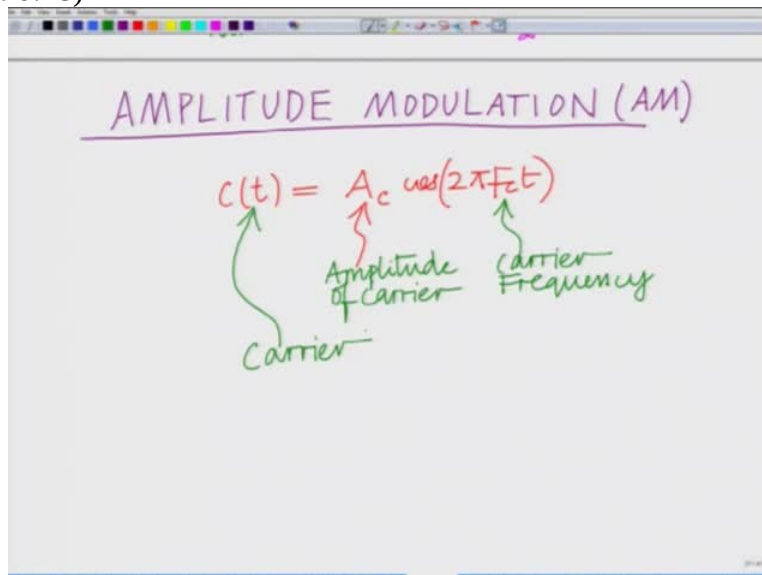


**Principles of Communication- Part I**  
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**Module No 2**

**Lecture 10: Introduction to Amplitude Modulation (AM), Modulation Index Envelope Distortion and Over Modulation**

Hello welcome to another module in this massive open online course. So today our in this module let us start looking at a modulations scheme or a communications scheme which is known as amplitude modulation, alright.

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A screenshot of a whiteboard with the title "AMPLITUDE MODULATION (AM)" written in purple. Below the title, the equation  $c(t) = A_c \cos(2\pi F_c t)$  is written in red. Green arrows point from the text "Carrier" to  $c(t)$ , from "Amplitude of carrier" to  $A_c$ , and from "Carrier Frequency" to  $F_c$ .

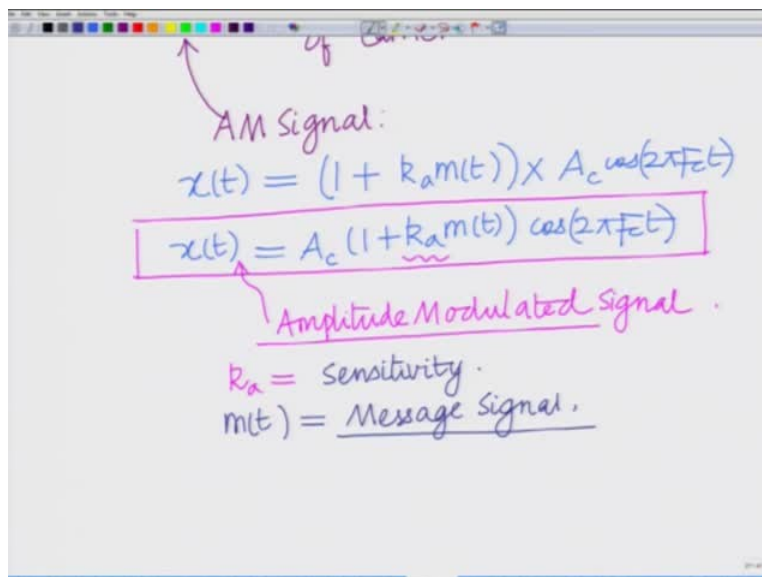
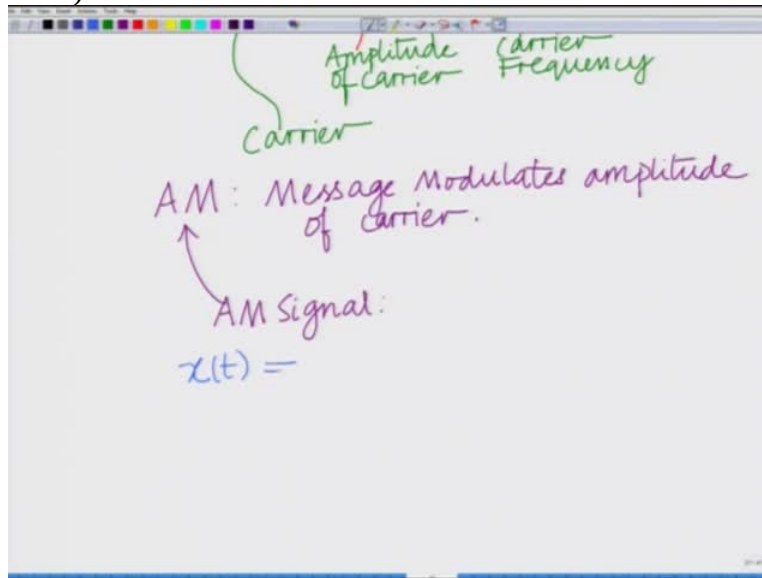
So let us start looking at amplitude modulation which is an analog communication scheme and one of the earliest communications schemes developed and widely used, this principles continued be used even today that is amplitude modulation often abbreviated as AM. Now any communication system or communications scheme involves a carrier signal which is a Sinusoidal signal, alright.

Let us denote this carrier signal by  $c(t)$  which is  $A_c \cos 2 \pi F_c t$ , this is the amplitude of the carrier and this is the carrier, so  $F_c$  is the carrier frequency  $A_c$  is amplitude of the carrier and this is basically your simply known as the carrier or the carrier wave. This is fundamental in every

communication signal a communication system and the signal that is the baseband signal or the message signal is modulated on this carrier, alright.

There are various ways to modulate the carrier. One of the techniques is amplitude modulation in which the amplitude of the carrier is modulated.

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$x(t) = (1 + k_a m(t)) A_c \cos(2\pi f_c t)$   
 $x(t) = A_c (1 + k_a m(t)) \cos(2\pi f_c t)$

Amplitude Modulated Signal  
 $k_a =$  sensitivity.  
 $m(t) =$  Message Signal.  
 Amplitude of carrier is varied or modulated according to message signal  $m(t)$ .

So in amplitude modulation so what we have in AM is basically your amplitude of the carrier. That is message modulates the amplitude of the carrier and an AM signal is given as the AM signal is described as  $x(t)$  which is equal to 1 plus  $k_a$  let me write the expression 1 plus  $k_a m(t)$  times the carrier that is  $A_c \cos(2\pi f_c t)$  which is also basically  $A_c$  plus 1 plus  $k_a$  times  $m(t)$  into  $\cos(2\pi f_c t)$ . This is the standard form or the canonical form of the amplitude modulated.

So this is the, this is your amplitude modulated this is the amplitude modulated signal, alright. And Let us explain of course we have already seen what is  $A_c$ ?  $A_c$  it is a carrier amplitude  $f_c$  is a carrier frequency  $k_a$  this constant  $k_a$  is also known as the sensitivity  $k_a$  is the sensitivity. Sensitivity of the sensitivity of a the sensitivity of this AM signal, of course  $m(t)$  is the this is the message to be transmitted. This is your message signal that is a message we want to communicate or the message that we would like to transmit, alright.

Now if you see what is happening in the amplitude modulation is we have a carrier, alright.  $\cos(2\pi f_c t)$ . Now when we multiply this carrier by the message, alright. Now the amplitude the amplitude of the carrier the un-modulated carrier  $A_c \cos(2\pi f_c t)$  which is constant and that is amplitude of the modulated carrier is simply  $A_c$ . Now once you multiply the message that is 1 plus  $k_a$  times  $m(t)$  the amplitude of this carrier varies according to the message, alright.

So this is this is this is basically the amplitude of the carrier is modulated, right. When the amplitude of the carrier varies according to the message another way to say this say phenomenon

is that the amplitude of the carrier is modulated by the message therefore this is also known as amplitude modulation, alright. So what we have here is basically to explain this thing the amplitude of the carrier you can see that the amplitude of the carrier is varied or modulated according to the message.

So we have to modulate, so modulation is nothing but the variations. So so what we have in when we multiply the message by when we multiply the carrier signal by the message the amplitude, one can think of it as if the amplitude of this carrier signal is being modulated by the message. Therefore this is termed as amplitude modulation. So amplitude of the carrier which is previously constant is now time varying in nature and it varies as per the message  $m(t)$ , alright.

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according to message signal  $m(t)$ .

Example:  $A_c = 1$   
 $k_a = \frac{1}{2}$   $F_m = \frac{1}{2}$   
 sensitivity  $m(t) = \cos(2\pi F_m t)$   
 $= \cos(\pi t)$

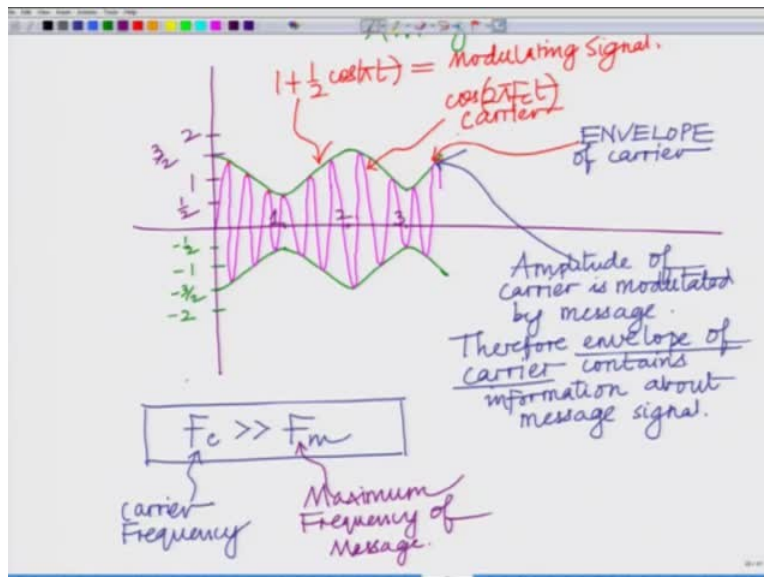
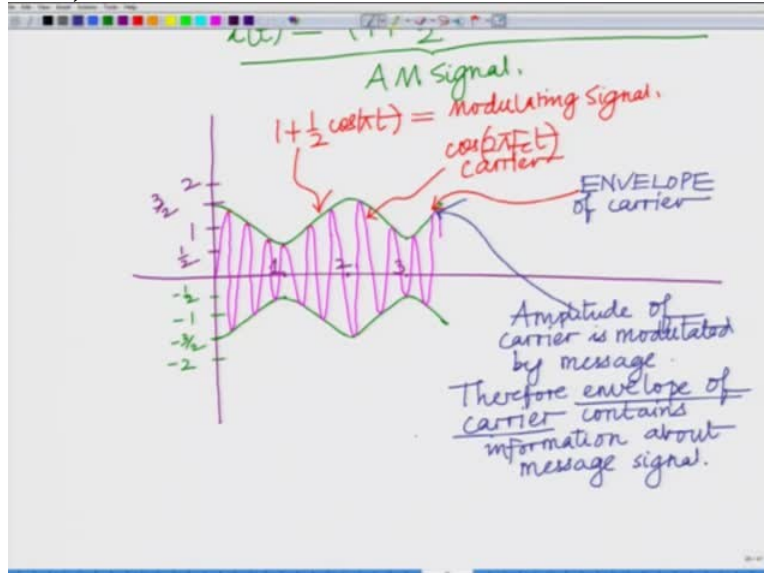
AM Signal:  
 $x(t) = (1 + \frac{1}{2} \cos \pi t)$

So to understand this let us look at a simple example, right. Let us consider a simple example, let us describe this by a picture. So I have, let us consider  $A_c$  that is amplitude of the carrier to be one, remember  $k_a$  is the sensitivity let us choose  $k$  equal to half and  $F_m$  the message frequency is equal to half. Let us choose the message to be a pure Sinusoid  $m(t)$  equals cosine  $2\pi F_m t$   $F_m$  equals to half. So this is simply cosine of  $\pi t$ .

This is your message signal. Therefore AM signal  $x(t)$  equals  $x(t)$  is given as  $A_c$ , that is 1 into 1 plus  $k_a$  that is half cosine  $\pi t$  times cosine  $2\pi F_m t$ . This is your amplitude modulated signal, alright. We have chosen the sensitivity  $k$  equals half. We have chosen the message to be a pure Sinusoid, that is cosine  $2\pi F_m t$  with  $F_m$  equal to half. So it is cosine  $\pi t$ , so the amplitude

modulated signal is  $1 + \frac{1}{2} \cos(\pi t)$  times  $\cos(2\pi F_c t)$  where we have chosen the carrier amplitude  $A_c$  also to be equal to unity.

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Now if we plot this will look something like this, let me first draw the time axis, correct? Okay and now if you look at this  $1 + \frac{1}{2} \cos(\pi t)$ , the maximum value of this is basically 3 by 2 when  $\cos(\pi t)$  equals 1 and the minimum value of this will be the minimum value of  $1 + \frac{1}{2} \cos(\pi t)$  will be half when  $\cos(\pi t)$  is - one. So this varies between this varies between this varies between half and 1 and a half that is 3 over 2.

So I have over here one, let us say I have over here 2 and this is half this is 3 by 2 . So now for instance when  $t$  equal to zero I have  $1 + \frac{1}{2} + \frac{1}{2} \cos \pi t$  equals 1 and a half that is 3 over 2 and that  $t$  equal to  $t$  equal to zero this is equal to 3 over 2 at  $t$  equal to one,  $1 + \frac{1}{2} \cos \pi t$  is  $1 - \frac{1}{2}$  that is half. So at  $t$  equal to 1 it is half and once again at  $t$  equal to 2 this is going to be equal to 1 and a half, at  $t$  equal to 3 this is going to be equal to half and so on.

So if I join these points now so if I joined these points I have some signal that looks like this, right? I have some signals that look like this. Now let us reflect this let us construct its mirror image about origin, so the mirror image about the origin will look something like this, so there is 3 by 2, it is - half, so the mirror image between will be between - 3 by 2 and - half and that is going to be something like this.

That is basically reflection of this about this, so I have this. Now the carrier modulated carrier basically the amplitude of the carrier is changed according is varied according to this, so I have a carrier whose amplitude varies according to this the modulating signal. So I am going to have something that looks like this, correct? So what is this? This is basically your  $\cos 2\pi F_c t$  this is the carrier. This is  $1 + \frac{1}{2} \cos \pi t$ . This is the modulating signal.

Now if you look at this the carrier, correct, The carrier basically the amplitude of the carrier is modulated by the message signal. Therefore if you look at the peaks of the carrier, the peaks of the carrier basically trace the message signal the peaks of the carrier, so the curve that is traced by the peaks of this carrier that is termed as the envelope of the carrier and that is very important because that contains the information.

So if you look at the peaks if I sample this at the peaks, right, the peaks follows the message signal. So this is termed as the envelope and this is an important aspect this is termed as the envelope of the carrier and this contains the envelope of the carrier this contains, so we have the amplitude of the carrier which is modulated the amplitude of the carrier is modulated by the message.

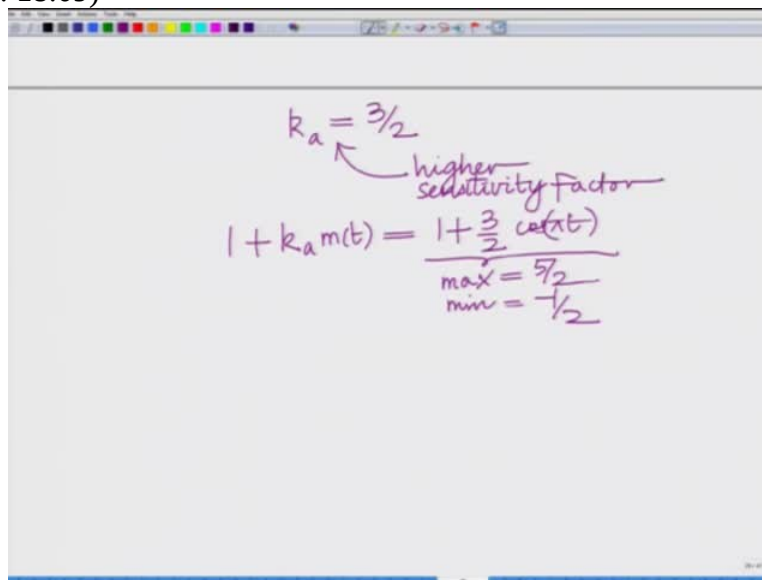
Therefore envelope of the carrier contains information. Therefore envelope of the carrier contains the information about the message signal and further if you look at this if you look at this we have implicitly assumed and this is true of all communication system that the carrier

frequency is much larger for this amplitude modulation or for any communication system to work we need the carrier frequency to be much larger than the message frequency, alright.

So the carrier frequency  $F_c$  is typically much larger than the message frequency  $F_m$ , alright. So carrier frequency. So  $F_c$  is much greater than  $F_m$ , correct?  $F_c$  is your carrier frequency of course we are considering a pure sinusoid, so  $F_m$  is the message frequency frequency otherwise you can also think of as  $F_m$  as the maximum frequency of the message maximum frequency maximum frequency component in the message that is the message is band limited between message has to be message is band limited between  $-F_m$  to  $F_m$ .

Therefore the maximum frequency component of this message signal has to be much smaller than the carrier frequency. The carrier frequency usually chosen is much larger than the maximum frequency component of the message frequency, alright. So you can see that the carrier frequency is much larger than the message frequency. Now another important implicit assumption here is the following thing.

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Handwritten mathematical derivation on a whiteboard:

$$k_a = \frac{3}{2}$$

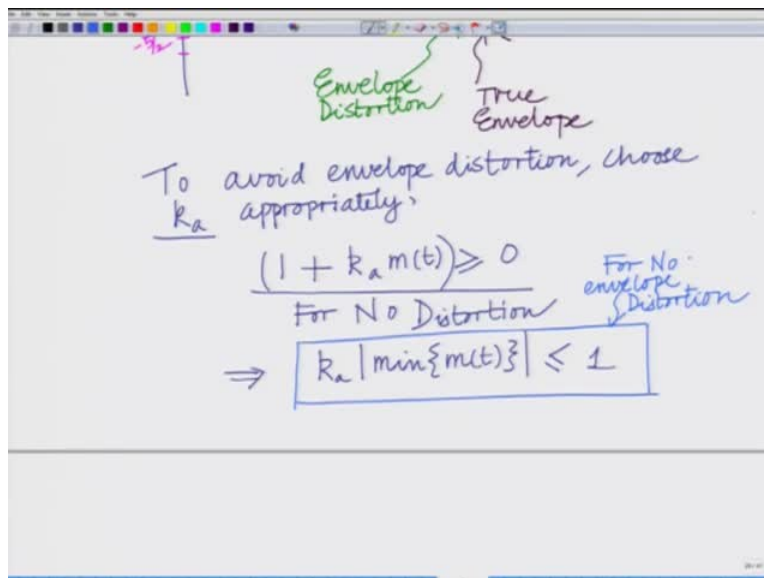
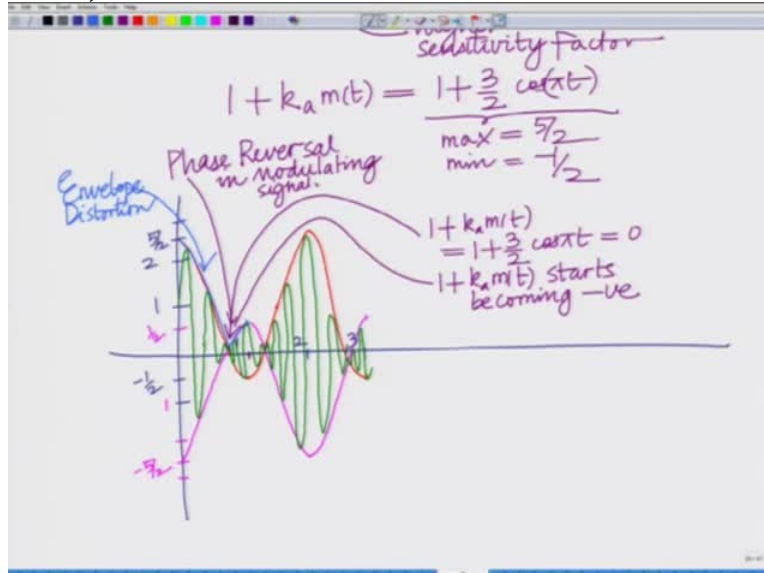
higher sensitivity factor

$$1 + k_a m(t) = 1 + \frac{3}{2} \cos(xt)$$
$$\begin{aligned} \text{max} &= \frac{5}{2} \\ \text{min} &= -\frac{1}{2} \end{aligned}$$

Let us which we will understood let us consider another example by choosing the same system when we choose  $k_a$  equals 3 by 2, that is a higher previously  $k$  equals half that is we are choosing a higher sensitivity factor. Now if you look at  $1 + k_a$  times  $m(t)$   $1 + k_a$  times  $m(t)$

equals  $1 + 3 \text{ by } 2 \cos \pi t$ , the maximum of this is  $5 \text{ by } 2$  and the minimum of this is  $-\text{half}$  when  $\cos \pi t$  is  $-1$  this is  $1 - 3 \text{ by } 2$  which is  $-\text{half}$ .

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Now if you plot the same thing that is the a signal that is modulated by this, so I have  $1, 2, 5 \text{ by } 2$  so this lies between  $-\text{half}$  to this lies between  $-\text{half}$  to  $-\text{half}$  to  $5 \text{ by } 2$ . So this is let us say your  $5 \text{ by } 2$  and this is let us say  $-\text{half}$ , again the same thing this is point this is  $1, 2$  that is time 1, time 2 time 3 and so on. At time  $t$  equal to zero this is equal to  $5 \text{ by } 2$ , at time  $t$  equal to 1 this is equal to  $-\text{half}$ , correct?



At  $t$  equal to 1 this is cosine  $\pi$ , so  $1 - 3/2$  is  $-1/2$ , at time  $t$  equal to 2 again this will be  $5/2$  and time  $t$  equal to 3 this will be  $-1/2$  and if I now join these points what I will get is something that looks like something that looks like this, right? And of course if I can draw the mirror image of this again the mirror image will be between  $-5/2$  and  $1/2$ . So this will be  $5/2, 1/2, 5/2, 1/2$ , so the mirror image will be something like this and now if I draw.

So this is basically your envelope and now if I draw again the carrier which is modulated by this, the carrier which is modulated by this message signal I have something like and this if you can look at this again if you can look at this you can see something very interesting at this point. At this point  $1 + k \cdot m(t)$  equals  $1 + 3/2 \cdot \cos(\pi t)$ , alright starts equal to zero and at this point it starts going negative.

So there is, so at this point at  $t$  equal to 1 if you look at this point at  $t$  equal to 1 at  $t$  equal to at this point, correct? So at this point it basically starts to go negative, correct? so this is your point 1 so at this point it starts to go negative that is message  $1 + k \cdot m(t)$  starts becoming negative, correct? So the so the envelope starts to become negative. So there is a phase reversal in the envelope, correct?

There is a phase reversal in the modulating signal, there is a phase reversal and therefore what you can see, if you track the envelope of this the envelope is going to be this, correct? So there is going to be instead of tracking the correct envelope you are going to track a distorted version of the envelope. So this leads to envelope distortion. So what is happening is because of because since  $1 + k \cdot m(t)$  is negative, correct?

And when you track the envelope you continue track the positive envelope and what and what happens eventually is that you will encounter is that you will have this leads to envelope distortion. So if  $1 + k \cdot a$  is much large, alright. If  $k \cdot a$  is much large that  $1 + k \cdot a \cdot m(t)$ , right can take negative values that leads to phase reversals of the carrier, correct? That leads to phase reversals and this leads to envelope distortion when you track the envelope of the carrier, alright.

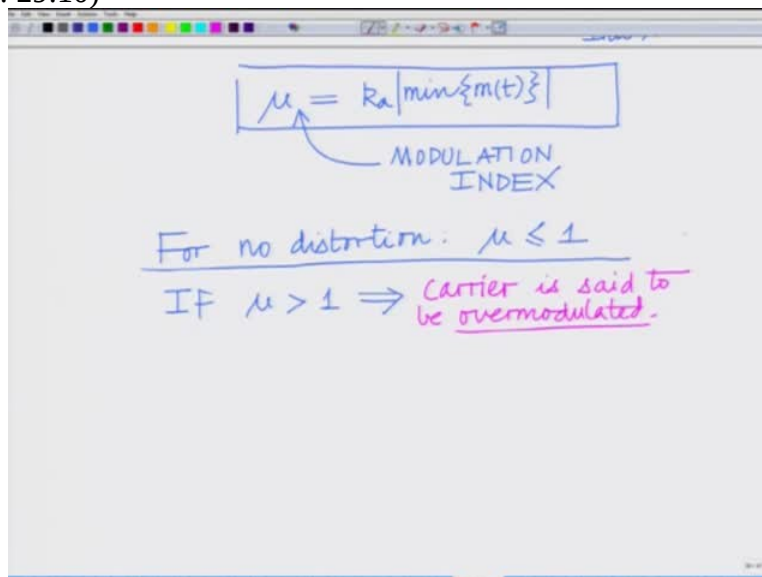
So eventually this was the true envelope, right? This was so the true envelope so to draw this again over here the true envelope is this. This is your true envelope and what you end up tracking? But what you end up tracking is something that is like this. This is your track envelope

and this leads to envelope distortion and this leads to envelope distortion. Therefore because it undergoes a phase reversal, correct? And therefore to avoid envelope distortion we can clearly see that the sensitivity factor  $k_a$  should be chosen appropriately to avoid envelope distortion.

Choose a sensitivity factor  $k_a$  appropriately in particular one must have  $1 + k_a m(t)$  to be greater than equal to zero, alright. This is the condition, correct? The envelope has to be greater than or equal to zero. This is the condition for no distortion which means and this implies that  $k_a$  times the magnitude of the minimum of  $m(t)$   $k_a$  times the magnitude that is the minimum that is when  $m(t)$  goes negative the magnitude of this minimum  $k_a$  times the magnitude of this minimum must be less than or equal to 1.

That is what we are saying is when  $m(t)$  becomes negative, if you look at the magnitude of that  $k_a$  times the magnitude of this trough  $k_a$  times the magnitude of this lowest point of  $m(t)$  or this negative peak of  $m(t)$  must be less than or equal to 1 for no envelope distortion, alright. This must hold for no envelope, this condition must hold for no envelope distortion and this constant  $k_a$  times magnitude of  $m(t)$  this is known as  $\mu$  the modulation this is known as  $\mu$  that is the modulation index, alright.

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The image shows a handwritten slide with the following content:

$$\mu = k_a |\min\{m(t)\}|$$

MODULATION INDEX

For no distortion:  $\mu \leq 1$

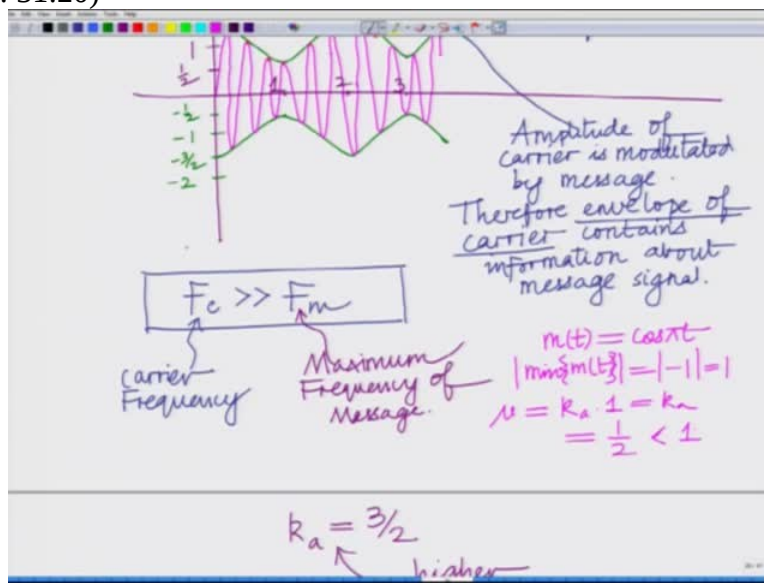
If  $\mu > 1 \Rightarrow$  Carrier is said to be overmodulated.

The modulation index of this amplitude modulated system  $\mu$  equals  $k_a$  times the magnitude of minimum of  $m(t)$  where  $\mu$  is the modulation index of these amplitude modulated system. Now

for no distortion, so for no distortion for no envelope distortion  $\mu$  is less than or equal to 1, correct?  $\mu$  has to be less than or equal to 1 for no distortion, if  $\mu$  is greater than or equal to 1 if  $\mu$  is greater than 1 then the carrier is said to be then there is going to be envelope distortion.

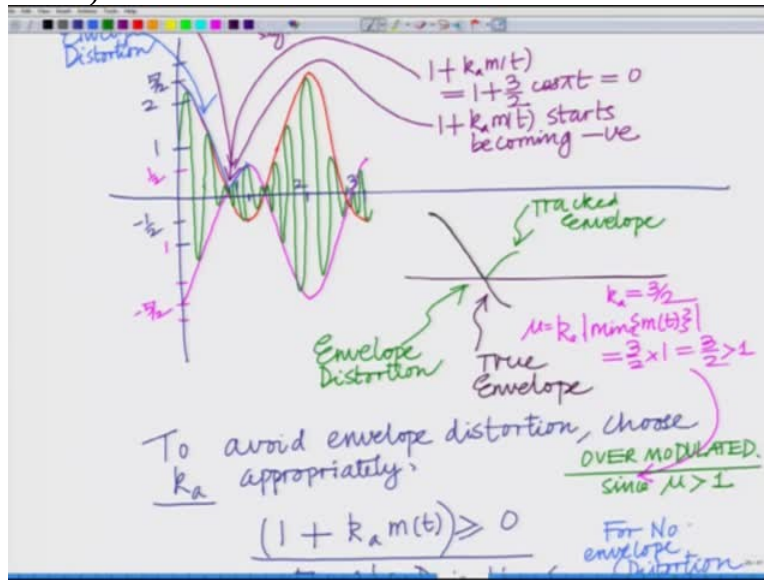
The carrier is said to be over modulated, so over modulation of the carrier leads to envelope distortion, so if  $\mu$  is greater than 1 this is basically over modulation carrier is when  $\mu$  is greater than 1 greater than 1 then carrier the carrier is said to be over modulated and over modulation leads to and over modulation leads to envelope distortion, alright.

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So if the carrier is over for instance in our 2 examples here, correct? Now look at these our  $m(t)$  is cosine  $\pi t$ , alright. Therefore minimum magnitude minimum  $m(t)$  magnitude of minimum  $m(t)$  is basically your modulus minimum of  $m(t)$  cosine  $m(t)$  cosine  $\pi t$  is - 1 magnitude of - 1 equals 1. Therefore  $\mu$  equals  $k_a$  times 1 equals  $k_a$  which is equal to which is equal to  $k_a$  is for this case  $k$  equals half, so this is equal to half which is less than 1 so therefore there is going to be no envelope distortion implies this implies no distortion, correct?

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However here let us if you look at the second example  $k$  in this case we have  $k_a$  equals 3 by 2  $k_a$  times magnitude of minimum of  $m(t)$  equals 3 by 2 into 1 equals 3 by 2. So this is basically your modulation index is greater than 1 therefore this is over modulated, so this is basically this is over modulated since the modulation index  $\mu$  is greater than one, alright. So we have seen 2 scenarios with cosine with a message signal cosine  $\pi t$  with  $k$  as maximum value that is magnitude of minimum  $m(t)$  is one.

So when  $k$  equals half this is under this is not a it there is no distortion, alright. However when  $k$  equals 3 by 2 this signal the modulation index  $\mu$  is is  $k_a$  times 1 that is 3 over 2 which is greater than 1, alright. So this the carrier resulting carrier signal is the resulting modulated signal the resulting carrier is over modulated and that leads to envelope distortion, that leads to phase reversals and envelope distortion, okay.

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Envelope Detection

For a sinusoidal message signal,

$$m(t) = A_m \cos(2\pi f_m t)$$

Amplitude of Message      Message Frequency

$$|\min\{m(t)\}| = |-A_m| = A_m$$
$$\Rightarrow \mu = k_a A_m$$

Modulation index = Sensitivity  $\times$  Amplitude.

Amplitude of Message      Message Frequency

$$|\min\{m(t)\}| = |-A_m| = A_m$$
$$\Rightarrow \mu = k_a A_m$$

Modulation index = Sensitivity  $\times$  Amplitude.

Sinusoidal Message Signal  $m(t)$

And finally for a Sinusoidal message signal, for a Sinusoid message signal, correct? We have  $m(t)$  equals  $A_m$  times cosine  $2\pi f_m t$  this is amplitude of your message, this is the message frequency, correct? This is the message frequency and you can see magnitude minimum  $m(t)$  the minimum value of  $A_m \cos 2\pi f_m t$  is  $-A_m$ , so magnitude minimum  $m(t)$  is  $A_m$  is positive that is magnitude minimum  $A_m$  which is  $A_m$ , correct implies  $\mu$  equals  $k_a$  times  $A_m$ , correct?

$\mu$  equals  $k_a$  times  $A_m$  which means the modulation index equals the sensitivity times the amplitude sensitivity times the amplitude for a Sinusoidal times the amplitude for a Sinusoidal

message signal. However it is important to remember this is only for a Sinusoidal is for a Sinusoidal message signal  $m(t)$ , alright. So the modulation index  $\mu$  equals the sensitivity  $k_a$  times amplitude  $a$ , alright.

So in this module we have started, alright. We have we have started looking at amplitude modulation which is one of which is one of the most fundamental I would like to say one of the most fundamental and 1 of the most 1 of the earliest schemes of transmitting that is transmitting and receive receiving communication signal or basically one of the amplitude modulation forms one of the one of the earliest technologies of communication, alright.

So we have looked at amplitude modulation that the structure of an amplitude modulated signal, correct? Examples of amplitude modulation, also the concept of the envelope of an amplitude modulated signal and also when is there and what is the envelope? When is there no envelope distortion and also when is the carrier the condition for over modulation of the carrier and the condition, alright.

The condition for no and when there is no envelope distortion and the condition when the carrier is over modulated which leads to envelope distortion, alright. So we will stop here and look at other aspects in the subsequent modules, thank you.