Electromagnetic Theory
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Lecture - 82
Quasi-Statistics - II

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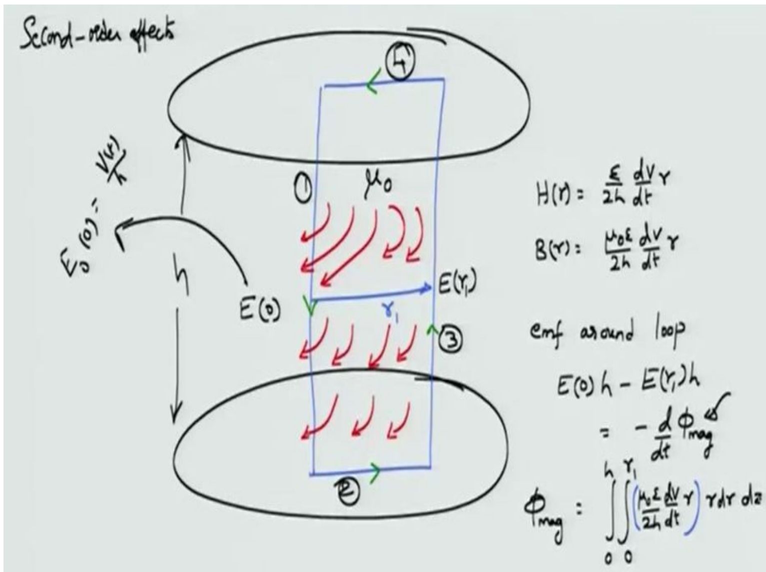

We were discussing in the last module about the quasi static analysis and we figured out that for the zeroth order only electric field E0 exist and magnetic field will be equal to zero. But we soon found out that that analysis is not correct so for the first order what we have obtained is the magnetic field. And that magnetic field is dependent on the derivative of the voltage so as long as dv by dt is nonzero then that magnetic field would exist.

But we assumed you know until this point that the electric field is uniform vertically down and it is varying with time but it is independent of this direction r right. So if that is the point of a uniform field so you take this top plate and the bottom plate. The electric field lines will all be uniform. This is what we had assumed but clearly such an assumption is not correct and there we enter into the so called second order effects.

In the second order the magnetic field that you found from the first order will induce a certain magnetic flux density which will be changing with time that will induce a certain EMF okay. So let us see that. We have considered one loop here of the length r1 and the height h because we have considered from top plate to the bottom plate and we are trying to apply an EMF Faraday's law essentially to this loop okay in the direction which I have shown by the green arrows.

The electric field at the centre is vertically down and it is given by E0. This was the same E0 that we found from the zeroth order. So which was E0 of zero and this was given by v of t by h right. So this is precisely the field that we had found earlier. But this electric field is no longer going to be the same. For argument sake let us assume that the field here is E of r1 had drop time dependence here but you also need to understand that there is time dependence but time is varying very slowly okay.

In other words, frequency is quite small so this E0 multiplied by h will be the contribution of this segment one. Segments two and four do not contribute anything to the EMF because here the electric field is vertically whereas the line or the field the loop that we have considered is horizontal segments for two and four. The contribution from one and two will be in the opposite direction because one is an integral from top to bottom. The other one is an integral from bottom to top.

Therefore, the EMF around loop will be given as E of 0h - E of r1 into h. What should this be equal to? This should be equal to the magnetic flux density d by dt of magnetic flux density okay which I am denoting as phi-mag so this is the equation. We will come back to the implication of this equation once we realize the right hand side. Once we know the right hand side. I know what is the magnetic flux density d?

And I know what is the area of this particular loop right. So if I want to find out what is the magnetic flux density hope for the same loop that I have considered? I need to integrate this one along say h you know say 0 to h in one direction that variable let us call this as z. This is a loop in which your r is changing and z is changing therefore the area itself would be along phi which is precisely the direction of the magnetic flux density vector as well right.

So put down the magnetic flux density Mu0 epsilon by 2 h and then dv by dt okay. Everything is a constant so you can easily move everything out but you have r here okay as part of the magnetic flux density itself. So this is part of the magnetic flux density but the loop itself since you are considering r and z the loop area will be r dr okay. So this would be r dr and the integration is from 0 to r 1 so r equal to 0 to r1 and integration along say dz.

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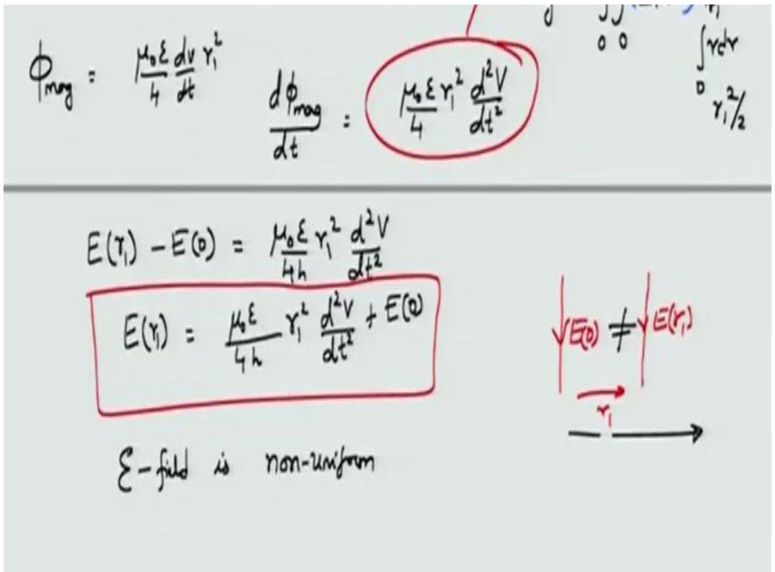

So clearly integration along z will give 1h that h will cancel the h in the denominator so the magnetic flux density that you will get will be equal to Mu0 epsilon by 2 okay. Dv by dt and you will have an integral of r dr from 0 to r1 which would be r1 square by 2. So you will have r1 square and there is a 2 in the denominator already so this becomes r1 square by 4. So this is the magnetic flux density.

And the time rate of change of magnetic flux density is clearly d phi-mag by dt is equal to Mu0 epsilon by 4 r1 square which is now a constant d square v by dt square correct. This quantity must be equal or the negative of this quantity must be equal to this E0 h - E of r1 which will allow me to write down E at r1 - E of 0 is equal to Mu0 epsilon by 4 h r1 square d square v by dt square.

In other words, E of r1 is equal to Mu0 epsilon by 4 h r1 square d square v by dt square + E of 0. This equation is a big change from the electric field that we had assumed. What it means is that the electric field at the centre which is E of 0 is not the same as the electric field at any other distance r1 correct. So this electric field where this is r1 the electric field is not the same at this point.

So these two are not equal which means that the field lines are not uniform as you go along the capacitor plates okay. So as you move from the centre and you keep going along the capacitor plates the electric field has become non-uniform so the E field is non-uniform okay. In fact, we really don’t know what is E of r1. We can actually obtain the relationship for E of r1 because I know that this equation can be applied at the boundary condition right. So at the edge and at the edge I know what is the EMF right?

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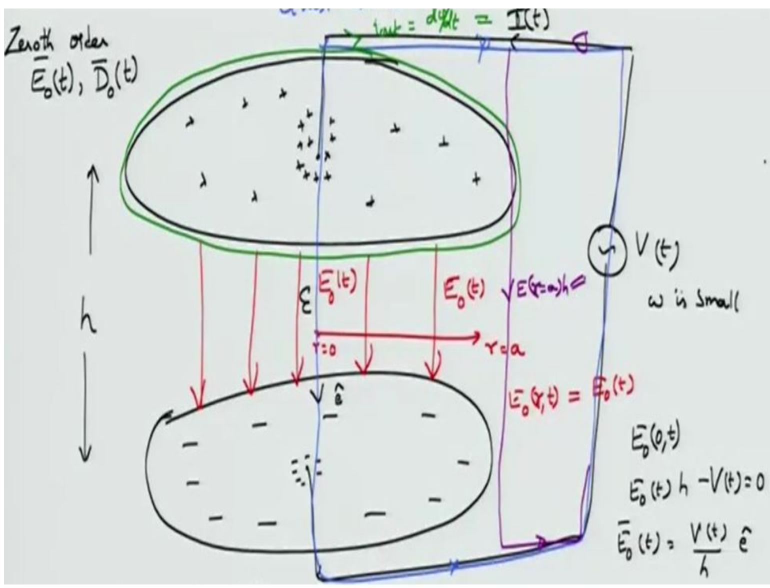

So go back to this first expression and then imagine yourself having a loop here okay. Imagine a loop at this case and there is no contribution here on the horizontal edges on the top and the bottom and here whatever the electric field that would be there would be the electric field at r equal to a times h must be equal to v of t.

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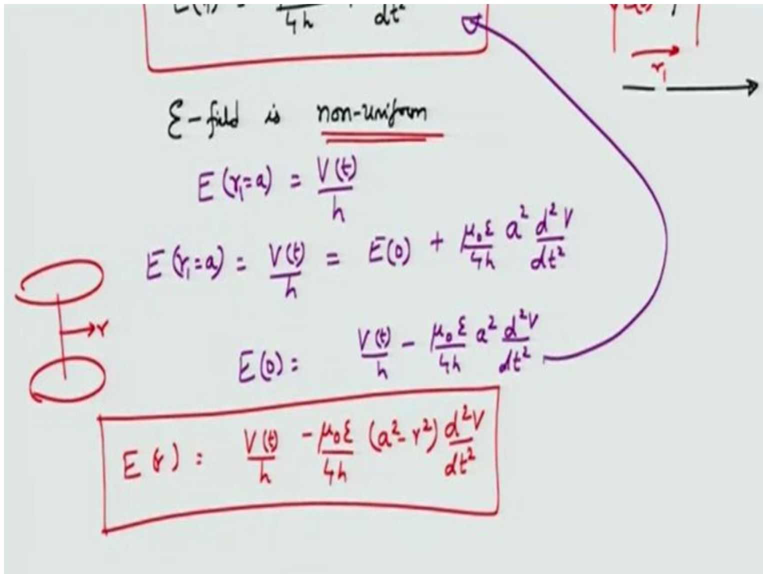

So now I already know how to obtain E of r1 so if I substitute r1 is equal to a then the total electric field must be equal to that v of t by h correct. So indeed the condition is that E at r1 is equal to a must be equal to v of t by h correct. So this will enable us to basically find out what is the electric field? So we will write down this as E at r1 is equal to a okay which is basically v of t by h that must be equal to E of 0.

This will actually enable us to find E of 0 so + Mu0 epsilon by 4 h and r1 is equal to a becomes a square d square v by dt square. So rearranging this equation you get E of 0 as v of t by h - Mu0 epsilon by 4h a square d square v by dt square. You can substitute that into this expression to obtain electric field E at any distance r you know along the capacitor you know the direction that we have shown is equal to v of t by h-.

And then I will have a square -r square okay times d square v by dt square. This is the electric field at any distance r from the centre of the capacitor plates. So this is the centre and if you go along the direction r as I have shown here. This would be the electric field and clearly this electric field is non-uniform okay. Now the only thing that we would like to know here is what is the EMF around the loop right or what is the EMF that is induced from the centre at axis r0 to the outer edge of the capacitor?

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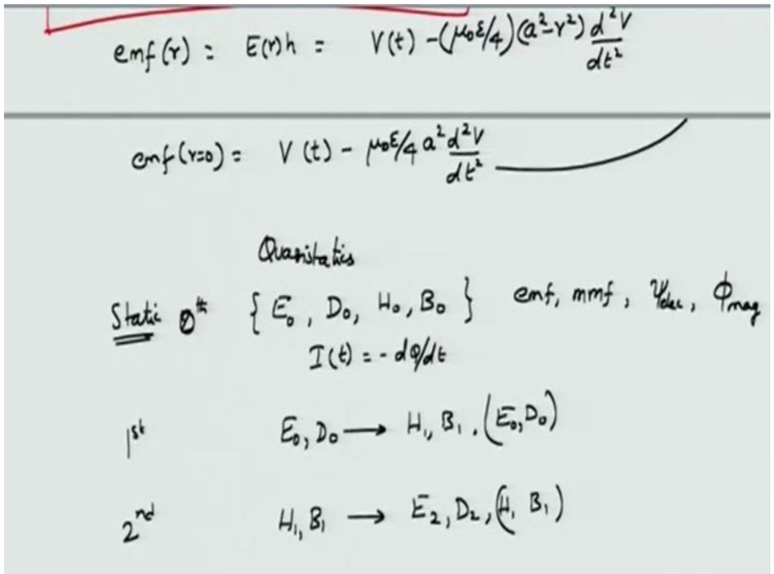

The EMF is in fact is given by E of r into h so we can write this a EMF at any r okay that would be given by the electric field magnitude E of r into h will be equal to v of t -Mu0 epsilon by 4 a square - r square times d square v by dt square. So as long as you have d square v by dt square changing with respect to time then you will have an EMF which is different okay. What is the EMF at the centre?

At the centre EMF at r equal to zero is given by v of t - Mu0 epsilon by 4 and r equal to zero will be just a square d square v by dt square. In fact, this is what we have obtained earlier. So everything checks out it this equation. The results of what we have done so far can be summarized like this. Quasi statics basically involves okay assuming static assuming that everything is static find out E0 okay. Find out D0, find out H0 and find out B0.

All the additional things like EMF or the magnetomotive force, electric flux density SI electric. Magnetic flux density Phi-mag can also be found from these set of equation once I know. Of course you also need to use the current continuity equation okay and this current continuity equation is dq by dt. Everything is varying with time but the time variation is quite slow okay. Once you know this then in the first order so this is my zeroth order.

For the first order, what I do is I use E0 and D0 which I know already to calculate H1 and B1 okay. The reason why you will have H1 and B1 which is different from H0 and B0 is because the moment you allow D0 to be varying with time there will be displacement current and that displacement current varying with time will create magnetic fields right so displacement current create magnetic fields and these magnetic fields now create H1 and B1 will create E2 and D2.

Because at this stage we still assume that E0 and D0 are the same so it will create E2 and D2 we will again assume H1 and B1 are the same okay. So this way you can successively built up to the different orders of electric magnetic field.

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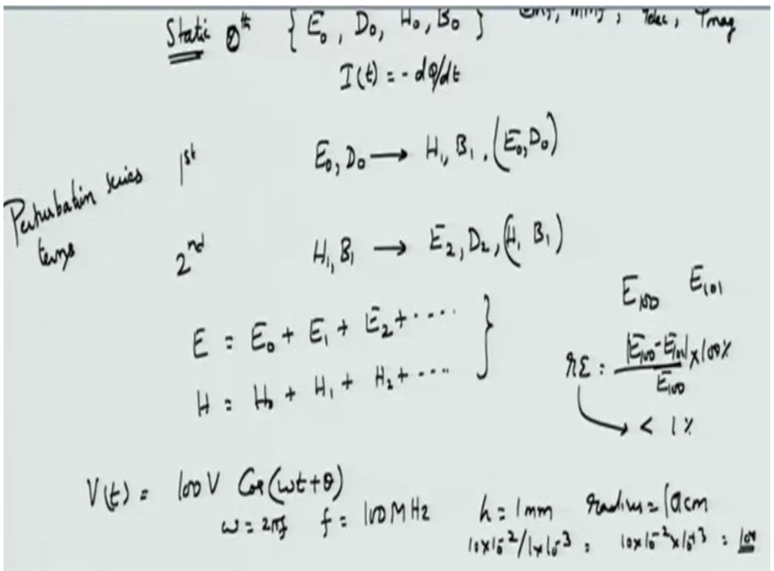

And in fact write down the total electric field as E0 + E1 + E2 and so on similarly for the magnetic field H as H0 + H1 + H2 and so on. Feel free to stop at any order in which you don't really find much use. For example, you have found out up to E100 let us say you find out what will happen at E of 101. That is the order 101 field but if the ratio of E101 to E100 okay is quite small.

If you express, this is percentage, you might express this in this way or you can actually express the difference between the two right. So E100 to E101 normalize with respect to E100 if this relative value is less than say 1% something that one is given already. So let us say if it is 1% or 0.1% you are putting on the problem that has been given to you. You can stop calculating the further orders.

The whole point of what I wanted to try and express to you was that if you understand quasi statics you will be able to build up to the solutions in stepwise manner or in series manner. In fact, formally this is called as perturbation series and this order terms that you are calculating are the perturbation terms okay. And you will have to stop the perturbation series because otherwise it will continue all the way to infinity.

Once you don't find any significant difference from say 100 to 101 or 200 to 201 and so on. There is a nice exercise that will evaluate the EMF for you if I have given you the values of v of t. Let me just mention the results of that exercise. Suppose I assume v of t to be some you know amplitude whatever the amplitude let us say 100 volt amplitude and it is a cosine wave varying as cos omega t + theta.

And we will assume that omega which is 2 pi F has a frequencies of 100 megahertz so this is quite clearly a very large frequency okay. The plate distances are say 1 mm and the plate radius is given by 10 cm. This is clearly quite a large ratio right. If you look at the radius which is 10 cm to the height which is 1 mm you will see that this would be something like 10 into 10 to the power - 2 my math has gotten rusty so, please forgive me this.

So this would be 100 times right. So since this is 100 times the area is 100 times or the radius is 100 times to the height. One can reasonably assume that fringing fields can be neglected and carry out the analysis that we have carried out.

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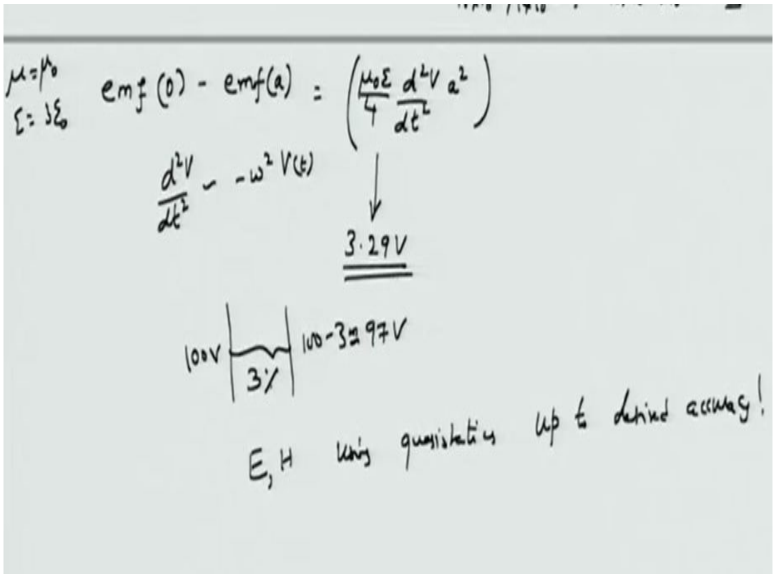

The goal here would be to try and find out what is the EMF okay? What is the EMF assuming that Mu is equal to Mu0 and epsilon is equal to epsilon 0. So you can assume everything to be air filled or let us say assume Mu is equal to Mu0. Assume epsilon to be equal to 3 times epsilon 0. So simply means that you are assuming the directory constant to be 3 okay. If you find the EMF okay? And find the difference between EMF at 0 and EMF at a?

Both of these terms can be obtained from the expression for EMF that I have. EMF at 0 is this fellow. EMF at r equal to a can be obtained by substituting R equal to a at which point the second term drops out and that would be equal to v of t. So the difference terms is simply equal to Mu0 epsilon by 4 a square d square v by dt square. So if you evaluate this difference which will turn out to be Mu0 epsilon by 4 d square v by dt square term right.

So that is the term that you are going to get times a square okay. So substituting all these values and knowing that d square v by dt square actually goes as -omega square times v of t itself. Because cos omega t once differentiation becomes omega sin omega t. Next time differentiation becomes omega square with an appropriate minus term. So the difference which would be this quantity actually turns out to be 3.29 volts.

The volt that you have assumed or applied voltage is 100 volts whereas the difference is turning out to be 3.29 volt across the capacitor. So the EMF here is 100 volts. The EMF here is 100 - 3 which you say 97 volts okay. So this difference is 3 which is approximating of course so this difference is 3. So what you see is that there is essentially about 3% of change in the actual EMF values okay.

So if this 3% is large then you are really not in the static regime but for your application if 3% happens to be small then you are in the quasi static regime. There are additional things that one can do. You know you have capacitor then you can show that they can actually also includes an inductor as its equivalent circuit because of these non-uniform effect but that is something that we will leave for an exercise okay.

The point here was that you can construct the solutions of E or H using quasi static up to the desired order. Up to desired accuracy that would be a better word to say. Thank you.