas loop the Costas receiver is is automatically, alright to this process will feedback through this loop through this feedback look at the there is a feedback in this loop, alright. So through this loop it is eventually able by appropriately adjusting the phase of the voltage control oscillator it is able to achieve phase synchronization between the locally generated carrier and that of the carrier wave component in the incoming signal.

So this Costas loop is very critical or has a key role in communication system and especially in a wireless communication system because remember we looked we (expla) it has been explained in the previous module that the wireless communication system synchronization this phase synchronization between the carrier of the incoming signal and the locally generated carrier the locally generated carrier is frequently lost Because of the constant motion of the user as the user is mobile the distance from the base station is changing hence the delay is changing and this gives rise to the phase offset between the carrier of the incoming signal and the locally generated carrier which can be adjusted, alright.

Which can that synchronization can be regained through using the Costas loop, alright. So will stop here and continue with the other aspects in the subsequent modules, thank you.