**Principles of CommunicationminusPart 1**
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**Module 1**
**Lecture No 1**
**Basicsminus Definition of Energy and Power of Signals**

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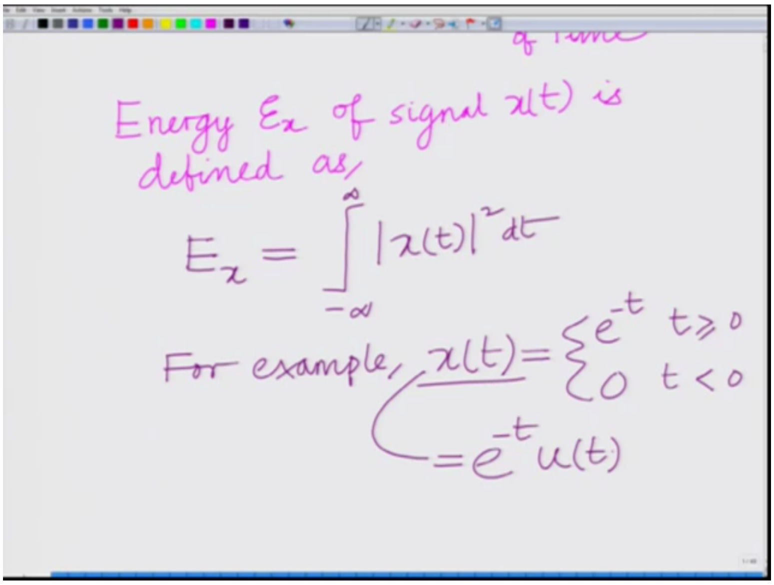


Hello, welcome to another module in this massive open online course. Let us start the discussion with the energy and power of a signal, alright. Let us start 1st by considering energy of a signal and consider for this purpose a signal x(t) okay, let us start by considering a signal x(t) and naturally this signal we are considering this is a function of time, correct. This is a function of time that is t denotes time. And the energy Ex of this signal x(t) can be defined as energy Ex of signal x(t) is defined as we can define this as Ex equals to integral minus inherit to infinity magnitude x(t) square dt that is integral minus infinity to infinity magnitude x(t) square dt alright.

For example, consider x(t) equals to e to the power of minus t for t greater than or equals to 0 and 0 for t less than 0 this is also termed as x(t) equals to e to the power of minus t into u(t) where u(t) is the unit step function that is u(t) is 1 if t is greater than or equals to 0 and u(t) is 0 otherwise this is known as the unit step function and these you should be familiar from a basic course on signal sins system.

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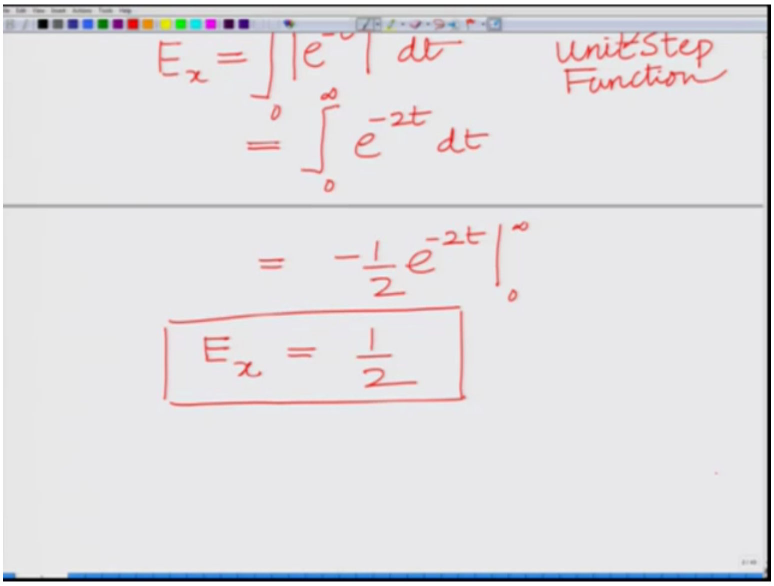


This is the unit step function, since this signal is nonminuszero only for t greater than equals to 0, Ex would be simply integral, since this is non0 only for x greater than equals to 0, so this is Ex integral 0 Ex is integral 0 to infinity, E to the power of minus t magnitude square dt that is integral 0 to minus infinity e to the power of minus 2t dt, which is equals to minus half e to the power of minus 2t 0 to infinity that is equals to half.

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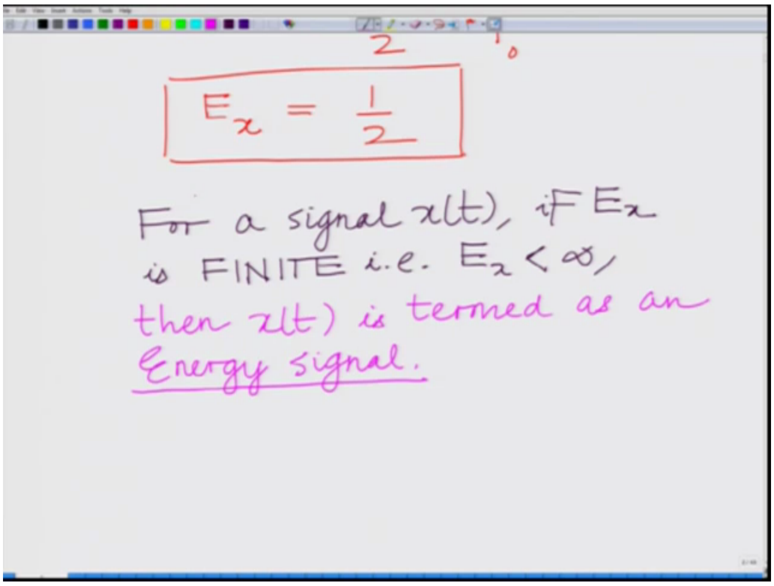


So we obtain a total signal energy that is Ex equals to half for x(t) equals to e to the power of minus t into u(t) okay, so that is energy of this signal, alright. Now, if Ex for a signal, for a signal x(t) if Ex is finite that is Ex is less than infinity that is strictly less than infinity, then x(t) is termed as an energy signal that is, if the energy is finite for instance similar to what we have seen above that is energy of the signal is half which is a finite quantity right. The signal x(t) is termed as an energy signal, okay.

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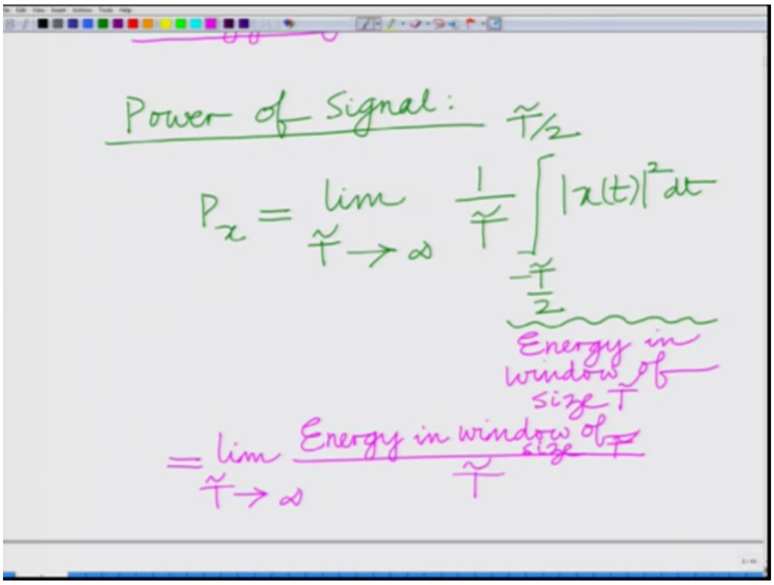


Now let us define the power of a signal x(t), the power of a signal x(t) is p of x equals to limit t tilde tends to infinity 1 over t tilde integral minus t tilde divided by 2 to t tilde divided by 2 the magnitude x(t) square dt which is basically the energy if you look at this quantity here this is the energy in a window of size t, this integral minus t by 2 this is energy in window of size t tilde. So this is basically your energy in a window of size t tilde divided by t tilde and the limit taken as t tilde tends to infinity. That is you take the energy in the window size t tilde normalize it divided by t tilde and take the limit of this quantity as t tilde tends to infinity that is the definition of the power of any signal x(t), okay.

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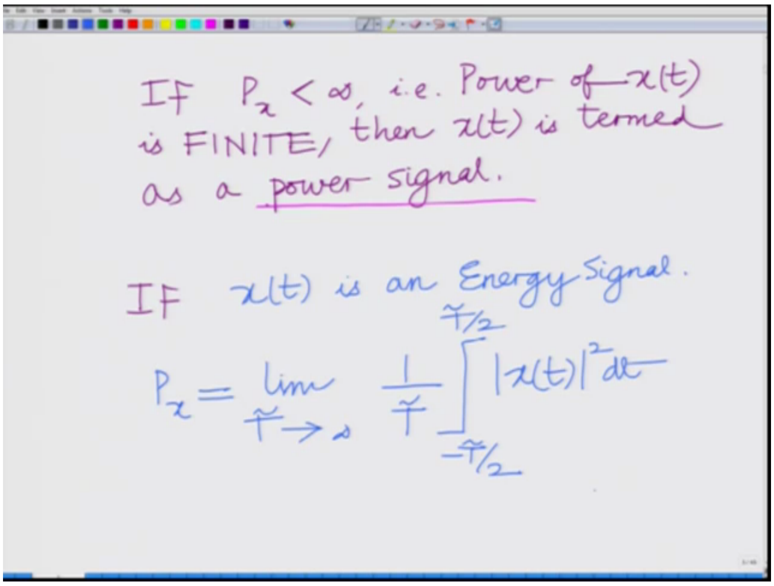


Now if Px is finite similar to the energy signal. If Px, now we have defined the power if Px is less than infinity that is power of x(t) is finite than x(t) is termed as a power signal for instance alright, so if so this is termed as a power signal. If the power of a signal is finite than the power of the signal x(t) is finite, then x(t) is termed as a power signal, okay. Now observe our interesting property if the energy is finite, if the signal is an energy signal. If x(t) is an energy signal then the power Px equals to limit t tilde tending to infinity 1over t tilde integral minus t tilde divided by 2 to t tilde divided by 2 magnitude x(t) square dt. Now this integral minus t tilde divided by 2 magnitude x(t) square dt is less than or equals to the integral from minus infinity to infinity because this is simply an energy in a window of size t tilde that is less than or equals to energy in the window of size, that is the energy in a window from minus infinity to infinity.

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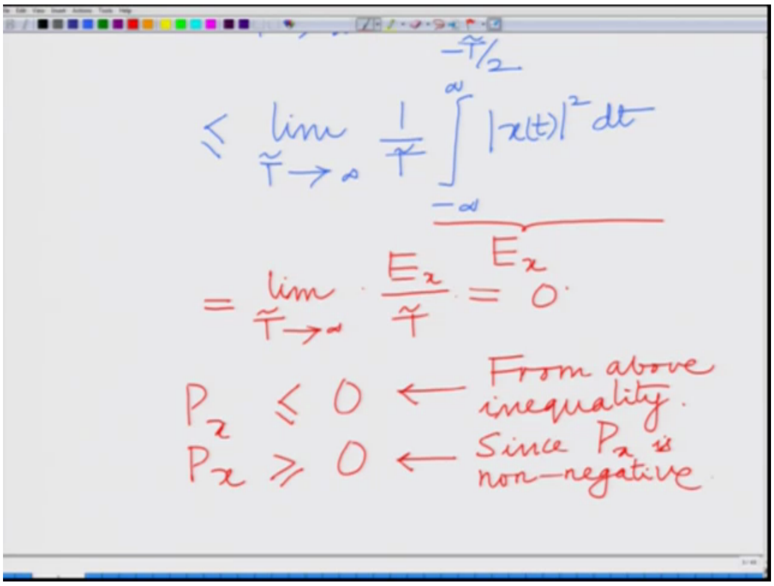


Therefore, observe that this is less than or equals to limit t tilde tending to infinity to 1 over t tilde minus infinity to infinity magnitude x(t) square dt. Now observe that this quantity is simply the energy of a signal which is finite because this is an energy signal, therefore this is equals to limit t tilde tending to infinity Ex divided by t tilde. So limit t tilde tend into infinity of Ex divided by t tilde, Ex is a constant divided by t tilde which is tending to infinity therefore, Ex by t tilde tends to 0, so this is equals to 0.

And therefore, what it means that the power of an energy signal Px that is Px is less than or equals to 0 from the above argument, this we have obtained from the above inequality and we also know that Px is a positive quantity, so Px is greater than or equals to 0 since Px is a positive quantity or a nonnegative quantity rather. Px is nonnegative because it is integral minus t tilde divided by 2 to 2 tilde divided by 2 magnitude x(t) square dt divided by t tilde. Everything is positive so this is everything is nonnegative, so this is a nonnegative quantity, so the only possibility is that Px is equals to 0. So Px is equals to 0 for an energy signal that is the power of an energy signal is equals to 0.

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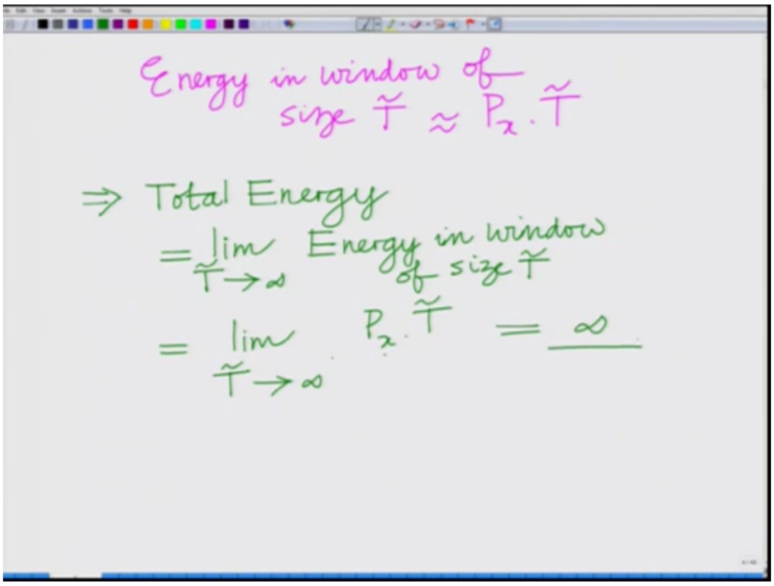


So this implies that basically that Px equals to 0 which basically implies that for energy signal that is power of an energy signal. So we get our 1st principle that is the power of the energy signal that is where signal x(t) is an energy signal that it is energy Ex is finite then its power is 0. Now, let us look at the energy of a power signal is x(t) is a power signal, on the other hand, if x(t) is a power signal , okay alright then the energy in a window of size t tilde, we know this is approximately equals to the power Px into t tilde because look at this we have the power that is Px equals to limit t t tilde tends to inifinity energy in window of size t tilde divided by t tilde which means the energy in a window of size t tilde is approximately t tilde times the power Px equals to them times the power Px which implies total energy equals to limit t tilde at tending to infinity energy in window of size t tilde, that is equals to limit t tilde tends to infinity Px times t tilde Px is a constant and t tilde tends to infinity, which means this is equals to infinity.

Therefore, how we have proved it is basically we have considered the energy in a window of size t tilde and we have said that is Px times t tilde therefore, as the window tends to infinity t tilde tends to infinity, right the window tends to infinity, the size of the window tends to infinity naturally Px into t tilde that tends to infinity because the power is a constant, correct. Therefore, the power in unit time is constant as time tends to infinity the total energy tends to infinity naturally; therefore energy of a power signal is infinity. This implies that energy of a power signal, the energy of a power signal is infinity, so we have that the energy of a power signal is infinity.

Now what kind of a signal is power signal, we have special we have power signals a special kind of a power signal other is a periodic signal, alright. So a periodic signal that is a good example of a power signal is a periodic signal. Let us therefore discuss, periodic signals are very important in the context of communication, alright and signal processing and several other applications in electrical engineering, so let us start this discussion about periodic signals, okay.

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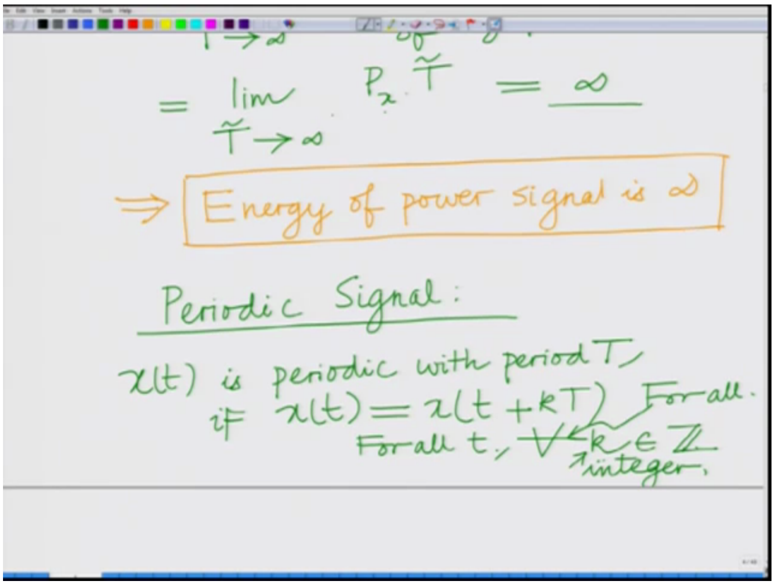


So let us consider a periodic signal, okay. Periodic signal, we know that x(t) is a signal with period to x(t) is periodic with period T if x(t) equals to x(t) + kT for all T that is and for all this is the symbol for all, for all integers that is for all that is we call x(t) to be a periodic signal with period capital T, if x(t) is equals to x(t) + some integer k times capital T, where capital T is the period for a for all times small t and for all integers K, alright. So basically it means something very simple that is if you take x(t) and if you shift it if you consider x(t) at any integer that is any integer multiple of capital T, later that t + k times capital T than the signal x(t) remains unchanged that is x(t) is a periodic signal, alright.

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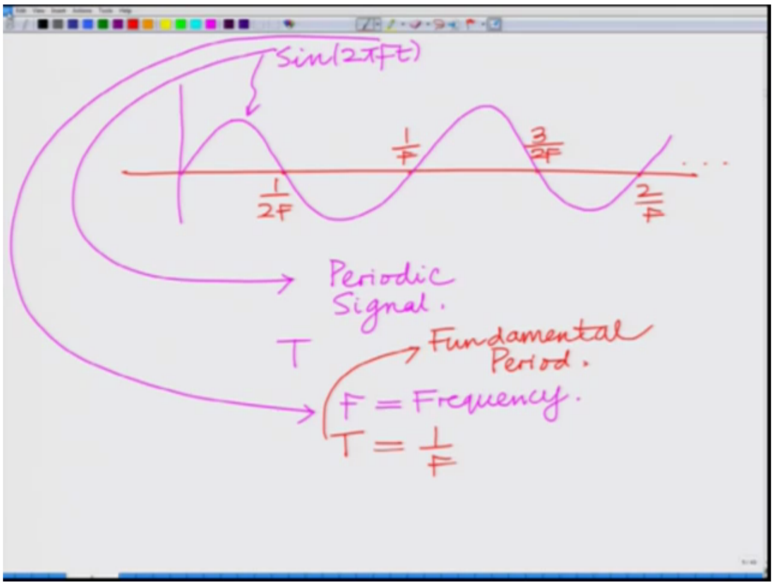


For instance some of the classic 1 of the very popular examples of a periodic signal is a sinusoidal signal, that is if you have sin 2 Pi Ft, this is a periodic signal, alright and what is the period we know that the period F is the frequency, F is the frequency of the sinusoidal signal, the period T equals to 1 over F. So this is basically your 1 over 2 F this point is 1 over F, this point is 3 over 2 F, this point is your 2 over F and so on, okay. So this is basically the Sinusoidal signal, alright okay and this is periodic with capital T and this is also termed as the fundamental period capital T equals to 1 over F is also termed as the fundamental period of the Sinusoidal signal.

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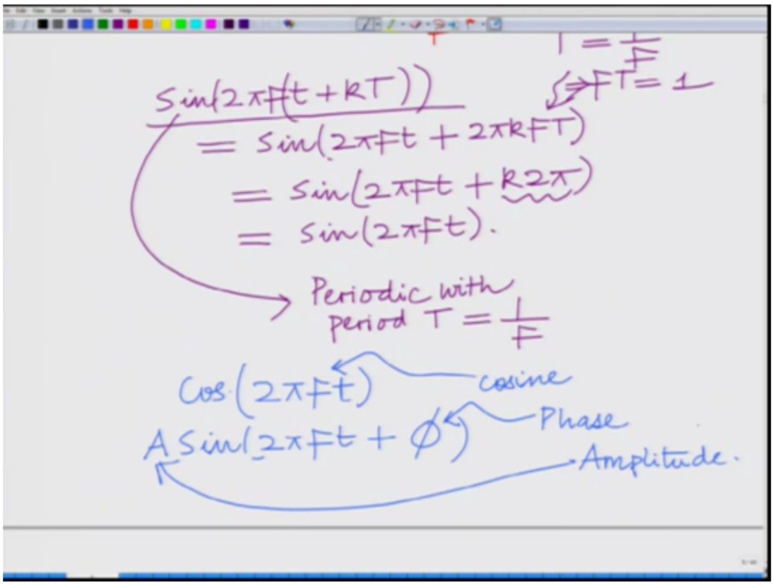

And you can see that it is periodic with T because sin of 2 Pi Ft equals to sin of that is if you consider sin of 2 Pi Ft + some multiple of the period, some multiple kT, which is equals to sin of 2 Pi Ft + 2 Pi kF into T, but we know T equals to 1 by F which implies F into capital T equals to 1, so this is basically sin of 2 Pi Ft + k times 2 Pi sin at any multiple that is k times 2 Pi that is sin of x + k times 2 Pi simply sin of x, so this is sin of 2 Pi Ft so this is basically periodic.

So we have shown this as periodic with period T which is equals to 1 over F. So the Sinusoidal signal is periodic with P equals to t capital T equals to 1 over F. Similarly, when we say a Sinusoidal signal does not necessarily mean only a sin signal, it means it can also be a cosine signal, cosine 2 Pi Ft also cosine which is a Sinusoidal signal and in general 1 can consider any sin 2 Pi Ft + Phi that is Phi is the phase, A is the amplitude.

So we can have a phase and an amplitude okay. So the Sinusoidal signal A sine 2 Pi Ft + 5 has phase Phi and amplitude A, okay. So this is a general Sinusoidal signal which we have shown that is Sinusoidal signal with frequency F as a pure Sinusoid with frequency F has a fundamental period that is a period of Sinusoidal signal is capital T equals to 1 over F that is the signal is periodic 1 over F, okay. Now what is the power of a periodic signal? How to find the power of a periodic signal?

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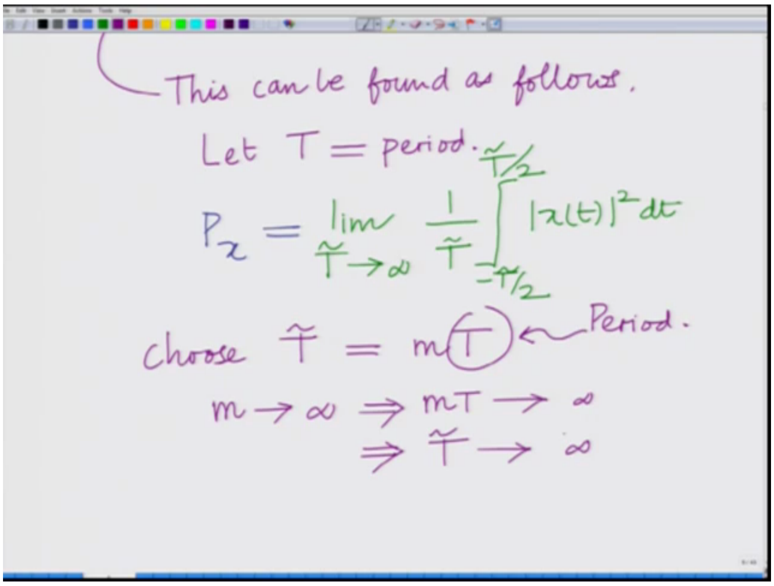


Now the power of a periodic signal this can be found as follows. Let T be the period of the periodic signal, then we have from the definition of power, we have Px equals to limit t tilde tends to infinity over t tilde minus t tilde divided by 2 to t tilde divided by 2 magnitude x(t) square dt, now what we are going to do is choose t tilde equals to m times capital T that is we choose t tilde to be m times capital T, so naturally as m tends to infinity t tilde tends to infinity. So instead of tending t tilde into infinity we can equivalently tend this integer m, so what we are doing is we are choosing t tilde to be an integer multiple m times T, where T is the period, right? So this is, realize note that T is the period of the signal, okay. So if t tilde, so if m tends to infinity, this implies t tends mT tends to infinity, this implies t tilde tends to infinity.

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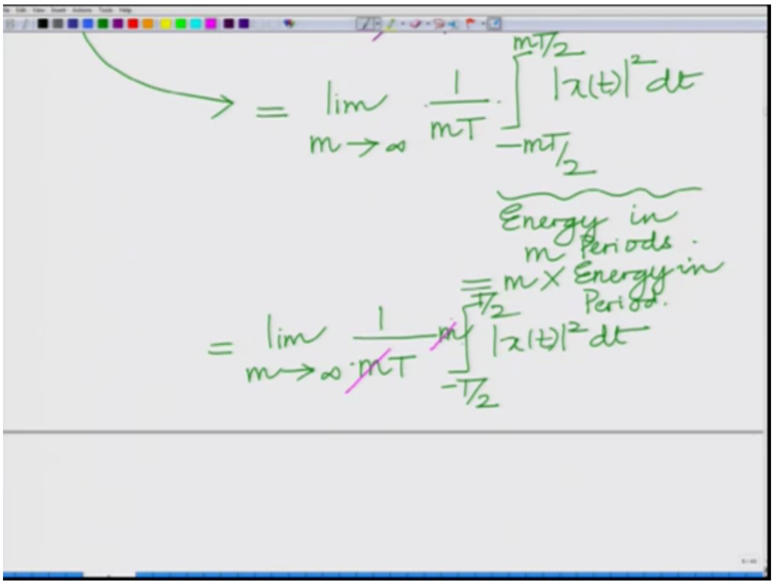


So naturally I can use this, so instead of limit t tilde tending to infinity I can here equivalent represent this as limit m tending to infinity 1 over mT minus mT divided by 2 to mT divided by 2 magnitudex(t) square dt, okay. Now at this, this is integral look from minus mT by divided by 2 to mT divided by 2 this contains m periods, right. The total duration is mtimes capital T, so it contains m periods of the signal that is this is the energy in m periods. Now this signal is periodic, so energy in m periods is m times the energy in a single period because this is a periodic signal, okay so that is the property we are going to solve.

This is basically energy in m periods equals to m into because the signal is periodic, energy in m periods is m times the energy in a single period, so this is limit m tend into infinity 1 over mT minus T by 2 to T by 2that is single period mtimes the energy in a single period. That is magnitude x(t)square dt, now you can see the m is canceled.

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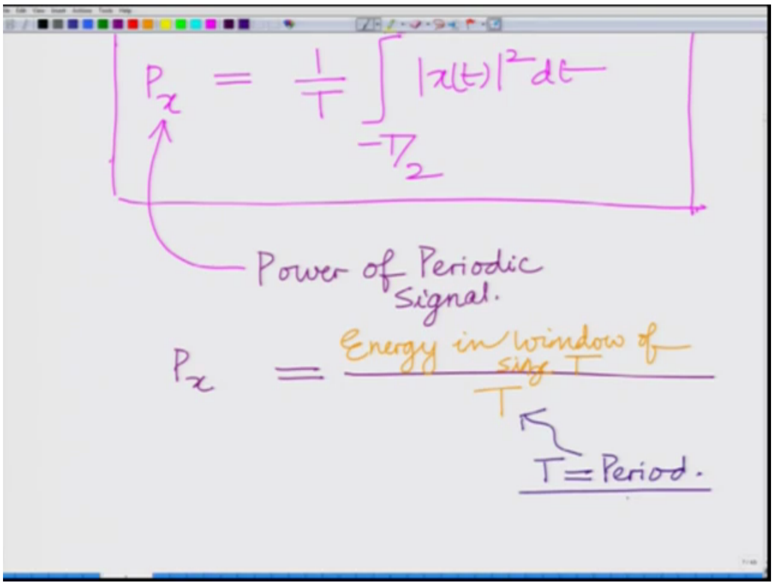


So this is equals to and therefore there is no m therefore limit m tend into infinity is simply 1 over T integral minus T divided by 2 to T divided by 2 magnitude x(t) square dt, this is the power Px of the periodic this is the power of a periodic signal. What is this? This is simply the power of a periodic signal x(t) is simply the energy in a single window that is the energy in a single window of size T that is energy in a single period capital T divided by the period T that is the size of the window capital T.

So the power of a periodic signal for the special case of a periodic signal this is energy in window of size T divided by T. Realize that this T is the period of the signal, so you simply take the energy of the periodic signal in 1 period divided by the period that is capital T and you get the power of the periodic signal that is Px, okay.

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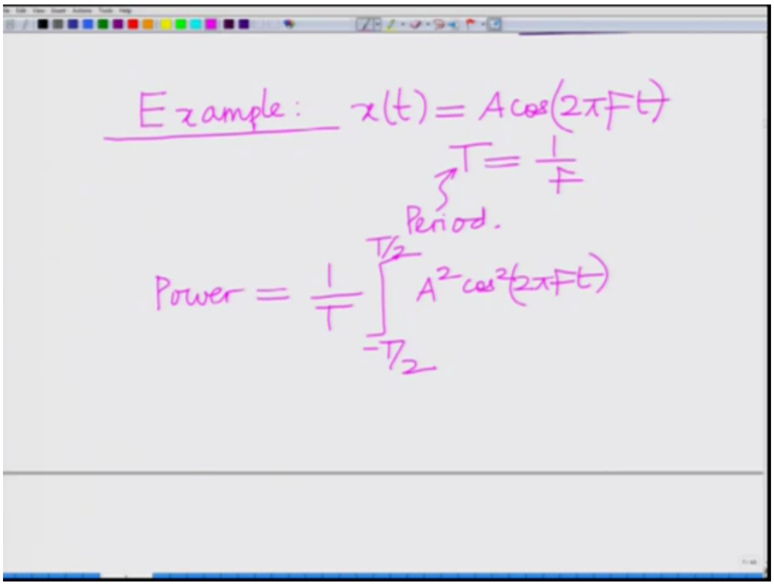


So let us take again a simple example, let us again go back to our standard periodic signal that is the Sinusoid example again, x(t) equals to A cosine 2 Pi Ft, let us consider this periodic this again we said the cosine signal with amplitude A phase Phi equals to to 0, we said cosine signal is also a Sinusoid, so this is Sinusoid and more importantly we know the period T, this is the period capital T which is equals to 1 over F, therefore the power of this signal power is equals to 1 over T minus T by 2 to T by 2, A square cosine square that is 1 over T 1 over the period capital T, where T equals to to 1 over F minus T by 2 the capital T by 2 a square cosine square 2 Pi Ft which is equals to, now I can bring the A square outside.

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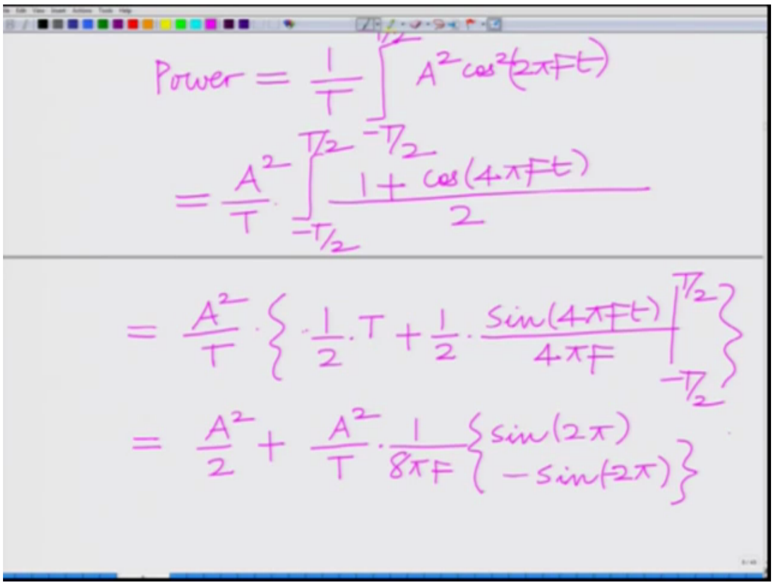

A square is constant, A square divided by capital T integral minus T by 2 to T by 2 cosine square 2 Pi Ft is 1 over cosine 4 Pi Ft that is cos square theta is 1 + cos 2 theta divided by 2, so cos square 2 Pi Ft is 1 + cosine 4 Pi Ft divided by 2. So this is now the integral of half between minus T by 2 to T by 2 that is straightforward that is basically half times T + half integral of cosine 2 Pi Ft is basically sine integral cosine 4 Pi Ft is sine 4 Pi Ft divided by 4pi F evaluated between minus T by 2 to T by 2, which is A square by T into half T. So that is A square by 2 + A square by + A square by T times 1 by 8 Pi F sine 4 Pi Ft at capital T by 2 F into T is 1, so this is sine 2 Pi minus sine 4 Pi Ft and t equals to minus capital T divided by 2 equals to sine minus 2 Pi and you can see that sine 2 Pi equals to sine minus 2 Pi because there is a difference of integer.

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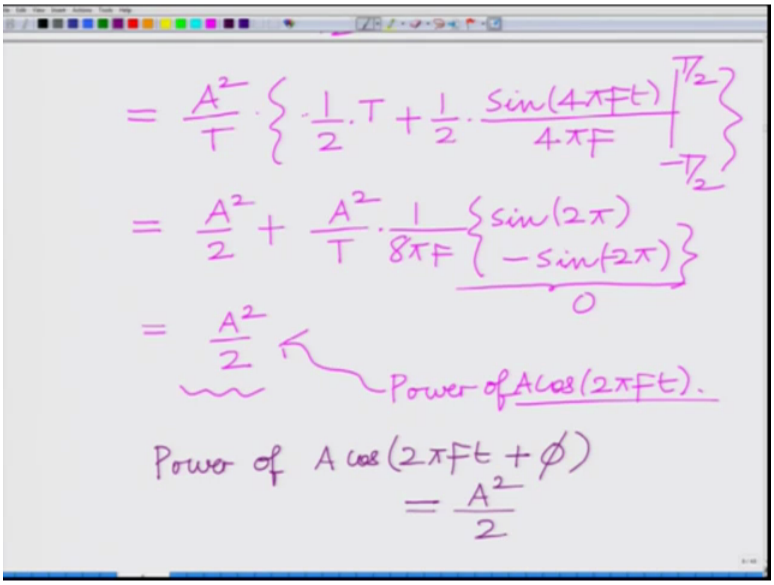


But anyway, integer multiple of 2 pi, so this is basically, so this quantity here equals to 0, so this is A square divided by power of the Sinusoidal signal of Amplitude A power of your cosine A cosine 2 Pi Ft that is Sinusoidal signal A cosine 2 Pi Ft is fine. In general you can show that the power of any Sinusoidal signal with amplitude A and phase five is equals to A square divided by 2. It can be shown that power of A, this is equals to A square by 2, where A is the amplitude that is it does not depend on the phase the power is simply a square divided by 2 where A is the amplitude of the Sinusoidal signal, alright.

So this is simple module where we have started with the definition of an energy, defined the energy of a signal, defined what is an energy signal? The power of a signal what is the power of a signal? Also looked at the periodic signal and defined the power of a periodic signal and illustrated how to compute the power of a simple or a very common and frequently used periodic signal and signal processing and communication that is the Sinusoidal signal, whose power is A square divided by 2 where A is the amplitude of the Sinusoidal, so we will stop here and look at the other aspects in the subsequent modules, thank you.