

Principles of Communication- Part I
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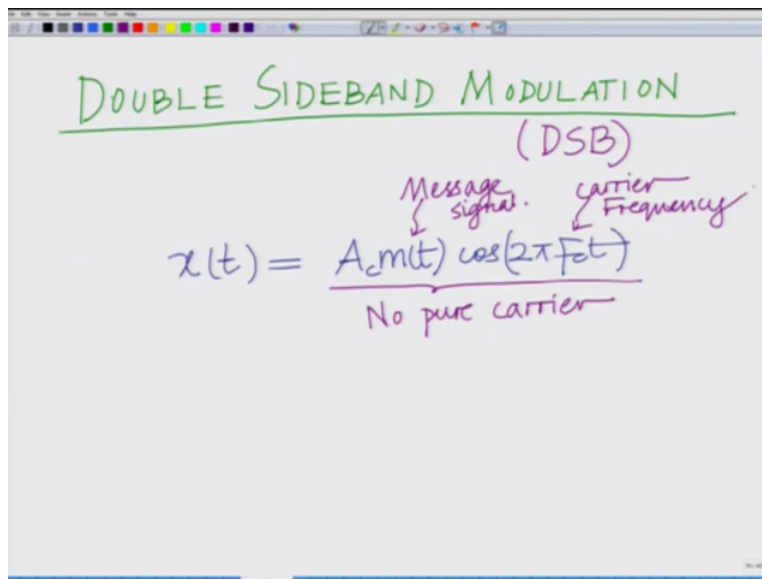
Module No 3

Lecture 14

Double Sideband (DSB) Suppressed Carrier (SC) Modulation, Spectrum of DSB-SC Signals and Coherent Demodulation

Hello welcome to another module in this massive open online course. So today let us start looking at double sideband modulation.

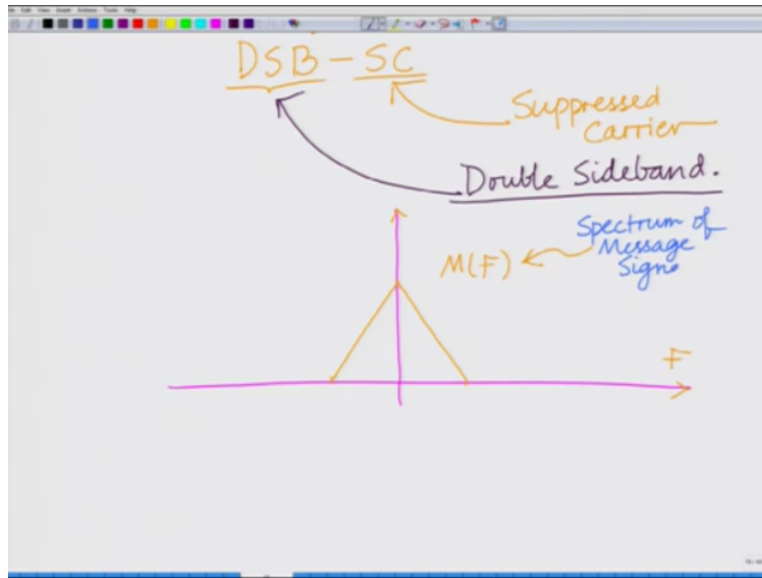
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The image shows a handwritten equation on a whiteboard. The title "DOUBLE SIDEBAND MODULATION" is written in green and underlined. Below it, "(DSB)" is written in purple. The equation is $x(t) = \frac{A_c m(t) \cos(2\pi f_c t)}{\text{No pure carrier}}$. There are handwritten annotations: "Message signal" with an arrow pointing to $m(t)$, "carrier Frequency" with an arrow pointing to f_c , and "No pure carrier" written below the denominator.

It is a, this is a different modulation scheme it is a form of amplitude modulation termed double sideband modulation also generated by abbreviation DSB. DSP stands for double sideband, okay. In the double sideband modulation scheme the modulated signal $x(t)$ is described as follows. $X(t)$ equals $A_c m(t) \cosine two pi F_c t$, now observe that in this amplitude modulated signal there is no pure carrier component, correct? We simply have the carrier that is modulated by the message $A_c m(t)$ is the message signal $\cosine two pi F_c t$ carrier at carrier frequency F_c , okay. So $m(t)$ is the $m(t)$ is your message signal and this F_c as we have already seen many times before this is the carrier frequency, okay.

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Now since there is no pure carrier, alright. This is also termed as DSB-SC where DSB as you would know already know this stands for double sideband SC stands for suppressed, correct? SC stands for suppressed and DSB stands for double sideband, DSB stands for double sideband, SC stands for suppressed carrier. Let us look at the spectrum of DSBSC signal. Let us start by considering a typical spectrum of the message signal $m(t)$, correct? Typical signal message spectrum $M(F)$ this is the frequency axis, alright. In the frequency domain this is the spectrum $M(F)$ is the spectrum of the message signal spectrum of the message signal.

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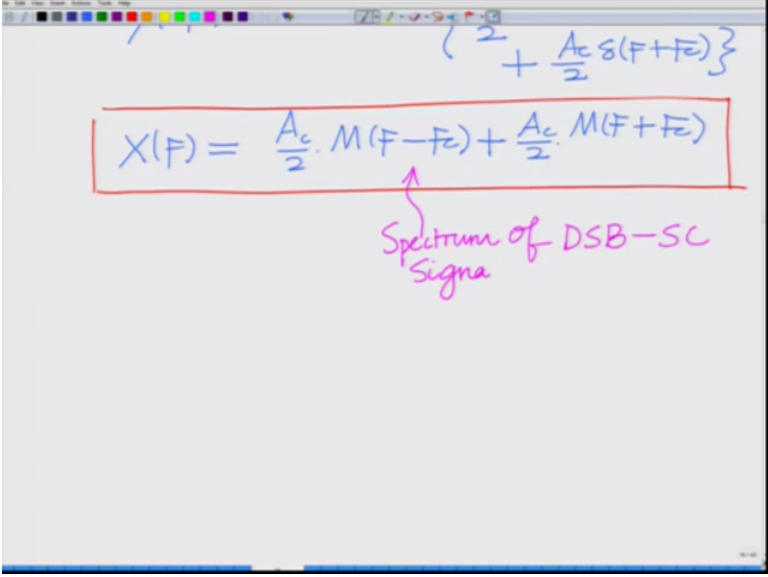
$$\begin{aligned}
 x(t) &= A_c m(t) \cos(2\pi f_c t) \\
 &= m(t) \times A_c \cos(2\pi f_c t)
 \end{aligned}$$

$$X(F) = M(F) * \left\{ \frac{A_c}{2} \delta(F - F_c) + \frac{A_c}{2} \delta(F + F_c) \right\}$$

$X(t)$ equals $A_c m(t) \cos(2\pi f_c t)$, okay. I can write this as the product of $m(t)$ with $A_c \cos(2\pi f_c t)$, correct? $x(t)$ is a modulated signal, correct? I can write this as a product of the message signal $m(t)$ with $A_c \cos(2\pi f_c t)$, it is a multiplication in the time domain which means the frequency response $X(F)$, right? Is the frequency response $M(F)$ convolved with the frequency response of the carrier $A_c \cos(2\pi f_c t)$.

So multiplication in the time domain is convolution in the frequency domain therefore we have, correct? We have already seen this before therefore $X(F)$ is equal to $M(F)$ the message spectrum convolved with the spectrum of, so $m(t)$ has spectrum $M(F)$ $A_c \cos(2\pi f_c t)$ has spectrum A_c by two $\delta(F - f_c)$ that is impulse scaled by A_c by two and shifted to f_c in the frequency domain plus A_c by two $\delta(F + f_c)$ that is impulse shifted to minus f_c and scaled by A_c by two. So this is the spectrum of $A_c \cos(2\pi f_c t)$ and convolution $M(F)$, alright.

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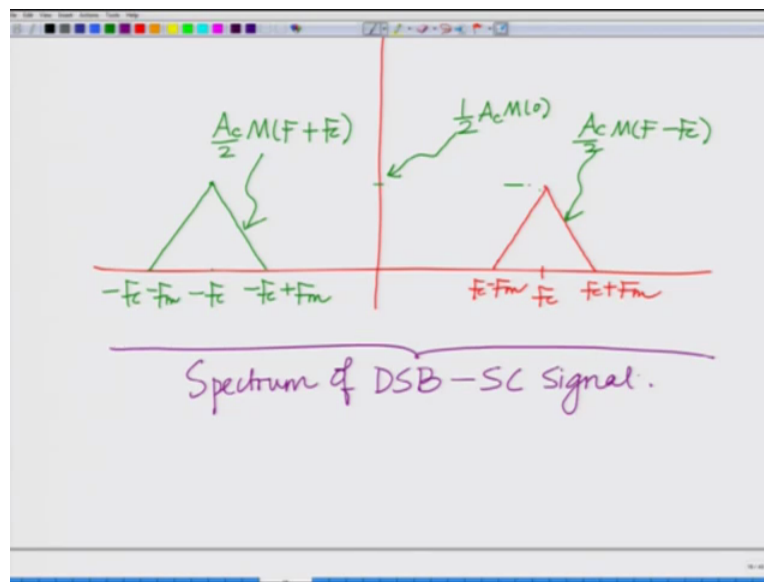


The image shows a handwritten equation on a whiteboard. At the top, there is a partially visible expression: $\left\{ \frac{A_c}{2} \delta(F - f_c) + \frac{A_c}{2} \delta(F + f_c) \right\}$. Below this, the main equation is boxed in red:
$$X(F) = \frac{A_c}{2} M(F - f_c) + \frac{A_c}{2} M(F + f_c)$$
. A pink arrow points from the text "Spectrum of DSB-SC Signal" to the boxed equation.

So just $M(F)$ convolved with $\delta(f - f_c)$ that is, alright. That is so it is A_c over two $m(t)$ convolved with $F - f_c$ that is $M(F - f_c)$ that is spectrum of $M(F)$ shifted to f_c plus A_c by two $M(F)$ convolved with $\delta(F + f_c)$ the spectrum $M(F)$ shifted to minus f_c that is $M(F + f_c)$, okay. So this is the spectrum of the DSBSC signal. This is the spectrum of the double sideband suppressed carrier. This is the spectrum of the DSB-SC signal, correct? Which is $X(F)$ that is A_c by two $M(F - f_c)$ plus $M(F + f_c)$ that is spectrum $M(F)$ shifted to f_c plus $M(F)$ shifted to minus f_c that is spectrum

$M(f)$ shifted to F_c is scaled by A_c by two and A_c by two $M(f)$ plus F_c that is spectrum of $M(f)$ shifted to minus F_c and scaled by A_c by two, alright.

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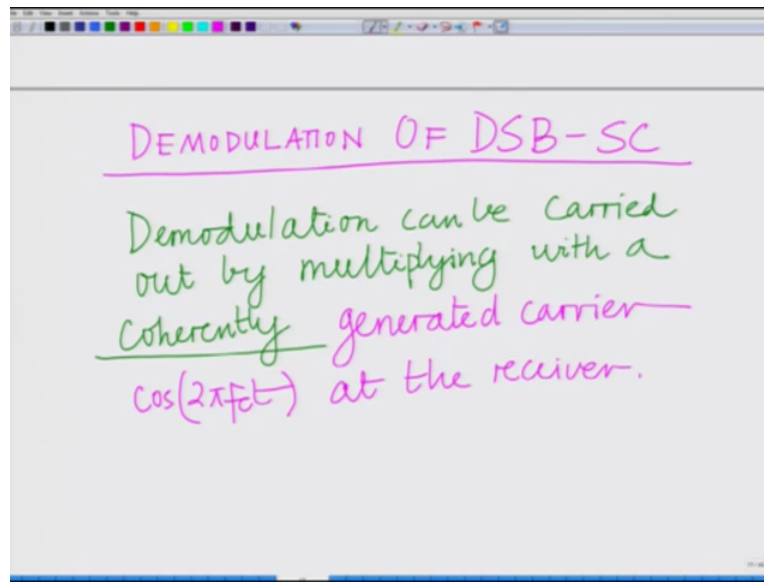


So let us now describe this schematically, I can draw this so I have at F_c , alright. I have F_c this is F_c plus F_m , F_c minus F_m , correct? In the maximum frequency component of the message signal we denoted by F_m minus F_m to F_m , correct? And then what we have is we have this point will obviously be half $A_c M$ of zero that is a spectrum at zero because this point remember this point is M of zero. So this point is half $A_c M$ of zero, correct?

M of zero is the spectrum at F equal to zero, correct? This is basically your A_c by two $M(F)$ minus F_c correspondingly on the negative will have an image of this on the negative frequency axis, correct? You will have an image of this on the negative frequency axis. This is minus F_c minus F_c plus F_m minus F_c minus F_m , correct? Correct, so this is the image and this is the and this is basically corresponds to A_c by two $M(F)$ plus F_c that is the spectrum $M(F)$ shifted to minus F_c and scaled by A_c by two.

Alright so you have two bands one from F_c minus F_m to F_c plus F_m and the other from minus F_c minus F_m to minus F_c plus F_m and as you can see there is no pure carrier component that is unlike the conventional AM signal where you had impulses denoting the pure carrier there are no impulses which means the pure carrier component is absent you only have the carrier that is modulated by the message, okay. So this is the spectrum of the DSB-SC signal, so let me characterize that so this is the spectrum of the DSB SC signal.

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Let us now look at the modulation of the DSB SC signal. Let us now look at the modulation of the DSB SC signal. The demodulation of a DSB SC signal before demodulation we can multiply it again by the carrier which is also termed as a coherent carrier I am going to explain this later. So demodulation can be carried out by multiplying by the by a coherent, coherently generated carrier cosine two pi Fct at the receiver, okay. So demodulation can be let us note that down demodulation can be carried out by multiplying with a coherently, this is an important term coherently generated carrier cosine two pi Fct at the receiver. So demodulation can be carried out multiplying by with a coherent carrier cosine two pi Fct at the receiver.

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A screenshot of a digital whiteboard showing the derivation of the received signal $r(t)$. The equations are written in purple and red ink. The first line is $r(t) = x(t) \cdot \cos(2\pi F_c t)$. The second line is $= A_c m(t) \cos(2\pi F_c t) \times \cos(2\pi F_c t)$. The third line is $= A_c m(t) \cos^2(2\pi F_c t)$. The fourth line is $= A_c m(t) \cdot \frac{1 + \cos 4\pi F_c t}{2}$.

$$\begin{aligned} r(t) &= x(t) \cdot \cos(2\pi F_c t) \\ &= A_c m(t) \cos(2\pi F_c t) \times \cos(2\pi F_c t) \\ &= A_c m(t) \cos^2(2\pi F_c t) \\ &= A_c m(t) \cdot \frac{1 + \cos 4\pi F_c t}{2} \end{aligned}$$

Let us look at that. Let us look at our signal $x(t)$ at the receiver we can express $r(t)$ equals $x(t)$ the modulated with the coherent carrier that is cosine two pi $F_c t$ which is equal to $A_c m(t)$ cosine two pi $F_c t$ times cosine two pi $F_c t$ which is basically you can write this as $A_c m(t)$ cosine square two pi $F_c t$ which is basically your A_c . Now cosine square two pi $F_c t$ can be written as one plus cosine four pi $F_c t$ divided by two, alright. Cosine square θ is one plus cosine two θ divided by two.

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A screenshot of a digital whiteboard showing the derivation of the baseband and carrier components of $r(t)$. The equations are written in purple and red ink. The first line is $= A_c m(t) \cos(2\pi F_c t) \times \cos(2\pi F_c t)$. The second line is $= A_c m(t) \cos^2(2\pi F_c t)$. The third line is $= A_c m(t) \cdot \frac{1 + \cos 4\pi F_c t}{2}$. The fourth line is $= \frac{A_c m(t)}{2} + \frac{1}{2} A_c m(t) \cos(4\pi F_c t)$. The first term is labeled "Baseband" and the second term is labeled "Frequency = $2F_c$ centered at $2F_c$ ".

$$\begin{aligned} &= A_c m(t) \cos(2\pi F_c t) \times \cos(2\pi F_c t) \\ &= A_c m(t) \cos^2(2\pi F_c t) \\ &= A_c m(t) \cdot \frac{1 + \cos 4\pi F_c t}{2} \\ &= \underbrace{\frac{A_c m(t)}{2}}_{\text{Baseband}} + \underbrace{\frac{1}{2} A_c m(t) \cos(4\pi F_c t)}_{\text{Frequency} = 2F_c \text{ centered at } 2F_c} \end{aligned}$$

And now we observe something interesting I have $A_c m(t)$ divided by two plus half $A_c m(t)$ into cosine four $\pi F_c t$ and if you observe this if you observe this is cosine four $\pi F_c t$ corresponds to a frequency that is the frequency is equals two F_c . This is of course your baseband that is centered at zero this is at frequency two F_c , so this is centered this component if you can look at this is centered because of the cosine four $\pi F_c t$ this is centered at two F_c , alright. So we have two components, one is $A_c m(t)$ divided by two which is in the baseband which is $m(t)$ scaled by A_c by two so naturally it is in the baseband that is from minus F_m to plus F_m , correct?

However $m(t)$ multiplied by cosine a four $\pi F_c t$ cosine four $\pi F_c t$ has frequency two F_c which means it is the spectrum of the spectrum of $m(t)$ that is $M(F)$ shifted to two F_c and minus two F_c , so this is far away from the baseband (14:50) and there this component can be removed by filtering to get recover $m(t)$.

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The diagram illustrates the frequency spectrum of a modulated signal. At the top, a horizontal line represents the frequency axis. On the left, a bracket labeled "Baseband" spans from $-F_m$ to F_m . On the right, a bracket labeled "Frequency = $2F_c$ " spans from $2F_c - F_m$ to $2F_c + F_m$. An arrow points from the text "centered at $2F_c$ " to the right-hand bracket. Below this, the equation $r(t) = x(t) \cos(2\pi F_c t)$ is written. An arrow points from this equation down to the label $R(F)$, indicating the spectrum of the modulated signal.

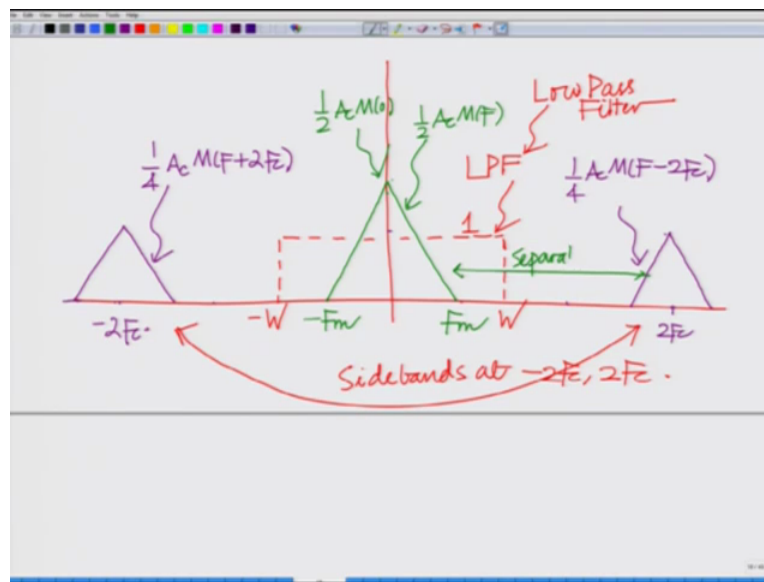
So let us look at the spectrum to understand this better let us look at the spectrum, so the spectrum of this that is $r(t)$ equals $x(t)$ cosine two $\pi F_c t$ multiplied by the coherent carrier and if I look at the spectrum of this that is $R(F)$ I can draw it as follows.

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$$\begin{aligned}
 r(t) &= \frac{1}{2} A_c m(t) + \frac{1}{2} A_c m(t) \cos(4\pi F_c t) \\
 &\quad \updownarrow \\
 &= \frac{1}{2} A_c M(F) + M(F) * \left\{ \frac{A_c}{4} \delta(F - 2F_c) + \frac{A_c}{4} \delta(F + 2F_c) \right\} \\
 &= \frac{1}{2} A_c M(F) + \frac{A_c}{4} M(F - 2F_c) + \frac{A_c}{4} M(F + 2F_c)
 \end{aligned}$$

Look at this $r(t)$ equals half $A_c m(t)$ plus half $A_c m(t) \cos 4\pi F_c t$ the spectrum of this half $A_c m(t)$ has the spectrum half $A_c M(F)$ plus of course I have $M(F)$ convolved with the spectrum A_c divided by four because I have $m(t)$ multiplied by A_c over two $\cos 4\pi F_c t$ which means in the frequency domain it is a spectrum $M(F)$ convolved with the spectrum of A_c over two $\cos 4\pi F_c t$ which is A_c over four $\delta F - F_c$ plus A_c over four $\delta F + F_c$ which is equal to half $A_c M(F)$ plus A_c over four $M(F - F_c)$ plus A_c over four $M(F + F_c)$ plus I am sorry $M(F)$ plus this has these are components at two F_c . I am sorry $\cos 4\pi F_c t$ has components at two F_c , so this is A_c over four $\delta F - 2F_c$ plus A_c over four $\delta F + 2F_c$ so this is A_c over four $M(F - 2F_c)$ plus A_c over four $M(F + 2F_c)$

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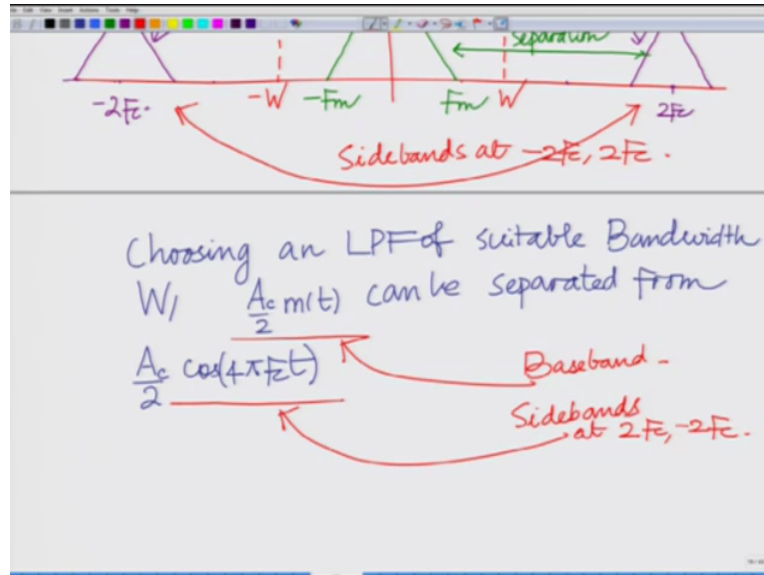
And therefore naturally when I draw the spectrum of this I am going to have the baseband component, correct? That is your half $A_c M(F)$ that is going to be there, this point this is your half $A_c M(F)$ this point is half $A_c M(0)$ and then you are going to have, so let us say this is F_c you are going to have bands at two F_c that is side bands at two F_c . And so this is at two F_c you have a component which is basically this component is your half or rather one over four one over four $A_c M(F)$ minus two F_c and similarly on the other side at minus two F_c you are going to have another component which is basically at minus two F_c , this is your one over four $A_c M(F)$ plus two F_c correct, so you have these two bands at two F_c and minus two F_c and therefore now I can have a low pass filter, correct?

I can have low pass filter because there is baseband is separated from the side bands at minus two F_c and F_c I can now have a low pass filter I can now pass this signal through a low pass filter that is minus W to W this is a low pass filter this is your low pass filter and these are the side bands, correct? These is the baseband and these are the side bands at minus F_c and two F_c , side bands at minus two F_c , F_c and the base band that is A_c over two $M(F)$ this can be separated from the side bands by passing it through a low pass filter between minus W and W .

And W can be chosen appropriately, alright because there is significant separation, alright because F_m is much that is maximum frequency F_m is much less than F_c , so naturally maximum frequency F_m is much less than two F_c and it is much less than two F_c minus F_m . So if you can

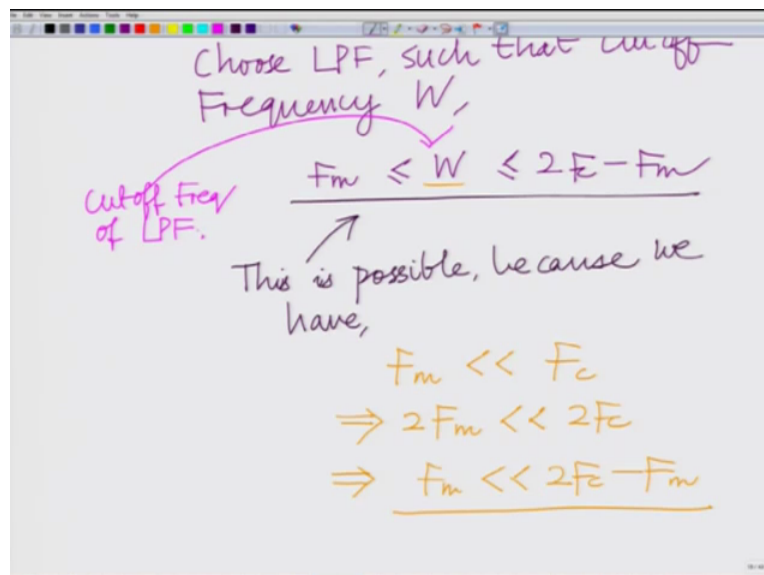
see there is a significant separation, correct? Here if you can look at this there is a significant separation.

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So choosing a low pass filter of bandwidth W LPF of bandwidth of let us say suitable bandwidth. Let us make it suitable bandwidth W $A_c \cos(4\pi F_c t)$ can be separated from $A_c \cos(4\pi F_c t)$, correct? Because this is your base band this is in the baseband and this is actually these are the side bands at two F_c , minus two F_c .

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And the LPF the low pass filter, correct? Low pass filter is chosen such that choose LPF such that cut-off frequency W , that is we must have F_m now as you can look at this, this is very interesting you have F_m should be less than two F_c minus F_m . That is we should have F_m less than equal to W less than equal to two F_c minus F_m and this is possible because because we have F_m that is maximum frequency component much less than equal to F_c which means two F_m much less than or equal to two F_c which means which implies automatically that F_m is much less than or equal to F_c minus F_m . So this because F_m the message frequency is much less than or equal to the carrier frequency F_c .

This is indeed possible and therefore you can choose an appropriate cut-off frequency W of the this W is termed as the cut-off frequency, alright. So that is the cut-off frequency cut-off frequency of the (Lp) LPF one can choose the cut-off frequency of the LPF appropriately, right. Such that its much greater than F_m much less than two F_c minus F_m . So the LPF blocks the the (bla) blocks the side bands at minus two minus two F_c and two F_c .

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cut-off freq of LPF.

This is possible, because we have,

$$F_m \ll F_c$$

$$\Rightarrow 2F_m \ll 2F_c$$

$$\Rightarrow F_m \ll 2F_c - F_m$$

LPE Blocks $\frac{m(t)\cos(4\pi F_c t)}{2}$ at $2F_c$.

So the LPF basically LPF blocks the component cosine two pi $F_c t$ $m(t)$ by two cosine four pi $F_c t$, this is the component at two F_c , right? The carrier has a common frequency cosine corresponds to cosine four pi $F_c t$ corresponds to the frequency two F_c , alright. So in this module what we have seen is we have seen a new form of amplitude modulation that is double sideband

modulation which has no pure carrier does not transmit the pure carrier therefore it is also known as DSBSC double sideband suppressed carrier, alright.

We have seen the spectrum of the DSB SC signal and also how to demodulate the DSBSC signal by multiplying with a coherently generated carrier $\cos(2\pi f_c t)$ at the receiver and finally filtering using an appropriate filter low pass filter with an appropriate cut-off frequency W such that you retain the baseband component while filtering out the side bands which are at $\pm 2f_c$. So we will stop here and continue with other aspects in the subsequent modules, thank you.