Electromagnetic Theory

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Lecture - 81
Quasi-Statistics - I

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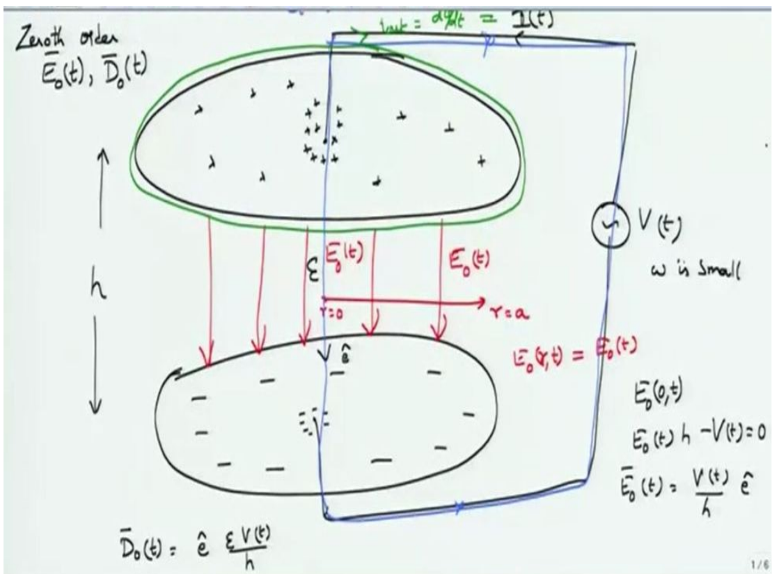

In this module we will discuss something that I promised to you a long time ago. What is called as quasi statistics? As the name suggested this subject does not really qualify as statistics nor it qualify as dynamics as we have studied. This subject lies somewhere in between statistics and dynamics. The idea behind quasi statistics is that we assume all the source very slowly. They will certainly be varying with time.

But we will assume that they are actually varying very slowly with time so that the field patterns actually exhibit that same kind of time variation. So the time rate of change must not be too rapid otherwise we will have to use the full Maxwell's equations and sometimes it is actually quite difficult to solve full Maxwell's equations. And in order to approach the solution at very high frequencies one can actually take successive steps.

Assume this quasi statistic nature prevails then make a correction to it. Make a correction to the correction and keep going until you are satisfied with the results that you are obtaining that is the kind of approach that quasi statistic takes to subjects and that is very powerful approach because it allows you to numerically do calculations and then keep making the corrections okay or you can analytically approach a problem which otherwise may not be approachable from Maxwell's equations at all.

Let us consider one prototypical problem in which we will see how quasi statistics helps us understand more about the solution. This is not about completely a problem that we cannot solve. We can actually solve this problem using Maxwell's equations but understanding that comes from quasi statistics perhaps depends our understanding of Maxwell's equations itself. So with that in mind let me show you what problem that I am considering.

I will assume that there is a large plate okay and one more large plate of conductors okay. So may be this way I can hopefully convey to you that these are two quite large plates and they are separated by a distances h okay. From centre to centre they are separated by a distance h. Please do not worry about the shape of the plate as long as these plates the conducting plates which we assume to be perfectly conducting is quite large then the shape by itself does not really matter.

If you really want to be specific you can always assume them to be two square plates okay or two circular plates. To these plates at the centre we connect a certain voltage source okay. So this is my voltage source V of t which is varying of course with respect with time right. This is what I have and in between let me put down a certain insulator which is described by epsilon okay. This voltage source creates a certain charging current and begins to charge the capacitor okay.

Now what physically happens when you connect such a source? What am I interested? I am actually interested in finding the electrical magnetic fields okay. In the first cut or in the zeroth order approximation we will find the fields which are of zeroth order okay which means that we will assume that the variation V of t with respect to time can be ignored or we will assume equivalently that the variations will be the same as the variation for the electric and magnetic fields okay.

We will assume that this variation V of t is quite small so which means that we are assuming that frequencies omega is quite small okay. So you may imagine that you take a voltage source of say one hertz or two hertz and which is quite small and then you take these plates which are quite large and then separate them with this one h okay. Because this voltage and the current are varying very slowly which of course will also depend on what is the characteristic as well as.

So we will assume that the voltage source is varying very slowly and also the charging current i of t is also varying very slowly okay. So the frequencies is quite small and the capacitance of the structure we will come to that one slightly later okay allows us to write down the fields in it is static form itself. You know this is the situation were you would consider it for example I have not taught you Maxwell's equations.

And you do not know anything about Maxwell's equation then you will say oh well forget about the problem let us simply assume an electric field which will not be varying with time or it would be varying with time at the same phase as the voltage source v of t okay. So this is called as synchronous variation and for the zeroth order we will assume that all fields are synchronously varying and because the variation is quite small with respect to time.

The only fields that would nonzero are the electric field E0 as well as the electric flux density D0 okay. We will so why this is the case in few minutes but we will assume that only these two quantities will be nonzero okay. That assumption we will justify very shortly. Physically what happens? Well, what happens is that ones you connect the source the current begins to follow and there would be charges around here.

So these charges which are being pulled into one end and pulled out from the other end will be in opposite polarities but the charge cannot be situated right at the ends where the wires are connected. The wire for this connection should come outside from the centre and then join back here okay. So this is how the connection should be made to the centre but obviously as the wire in the below is pulling out the charges right.

While the charges of the opposite polarity are being pushed from the upper wire these charges cannot remain near the nodes right. Because they are deposited on the same conductor and the conductor essentially is acting like an equipotential surface. Why is it acting like an equipotential surface? Because we have assumed that this is more or less a static scenario right. The zeroth order is static scenario everything is static.

And in static case the plates will act like the equipotential surface. They would repel these charges and therefore the charges actually get distributed. Because any other charge that is remaining would start to repel so essentially what happens as you bring in the charges these charges will spread all over the top plate. Similarly, the charges that are being pulled also come from a uniform distribution of the charges on the opposite polarity on the bottom plate.

So with the idea that charge distribution on the plates is all quite uniform we can try to find out what would be the electric field. To find that one we will not look at any other thing. What we will do is? We will try to write KVL kind of an equation. Or we will go back to the definition of the electromotive force which would be written in terms of integral of the electric field okay. Because the charge distribution is essentially uniform.

You might imagine that the field line would all be directed vertically downward from the top to the bottom plate. And these field lines will be the ones which are the zeroth order fields they would be varying with time but that variation is synchronous and this E0 which is down on this side will induces a certain EMF in the path. What is an EMF equation? Well if you want to integrate from the centre to centre right.

In fact, we should may some provision and say that the fields at the centre at some distance. For example, if this is you know measured in terms of r okay. This r is equal to zero corresponds to this point and r equal to say a corresponds to the edge of the conductor plates. But we will assume that the fields are all uniform we will neglect any fringing fields that would exist which means that electric field is zero is function of both r and t but it would be function of both r and t but this is actually function only of t.

Because of that reason I will drop the dependence on the radical distance r but I will simply assume that this would be the same electric field and that is justified because the plate areas are quite large and we are in the static region the charge resolution is all uniform. Because of all these condition the electric field to the zeroth order will be the same along sets the uniform electric field.

So writing down the expression for KVL or writing down the simple definition for the EMF. We know that EMF is integral of E right so over this particular closed path which begins okay. This is my closed path okay which is shown here in this blue line okay. Will there be any electric field configuration or any contribution to the EMF from these horizontal wires? There won't be any contribution because the wires are horizontal. There are conductors but the electric fields are directed vertically down.

EMF measures the electric field along the loop. Since there is no tangential electric field there won't be any contribution from the top wire as well as from the bottom wire. Therefore, whatever the EMF drop that must occur across this structure must be exactly equal to V of t. So in the equation what we have is E0 which is the electric field which at the centre is zero and depending on time t we might approximate this one simply as E0 t times h.

Because if you integrate a uniform electric field over the distance h it would simply factor this h out - V of t that we have connected must be equal to zero. This allows us to write down what E0 of t? E0 of t is simply v of t by h. So for there seems to be no surprise but this will immediately allow us to write down what is D0? D0 of t will also be and let us also put down the direction for the electric field so let us say this is the direction along E.

So where E is directed vertically downwards okay. Similarly, D0 of t will be also directed vertically downwards and that is simply given by epsilon v of t by h okay. Now there is one other thing that we need to introduce. This other thing is the current that is associated with this particular loop right. So what is the current that is associated with or not with this loop? What is the current that is associated with the surface that we are going to consider?

Assume that you have a nice surface which will just hug the top plate okay. If it just hugs the top plates, there are charges here and this I of t represent the direction of current that is coming in. Now if the charges within this region are decaying right. If the charges within this are decaying, then there must be a current out and if you take the current out that current out will be in this direction.

So the current out okay which must be equal to the charges that is decreasing dq by dt. This must precisely be equal to the current that is coming in right. We assume the current coming is given by i of t so this i of t must be exactly equal to the current that is coming in right. So because of this current continuity equation you know the charge that is decreasing must be equal to the current that is out but this current is equal to - i of t.

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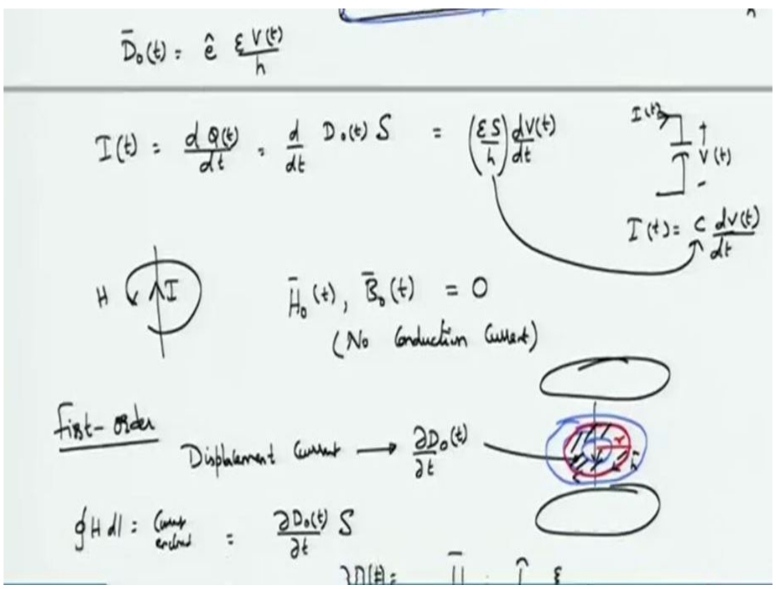

And cancelling the minus and minus signs what we obtain is the current incoming on to the top region is actually given by dq of t by dt right. But what is the charge that is enclosed? Charge enclosed is d by dt okay the d by dt actually comes from the charge enclosed is this D0 of t times whatever the surface area right. So if I call the surface area as say S which would be the surface area so if you look at and since d is also uniform.

The charge that is enclosed will simply be D0 of t into S. The magnitude of D0 of t is epsilon V of t by h. So we have d by dt of D0 of t S which we can rewrite this as epsilon S by h dv of t by dt. What does this equation remind you of? If you take capacitor right and then you charge the capacitor with the current, I of t were in there is voltage V of t across this the capacitor current I of t is given by C dv by dt correct.

So this variation dv by dt time C will be the current through the capacitor and this equation is precisely tell you the same thing except that the capacitance parameter C is given by epsilon S by h okay. So this is clearly the capacitor equation and that is what you will have right. So everything seems to check out here and if we did not know any better we would have simply stopped at this point and gone home happily knowing that we have solved the capacitor problem.

But unfortunately we know little more than what we know right we know that this is not the end of the story. I need to know what is the magnetic field? What is the magnetic field here? Now remember how the magnetic fields are generated. There is a current carrying conductor okay and around that there will be the magnetic field h. Now what is the current that is being carried? Well there is an insulator sitting in between right.

Since the insulator is sitting in between there is no option for us to have any current conduction current through from one top to the bottom one. Therefore, for this zeroth case there won't be any magnetic field okay so the zeroth order magnetic fields H0 as well as B0 the magnetic flux density both must be equal to zero because no conduction current okay. There is no conduction current.

This completes our analysis of the zeroth order. Except that zeroth order analysis is completely wrong why? Because we completely neglected one other type of current, we suggest that there is no conduction current. And hence there is no magnetic field but we know that it is not conduction current that exist in the capacitor but it is displacement current right. So we know that it is actually displacement current that exists and what is the displacement current? Del D by del t.

Now what should be the D that we need to use, well the only D that we know so far is D0 so let us use this D0 of t as the source of the magnetic field. D0 we have already calculated and using this displacement current we can calculate what the magnetic field is? What direction is this displacement current? It is actually coming down vertically right. So this is the top plate and this is the bottom plate. The displacement current is coming down in this way.

This is the displacement current quantity so the magnetic field lines because the current is coming down must curl around okay in this nature okay. So the magnetic fields must curl around and there would be actually maximum near the centre and their density starts to decay or the magnitude starts to decay as you go away from the centre okay. If the magnetic field lines are all curling around and you take a particular curve which is at a radius r okay.

Calculate what is the magnetic field? You will find that because the field lines are all going in a say direction h okay. And h is the azimuthal direction or the direction of the magnetic fields and we assume that the displacement current through this loop is all uniform. It is reasonable assumption because D0 is uniform therefore del D0 by del t is also uniform therefore you pick a particular radius r then the enclosed current will be given by del D0 of t by del t.

That is the displacement current okay times the surface area of this particular loop right that must be equal to the current times this one right. What is ampere's law? Ampere's law states that integral of h dl must be equal to total current enclosed right. So total current enclosed is this fellow so h dl integral over h dl will give you h along the direction h right 2 pi r is equal to del D0 t by del t S.

So the magnetic field component h will be given by along the direction h okay which is you know in the case here if you consider that could be 5 or anything but this is a general direction that I am considering what is del D0 of t? D0 we already know it is epsilon v of t by h.

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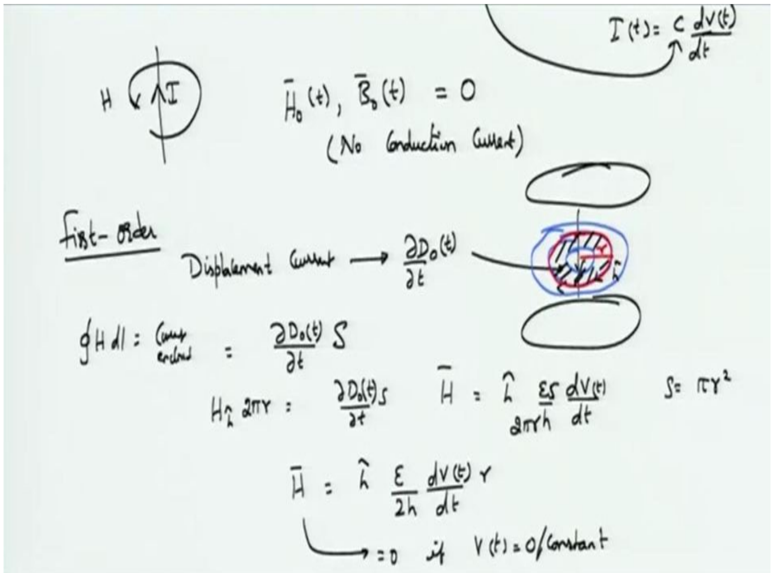

Therefore, the differentiation of that one will correspond to epsilon by h okay and you have the area here so no epsilon by h into V of t so that would be dv of t by dt. There is S somewhere here because of this one and there is also 2 pi r right. So magnetic field component h is given by h times and we can actually rewrite this equation by writing down the expression here. What is the surface area S? Surface area S is pi r square because this is the surface area of this loop which is at a radius r.

Therefore, this would be pi r square. There is an r in the numerator. There is an r in the denominator that would cancel. There is a pi and pi that would and what I obtain is epsilon by two h dv of t by dt times r okay. So please take a look at this equation convince yourself that now h which is circulating displacement current actually vanishes this fellow will be zero if v of t is equal to zero. Does it make sense? Absolutely yes.

Why? If you take a voltage v of t is equal to zero, then there is or if you assume that v of t is equal to zero or constant either for dc or for zero voltage which is again a constant voltage there won't be any time variation. There is no way you can have the time varying fields inside the capacitor which means that there is no displacement current and hence the magnetic field will be equal to zero right.

So the magnetic field will exist only when you have some time variation inside here. So this magnetic field is also function of the radial distance right. So at r equal to zero this fellow would be zero but as r increases the magnetic field would actually start to increase linearly okay. So this is the magnetic field but one of the things that we will have to look at now is that is this the end of the story? Is this the story that is ending but unfortunately that is not.

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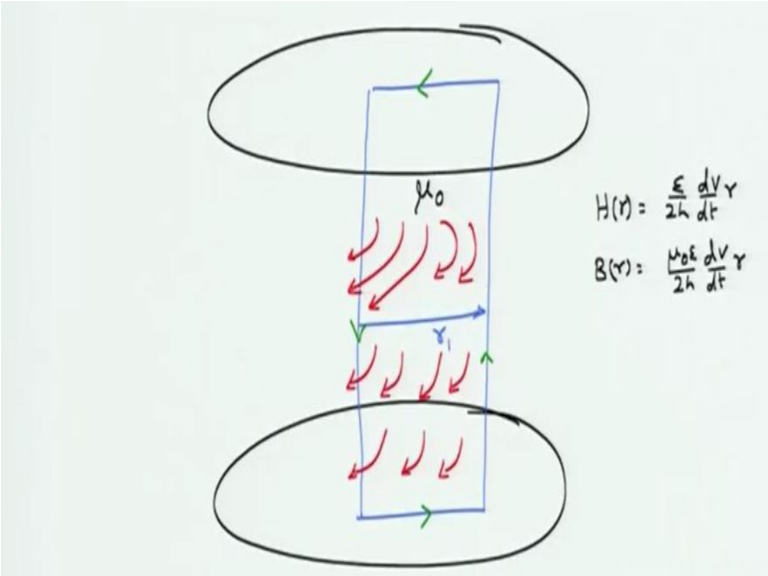

Because now there is a time varying magnetic field and you that very well if a time varying magnetic field exist then it should induce an EMF right. So we will see what happens to that particular case okay. Again go back to the top plate and the bottom plate condition okay. So you have a top plate and the bottom plate I know that I am going consider a loop. I consider a loop from centre to centre and this distance that I am considering will be say r okay.

This is say r or r1 so at which point there is magnetic field. Now how would the magnetic fields exist? The magnetic fields would all be curling around right. So they would all be curling around like this. So clearly the magnetic fields around this loop which is also time varying must induce an EMF according to Faraday's law if you take an EMF around this path there would be this EMF because of the changing magnetic field right.

So we can actually find that one out okay by going back to expression for magnetic flux density and rate of change of magnetic flux density that is linking to this loop must give us the EMF.

What would that be? First let us write down the expression for h, h we have written down. We already know h of r is given by epsilon by 2 h dv by dt times r which would also mean B the magnetic flux density is also function of r will be Mu 0 epsilon by 2 h.

This is we are going to assume that the medium is all non-magnetic and is characterized by Mu 0 right. So this is Mu 0 epsilon by 2 h dv by dt times r okay so for this loop, if you were to integrate this loop along this particular direction okay. So let me integrate this loop in this direction go from the top to bottom, sideways and bottom to top and again sideways. So if I integrate this EMF around the rectangle what I will get is very interesting.