Electromagnetic Theory
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Lecture - 09
Transformation & Electric Field-II

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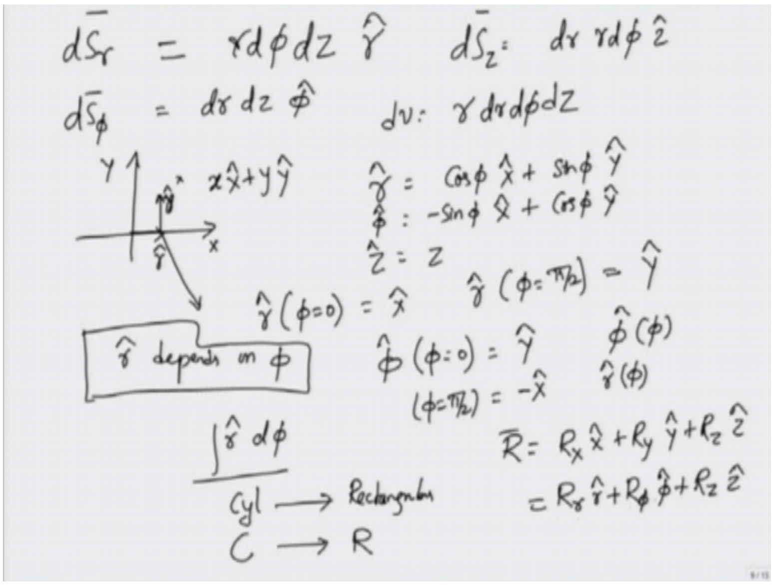

Suppose that vector is R and I can write this vector R as consisting of 3 vectors Rx, I can resolve this vector R as Rx x hat plus Ry y hat plus Rz plus z hat. The same vector in the cylindrical co-ordinate system can also be written as Rr r hat where Rr will be the component of the vector R along R hat vector along the radial component plus R phi phi hat plus Rz z hat.

You can guess from intuition that Rz will be equal to Rz import cases and you will be right. However, we are now interested in Rx, Ry, Rr and R phi.

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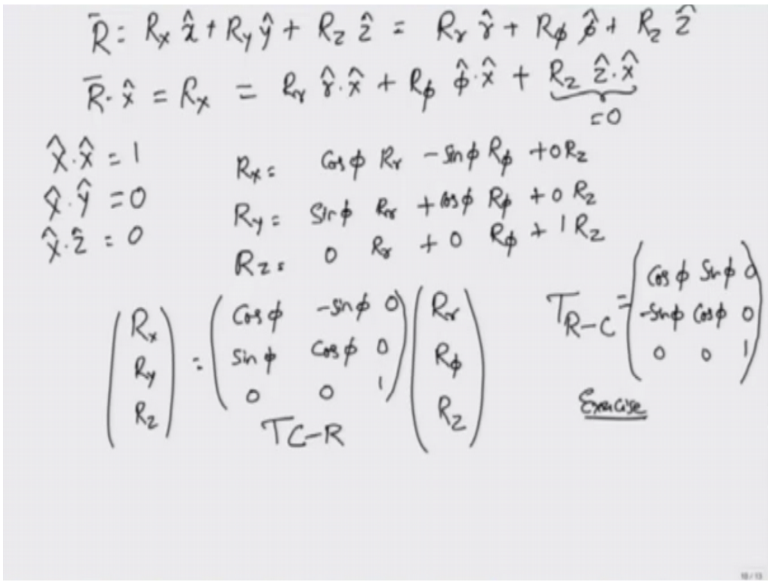

So I know that R is equal to Rx, x hat plus Ry y hat, plus Rz z hat. This is also equal to Rr r hat plus R phi phi hat plus Rz z hat. Now if I try to find out the component of the R vector along the x hat vector that is along the x axis, I would be obtaining, I can do that 1, by finding out the dot product of car along x. So if I do this 1 I am going to get the component of R along x, correct? So if I do this operation I am going to get Rx. Why?

What would happen when you take x dot with respect to this 1. it stands out that, x hat, dot x hat will be equal to 1 because the length of 1 vector upon itself will be giving the length of the vector itself. However, what happened to x hat dot y hat. This became equal to 0 because x is perpendicular to y and dot product will be equal to 0 when theta ab is equal to 90 degrees, right?

So this is another definition of perpendicularity or normality or orthogonality. Two vectors are said to be perpendicular to each other or normal to each other or orthogonal to each other when the dot product between the two vanishes. Similarly, x hat dot z hat will also be equal to 0. In fact, the corresponding vectors x, y and z they form mutually perpendicular set of vector, something that we looked at in the last class.

R dot x hat will give you the Rx component. Now I have to dot this R vector in the cylindrical co-ordinate by the x hat vector, right? So if I do that 1, this would also be equal to Rr r hat dot x hat plus R phi phi hat dot x hat plus Rz z hat dot x hat. This component is equal to 0 or this value is equal to 0. So Rx will be equal to, what is R dot x, this is nothing but cos phi.

So have cos phi Rr and what about phi dot x, in the previous slide we have already seen that this is equal to minus sin phi R, so therefore this is minus sin phi multiply this 1 by R phi. What will be Ry, you can show similarly by considering the dot product of R with y. This will be equal to sin phi Rr plus cos phi R phi plus 0 times Rz plus 0 times Rz just to complete the equations.

I can also write down Rz as 0 times Rr plus 0 times R phi plus 1 time Rz, okay? Now this might be looking very suspiciously like a set of linear equations and these are the set of linear equations and these are the set of linear equations. We can obtain Rx, Ry and Rz, if I know the component value of Rr, R phi and Rz by a transformation matrix which takes me from cylindrical to rectangular co-ordinate systems.

This is the cylindrical to rectangular co-ordinate system and the matrix is cos phi minus sin phi 0, sin phi cos phi 0 1 0 and 0, this is a 3 by 3 matrix which transforms any vector which is in the cylindrical co-ordinate system into rectangular co-ordinate system. If you are interested in finding what would be the corresponding transformation from rectangular to cylindrical you can actually show that this transformation is nothing but the inverse of this matrix T CR.

And you can show that this will be equal to cos phi, sin phi, 0, there is a 0 here, 1 here, minus sin phi, cos phi, 0 and 0, okay? I will leave this as an exercise to you to show this 1. Now these two matrices are very useful to you. In fact, you can take a look at any electromagnetic text book, they are going to give you these kind of formulas which will allow you to convert of vectors in 1 co-ordinate system to another co-ordinate system.

I will give you 1 simple example of conversion, how to do this particular conversion and I will point out 1 very important aspect of cylindrical co-ordinate systems to you. What is that problem?

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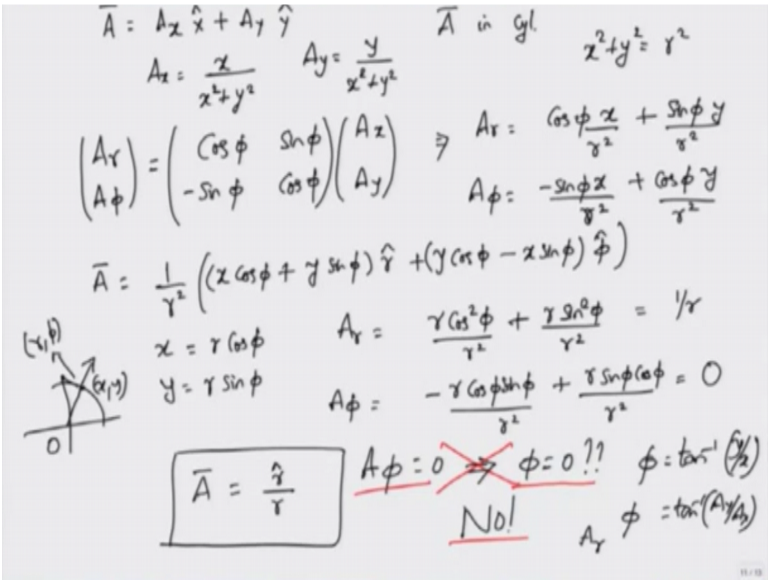

I have a vector A which is described in the rectangular co-ordinate systems having components Ax and Ay, okay? Ax and Ay where Ax is equal to x by x square plus y square, Ay is equal to Y by x square by y square. Now I want you to find out the vector in the cylindrical co-ordinate systems. How do I find the vector in the cylindrical coordinate system? I have to use the formula that I developed earlier. So what is that formula?

I have to use this transformation matrix. I know Ay, Ay, I know Az. A z in this case is 0 and then I have to find out Ar, A phi and Az. Okay, let me proceed to find that 1 out. Ar, A phi will be equal to because there is no z component there, I am not going to write down the z component for you, I mean because there is no point in writing down that 1, this will be equal to cos phi sin phi minus sin phi cos phi times Ax, Ay. Hopefully this is alright.

So let us go back to the matrix and check. Cos phi, sin phi minus sin phi and cos phi so we are on the right track. Now substitute for Ax and Ay. If you substitute for Ax and Ay you are going to get Ar will be equal to cos phi times Ax and sin phi times Ay, but I also know that x square plus y square is actually equal to r square in cylindrical co-ordinate systems and substituting therefore Ax as x by r square and for y as y by r square what will be Ar?

Ar will be equal to cos phi x by r square plus sin phi y by r square, okay? But what is x cos phi and y sin phi and what is A phi? A phi is minus sin phi x by r square plus cos phi y by r square, okay? Now let us right down the vector in the cylindrical co-ordinate system. This vector in the co-ordinate system will be equal to 1 by r square is a common factor that is coming out everywhere, so I can take that common factor out.

So I have x cost phi plus y sin phi along the radial direction plus cos phi, y cos phi minus x sin phi along the phi direction, okay? But I also know that x is equal to r cos phi, y is equal to r sin phi. So I can substitute for x and y in this expression. So if I substitute for that 1, this expression, x cos phi plus y sin phi that is Ar term will become x is equal to r cos phi, so that becomes r cos square phi by r square plus y is r sin phi.

Therefore, this becomes r sin square phi by r square which is equal to 1 by r and A phi is equal to minus x is nothing but r cos phi. So minus r cos phi sin phi by r square plus y is r sin phi cos phi by r square which is equal to a big 0. So the vector A in cylindrical co-ordinate system is simply given by r hat divided by r. is this surprising to you. If you are not surprised, then you should be surprised.

Why you should be surprised, because in order to represent a point which is originally represented in Cartesian coordinates as x and y or x by x square plus y square and y by y square, x square plus y square, we had to give the values of x and y, both the values of x and y. To represent the same point in cylindrical co-ordinate system, I still have to give the values of r and phi.

I have to tell you what distance that particular point is located and what is the angle with respect to the x axis that I have to locate that point on. If I do not give you r and phi you will not be able to pin point that this point in the cylindrical co-ordinates will correspond to the same point in the rectangular co-ordinates, right? There are an infinite number of choices. Yet, my vector A which was originally Ax x hat plus Ay y hat is actually giving me component only along r.

There is no mention of phi component. The phi component is 0 in this case. Z of course is also 0 in this case. Does it really mean that if phi hat is equal to 0 or if A phi is equal to 0, that is any phi component is equal to 0, does this imply that phi is equal to 0? No. Phi is not 0, just because there is no phi component it does not mean that the phi component is also equal to 0, in fact the phi component is given by inverse tan of y by x.

If you know what is the value of Ax and Ay or this is equivalent of writing this as tan inverse of Ay by Ax, if you know what is Ay and Ax you should be able to find out what is the value of phi, similarly you know what is the value of r, or the Ar component, you should be able to find phi and you will see that this phi will not be 0. So this is a very important step for me to highlight to you.

Just because phi is equal to 0 it does not mean that the A phi component is 0, it does not mean that phi is also equal to 0. This is not correct. Do not think that A phi component is 0 means phi is also equal to 0. But this makes sense physically because after all where is my vector pointing, on this particular radius, let us say this is the point where I had originally in Cartesian co-ordinate system represented as x and y component.

And I had set up a particular vector also. The corresponding vector in the cylindrical co-ordinate would be directed from the origin passing through this particular point, right? This point no doubt is given by r and phi values, but the direction of the vector is only along r direction because there is no component of phi direction or there is no component of the vector along phi direction, okay?

This you keep in mind; it is very important. Now we have seen this result, let us put this result into good use by resolving the same problem of infinite line charge, okay?

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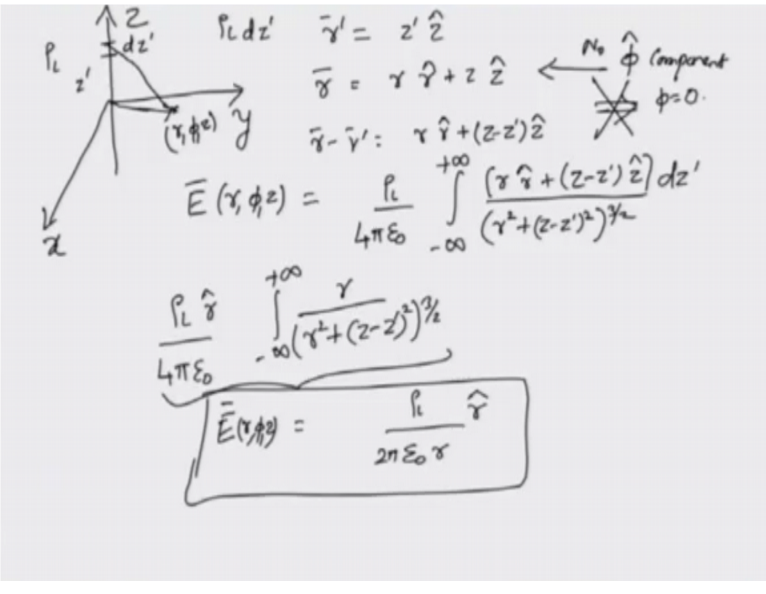

We consider the infinite line charge problem earlier, we are going to revisit the same problem and learn how to use the cylindrical co-ordinate systems. This is sometimes called as circular cylindrical co-ordinates. It does not really matter. I have again a uniform line charge of line charge density rho L kept on the z axis, going all the way from minus infinity to plus infinity. I again consider a line segment at a height of z prime from the z is equal to 0 plane, starting from the horizontal plane and I consider a small segment dz prime.

The total charge there will be rho L multiplied by dz prime, okay? Now I have to find out the field at any point in the plane which I am now going to consider any general r phi and z point, okay? I am not looking for z is equal to 0, although you can solve the problem with z is equal to 0. Let me consider any z value and see what happens. You will see that the problem does not become complicated at all. So what is the unit vector for this 1?

Okay, what is the unit vector for the field point and what is the unit vector for the source point? The vector for the source point is r prime is z prime, z hat, this is at a height of z hat and what is the vector for the field point. It turns out that the vector for the field point has no phi component. Again remember just because there is no phi component it does not mean that phi itself is 0. So I have this as r, r hat plus z, z hat.

Again remember, there is no phi component, no phi component, does not mean phi is equal to 0. In fact, phi will be non 0. We will later see why phi does not enter into picture over here. So what is this vector now from the source the field point, the vector is r minus r prime which is r r hat plus z minus z prime along z, okay? Now we use the expression for electric field. We integrate in the appropriate dimensions.

I am looking at the electric field at a particular point r phi z in the plane. This will be equal to rho L by 4 pi epsilon 0. I am going to take this constant anyway outside. So I am going to put that constant outside of the integral first. Integral is going along z direction from minus infinity to plus infinity and inside I have r minus r prime by magnitude of r prime to the power 3, right?

So I have r r hat plus z minus z prime multiplied by z hat divided by r square by z minus z prime square to the power 3 by 2 and this entire thing is actually integrated along z prime. Now I still have an integral which have this component of 3 by 2, but I know how to solve that integral. But what is the big change I have done. If you see what is the big change what has happened, out of the 3 integrals 1 integral has dropped out.

Now there are only two integrals for me to work around. However here I have to be careful a little bit. Again the integral will be the sum of the integral for r and sum of the integral for z because there is an r hat inside the integral, z hat inside the integral. However, in this particular case, it turns out that at any point on the z axis I might be, r hat does not change or does not depend on z, z also does not depend on z.

We remember that r hat and phi hat depend only on phi and not on z. They also do not depend on r, so in this case there is no integration of r. So r hat does not depend on z, therefore that also becomes a constant and can easily come out of the integral. So when it comes out of the integral you can use the same ideas that we develop in the last class to solve for this integral in the last class and you can basically show that this integral.

This entire integral will turn out to be rho L by 2 phi epsilon 0 r in the r direction. Okay? So the fields are in the radial direction, radially outward if it is a positive line charge, radially inward if it a negative line charge density. Okay, so I have rho L by 2 pi epsilon 0 r, r hat and there is no phi component, also the integral for the z component drops out. So this is the electric field at r phi and z.

This is completely independent of z, depends only on r and they are only going away along r direction. Let me give you briefly why this particular electric field has come out the way it has come out. See we have chosen a proper co-ordinate system. But what we have not talked about is the symmetry of the problem. Symmetry is a very very very important concept in electromagnetic.

If you look at the textbook that we are using for this course, electromagnetic engineering electromagnetic by Hayt, the way he solves this problem, the infinite line charge problem is that he gives you the first statement of the solution saying that in solving such problems, you have to look for symmetry because symmetry simplifies calculation. There are two things that we have to do which are paramount important.

One is choosing the right co-ordinate system and then exploiting the symmetry. So if you choose the right co-ordinate system in this case, instead of 3 integrals I am now down to two integrals and if I had actually exploited the principle of symmetry, I would not even have worried about the z component. Let us see, symmetry what does it do. Symmetry asks two kinds of question.

It first says, at any given particular point which way the electric field will be pointing and then it will also ask on what variables, what co-ordinates does this electric field depends on. Imagine that there is no infinite line charge out here and then you are at a particular distance, may be you are in that horizontal plane, you are at a particular distance r and you look around this charge and you move along the phi direction.

So imagine yourself moving along the phi direction, the line charge is uniformly infinitive and the line charge basically does not have any dependence on the phi. So as you keep moving along phi direction, you will not see any difference in the line charge. So the line charge looks the same no matter what value of phi you are in. Therefore, there is clearly no way the electric field is going to depend on phi. It does not matter if the line is infinity long or if the line is finitely long.

As long as the line is symmetric with respect to the phi axis or the azimuthal direction, there will be no dependence on the phi component. Similarly, for an infinite line charge we can move up and down. There will again be no z dependence, because you move up you are going to see the same charge, you move down you are going to see the same charge. You can move up and down and you are going to still see the same uniform line charge.

Therefore, there cannot be any dependence on the z component. The only way you can have your electric field magnitude decrease is if you move away from the charge. So you have the charge and you keep moving away from the charge and you can expect that as you move away from the charge there will be some drop in the magnitude of the electric field. So that is why the electric field is dependent only on the r co-ordinate.

Only along the radial distance from the charge. Suppose you are at a particular point and now you say can there be a phi component in the electric field, the answer is no because to have a phi component remember electric field is a force, essentially in some sort of a force. You can place the test charge at a point and then see if there is a to be a component along phi, there has to be some other charge which has to be giving you the force in the phi direction.

There has to be a charge in the phi direction. But we have no such charge, we have only 1 charge and that is along the z axis and that would not give you the phi component. So there is no phi component in the electric field. Can there be a component along the z direction? For an infinite line charge, if I have to say that okay if there is some non 0 z hat direction, it also means that there is a charge somewhere which is giving you a field along z axis, but that is not the case here.

The only electric charge that we have, charge distribution that I have is sitting on the z axis and it is sitting along all the way from minus infinity to plus infinity. If at this distance if this is the charge and therefore this distance I need to have a z component then I need to have a force somewhere over here which is pushing the charge, test charge around in this upward direction or in the downward direction, right, if I have a negative component.

So there is no z component, there is no phi component, there can only be r component. Because for the r component, there is a charge here. It will exert a force up there. So if you had actually exploited all these symmetry you would have concluded that we need not even have perform the integral for the z axis. I did not introduce this for a very specific reason. There is another law which we are going to study very shortly called as Gauss's law.

Gauss's law is specifically meant for exploiting symmetries. The problem that we took for two classes, to solve two classes can actually be solved very easily if you apply Gauss's law and we are going to do that 1 once we get acquainted with that particular law. So for now we are not putting any symmetry arguments, however we are only going to choose the appropriate co-ordinate systems. That is what I have done here.

There are couple of other problems that you might be interested and these are very important as well. At this point I will give them as an exercise and therefore you can work them out and if time permits we will come back to those distributions. I will like to move on from solving these type of charge distribution problems to introduce you to a different concept in our study of electromagnetic and this concept is called potential.