Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology - Kanpur

Lecture - 78
Radiation and Antenna

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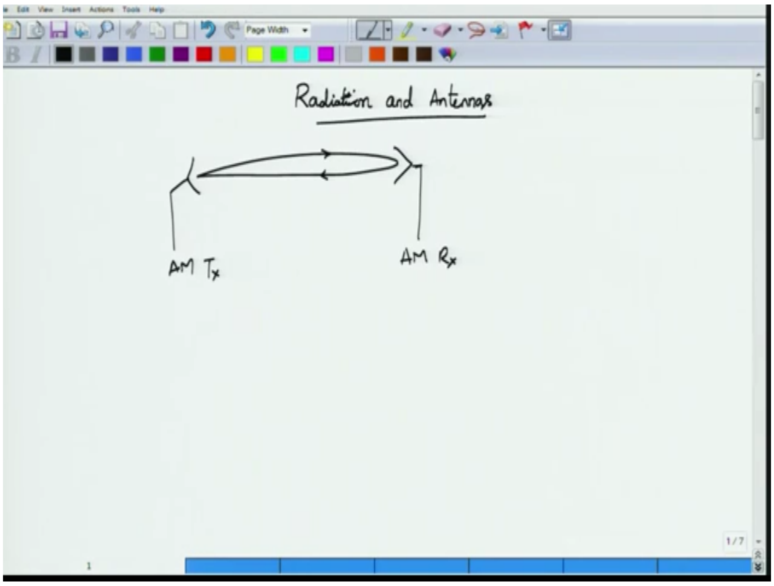

In this module, we will study briefly the concept of radiation and antennas. In this and the next module, we will be concentrating on the fundamental principles of antennas and we will discuss only one antenna, which is called as linear thin wire antenna. The reason why we will use antennas is probably not required to even point out. If you want to send electromagnetic energy from an AM transmitter to a receiver that is located far away.

And these two are not connected by a wire then you have to use an antenna at the transmitter to properly consider the electromagnetic energy and put it out in a particular direction. Of course, there are certain antennas, which would spew the energy in all the directions in an equal manner or in a slightly equal manner. These antennas are called as isotropic antennas. Isotropic antennas are one in which electromagnetic energy basically is the same.

They transmit energy or receive energy in an equal measure around all of that one. So, there is no directional dependence on the power that is being transmitted, but such antennas are rarely useful for us. You most likely want an antenna, which has a specified you know, direction of transmission. For example, if this is an antenna that we have and this is a receiving antenna here we have, okay.

For some purpose, this could be an AM transmitter. This could be a mobile AM receiver, okay. So, this is an AM transmitter and this is a mobile receiver. You want almost all of its energy to be concentrated in this particular narrow band of region, right. So, you want some sort of directionality to your antenna, which means that antenna spews out energy in a particular direction.

Regardless of what an antenna and how you know all these different parameters are connected, there are some fundamental principles behind which these antennas operate. And in this short two module class that we can have it will not be possible for us to go into details about antennas. Antennas are you know such a wide subject that it is perhaps useful for you to consider this as a separate course rather than considering.

I mean, it is necessary for you to take up a separate course on antennas to really understand different types of antennas, okay. So, we will not attempt to be anywhere exhaustive in terms of antennas.

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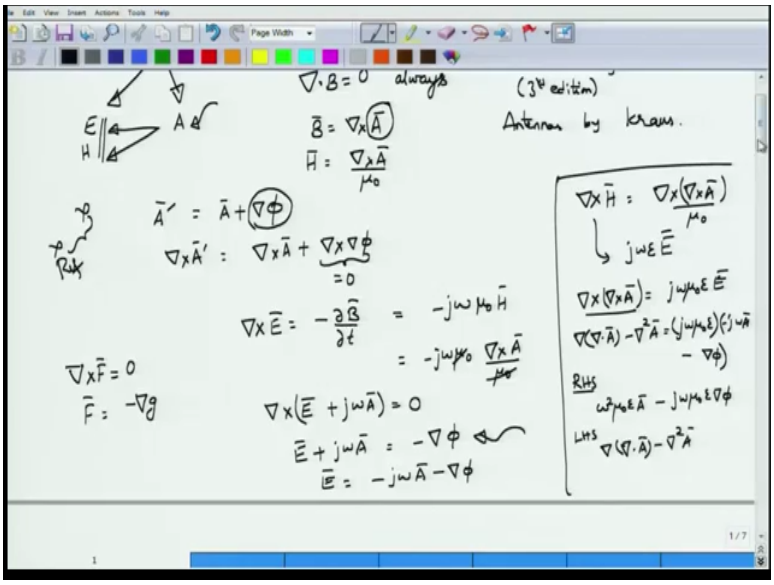

Our idea would be to see if you can describe the fundamental characteristics of the antennas with the help of a very simplified antenna that we will consider and then later we will consider one example of an antenna that is kind of practical. So, the first thing that we are going to discuss is somuhat theoretical antenna, but that is sufficient for us to understand some of the fundamental characteristics of antennas, okay.

At this point, I would like to point out that there are very good references to antennas. The text book that we have followed in the course would not really be enough for you to understand more about antennas. One of the good reference to antennas is Antennas by Balanis. This particular book is in its third edition, if I do not, if I am not wrong, so you can refer to this or if you want slightly more practical flavor.

But at an older literature level, we can also look at Antennas by Kraus, okay. Having said this let us consider some preliminary stuff that we need to perform in order to go ahead with considering what radiation and antennas are. How radiation is fundamentally different from so far the methods we have used to guide energy. Then in a very broad sense, even an antenna can be considered as a transmission line except that you know you do not have a wire or something connecting the source and the load end.

Everything is done in free space or in a medium with no connection between. But there is the concept of energy being transmitted from one point to another point that is sufficient for us to characterize antennas as a transmission line. Traditionally, of course antennas have been considered as radiators because they kind of radiate energy from one point to another point in space and by titling the antenna.

You know you can imagine that there is an antenna, which is giving out energy in a particular direction, but if you tilt that antenna, the energy direction would change, okay. So, because of this antennas are studied as a separate category. They are studied under the general heading of radiations. So, radiations from antennas is perhaps a better fitting title for this module. So, before we can even discuss radiation, there are certain things that we need to do.

First, there are two ways of discussing radiation from antennas, one method begins with E and H, which are the quantities that are already present in Maxwell's equation. They do not introduce any additional vectors for discussion, whereas more common method of analysis of radiation begins by auxiliary potentials, okay. The primary auxiliary potential that we use is called as the magnetic vector potential.

Sometimes there is also a different type of potential that is used, which is the scalar potential, but scalar potential is not commonly used. It is the magnetic vector potential that is quite commonly used, and the reason why it is used is because it can be used to relate both E and H. So, A, the magnetic vector potential can be related to electric field E and H. We have of course seen already how to use.

I mean we have already seen how this magnetic vector potential is defined and we have also obtained an equation for this A, but we are going to consider slightly different picture. So, it is worthwhile to go back to the definitions of A and see how the expressions for E and H are related to A. So, the definition for A is where we began even in one of the earlier modules is to recognize that del dot B is equal to zero always.

Because, there are no magnetic monopole for you to have a divergence, non-zero divergence for the magnetic field B. So, because del dot B is equal to zero, it is possible to actually represent B as curl of some other quantity. This some other quantity is the magnetic vector potential A. So, we write down B as del cross A, since B and H at least in free space are connected just by the permeability of the free space.

We can also write down H in terms of curl of A as curl A divided by mu zero. So, at least one equation we have obtained, in which we see the H is curl of A. Is this sufficient? Is this sufficient to completely characterize my H and A relationship? Unfortunately, no. Because consider what happens, if I consider A prime, okay. A prime is related to A by this particular operation.

So there is A, which is the magnetic vector potential that we have considered, to that we have added a gradient of some scalar function, okay. This is a gradient of some scalar function phi. Now consider what happens to del cross A prime. This del cross A prime, because del cross can be applied individually to these two elements will give you del cross A plus del cross gradient of phi.

But we already know that curl of gradient of phi is equal to zero uniformly, which means that curl of A prime is equal to curl of A. So, there is some sort of an ambiguity in defining A, we get around this activity in the same way we got around the ambiguity for scalar potential. We said that in scalar potential case, there could be some ambiguity, but we are not interested in the actual scalar potential or potential at a point.

We are only interested in the potential difference between one point to another point, because those are the meaningful results that we can have. So, you have voltage at one point, by itself, that does not tell you anything, but voltage difference between two points is what we are interested. One point we call as reference, the other one is the voltage at that point we are interested in. So, because of that there is this ambiguity in A which we understand, okay.

But we do not really do anything about this except saying that there is ambiguity of gradient of phi, at least for now let us accept this. So, we have a relation for H and A, we still require a relation for E. How can we obtain the relationship for E? I already know that curl of E is equal to minus del B by del t, this is from one of the Maxwell's equations. This is in fact Faraday's law, but since we are considering E and B all as, E and B as phasor quantity.

So, in terms of phasor, this del by del t will be replaced by minus j omega and B can be replaced by mu zero into H. So, I have this relation, but I also know that H itself can be written as del cross A divided by mu zero, cancel mu zero on both sides. What you get is curl of E plus j omega A must be equal to zero. Now this equation looks very nice there is something that you are saying as curl of quantity is equal to zero, which means that I can express since curl of a quantity is equal to zero.

I can actually express this quantity F in terms of gradient of some scalar function, right. So, I can write down this as E plus j omega in terms of gradient of some quantity phi and in fact, you can use this as a definition of phi because I know E, I know A. I have already defined A in terms of B. So, I can obtain the definition for phi. It is electric scalar potential, not in the static case because we are already in the time varying case.

E can be written in terms of A and phi, A being the vector potential, phi being the scalar potential as minus j omega A minus gradient of phi. So, in defining this gradient of phi or introducing this gradient of phi, we are able to now relate both E and H to this quantity A. We in fact remember that we went one step ahead. We said that we can actually put this equation over here, so we have this del cross A here, right.

So, we have this particular equation and we can obtain a wave equation for this one. We can actually try and see what we will get for del dot if we take del cross B. So, we will take the del cross B over here, right. So, I have del cross H, let us take del cross H because that is simpler. Again del cross del cross A divided by mu zero and del cross H can be written as in terms of Maxwell's equation.

This would be j omega epsilon assuming that there are no surface currents. I can write this as j omega epsilon E. So, I have del cross del cross A is equal to j omega mu zero epsilon E and I can substitute for E. E is nothing but minus j omega A minus gradient of phi, but I can also write down this del cross del cross A itself as del of del dot A by expanding it minus del square A.

This would be equal to j omega mu zero epsilon times minus j omega A minus gradient of phi. So, it is kind of getting difficult to write down this one, so I will only write down the RHS term here, expanding this I get omega square mu zero epsilon A that is, this terms minus j omega mu zero epsilon gradient of phi. What happens to the left-hand side term? The left hand side term still remains del of del dot A minus del square A.

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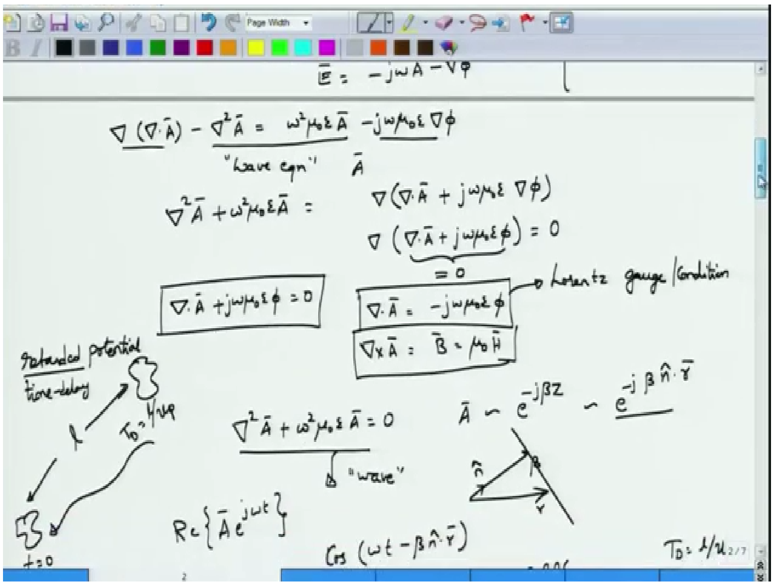

Equating this left hand, right-hand side terms, what we get is del of del dot A minus, so this is the equation that we have, right. So you have a gradient of something over here, you have something here as well. This kind of reminds you of a wave equation except that your wave is now A, right. This is a wave equation type equation, okay, which is now A. A is the one, which is waving, so if it is possible for me to somehow cancel these terms.

Then it would be nice then I can actually have a nice wave equation, whose solutions I can easily write down because we have now enough experience in solving these type of wave equations, right. So, we have harmonic solutions and we can easily write down the solution for a given coordinate system and then do whatever analysis that we want to do. So, if it is possible for me to somehow cancel these two term.

Then it would be nice for me to have a wave equation. So in order to do that one, let me do something interesting. So, let me take this term minus j omega mu zero epsilon del phi to the left hand side and push this del square A to the right hand side. So, if I do that and again switching back left and right hand sides, I get del square A plus omega square mu zero epsilon A must be equal to gradient of del dot A plus j omega mu zero epsilon gradient of phi.

If I now write down this del dot A, I can take the gradient operator out, so what I get is del dot A plus j omega mu zero epsilon phi gradient of the whole thing that must be equal to zero. I can do this because gradient is independent of omega mu zero and epsilon. I am of course assuming that epsilon itself is a constant, which is valid for air or for any other medium in which epsilon is constant.

So, this is a homogeneous medium that we are considering. We are of course also considering the medium to be isotropic and constant medium, time invariant medium as well. So, I have gradient of some quantity equal to zero, the way I can make this equal to zero is to set this fellow equal to zero. So, I have del dot A plus j omega mu zero epsilon phi equal to zero, which gives me a relationship between A and phi itself.

I already have defined phi, the defining equation for phi is to say this as minus gradient of phi is equal to E plus j omega A, but now I also have a nice relationship for del dot A and phi. If I choose del dot A as minus j omega mu zero epsilon phi then I also know that curl of A is equal to B or equal to mu zero H accordingly, then I can specify this A completely.

This condition in which the divergence of A is chosen as this is called as the Lorentz condition or sometimes called as Lorentz gauge, okay or Lorentz condition might be something that we can write down for this course without really bothering what gauge is and, we know from certain vector analysis theorems, you know from your mathematical analysis that if I have a vector field and I specify its divergence and its curl.

Then I have completely specified the vector field. So, the way to specify vector field A for me would be to take this divergence to be equal to minus j omega mu zero epsilon phi. If I do that substitution, if I take this Lorentz condition, then I get a wave equation for A. So, I have omega square mu epsilon A is equal to zero. Well, this is very interesting because I can solve this A.

I know that A must now be in the form of e power minus j beta z assuming that the wave is waving along z, then it would be to the power minus j beta z. In case it is waving in any arbitrary direction then this would be e to the power minus j beta n hat dot to r, where n hat will be direction of beta. So, if beta is in this particular direction, then define normal n along the direction of beta and then for any other direction of r.

You know the wave is waving in this particular direction and the component or the phase of that wave would be beta n hat dot r. This is not the only thing that happens. This is just a phasor part. What about the time part? So you go back to the time by multiplying the e power j omega t and then taking the real part, right. So, for now, you just concentrate on what happens to this omega t and beta n dot r, okay.

So, what is happening here is when you take the real part you get cos omega t minus beta n hat dot r. What you see here is that this is exactly equivalent to a traveling wave omega t minus k z is what the uniform plane wave that we considered or in any other wave that was actually waving along z direction. So, what we have in fact shown is that this is a wave equation and A will wave, okay.

So, A will wave along a particular direction beta, okay and when it waves there is a corresponding beta that we can define beta or the propagation constant that we can define and the wave because of this wave nature, we also know that if there is a disturbance of something at t equal to zero, right. If A changes suddenly from one value to another value t equal to zero.

Then this disturbance will not be visible to me if I am sitting at a distance of l from the source. So, if something happens at t equal to zero, then I will be able to see this one only at l by up where up is the phase velocity for that particular frequency component or for that matter it is just a time delay. So, there is some amount of time delay that is involved. Again, this is also not something new to us.

We have already seen this one in transmission line. We know that if you connect the switch of the transmission line, it will not be able to see the load immediately. There has to be some propagation delay. All that this mathematics has brought to us is that when A waves you know or A changes at t equal to zero at say z equal to zero then at z equal to l this change will be visible after a delay of l by u, where u is the phase velocity, okay.

So, this character that something is changing that t equal to zero, but the effects appear later is what is called as retardation or retardation in time and this potential is called as retarded potential. Retardation simply means time delay, okay. So, there is a certain time delay associated because of the waving nature, so any change that happens at t equal to zero will not be visible immediately.

Obviously, since the velocity of propagation is finite, so it will not be visible immediately, but it will take some amount of delay. So, whatever you're looking at here, at your side at a distance l from the source is actually how the wave looked some delay back. This is equivalent of you looking at a star. The star that is shining or the star thing that you look at is not how the star is now.

It is actually equal to the time delay, which has taken for the light to arrive from the star, which is very distant all the way to earth and to your eyes. So, this kind of time delay means that things that you are looking at is not now, but it is at the retarded time.

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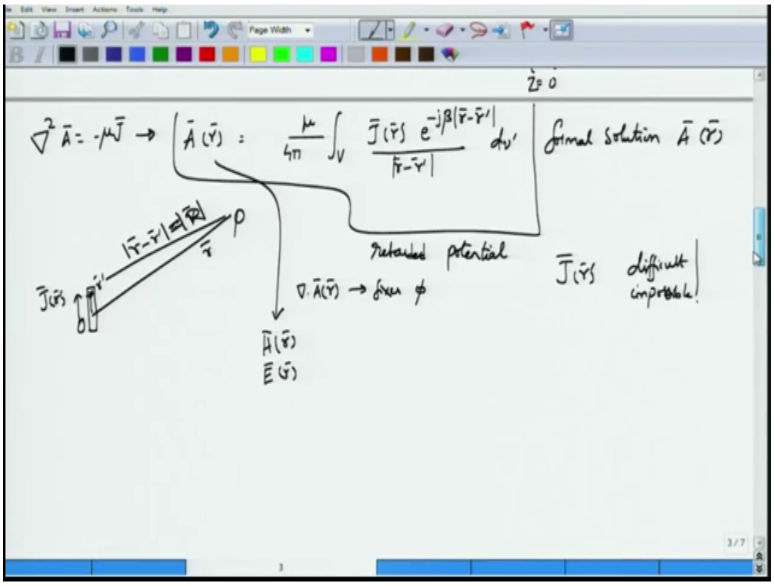

Now, we also have seen because of the similarity of this equation del square A is equal to some mu j, right. When the right hand side was not equal to zero, you could show that the solution for this del square A equal to mu j was given by A of r in terms of this Laplacian thing, right. So, you had this equivalence to a Laplacian form, del square A is equal to minus mu j had a solution for A as one by four pi or rather mu by four pi.

Because this is one by four pi epsilon, but epsilon needs to be replaced by one by mu. So, it would be mu by four pi integral over the volume J of r prime, which is the current that is changing at r equal to r prime times e to the power minus j beta r minus r prime. This beta r minus r prime simply incorporates the delay that happens, so e power minus j beta r is the phase factor that a wave would pickup while it has propagated to a distance of r minus r prime divided by r minus r prime, right, the magnitude of this one of course.

So, this is the solution, formal solution of, formal solution of the vector potential A at any region in space r, so you have some current, which is being carried. There is a current density that is appropriately defined by j of r prime and from here, so this is r prime. This is the source point, and you are at a distance r along a particular direction, you are at distance r, here and observing what is happening here.

So let us call this observing point as P and the origin of this vector point as zero. So, this r minus r prime will give you the distance between the source and the observation point, right and this r minus r prime would be approximately equal to r when this length of the antennas or length of this wire would be very small compared to the distances that you are considering. Otherwise, you can simply call this as equal to r.

And this r magnitude would be the one that would determine the time delay between something happening at the source and what you are going to observe. So, this formal solution is what we call for A of r is what we call as retarded potential, retarded vector potential, but with that vector thing is so understood that we simply call this as retarded potential.

And all our antenna problems can be boiled down to find A from A you find out H at that particular point and also find out the electric field at that particular point assume that pi. Phi of course, also has a similar liquidation, but if there are no free currents than phi can be taken to be zero, and in any case, we take phi as not of a consequence because we can fix this phi by writing this del dot A of r condition. So, this actually fixes this scalar potential phi.

So because of this, almost all of the antenna problems that are studied under this particular formalism would simply consist of solving this equation. Now you might question, hi what is the big deal about this? If I know what the current density j of r prime is, this equation seems to be quite simple. There is a dv prime. This is the integral over the volume in which the current density is present. So, you can say what is the big deal here?

I can always solve this equation, if not analytically always by numerical methods when I know what is the current density j of r prime. But the problem with antenna analysis is that, this quantity j of r prime is notoriously difficult, or in fact, in many cases impossible to specify and in fact this is the fundamental problem with antenna analysis. You do not know the current distribution on the antenna.

If you knew the current distribution, then all the analysis could have been simplified. So, because this current distribution is not known, and it is not very easy to measure that this is where we have this problems of antenna analysis. Luckily, we do not go into so much of detail in this course, but we will see some flavor of this and when we consider a thin half-wave dipole or linear antenna.