

Principles of Communication- Part I
Professor Aditya K. Jagannathan
Department of Electrical Engineering
Indian Institute of Technology Kanpur

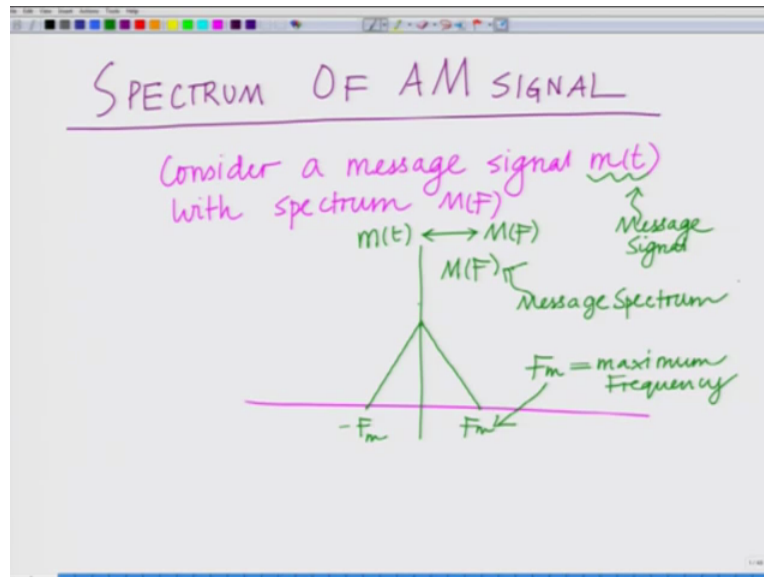
Module No 3

Lecture 11

Spectrum of Amplitude Modulated (AM) Signals and Introduction to Envelope Detection

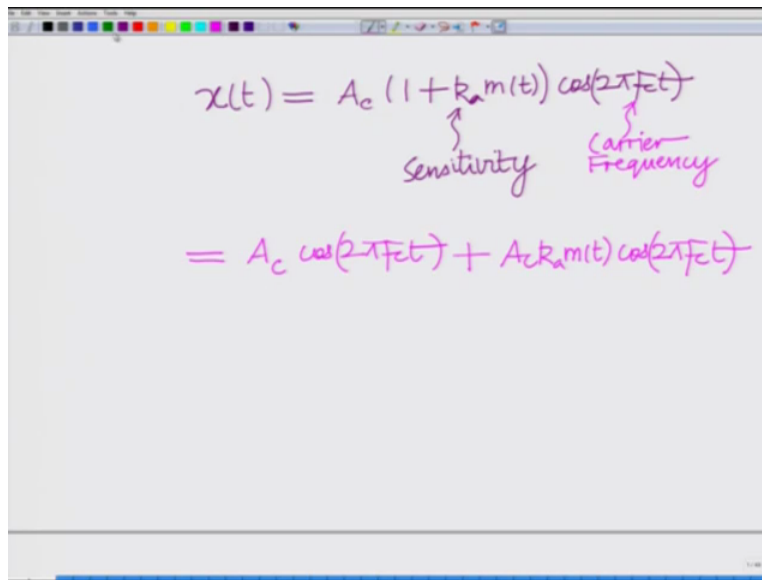
Hello welcome to another module in this massive open online course, so we are looking at amplitude modulation, alright. We have said amplitude modulation, an amplitude modulated signal is where the amplitude of a carrier, right? A carrier since $\cos(2\pi F_c t)$ at carrier frequency F_c is modulated by the message signal $m(t)$, alright. So today and we have looked at various concepts such as modulation, right? Modulation index over modulation etc. So today let us look at the spectrum of amplitude modulated signal, okay.

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So let us look at the spectrum of an Am the spectrum of an Am our amplitude modulated signal and our (consid) consider a message signal, so consider consider a message signal $m(t)$ with spectrum $M(F)$, correct? So let us draw a typical spectrum $M(F)$, so let us say this is our spectrum, okay. So let us say this is minus F_m this is F_m where F_m equals the maximum frequency frequency of $m(t)$, correct? And this is our spectrum $M(F)$, okay. So this is the message spectrum $M(F)$ so we have $m(t)$ spectrum $M(F)$ this $m(t)$ is our message signal, correct? $M(F)$ is the $M(F)$ is the message spectrum, okay.

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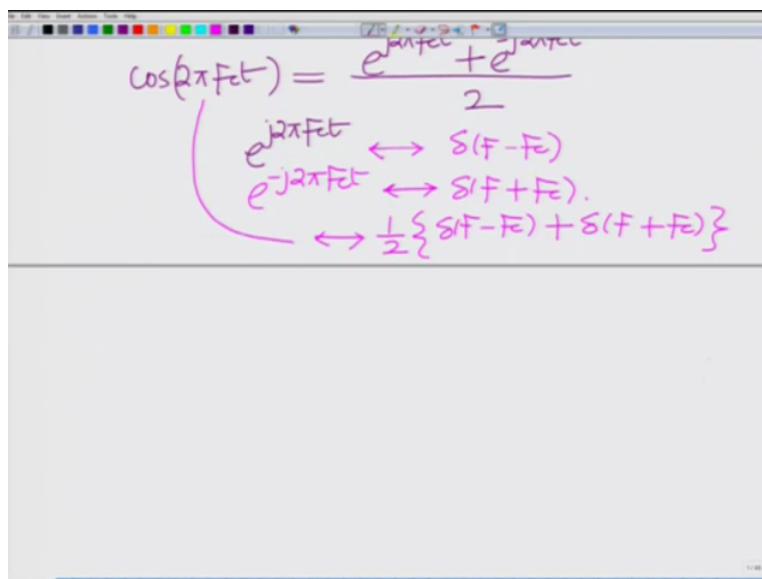

$$x(t) = A_c (1 + k_a m(t)) \cos(2\pi f_c t)$$

Sensitivity Carrier Frequency

$$= A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t)$$

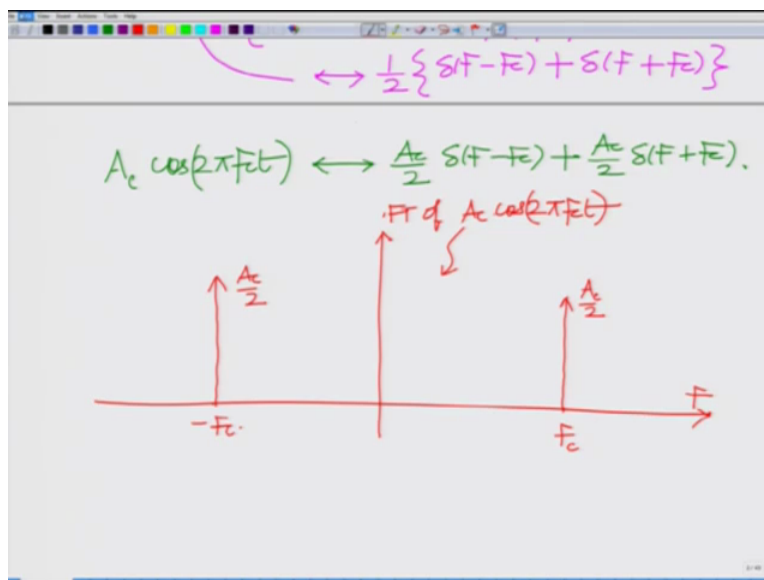
And now our amplitude modulated signal, remember $x(t)$ equals A_c one plus $k_a m(t)$ into cosine two pi $F_c t$ this F_c is cosine two pi $F_c t$ is the carrier or A_c cosine two pi $F_c t$ is the carrier remember k_a is the sensitivity. This F_c is the carrier frequency, okay. So we have our amplitude modulated signal which is given as A_c times one plus $k_a m(t)$ into cosine two pi $F_c t$. This can be expanded as A_c into cosine two pi $F_c t$ plus, correct? $A_c k_a m(t)$ cosine two pi $F_c t$ now let us look at the spectra of these two components.

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$$\cos(2\pi f_c t) = \frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}$$
$$\begin{aligned} e^{j2\pi f_c t} &\leftrightarrow S(f - F_c) \\ e^{-j2\pi f_c t} &\leftrightarrow S(f + F_c) \end{aligned}$$
$$\leftrightarrow \frac{1}{2} \{ S(f - F_c) + S(f + F_c) \}$$

Now consider cosine two pi Fct I can write this as e to the power of the j two pi Fct plus e to the power of minus j two pi Fct divided by two, correct? Cosine two pi Fct is e to the power of j two pi Fct plus e to the power of minus two pi Fct divided by two and the Fourier transform of the exponential, of the complex exponential is given by the impulse that is e to the power of j two pi Fct has Fourier transform delta F minus Fc, e to the power of minus j two pi Fct has Fourier transform delta F plus Fc. Therefore cosine two pi Fct this has the Fourier transform which is half Fourier transform of e to the power of to j two pi Fct that is delta F minus Fc plus delta F plus Fc.

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And therefore A_c if I multiply this by A_c that is $A_c \cos$ two pi Fct this has a Fourier transform $A_c \cos$ two pi Fct has the Fourier transform A_c by two delta F minus Fc plus A_c by two delta F plus A_c Fc and therefore if you draw the spectra of this part that is the carrier it comprises of two impulses A_c by two at Fc and another impulse scaled by A_c by two at minus Fc, correct? So this is the Fourier transform or Ft of A_c , this is the frequency axis, okay.

This is the Fourier transform of $A_c \cos$ two pi Fct that is it (consi) it is an it has an impulse of scaled by A_c by two at Fc impulse scaled by A_c by two at minus Fc which can be expressed as A_c over two delta f minus Fc plus delta f plus Fc, okay. So that is the spectrum of the first component that is a carrier component, okay.

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carrier component

$$= A_c \cos(2\pi F_c t) + A_c k_a m(t) \cos(2\pi F_c t)$$

$$\cos(2\pi F_c t) = \frac{e^{j2\pi F_c t} + e^{-j2\pi F_c t}}{2}$$

$e^{j2\pi F_c t} \leftrightarrow \delta(F - F_c)$
 $e^{-j2\pi F_c t} \leftrightarrow \delta(F + F_c)$
 $\leftrightarrow \frac{1}{2} \{ \delta(F - F_c) + \delta(F + F_c) \}$

$$A_c \cos(2\pi F_c t) \leftrightarrow \frac{A_c}{2} \delta(F - F_c) + \frac{A_c}{2} \delta(F + F_c)$$

FT of $A_c \cos(2\pi F_c t)$

So we are calling this as the A_c this as the the pure carrier, okay. This is the pure carrier.

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Multiplication in Time Domain

$$\frac{A_c k_a m(t)}{\quad} \cdot \frac{\cos(2\pi F_c t)}{\quad}$$

\updownarrow
 $A_c k_a M(F) * \frac{1}{2} \{ \delta(F - F_c) + \delta(F + F_c) \}$

CONVOLUTION in Frequency Domain

Now let us look at the other components that is $A_c k_a$, so observe $A_c k_a m(t)$ times cosine two pi $F_c t$. Now this is a multiplication in the time domain, now Fourier transform of A_c of this part will be $A_c k_a M(F)$ Fourier transform of this part will be half delta F minus F_c plus delta F plus F_c Fourier transform of cosine two pi $F_c t$ is half delta F minus F_c plus delta F plus F_c plus the two signals are multiplied in the time domain which means the respective spectra will be

convolved in the frequency domain, alright. So $A_c k_a M(F)$ will be convolved that is multiplication in time domain becomes the convolution, this becomes the convolution in the frequency domain.

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$$A_c k_a M(F) * \frac{1}{2} \{ \delta(F - F_c) + \delta(F + F_c) \}$$

CONVOLUTION in Frequency Domain

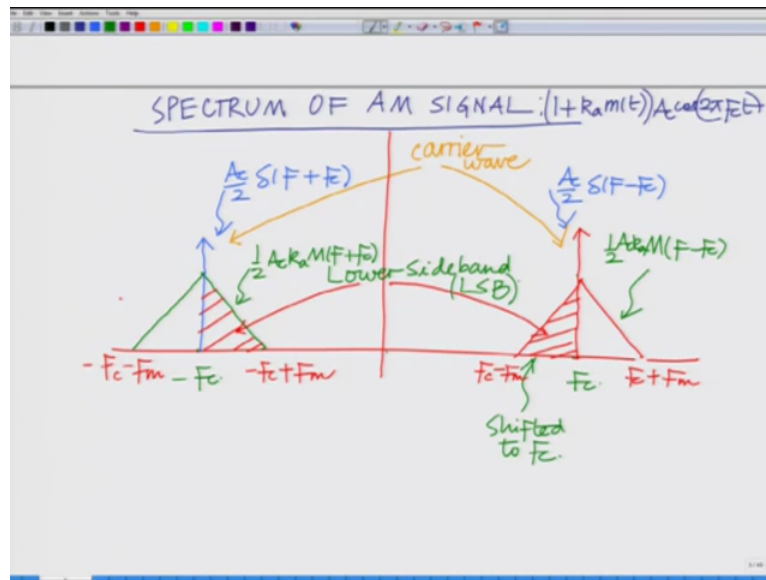
$$= \frac{A_c k_a}{2} M(F) * \delta(F - F_c) + \frac{A_c k_a}{2} M(F) * \delta(F + F_c)$$

Shifts $M(F)$ to F_c

Shifts $M(F)$ to $-F_c$

Therefore this is now, I can write it as two separate convolutions $A_c k_a$ over two $M(F)$ convolved with $\delta(F - F_c)$ plus $A_c k_a$ by two $M(F)$ convolved with $\delta(F + F_c)$. Now $M(F)$ convolved with $\delta(F - F_c)$ simply shifts the spectrum $M(F)$ to F_c that is if the if $M(F)$ that is $M(F)$ convolved with $\delta(F - F_c)$ shifts the spectrum to F_c , $M(F)$ convolved with $\delta(F + F_c)$ shifts the spectrum to minus F_c . So this from the properties of the Delta function you can check that this is simply shifts $M(F)$ to F_c and this is plus F_c $\delta(F + F_c)$, this shifts $M(F)$ to minus F_c and further we have the scaling by this $A_c k_a$ by two this is simply a $A_c k_a$ by two is simply a scaling factor.

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So what we are going to have net is basically now we are going to have the spectrum which looks as follows we are going to have at F_c we are going to have $M(F)$ minus so this is basically your $M(F)$ shifted to F_c that is $M(F)$ minus F_c half A_c ka that is this is basically your shifted to F_c , correct? Correspondingly over here we are going to have the image that is shifted to minus F_c , okay. So this is half A_c this component is half A_c $M(F)$ plus F_c plus if I add the spectra of the carrier.

Now I am going to have two impulses, so this is your impulse A_c by two that A_c by two A_c by two δF I believe δF minus F_c this is your impulse and this is your impulse A_c by two δF by F_c , so this corresponds to the carrier we have added the, so these two impulses corresponds to the correspond to the carrier wave these two corresponds to these two triangular bands correspond to the bands of the that is spectra of the original message shifted to F_c , shifted to minus F_c and this is the complete spectrum of the $A_s A_m$ signal that is a message spectrum that is a shifted message spectrum plus the carrier component, okay.

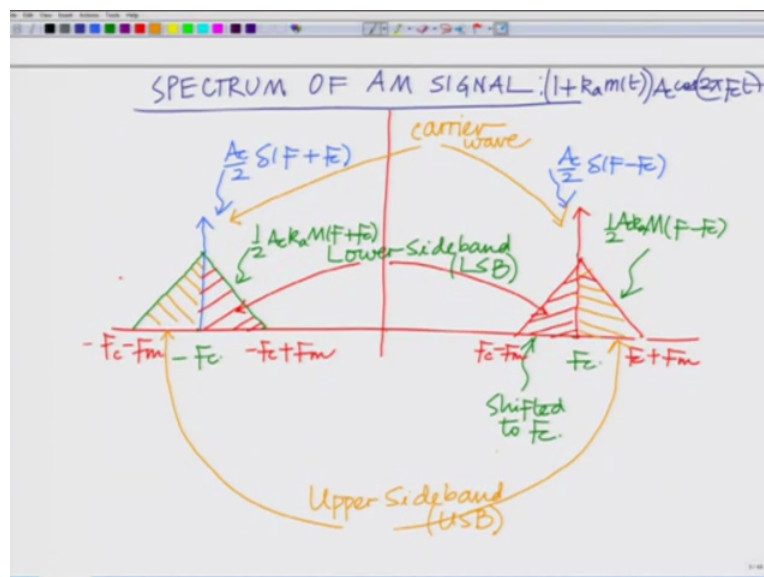
So this is the your complete that is your spectrum of the AM signal, so this is your spectrum of the AM signal. This is the spectrum of the AM signal, okay or the amplitude modulated signal which is given as remember the AM signal which is given as one plus $k_a m(t) A_c \cos(2\pi f_c t)$ Fct this is the spectrum, alright. Look at look at this this is symmetric about the origin, correct? It contains two bands one shifted on the one on the positive frequency side one on the negative

frequency side, correct? One corresponding to the $M(f)$ shifted to f_c , $M(f)$ shifted to minus f_c , correct?

Now these two over here these two bands over here these two over here that is minus f_c if I look at this this is minus f_c plus f_m this is minus f_c minus f_m this is f_c plus f_m f_c these two bands between f_c to $f_c - f_m$ to f_c and minus f_c two minus f_c plus f_m these two bands are known as the lower side band or the LSB let me just write this (compl) a little clearly these two bands comprise the lower side band or LSB and these two bands if you look at the bands between minus f_c minus f_m to minus f_c that is one being shaded in orange and f_c to $f_c - f_m$ f_c f_c to f_c plus f_m that is if you look at if you look at, these two if you look at between f_c f_c minus f_m to, minus f_c and f_c to f_c plus f_m , if you look at these two bands these two bands are known as the comprise the upper side band or the USB.

That is the outer side band that if you look at the outer side band of this modulated AM signal they comprise the upper side band the inner side bands of this AM modulated (conv) signal comprise the comprise the lower side band and these are going to play an important role later on when you talk about single side band modulated signal that is when we are going to transmit only one of the side bands either the upper side band or the lower side band that is a different modulation technique which is known as single sideband modulation and that has an important role in communication because it results in a savings of bandwidth, correct?

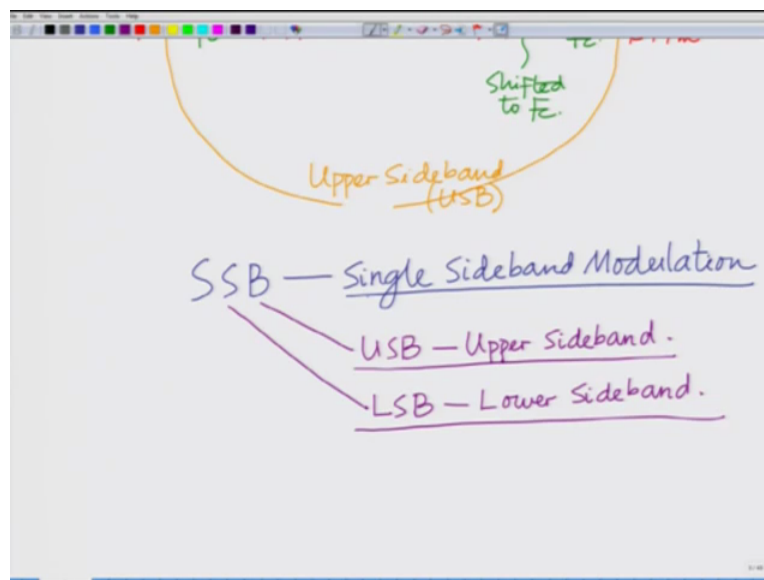
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Alright, so this is basically the complete spectrum of the AM signal which we can see (compo) comprises of the carrier wave component which is represented by the two impulses at $\pm F_c$ and also the two shifted versions of $M(F)$ that is $M(F) - F_c$ at F_c and $M(F) + F_c$ at $-F_c$, alright and these are appropriately scaled the impulses are scaled by A_c by two and the spectra that is the shifted bands of the message are scaled by half A half A_c , okay. So this is the complete spectrum this is the complete spectrum of the AM signal.

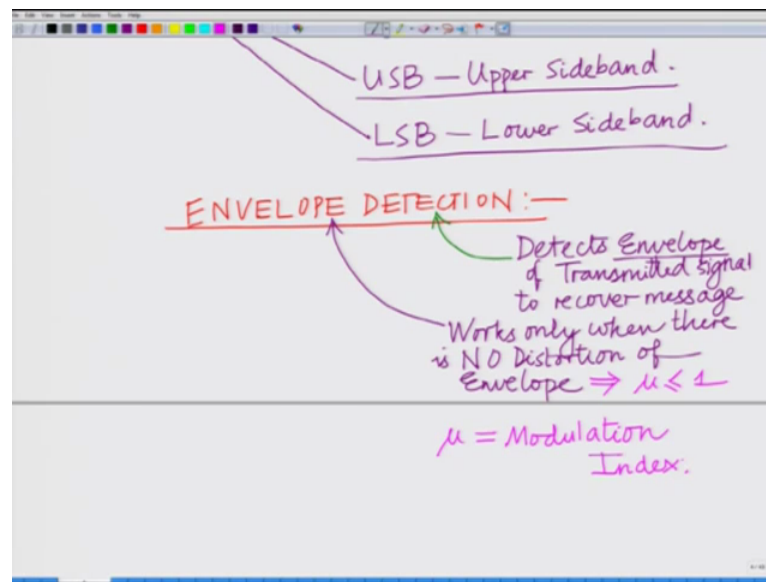
And in future we will (exba) examine and we have also and we have also noted the upper side band of this message (19:44) spectrum and the lower side band and in future we are going to examine all we are going to explore single sideband modulation that is where we transmit only the upper side band or the lower side band.

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((19:53)) also make a note that the note of that that is that is we have single sideband modulation which transmits only the either the USB that is the upper side band or the LSB of the signal that is the lower sideband, okay. We will explore these different modulation techniques in the future where we transmit only the upper side or the lower sideband, okay.

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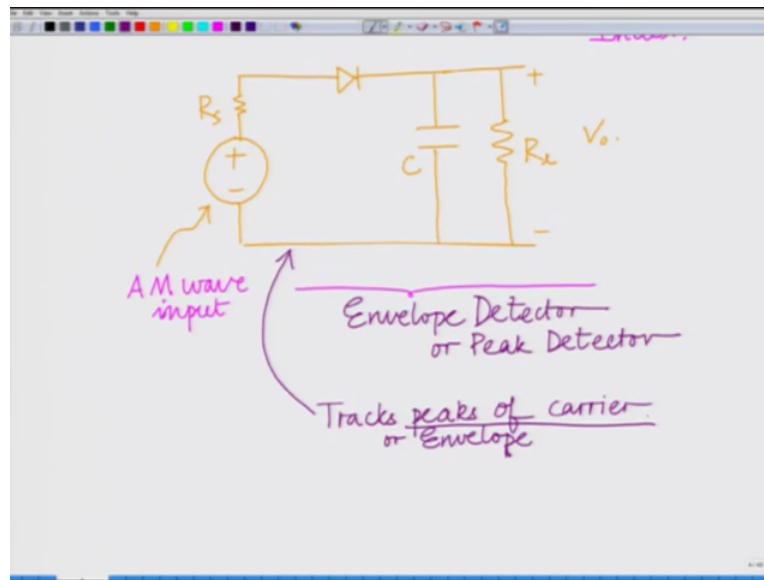


Now let us look at detection for this AM signal or amplitude modulated signal and one of the most popular schemes for detection is what is known as envelope detection and as the name implies as the name implies envelope detection detects the envelope, correct? This detects the envelope this detects the envelope this detects the envelope of the transmitted signal to recover the message.

This detects the envelope of the transmitted signal to recover the message, correct? And therefore by default it implies that you have to detect the envelope then there should be no distortion in the envelope therefore this works only when there is no distortion in the envelope which means the modulation index μ must be less than or equal to one, alright. That is the condition that we have seen before if you go way back if you go to our previous if you go to the if you go to the previous module what we have seen is that for no distortion that is the modulation index the modulation index μ must be less than or equal to one, correct?

The modulation index μ must be less than or equal to one for no distortion, okay. So this is what we have seen in the previous module, alright. So for this that detects the envelope so for this there should be no distortion works only when there is no distortion of envelope this implies μ has to be less than or equal to one where μ remember μ is the modulation μ is the modulation μ is a modulation index and what is envelope detection?

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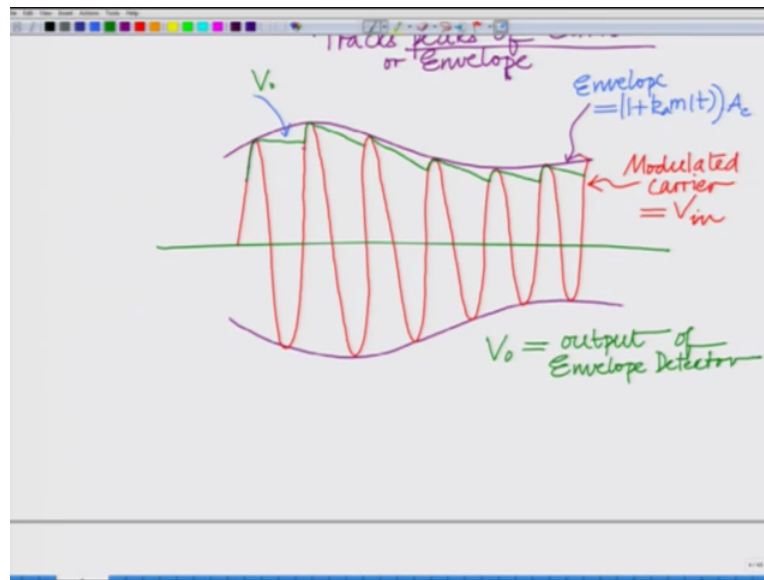


Envelope detection employs a single circuit which is known as the envelope detector and that can be explained as follows I have an input here, alright. And this is my source resistors R_s I have a diode, correct? A capacitor with charges and discharges and a load resistance R_L and I measure the voltage V not across the capacitor, correct? So no this is known as this circuit and the AM wave is given as input, the AM wave is given as input to this circuit, alright.

This circuit is known as an envelope detector or a peak detector this circuit is known as envelope detector or peak detector as you can see it is a simple (circuit) it is a simple circuit, correct? This envelope detector it is a very simple circuit which consist of diode, alright and a capacitor in parallel with resistance there is a load resistance R_L , alright.

And the capacitor charges and discharges and this tracks basically we are going to show how this track basically the envelope of the incoming signal and as we have already seen the incoming signal or the AM wave the amplitude modulate the signal is fed as input to this envelope detector, alright. And this circuit we can see this tracks the peaks of the carrier or the envelope this tracks remember the waveform formed by the peaks of the carrier that is nothing but the that is nothing but the envelope of the signal, alright.

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And how is this achieved, let us look at this let us look how this is achieved let us draw again a simple AM modulated signal and remember also we are considering that the modulation index is less than one therefore there are no of this is the carrier this is an approximate so this is your envelope, correct? The actual original envelope which is basically your one plus k_a times $m(t)$ into into A_c this is the actual envelope now this is the modulated carrier, correct? This signal over here this is your modulated carrier which is fed as your input, so this is the input V in which is basically your modulated carrier.

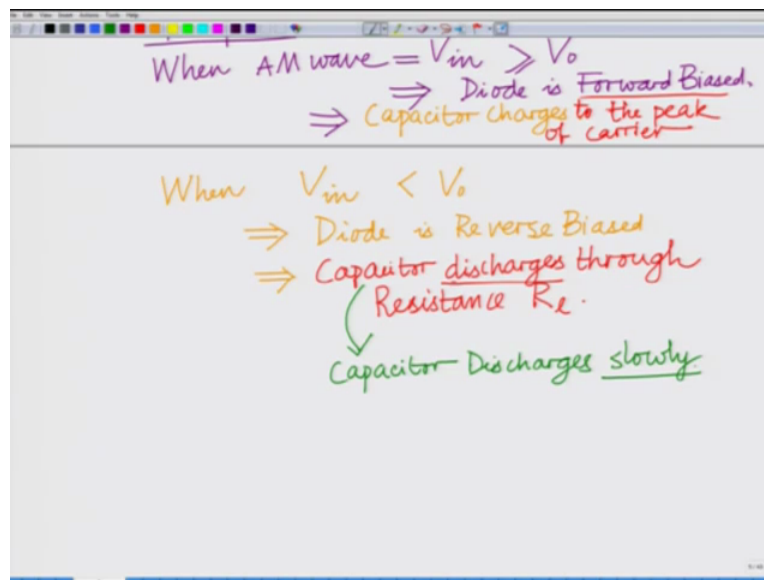
Now let us say the output voltage starts from (ze) zero now when the input voltage is greater than the output voltage the capacitor charges to the input voltage and once it reaches the peak, correct? Because the diode becomes reverse biased, alright. The capacitor is cut off and it starts discharging it starts discharging until again V_{in} becomes greater than V_{out} at which point it starts charging and again once it reaches the peak the diode becomes reverse biased as V_{in} starts falling the diode becomes reverse biased so the capacitor charges and discharges and as you can see it traces a path that is somewhat close to the actual envelopes.

Let me draw it for the different colors so the capacitor charges and at this point V_{in} starts to decrease that is V_{out} is greater than V_{in} so the diode is reverse biased the capacitor starts to discharge again capacitor charges and capacitor discharges and you can see this is the, so this is the output this is the output V_o which is close to the actual envelope, so V_o is the output which is

basically output of the envelope detector V_o is the output of the envelope detector and you can see it is fairly close to the envelope of the input signal that is envelope of the incoming amplitude modulated wave and it works on the following principle that is when the input voltage is greater than the output voltage the capacitor charges.

So the capacitor rises to the peak once the capacitor voltage reaches the peak of the carrier at that point, alright and V_{in} starts to fall the capacitor than the diode becomes reverse biased so the capacitor is cut off and the capacitor starts to discharge from the load resistance R_L . So basically it reaches the peak and discharges slowly until the next time instant when V_{in} again becomes more than V_o at which point The V_o the output voltage the capacitor again charges to the peak and this process keeps continuing whereby the capacitor keeps charging to the peak and discharges slowly thereby approximately traces the it traces the wave that corresponding to the peaks of the incoming carrier wave and that is basically nothing but the envelope of the incoming incoming amplitude modulated signal.

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So the so the envelope detector works on the principle basically the principle of envelope detector a is when a when your AM wave that is equal to V_{in} is less than V_o this implies diode equals or when V_{in} is less less than V_o or when V_{in} is greater than V_o the input voltage is greater than or equal to V_o . Diode is forward biased, so this is your so this is your diode let us

denote this this is basically this is your diode D so V in greater than V_o implies D is forward biased and V in less than V_o implies D is reverse biased.

So when V in greater than equal to V_o the diode is forward biased, okay. The diode this implies that the diode is forward biased, correct? This implies your capacitor charges charges, okay. On the other hand when V in less than V_o ? This implies diode is reverse biased this implies capacitor is cut off and basically capacitor discharges capacitor discharges to R_L .

So when the diode is forward biased capacitor is charges capacitor charges and capacitor charges to the to the league of the carrier and capacitor discharges and remember it does not discharges very fast the capacitor discharges slowly. If I might add capacitor the key here is to realize that the capacitor discharges slowly because if the capacitor discharges very fast and it reaches the peak than it is almost going to fall down immediately, correct?

So we want the capacitor to charge to the peak and then hold to the peak and fall discharge very slowly so that it can again charge to the peak at the next time and thereby trace the envelope rather than trace the carrier we would like the capacitor to charge the peak and then hold to the peak discharging slowly. So that it is only tracking the message signal but not discharging again with the carrier, alright. So we want the capacitor to discharge slowly.

So in fact the time constant of this circuit that is the time constant of the capacitor that is the discharge circuit that is C into R_L that is τ which is c times this time constant has to be chosen appropriately so that, alright. So that the capacitor does not discharge too fast or the capacitor does not discharge too slowly, alright. So we will see, alright we will see that the capacitor that is the time constant τ has to be chosen appropriately so that the rate of discharge of this capacitor has to be appropriate so that it tracks the envelope of the incoming amplitude modulated signal.

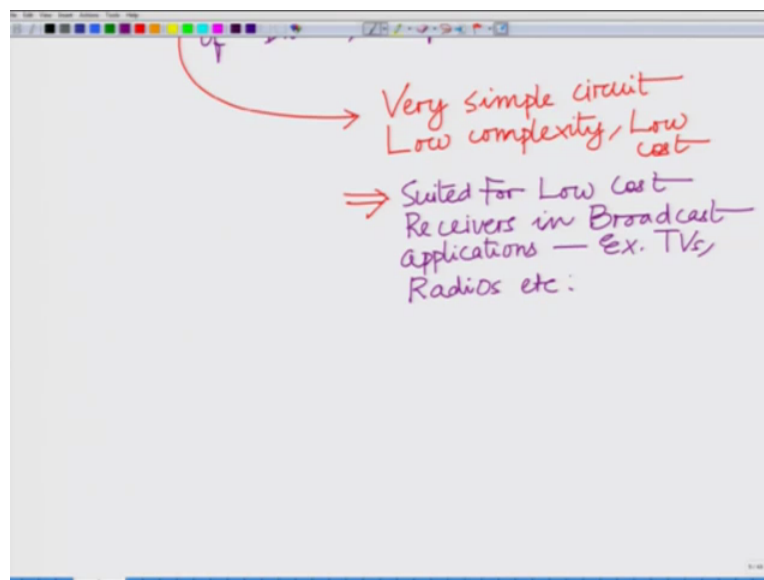
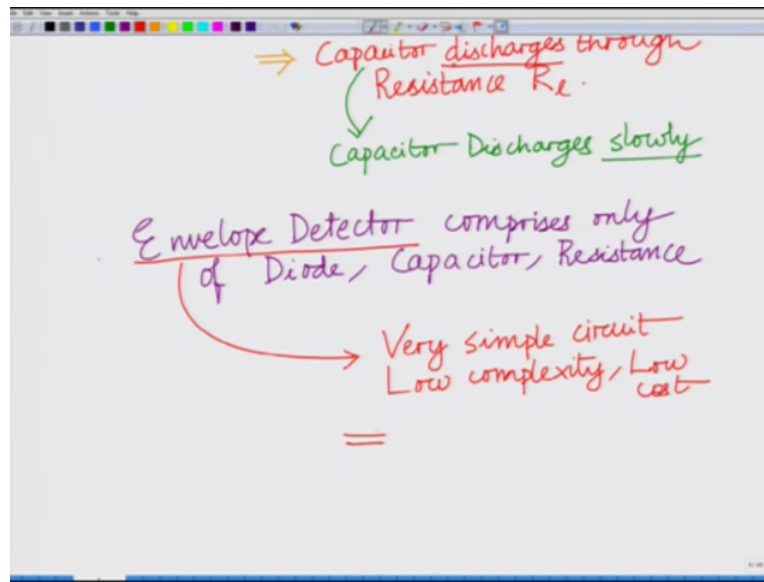
And we can see that once that happens the output voltage of the across the capacitor is basically a close replica of the envelope of the (incom) envelope of the incoming amplitude module incoming AM signal which is basically proportional to the message signal because remember the envelope of the amplitude modulated signal if there is no envelope distortion is basically proportional to the message signal. It is it is in fact A_c times one plus $k_a m(t)$, so that is how basically this simple envelope detector which comprises of a diode and a capacitor and a load

resistance is able to recover, alright able to recover the envelope of the incoming amplitude modulated signal and from this the message signal $m(t)$ can be recovered, alright.

And the advantage of this circuit the advantage of envelope detector is it is a simple circuit it (com) comprises only of a diode, a capacitor and a resistor so it is a very low complexity and it is a very low-cost or a low-budget circuit and therefore it makes implementation of an AM of the receiver or an AM receiver makes a implementation of an AM receiver very low-cost and very simple and therefore it is very the envelope detector although is not optimal although does not yield exact recovery as we can see the envelope is a close approximation of the envelope of the incoming signal but not an exact replica of the envelope of the incoming signal therefore even though the approximation is score the envelope detector envelope detection is one of the most popular schemes for demodulation of or detection of Am signals, correct?

Detection of AM signals because envelope detector is a very low cost a very simple low-cost circuit which makes the receiver implementation that is like which makes the AM demodulator implementation the AM detector implementation very simple and (())(38:36) it enables low-cost it enables low (co) implementation of low-cost receivers which is especially suitable for broadcast applications for instance applications such as TV transmission, radio transmission where the broadcast equipment can be very high cost but the receiver such as the televisions or the radios have to be very low-cost that is the advantage it helps build receivers with very low-cost because of the low complexity of the circuit.

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So this is envelope detector to add the envelope detector comprises only of a diode, a capacitor and resistance, correct? And therefore it has a simple circuit suited its very simple circuit and it has a low complexity and low-cost more importantly it has a low complexity and therefore this is suited for development of low-cost low-cost receiver low-cost receivers such as televisions or radio sets in especially broadcast applications, examples being your TVs or transistor sets or radio sets etc.

So it is the the attractive feature of the envelope detector is the the simplicity or the low complexity of the circuit which enables low-cost implementation and therefore helps built

receivers with a very (low) cost and suited for broadcast applications. So in this module we have seen a couple of things we have seen the spectrum of the amplitude modulated signal, alright. Seen the various components the upper side and on the lower side band of this amplitude modulated signal and we have also seen a simple technique for detection or simple technique for detection of the envelope or simple technique for demodulation of the AM signal, alright.

That is the envelope detection and we have seen the circuit and we have seen how this circuit works to yield a replica or to yield a close approximation of the envelope of the incoming amplitude modulated signal, alright. So we will stop here and continue with the other aspects in the subsequent modules, thank you.