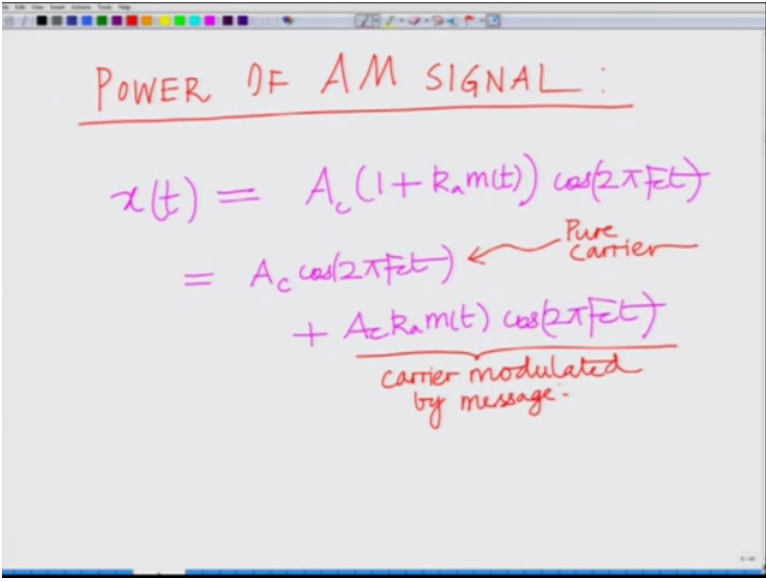


Principles of Communication- Part I
Professor Aditya K. Jagannathan
Department of Electrical Engineering
Indian Institute of Technology Kanpur
Module No 3
Lecture 13

Power of Amplitude Modulated (AM) Signals and Power Efficiency of AM Signals

Hello welcome to another module in this massive open online course. So today let us look at the power of an AM signal.

(Refer Slide Time: 0:31)



The image shows a whiteboard with handwritten text in red and purple ink. At the top, it says "POWER OF AM SIGNAL:". Below this, the equation for an AM signal is written: $x(t) = A_c(1 + k_a m(t)) \cos(2\pi F_c t)$. This is then expanded into two terms: $= A_c \cos(2\pi F_c t) + A_c k_a m(t) \cos(2\pi F_c t)$. A red arrow points from the text "Pure Carrier" to the first term, $A_c \cos(2\pi F_c t)$. A red bracket under the second term, $A_c k_a m(t) \cos(2\pi F_c t)$, is labeled "carrier modulated by message".

Power of amplitude modulated that is the AM signal and we know that a typical AM signal is given as $s(t)$ or $x(t)$, let us use the notation $x(t)$ equals A cosine or basically A_c one plus $k_a m(t)$ into cosine two pi $F_c t$ which is the carrier component A_c cosine two pi $F_c t$ plus the message component that is $A_c k_a m(t)$ cosine two pi $F_c t$, correct?

Now this is the pure carrier component, correct? And this is carrier modulated carrier modulated, so we have two components one is the pure carrier A_c cosine two pi $F_c t$ and the other is the carrier modulated by the message that is $k_a A_c m(t)$ cosine two pi $F_c t$, okay. So let us try to let us now to compute the power of the AM signal that is $x(t)$, let us compute the power of each of these components separately and add them to get the power of the AM signal.

(Refer Slide Time: 2:47)

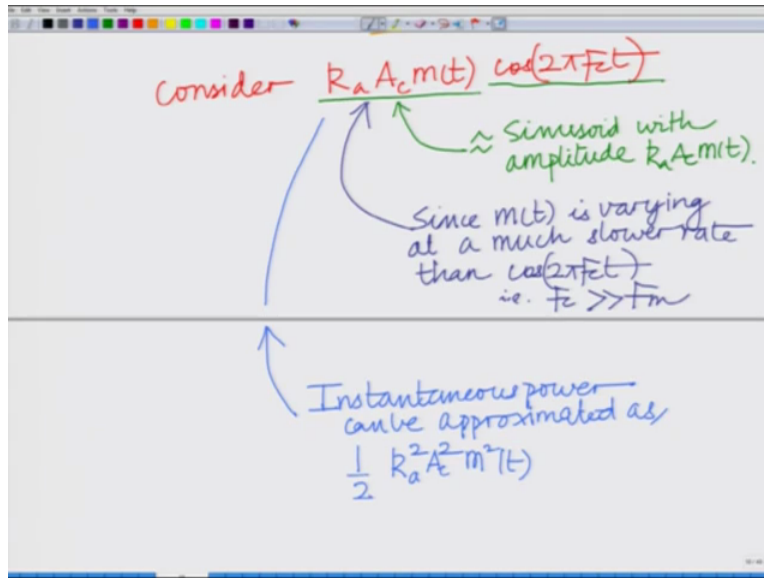
$$\begin{aligned} x(t) &= A_c(1 + k_a m(t)) \cos(2\pi f_c t) \\ &= A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t) \end{aligned}$$

Annotations in the image:

- An arrow points from the text "Pure Carrier" to the term $A_c \cos(2\pi f_c t)$.
- An arrow points from the text "carrier modulated by message" to the term $A_c k_a m(t) \cos(2\pi f_c t)$.
- A bracket groups both terms, with an arrow pointing to the word "com".

So we compute power of each of these two components compute power of each component separately, okay. Now the power of the carrier, well that is straightforward and we already know this thing this is a Sinusoidal signal. So the power of this of this that is $A_c \cos$ two pi $F_c t$ is amplitude square that is half A_c square. So the power of the carrier, correct? The power of the carrier component we already know that that is a Sinusoidal there is a pure carrier. That is a Sinusoid signal, so the power is half amplitude square that is half a six let us now compute the power in the carrier that is modulated by the message.

(Refer Slide Time: 3:59)



So now consider the other component that is $K_a A_c m(t) \cos(2\pi f_c t)$ now you can think of this as the carrier and this as the amplitude that is you can think of this although this is not exactly correct you can think of this as Sinusoid with amplitude that is I am writing using the approximate signs Sinusoid with amplitude $K_a A_c m(t)$. Well, correct? Although strictly speaking that is not true because when you multiply $m(t)$ time varying signal $m(t)$, correct?

With a Sinusoid it is no longer a Sinusoid but since $m(t)$ message signal is varying much because the carrier frequency f_c is much higher than f_m that is the maximum frequency component of the message therefore $m(t)$ that is this envelope or this amplitude of this (Sinusoid) this $m(t)$ is slowly much varying at a much slower rate than the Sinusoid, correct? $\cos(2\pi f_c t)$. One can approximately think of this as the Sinusoid $\cos(2\pi f_c t)$ with instantaneous amplitude given as $K_a A_c m(t)$.

This so let us write this justification also since this is not always true this approximation is valid since $m(t)$ is varying at our much lower rate than $\cos(2\pi f_c t)$ that is your f_c is much greater than f_m that we already know, alright. The carrier frequency is much greater than the message frequency therefore the power of this can be (approx) instantaneous power.

(Refer Slide Time: 7:34)

Instantaneous power can be approximated as

$$\frac{1}{2} R_a^2 A_c^2 m^2(t)$$

Average Power

$$= E \left\{ \frac{1}{2} R_a^2 A_c^2 m^2(t) \right\}$$

Expectation
Average value

$$= \frac{1}{2} R_a^2 A_c^2 E \{ m^2 \}$$

Again can be approximated as half this amplitude square that is half k_a square A_c square m square t , this is the instantaneous power and therefore average power from this is your average power can be obtained the average power can be obtained as that is expected value of this quantity half that is average, remember E stands for the expected value expected value of a random variable or a random process in this case $m(t)$ is a time varying signal, so it is a random process. E stands for expected, this is the expectation or the average at the average value and therefore this is equal to I can bring the constants outside half k_a square A_c square average value of the quantity m square t .

(Refer Slide Time: 8:47)

The image shows a whiteboard with handwritten mathematical equations. The first equation is
$$= \frac{1}{2} K_a^2 A_c^2 \overline{E \{ m^2(t) \}}$$
 with a bracket under $\overline{E \{ m^2(t) \}}$ labeled "= Power of Message signal" and "= P_m ". The second equation is
$$= \frac{1}{2} K_a^2 A_c^2 P_m$$
 with a red underline under the entire expression. A red arrow points from the text "Power of message signal $m(t)$." to the P_m term in the second equation.

But now realize average value of this quantity $m^2(t)$ is nothing but the power of the message signal we can denote this by the quantity P_m , okay. So this average quantity this is nothing but the power expected value of therefore let us say this is denoted by P_m , so this is half K_a square A_c square P_m where P_m is the power of the this is the power of your message signal where P_m is the power of the message signal $m(t)$, alright.

So we have derived the power in the pure carrier component and now we have derived the power in the component that is a carrier which is modulated by the message $m(t)$ and we are saying that power is a proximity half k_a square A_c square into P_m where P_m is the power of the modulating signal that is the message signal signal $m(t)$.

(Refer Slide Time: 10:00)

The image shows a digital whiteboard with handwritten mathematical derivations for the power of an AM signal. At the top right, it states: $\text{Power of Message signal} = P_m$. Below this, the sideband power is derived as $= \frac{1}{2} k_a^2 A_c^2 P_m$. An arrow points from the text "Power of message signal $m(t)$ " to this equation. At the bottom, the "Total Power of AM signal" is boxed as $= \frac{1}{2} A_c^2 + \frac{1}{2} k_a^2 A_c^2 P_m$.

$$\text{Power of Message signal} = P_m$$
$$= \frac{1}{2} k_a^2 A_c^2 P_m$$

Power of message signal $m(t)$.

Total Power of AM signal

$$= \frac{1}{2} A_c^2 + \frac{1}{2} k_a^2 A_c^2 P_m$$

And therefore now the total power is the sum of these two powers the total power of your Am signal equals half A_c square plus half k_a square A_c square into P_m . This is the total power this is the total power of the AM signal that is half A_c square plus half k_a square by A_c square into P_m , okay. So this is the total power of the AM signal. Now let us define another quantity that is the efficiency, the efficiency of this AM signal is basically the power in the message signal divided by the total power that is a power in the carrier class plus power in the power in the carrier modulated by the message divided by the power in the pure carrier plus divided by the total power, right.

Because remember the only (udh) useful part of the amplitude the AM signal amplitude modulated signal is the part that corresponds to the carrier modulated by the message because that is actual component that is carrying the message signal which is conveying the information the pure carrier is only being transmitted for to enable envelope detection, alright. Otherwise by itself it does not convey any information therefore the efficiency of this scheme can be denoted by the power in the message component. That is power in the carrier modulated by the message divided by the total power of the AM signal, okay.

(Refer Slide Time: 11:42)

$\eta = \text{Efficiency of AM}$

$$= \frac{\text{Power in carrier modulated by message}}{\text{Total Power}}$$

Because pure carrier component $A_c \cos(2\pi F_c t)$ does NOT convey any information.

So the power η equal to efficiency of AM and this is equal to power that is power in carrier modulated by message divided by the total power and this is because remember the justification this is because a pure carrier component which is $A_c \cos(2\pi F_c t)$ does not any information. This does not convey any information does not convey any (inf) the information is contained in the message signal $m(t)$ therefore the pure carrier is only being transmitted for envelope to enable envelope detection, alright. That is the reason why the carrier component is being transmitted, alright.

(Refer Slide Time: 13:39)

The image shows a digital whiteboard with a toolbar at the top. The title 'detection at Receiver' is written in green at the top right. The main derivation is in purple ink. It starts with the equation for efficiency η as the ratio of the power of the modulated signal to the total transmitted power. The numerator is $\frac{1}{2} A_c^2 k_a^2 P_m$ and the denominator is $\frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 k_a^2 P_m$. To the left of this equation, the text 'efficiency of AM system' is written in orange with a checkmark. Below this, the simplified equation $\eta = \frac{k_a^2 P_m}{1 + k_a^2 P_m}$ is enclosed in a purple rectangular box.

$$\eta = \frac{\frac{1}{2} A_c^2 k_a^2 P_m}{\frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 k_a^2 P_m}$$

efficiency of AM system ✓

$$\eta = \frac{k_a^2 P_m}{1 + k_a^2 P_m}$$

So you realize that if there is no information why is being transmitted? The pure carrier is being transmitted simply to enable is being transmitted simply to envelope detection at the simply to enable envelope detection at the receiver and therefore eta that is the efficiency is the power in the carrier that is carrier modulated by the message that is half A_c square k_a square P_m divided by half A_c square plus half A_c square k_a square P_m which is basically that is eta equals, well that is your cancelling A_c square this is k_a square P_m , remember k_a is the sensitivity P_m is the power in the message signal divided by one plus k_a square P_m .

So this is the efficiency of the AM system, correct? So I can write this as the efficiency this is the efficiency of the amplitude modulated system that is k_a square into P_m divided by one plus k_a square P_m where k_a is the sensitivity and P_m is the power of the message signal. Let us try to compute this efficiency for a Sinusoidal message signal.

(Refer Slide Time: 15:59)

The image shows a handwritten derivation on a digital whiteboard. At the top, the efficiency η is given by the formula $\eta = \frac{R_a P_m}{1 + R_a^2 P_m}$, enclosed in a purple box. Below this, it says "consider a sinusoidal message signal." followed by the equation $m(t) = A_m \cos(2\pi f_m t)$. A purple arrow points from this equation to the power formula $P_m = \frac{1}{2} A_m^2$.

$$\eta = \frac{R_a P_m}{1 + R_a^2 P_m}$$

consider a sinusoidal message signal.

$$m(t) = A_m \cos(2\pi f_m t)$$
$$\Rightarrow P_m = \frac{1}{2} A_m^2$$

Consider now the specific case of a Sinusoidal message signal, consider a Sinusoidal message signal $m(t)$ equals A_m this is our message signal with amplitude A_m cosine two pi $F_m t$, alright. Therefore this implies that the power of the message signal P_m equals half A_m square.

(Refer Slide Time: 16:59)

The image shows a handwritten derivation on a digital whiteboard. A purple arrow points from the power formula $P_m = \frac{1}{2} A_m^2$ to the modulation index formula $\mu = R_a A_m$. Below this, the average power of the modulated signal is calculated as $\frac{1}{2} \mu^2 = \frac{1}{2} R_a^2 A_m^2 = R_a^2 P_m$. Finally, the efficiency η is given by the formula $\eta = \frac{R_a^2 P_m}{1 + R_a^2 P_m}$.

$$\Rightarrow P_m = \frac{1}{2} A_m^2$$
$$\mu = R_a A_m$$
$$\frac{1}{2} \mu^2 = \frac{1}{2} R_a^2 A_m^2 = R_a^2 P_m$$
$$\eta = \frac{R_a^2 P_m}{1 + R_a^2 P_m}$$

This is the power of the message signal also realize that μ the modulation index equals remember modulation index this is defined we have defined the modulation index and the modulation index for this Sinusoidal signal is basically μ equals k_a times A_m basically we have

defined μ in one of the earlier modules and we have shown that for a Sinusoidal (modul) for a Sinusoidal message signal μ equals k_a times A_m , correct?

(Refer Slide Time: 18:56)

The image shows a whiteboard with handwritten mathematical derivations for the efficiency of an AM signal. At the top, the equation $\frac{1}{2}\mu^2 = \frac{1}{2}k_a^2 A_m^2 = k_a^2 P_m$ is written in orange, with a curved arrow pointing from $\frac{1}{2}\mu^2$ to $k_a^2 P_m$. Below this, the efficiency η is defined as $\eta = \frac{k_a^2 P_m}{1 + k_a^2 P_m}$ in orange. Then, it is rewritten using μ as $\eta = \frac{\frac{1}{2}\mu^2}{1 + \frac{1}{2}\mu^2}$ in purple. Finally, the simplified formula $\eta = \frac{\mu^2}{2 + \mu^2}$ is boxed in purple.

$$\frac{1}{2}\mu^2 = \frac{1}{2}k_a^2 A_m^2 = k_a^2 P_m$$

$$\eta = \frac{k_a^2 P_m}{1 + k_a^2 P_m}$$

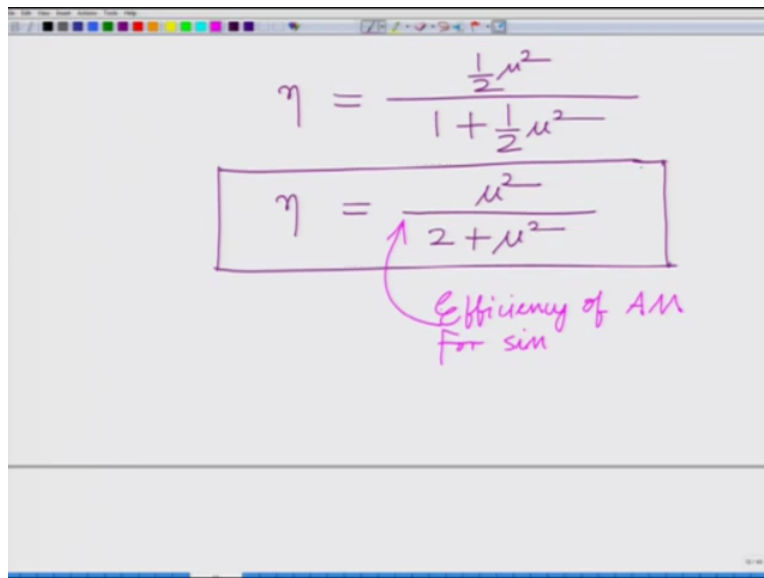
$$\eta = \frac{\frac{1}{2}\mu^2}{1 + \frac{1}{2}\mu^2}$$

$$\boxed{\eta = \frac{\mu^2}{2 + \mu^2}}$$

And therefore now we can also write half, correct? μ square equals half k_a square A_m square but half A_m square equals P_m , so this is equal to k_a square into P_m which means the efficiency η the efficiency η of the AM signal, correct? For the Sinusoidal modulating signal that is remember that is half k_a half remember that is k_a square P_m which is now I can write this half μ square divided by let me just write it completely this is k_a square P_m divided by one plus k_a square P_m which now for this Sinusoidal modulating signal is half μ square, well k_a square we have just shown that half μ square equals k_a square P_m .

So this is half μ square divided by one plus half μ square this is equal to, well μ square divided by two plus μ square and this is another important relation of the efficiency of amplitude modulation for a Sinusoidal modulating signal, correct?

(Refer Slide Time: 19:24)



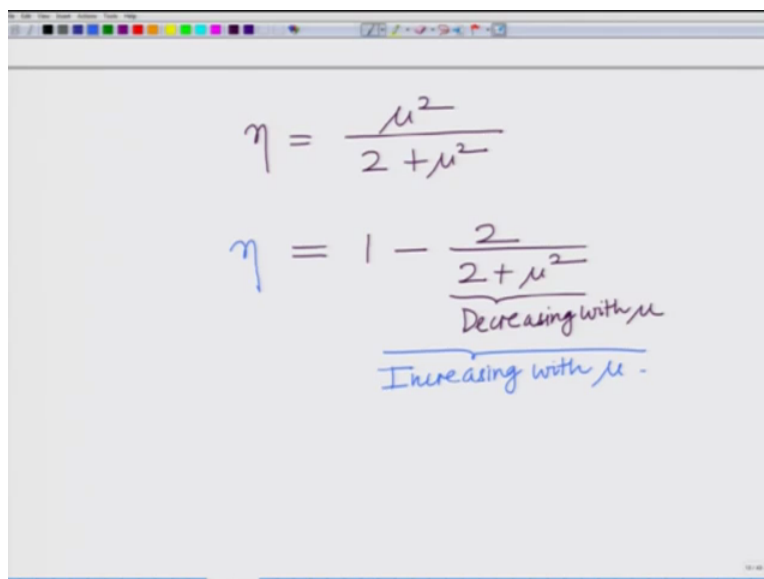
A screenshot of a digital whiteboard showing the derivation of the efficiency of AM. The first equation is $\eta = \frac{\frac{1}{2}\mu^2}{1 + \frac{1}{2}\mu^2}$. Below it, the same equation is boxed and simplified to $\eta = \frac{\mu^2}{2 + \mu^2}$. A pink arrow points from the boxed equation to the handwritten text "Efficiency of AM for sin".

$$\eta = \frac{\frac{1}{2}\mu^2}{1 + \frac{1}{2}\mu^2}$$
$$\eta = \frac{\mu^2}{2 + \mu^2}$$

Efficiency of AM for sin

This is the expression for efficiency of AM for a efficiency of AM specifically for a Sinusoidal modulation where mu is the mu is the modulation index. This is specifically the efficiency of a Sinusoidal modulating signal where mu is the modulation index, alright. Correct?

(Refer Slide Time: 20:04)



A screenshot of a digital whiteboard showing two alternative forms of the AM efficiency formula. The first is $\eta = \frac{\mu^2}{2 + \mu^2}$. The second is $\eta = 1 - \frac{2}{2 + \mu^2}$. Below the second equation, there are two annotations: "Decreasing with μ " under the denominator, and "Increasing with μ " under the entire fraction.

$$\eta = \frac{\mu^2}{2 + \mu^2}$$
$$\eta = 1 - \frac{2}{2 + \mu^2}$$

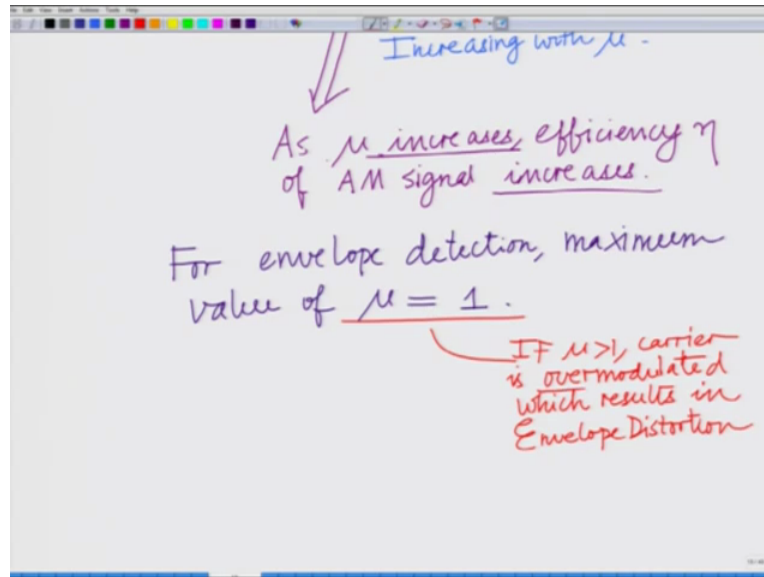
Decreasing with μ

Increasing with μ

And also now realize that if you look at this expression eta again look at this expression eta equals mu square divided by two plus mu square, I can write this as one minus two over two plus mu square look at this two over two mu square this is decreasing with mu and therefore one

minus two over two plus mu square this whole thing is increasing, so I can write the efficiency η as one minus two over two plus mu square. Two over two plus mu square is decreasing with mu as mu increases this quantity is decreasing therefore one minus two over two plus mu square is increasing with mu which means as mu increases the efficiency η of the modulating of the Am of the Am signal improves, correct?

(Refer Slide Time: 21:15)



So now you observe this implies as μ increases efficiency η of AM signal increases this is important to remember as μ increases the efficiency increases. Now however interestingly μ remember for envelope detection the modulation index μ has to be less than or equal to one if μ is greater than one the signal becomes over modulated therefore for envelope detection μ has to be the maximum value of μ can take is one.

For envelope detection for envelope detection maximum value of μ equals one otherwise if μ is greater than one then the carrier the lead becomes over modulated, correct? At this leads to envelope (dis) distortion remember, correct? Since if μ is greater than one carrier is over modulated this is termed as modulated which results in envelope distortion carrier becomes over modulated this results in envelope distortion put these two things together now you can see that efficiency is increasing with μ maximum value of μ is equal to one for envelope detection therefore maximum efficiency occurs for μ equal to one.

(Refer Slide Time: 24:02)

value of $\mu = 1$.

If $\mu > 1$, carrier is overmodulated which results in Envelope Distortion

Maximum Efficiency occurs for $\mu = 1$

$$\eta_{\max} = \frac{\mu^2}{2 + \mu^2} \Big|_{\mu=1} = \frac{1}{2+1}$$
$$\boxed{\eta_{\max} = \frac{1}{3}}$$

Maximum efficiency occurs which means η_{\max} equals μ^2 by two plus μ^2 evaluated at μ equal to one which is one by two plus one which is one over three, alright. So this is η_{\max} .

(Refer Slide Time: 24:50)

I_{\max}

Therefore maximum efficiency of AM with No envelope Distortion

$= \frac{1}{3}$ or 33%

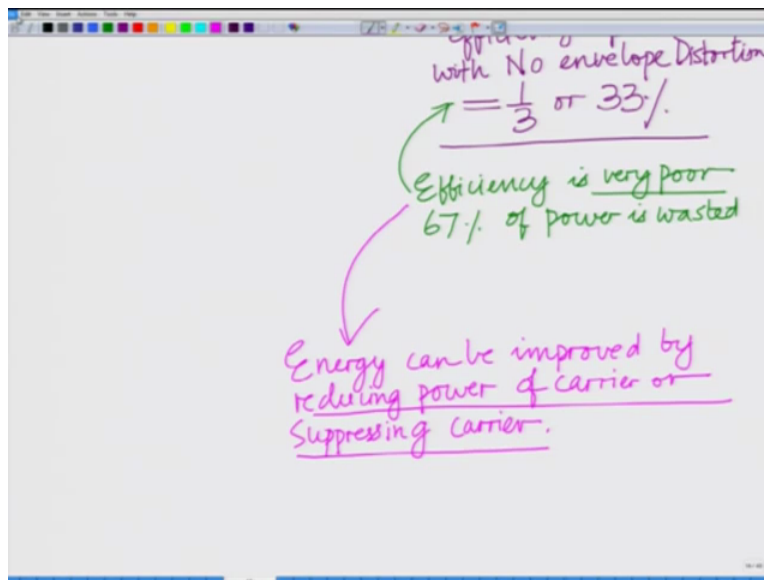
Efficiency is very poor
67% of power is wasted

Maximum efficiency of AM with envelope detection, if envelope detection has to be allowed therefore maximum efficiency of AM with no envelope distortion equals one by three or only thirty-three percent which is a very surprising result and this tells us that the efficiency of

amplitude modulation is very poor. That the maximum efficiency that is possible is only thirty-three percent which means close to sixty seven percent of the energy is basically wasted, correct? So this is very poor efficiency and realize that this is arising because we want to enable envelope detection at the receivers for which purpose is we are transmitting the pure carrier.

So the efficiency can be improved but that means the power of the carrier can be (dec) should be decreased or the carrier should not should not be transmitted altogether and that is what we will discover in the subsequent (modu) subsequent techniques of amplitude modulation that is double side band suppressed carrier etc. So therefore to conclude this efficiency is very poor to improve efficiency that is sixty six percent or sixty seven percent of power is wasted.

(Refer Slide Time: 26:46)



Therefore now efficiency can be improved by reducing the power of the carrier or suppressing power of the carrier energy can be improved or reducing power of carrier or suppressing the carrier. So the energy can be improved by reducing the power of the carrier or suppressing the (co) power of the carrier this is what we are going to look at in the subsequent techniques that is when we look at DSBSC double side band suppressed carrier that is a much higher efficiency because the carrier is suppressed there is no power is wasted on transmitting the pure carrier, alright.

So in this module we have calculated the power the power (effi) power of an AM signal, the power efficiency and also the maximum power efficiency with amplitude with envelope detection we have seen that this modulation index μ is less than or equal to one the maximum efficiency possible is thirty three percent. So let us stop here.