Electromagnetic Theory
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Lecture - 80
Hertzian Dipole Antenna (contd.)

So, we will continue our discussion of antennas in this module. We already have seen the basic antenna expressions; I mean the expressions for a very simple antenna called as a short wire antenna. If you recall what a short wire antenna was, a short wire antenna is one whose length is very, very small compared to the wave length, which is exciting the antenna.

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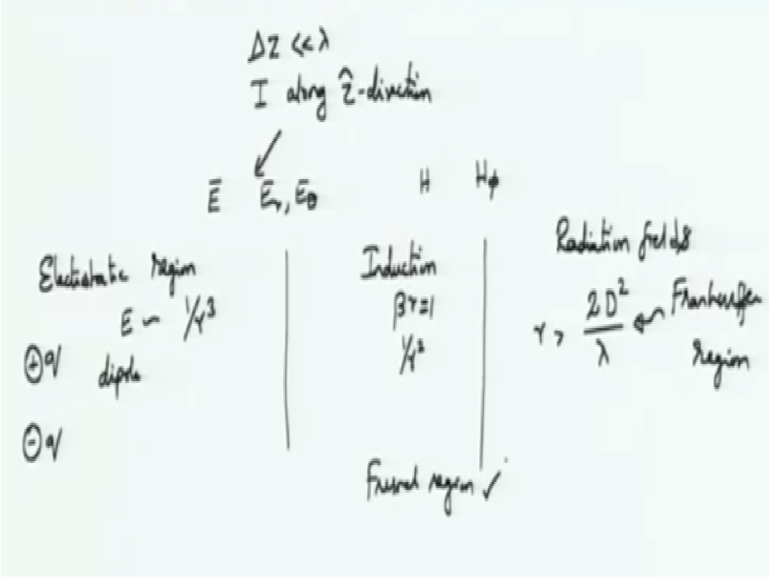

Moreover, this short length antenna when we looked at its field pattern, we assumed a uniform current distribution, which allowed us to simplify the expressions for magnetic vector potential and from vector potential, we derive the expressions for electric and magnetic fields. So, we assumed a uniform current along z direction the antenna was oriented along z direction and then we obtained components for electric field.

We saw that electric field components would be Er & E theta, whereas the magnetic field component would be H phi. We also saw three zones of regions, you know three zones of operation around the antenna. So, first was electrostatic region, so in the electrostatic region or in the electrostatic zone, what we have is electric fields would go as one over r cube and these fields would resemble the fields of a short dipole.

You know, you have two dipoles with opposite charges of equal charge magnitude and the fields for the electrostatic region around the short wire antenna or the short dipole antenna would be equivalent of the dipole field, okay. And we also saw that as you go away from antenna at a certain distance where beta r becomes equal to one, okay. At this approximate distance, you have what are called as inductive fields.

Inductive fields go as one by r square and they do not really radiate or they do not dissipate power. The power is actually stored in the form of the fields around it, the reactive fields around the antenna. However, these reactive fields can be exploited to make some work for us. You can do that and there are many applications of that and one of them being induction cooker that you might have seen at your home.

This appliance basically works on using reactive powers and then trying to convert that reactive power into the real power. But, by far, we are mostly interested in the region what we called as radiative region. And the fields in that region are the radiation fields, this typically happens, okay, as a very thumb rule, you know that it is not necessary that this is exactly border at point it would happen.

The thumb rule is that if d is the maximum dimension of an antenna, then radiation fields would exist for distances greater than, okay, for distances greater than two d square by lambda. Sometimes, this coming from the optics region is also called as the Fraunhofer. I might have got the spelling wrong, but Fraunhofer diffraction region, or Fraunhofer diffraction corresponds to this particular criteria.

So, the fields which are within this region are called as Fresnel region, okay are the fields are within this so called Fresnel region. But radiation actually happens in the Fraunhofer region. So since this is not diffraction per se pursue we can call simply this as Fraunhofer region. But remember these two words Fresnel and Fraunhofer are not, I mean they actually come from optics where we, I mean when we are analyzing diffraction patterns of certain objects.

Then we will label Fresnel and Fraunhofer regions. There again the idea is that in Fresnel region there are fields which are, whose characteristics are quite different from the fields at the radiation. Again I would like to emphasize that these straight lines which I have drawn do not actually represent a proper boundary, okay. You do not have antenna and then so, okay, this is the electrostatic region, this is the inductive region and this is the radiative region.

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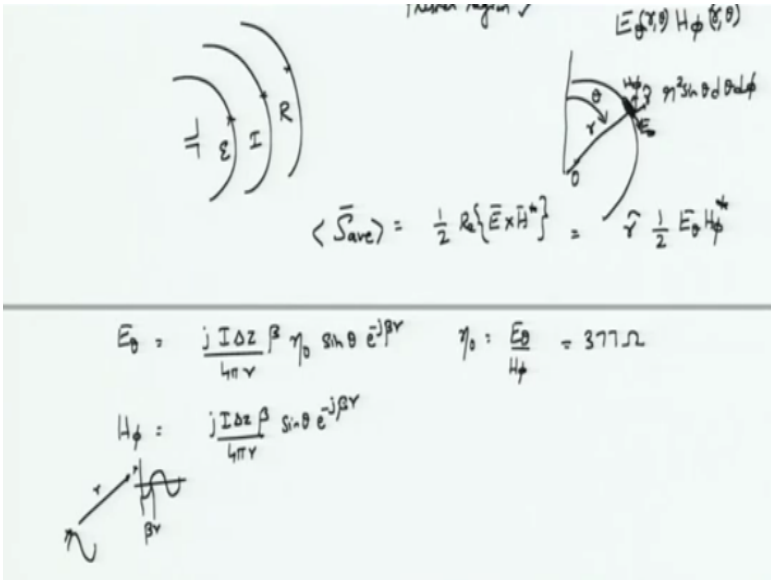

There is no definite year marking of this boundary is there kind of fuzzy and one basically looks at the contribution of the different components of the electric field to arrive at whether we are operating the electrostatic induction or radiation fields, okay. These are not really set in stone or given by exact formula. So, please do not rely on this for all your work, these are just thumb rules, okay.

And as we said radiation fields are the ones which we interesting, because they actually correspond to power being carried away from the antenna. Where actually the power being carried away, who is carrying this power. Well, if you look at the fields in the radiation region, we see that for the electric field there is E theta component and for the magnetic field is H Phi component, both for our antenna the fundamental antenna that we are discussing are functions of only r and theta, correct.

But if you were to work fix this r and then look at what is power coming out of a short patch of region here, okay. So let us say this is my origin and the radius r I have fixed, so this short patch that is there will have surface area of r square sin theta d theta d phi, where theta would be something that you measure from the x axis and this d phi would correspond to the projection of this patch on to the xy plane and then measuring the phase angle from there.

We assume of course, the patch area is very, very small. Now if you ask what is the power that is actually crossing this imaginary surface whose radius is r and you assume that r is in such a way that your fields are mainly radiation fields, then what would be the electric field. Electric field will be one component along E theta and the other component along H phi, right.

So, you have electric field components along theta and magnetic field component along phi, and we know that the average power density, right the average power density, is given by half real part of E cross H complex conjugate. This is pointing average power density, so let us also put that square brackets, sorry these angle brackets to indicates that this is actually average power density, okay.

If you are interested of course in the power that is going out you see that, since E is along theta H along phi the power is actually radiated along r direction, which is something that this comforting to us, because the energy is actually radiated away from the antenna in the radial directions, correct. So, if you evaluate what is this quantity the average power density for this particular antenna, we will see that since E is E theta H is phi.

We have the direction of power being r, so let me write down this r, and then you have half E theta H phi complex conjugate. But what is E theta H phi complex conjugate, let us recall what E theta is. If you remember E theta was Ij, I delta z beta divided by four Pi r, then there is a constant eta zero sin theta e to the power minus j beta r. You can see that all these things are essentially going to be constant for a given value of r.

The only dependence on the variable would come from a sin theta dependence, for E theta. What is eta zero, eta zero is the impedance of the free space medium, and it is actually given by E theta by H phi. This is the free space intrinsic impedance, and this is equal to 377 ohms. We have of course assumed tacitly, that the antenna is actually kept in air, and the medium surrounding the antenna is also air.

If it is not, you just have to replace the corresponding impedance of the medium. And, we have used eta zero here, because E theta can be related to H phi, right. So if you take E theta and then divide by the expression for the H phi, then the result should be equal to eta zero.

So, clearly H phi must be equal to j I delta z beta divided by four Pi r again one sin theta e power minus j beta r. Remember this e power minus j beta r is the phase retardation.

So if this is of cosine wave that is changing at the antenna terminals, at this point which is radius r away, it would be cosine or a sine wave, but with an extra phase. This extra phase is the result of the antenna taking some time, or the fields taking some time to go from the antenna output terminals to the point where you are observing them, okay. So, this is the phase factor, and these fields are actually retarded fields, okay.

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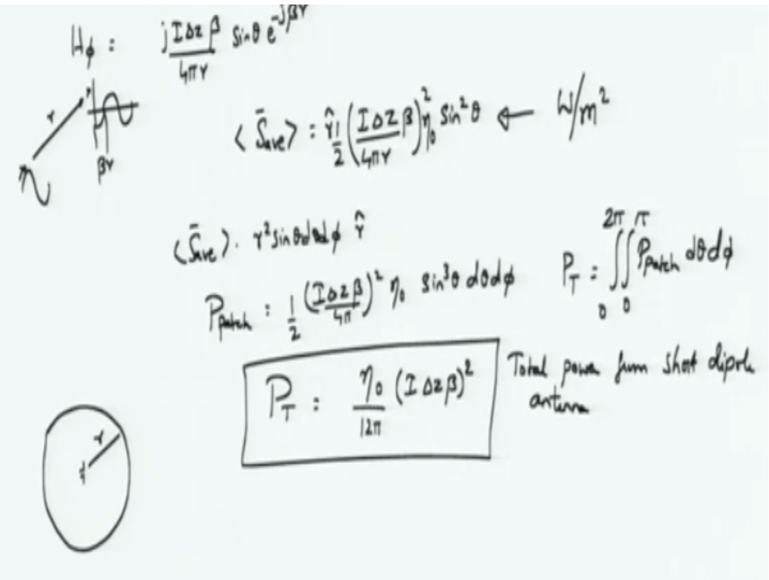

So, what we were discussing, we were discussing this average power density, so the average power density will be along r direction, and then I have E theta, H phi complex conjugate, right. So, if I take H phi complex conjugate what I see here is that j becomes minus j, so minus j into j becomes plus one. So the power that you get or the power density that you get will be completely real.

And it would be equal to, so the average power density is equal to I delta z by four Pi r beta whole square, then you have there is eta zero sin square theta and e to the power minus j beta r becomes e to the power plus j beta r, because of the complex conjugation. That when you multiply by e power minus j beta r vanishes, okay. So, this is what the power density that you are going to get, and how do you measure this power density.

Well, power is measured in watts, and you are looking at pointing vector, which is supposed to measure power per unit area, right. Electric field is volt per meter, magnetic field is I or current per meter, so V into I would be the power, power per meter square is what the average power density vector is. So W by or W per meter square is the units for power density.

And, going back to this patch, which we were considering whose area was r square sin theta d theta d phi, the power coming out of that particular patch will be the average power density dot r square sin theta d theta d phi, this would be along r direction since, average power is also along r direction, sorry let me put that back, and the patch also has an area along directed along r prime.

This r dot r will be equal to one, and what you get here is the power coming out of that patch will be equal to I delta z beta by four Pi whole square, okay, there is eta zero there is also a half here, yeah, we forgot the half here, so there is half and then sin square theta becomes sin cube theta r square will cancel with each other and then you have d theta d phi. This is the power that is coming out of that particular patch.

Of course, if you want to find out what is the total power, you need to integrate this power coming out of that small patch over d theta and d phi. Of course, the integration over d theta and d phi cannot be done with respect to these angular variables alone, you need to multiply them by, so actually we have already done that r square sin theta should have been multiplied, and we have already done that one.

So you integrate this patch power over d theta and d phi, okay. You know theta goes from zero to Pi, where as phi goes from zero to two Pi. So, if you do this calculation I will leave this as a simple exercise of integration for you to do this. The power that is transmitted or the power that is radiated, is given by eta zero divided by 12 Pi I delta z beta whole square. We have of course, assumed that delta z is a positive number, beta is a positive number.

The current is positive, r even if the current is negative going along minus z direction. The power will essentially be independent of sin of I, because it is I square. And this is the power that is radiated from our short dipole. This is the total power that is radiated from the short dipole. Now, look at this expression for the power, okay. Look at the power expression and due you find any dependence on phi, no, in fact, if you were to fix the radius, know in imaginary surface around the antenna, having a certain radius r.

Then you see that, the total power would be independent of any of these. And the power that is coming out itself will be in the form of, or the power density will be in the form of sin square theta, okay. So, the total power does not depend on theta, the total power does not depend on r, and the total power does not depend of phi as well, in the radiation region. And this is something that our intuition would agree.

Because where is the power coming from this. Suppose, you turn around the situation, you are no longer interested in the transmission property of an antenna. Let us say, this is the antenna, and you are actually sending in some electromagnetic wave, so you are actually putting in some electromagnetic wave. What would be the power that is received by the antenna.

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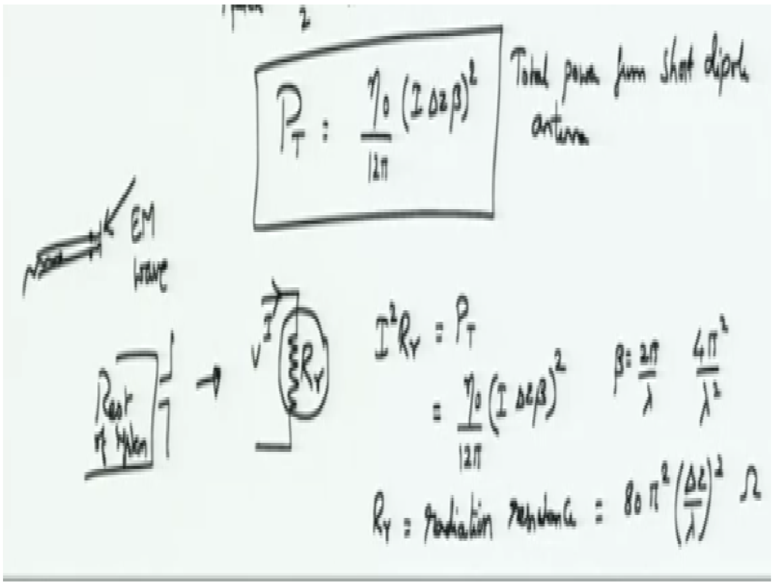

Actually reciprocity tells us that, an transmitting antenna will have the same characteristics as a receiving antenna. That is you take a short dipole antenna, you operate it as a transmitter by connecting it to a transmission line, okay. When you do that one the power that you are putting in would be converted and the power would be going out of the antenna.

On the other hand, if you have some power coming in, in the form of or the power density coming in, in the form of EM wave, then the power that would be received by the antenna would also be the same as if the power would be transmitted by the antenna, okay. This is very important. The same antenna can be used as receiving as well as for transmitting, in fact antennas are passive devices.

This gives us a certain analogy with a resistor, okay, because the power is being, so if you consider the passive linear devices of r, l and c. You will see that it is the resistor which dissipating power l and c only have reactive powers. Therefore, the circuit model for an antenna would be something like a resistor, okay, with some current and a voltage, so that the power is actually being dissipated by this resistor.

What would be the power dissipated by the resistor, it would be the current times the radiation resistance. This must be equal to the total power radiated by the antenna, which means that this must be equal to eta zero twelve Pi I delta z beta whole square. In fact, you can expand this and show that I square will cancel on both sides and then delta z is there. Beta can be expressed in terms of its lambda, so beta is two by five lambda.

So beta square will be four phi square by lambda square okay, so you can show that Rr the radiation resistance. This is called as radiation resistance and this is the equivalent resistance of an antenna. That is, for a circuit engineer, if you go tell them that this is the antenna, what the circuit engineer would like to know is that, can I find out what would be the power that is delivered to the antenna or the power that is received by the antenna.

And the answer is yes, you can treat this antenna as a resistor and then calculate how much power is being delivered or how much power is received. This circuit property of antenna also helps us design matching networks for the antenna, so if I know what is the radiation resistance, okay then I can actually design the matching network, so as to match the other connecting components may be a wave guide, may be a (()) (16:08) or may be a transmission line.

So I can connect any of these components to the antenna system, so the rest of the system can be made to have its impedance match to this Rr in order to have maximum power delivered across the antenna. So for the short dipole or the short wire antenna that we have been considering, the radiation resistance turns out to be 80 pie square delta Z by lambda whole square ohms.

We will now discuss some of the characteristics of antennas, which are quite commonly used across all antenna types okay.

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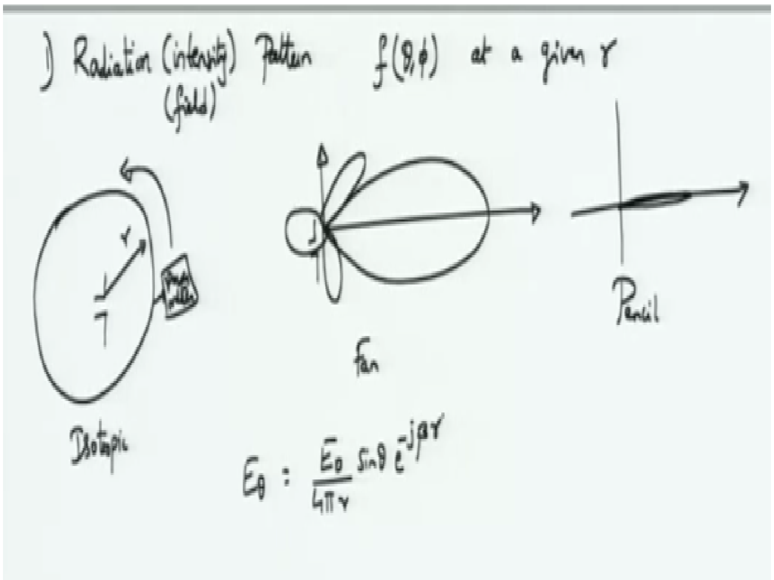

So these characteristics are widely used to characterize any antenna and one of the main characteristic that will be interested is, how the radiation when we actually say radiation be more or less mean by radiation intensity, sometimes we were interested also in the radiation field that is in the form of electric field or magnetic field, but intensity or power per unit area is what were are mostly interested in.

So radiation intensity pattern is something that is very interesting for us because it will serve to differentiate different types of antenna may be you have an antenna, which will radiate power equally in all directions such an antenna is called an isotropic antenna, so isotropic antenna if you were to put a power meter here okay and you move this power meter around the antenna, but the certain radius r of course.

The power meter will always register the same power, so the power is actually independent of theta and phi such an antenna is called as isotropic antenna or sometimes called as an isotropic radiator and it is used to serve as a reference to characterize other antennas okay. Moreover, this isotropic antenna that is not something that can be fabricated in real life. This is more or less theoretical.

This is actually a theoretical construct, which allows us to use this as a reference for characterizing other antennas okay. Now as I said, we have radiation intensity pattern or radiation field pattern, so one antenna could be isotropic, the other antenna could do something very interesting you know, it will may be power going only in a particular direction okay.

So for this antenna clearly, the power will be maximum along this particular direction and the power would be minimum or in fact zero along this direction along the wire of the antenna, so if this is the antenna wire, then there would not be any power along that direction whereas maximum power is there in the along the direction that is perpendicular to the wire axis, maybe this is one antenna that you are interested in.

In practice, you will also find what are called as additional lobes or side lobes. There is also something called as a back lobe, so that antenna actually radiates backwards as well, so this type of an antenna is clearly not the same as an isotropic antenna and in fact depending on what the beam and the number of side lobes, one can further characterize these antennas into fan beam and pencil beam okay.

These are called as beam patterns or beams of an antenna, so the pencil beam would basically have its radiation along a very, very narrow angles okay, so this is what we actually mean by radiation intensity pattern, the power being a function of theta for a given value of R and the power being function of phi is called as the radiation intensity pattern.

Sometimes you are also interested in the radiation field pattern in which case you are interested in electric or magnetic fields and accordingly you will have electric field pattern and you will have a magnetic field pattern. Again these field patterns are simply some functions of theta and phi okay at a given r, so you fix r, the radial distance from the antenna and then you move around with the power meter or a field meter or a old meter to be precise.

Then you look at or you look at what is the readings of these meter powers or the volt meters and we assumed that the volt meters will read the electric field okay and map that out, you will get the radiation pattern. For the short dipole antenna, we know that E theta in the radiation region will be equal to some constant E zero divided by 4 phi r, sin theta e to the minus j beta r correct.

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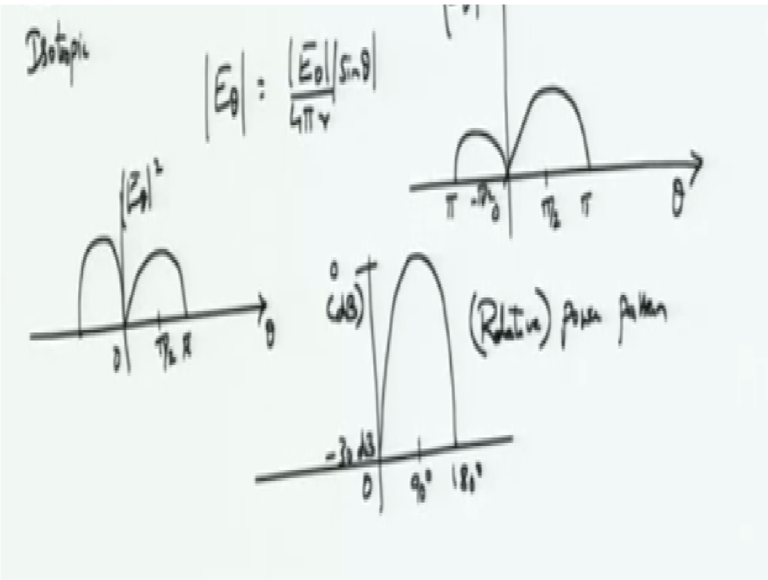

So this is the electric field and if you are interested of course in the magnitude of the electric field, this e power minus j beta r will drop out and then E0 magnitude let us assume to be one, so if you look at the magnitude obviously the sin theta magnitude you have to consider and if you plot this quantity E theta magnitude, as a function of theta you will get a graph that would look something like this, so this is zero, this is Pi by two and this is Pi right.

You can also go in the other region to go from zero minus Pi by 2 and Pi, so this is the magnitude field pattern. What could happen to the power density pattern or the radiation intensity pattern as a function of theta. We know that E theta square will contribute to power pattern, therefore the power pattern will go as sin square theta okay, so it would go as sin square theta, again it would go as a zero and it would reach its maximum and again go back to zero at phi okay, but it would actually goes as power.

It is actually rather interesting to talk of power pattern in terms of db scale, so you can actually have a db scale pattern, in which you go from say zero to pi okay or 180 degrees and this is how the power pattern would be at ninety degrees where you have the maximum power right at Pi by two you have maximum power, because sin theta reaches maximum there and at the other regions, it would assume vertically approach to some minus thirty db.

So this is your relative power pattern okay, relative to the maximum value okay. Normalized power pattern, this is sometimes also called as normalized power pattern, so you just take the power and then normalize it to the maximum value that you get that would be the relative power pattern or normalized power pattern okay, but these are not the way in which you would have found antenna literature to give you the power patterns or the field patterns.

The reason is that you want a sort of a 3D picture, but you cannot draw a 3D picture on a 2D plane or at least not without stretching your visual limits real in a sense, so you do not want to visualize too much and then try to plot a 3D picture, so what we do is we provide patterns at different cuts, so one of the cuts that we will make would be at x equal to 0 and then you are looking at z and y planes okay.

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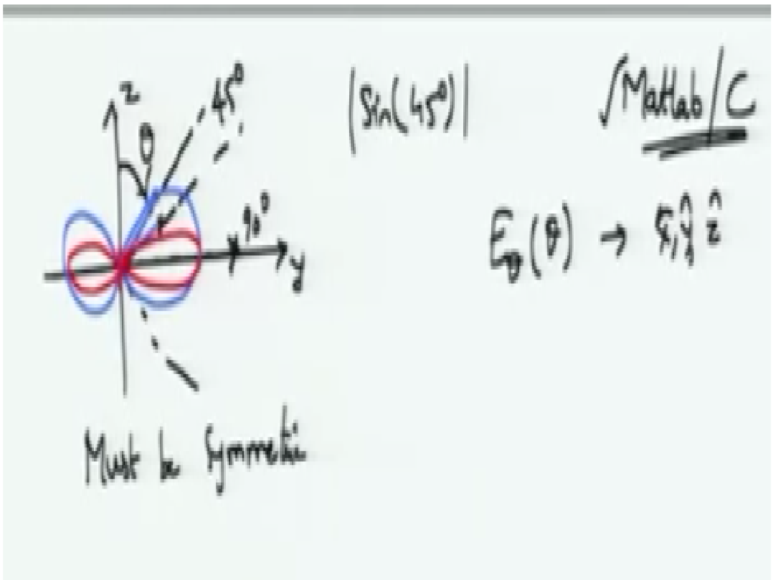

So this is the y planes and this is z plane and we have kept our antenna at the centre, the antenna itself is quite short and now what we do is, we know that theta is measured from the z axis right, so this is how theta is measured and what we start doing is we imagine that we have different thetas over here, so theta at along y axis will be measured in the y axis and then corresponding minus theta would also be measured here okay.

Then for this theta, let us assume that this theta is 45 degrees, we mark a point okay, along the y axis where theta is equal to 90 degrees, you mark a point, which would correspond to the maximum length, then along this line, you cut the corresponding sin 45 degrees, because we are trying to plot the magnitude patterns, so you find what is sin 45 degree, take its magnitude and cut that particular distance.

So whatever the distance that is there mentioned in this blue line is actually this guy, okay, so it is this sin 45 degree magnitude okay, so you keep doing this for different angles, and then keep cutting different lengths okay and then eventually you join all of them okay, what you find is a pattern that would look like this okay. So this is the pattern that would look obviously that has to be symmetric is just that my diagram does not show the symmetry okay.

So let me write down this must be symmetric okay, so this is how the pattern would be looking, if you were to look at it from x equal to 0 plane or at an x equal to constant plane okay. Obviously the full pattern would be coming in along the x direction as well, so the 3D plots one can obtain very easily these days using a personal computer and writing a program in one of the languages such as MATLAB or C.

I encourage you to use MATLAB and just plot this 3D pattern you know, you have this E theta, which is the function of theta, you assume a certain value of R okay and then you calculate these values and then convert this e theta of theta into xyz vectors and then appropriately plot the pattern. There is another way in which you can plot the pattern, in which you are looking down from the antenna.

So you are actually imagining yourself along the z axis and you look down on the antenna, what you see is that an antenna would look like a point and then the power would be still around that particular antenna, okay. So we have all these different ways of writing the antenna pattern. It is not probably very important for us to go into all these different pattern, but keep in mind that you can plot both power as well as intensity okay.

So if you were to plot the power or intensity, the intensity pattern would also look like this sorry, which has to be the same thing, but it would be much more narrower than what you have, okay. Because this is sin square of theta magnitude alright.

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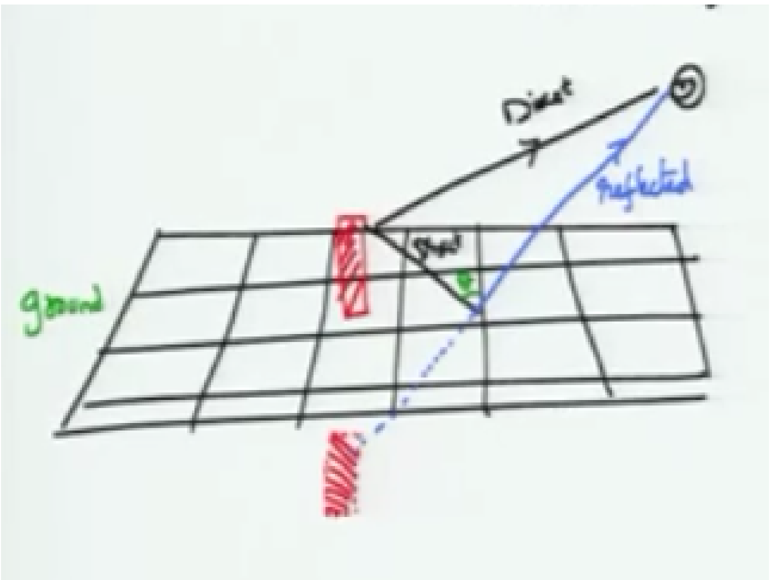

Let me briefly comment something about the effect of ground okay. So let us say, I have an antenna, and then this is my ground okay, so I have a ground here and on top of this, I keep my antenna okay, so this antenna is assumed to be short, do not be fooled by the length that I have drawn that is supposed to be quite short okay.

Then what happens is if the ground is nicely conducting, we know from image theory that we discussed in the electrostatic case that if you have a charge at the top and of course you need to have charges in ordered to have current and currents are the one which are generating these fields, so you have these charges on the short wire above the ground, there would be equal and opposite polarity charges induced in the bottom.

Now if you were an observer over here okay, so you are an observer at this point. If you look at the field lines that are coming to you, you could actually obtain one wave, which would be direct wave, but because the antenna radiates in all directions there would be radiation down, so this would also be a direct wave, but this direct wave goes and hits the ground and what is the property of the ground, from the ground it actually has to reflect right.

So this is the reflected field. This reflected field if you pull it back actually looks like it has come off from an image okay. So it is look like, as though it is off from the image and this would be the wave that you are actually receiving okay. If you look at the angle of incidence okay, call this angle may be you know, call this angle as an angle theta. Depending on these angles, the reflected wave and the direct waves may constructively or destructively interfere.

The point here is that if you have a ground plane, okay, this is your ground plane, if you have ground plane anywhere near the antenna, then the field pattern and power pattern of the antenna gets modified. Sometimes, you might use this concept of having a ground to your advantage okay. We will see one example when we discussed half wave dipole, so instead of having a half wave or half wave length lambda by two dipole on the top okay, okay.

You can actually use this idea of the image antenna by cutting down the length from lambda by two to lambda by four and what is called as lambda by four monopole antenna over the ground. It will simulate the effect of the lambda by two antenna, but of course it will also have some other factors that we will not discuss at this point, but the point is that ground planes will disturb the antenna pattern, keep this in mind alright.

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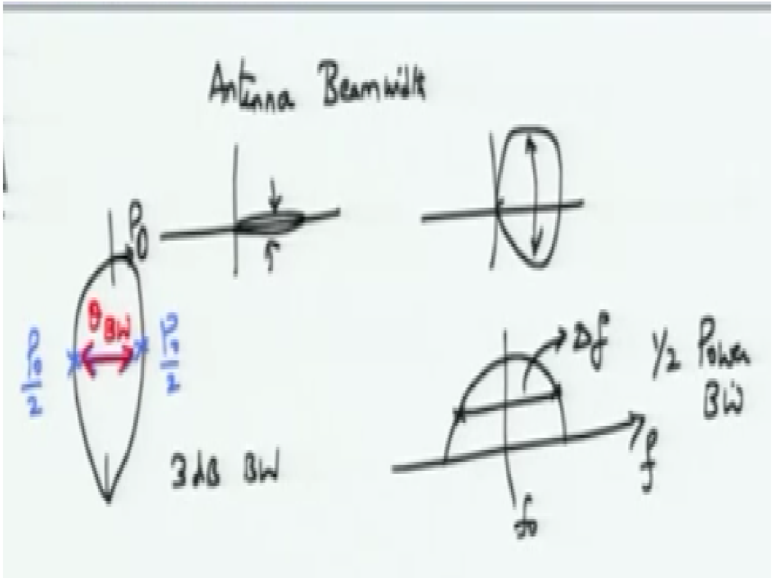

So we will close our discussion of characteristics of the antenna by looking at two additional factors, one of this is again related to the pattern itself. This is called as antenna beam width and as I said, antenna beam width is one of the characteristics that separates one antenna from another antenna. For example, this is a pencil beam, then you will also have a fan beam okay.

So this antenna obviously radiates much larger and the width over which this radiates will be much larger than the width over which this particular thing would radiate okay and beam width is defined in terms of in a very similar way as that of a band width, okay in which if this is the antenna pattern along a particular direction, okay we look at the power that is here okay.

So call this as P0 that would be the maximum power that could be radiated away from the antenna okay and then we mark two points, at these points, the power is only half compared to the maximum power P0 okay and the length or the width between these okay, the width between these is called as the antenna beam width and is denoted sometimes by theta bw and it is measured in degrees between the two points, where the power is reduced to half.

Sometimes for this same reason, this is also called us 3dB beam width okay. A similar characterization for an antenna can be carried out in terms of its frequency okay and you see what is the radiation pattern of the antenna in terms of as a function of frequency mark off two points and then say with respect to the centre frequency okay, what would be the width of this particular frequency band or what is the width of this band at which the power is reduced to half okay.

This is called half power bandwidth. This is called us half power bandwidth. BW stands for both beam width as well as for bandwidth, but the context should obviously make you differentiate between the two.

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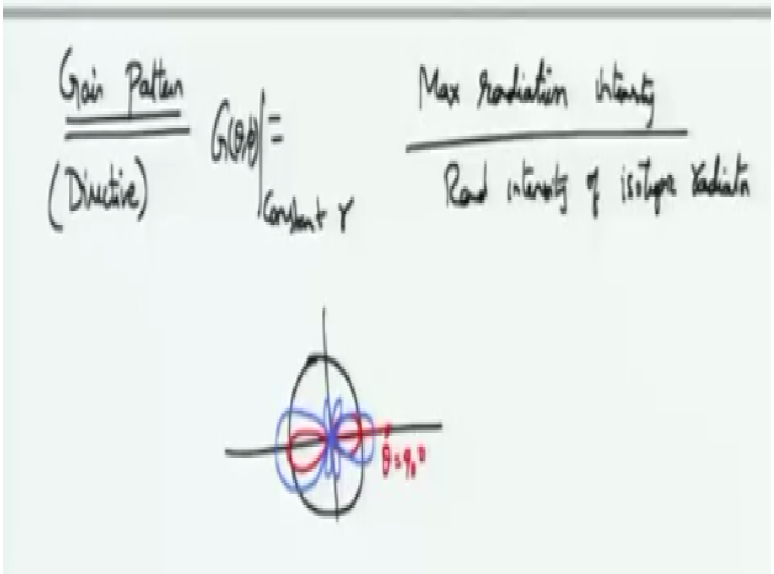

Gain Pattern

(Directive)

$$G(\theta\phi)\Big|_{\text{Constant } r} = \frac{\text{Max Radiation intensity}}{\text{Rad intensity of isotropic radiator}}$$

One final one, sorry I said I will only finish with two, but this is something that I would just like to introduce. This is called as gain pattern of an antenna. Now you might be surprised by knowing that an antenna can have a gain pattern. I mean, after all we will just said that antenna is a passive device and how can it have a gain, well what happens is that it is not really the gain in the absolute terms but gain only in relative terms.

So this gain pattern is most likely we actually mean by directive gain pattern because most antennas actually are directive that is they have beams, which are either pencil like or fan like or somewhere in between. They are not isotropic at all okay, so we are interested in directive beam patterns, because that is the one which will give you, you know the communication angles.

The antenna pattern is directed in a particular direction, then you can put the receiving antenna there and all of the energy would be received at that particular angle. So the gain pattern of an antenna is also sometimes called as directive gain pattern. It is a quantity that would be dependent on theta and five at a constant r okay at a constant value of r.

And this is defined as the ratio of maximum radiation intensity or radiation power density okay, divided by radiation intensity of the reference antenna or the radiation intensity of an isotropic antenna or an isotropic radiator. Why is that chosen, well. If you look at the radiation pattern of an isotropic antenna, it would be a nice big circle right whereas for most directive antennas the radiation pattern would look something like this.

It would be directed in a particular direction, so how much the power is actually being radiated in this narrow region or along this particular direction to the power that would be radiated by the overall you know by the isotropic antenna will define the gain pattern. Sometimes you can also talk of maximum gain. For example, maximum gain occurs here in this case for theta equal to 90 degrees and it would be equal to this ratio.

This one divided by the isotropic gain at that point okay. Of course, you can also have an antenna whose pattern would look like this, then you will have a gain pattern that also suitably defined okay. So this is all about the characteristics of an antenna that we wanted to discuss. In the next module, we will discuss another antenna, which is called as half wave dipole antenna that is more practical because this antenna that we discussed the short dipole at the short wire antenna cannot really be used in any practical scenario. Thank you.