Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture - 76
Dielectric Waveguide

In this module we will discuss a different kind of waveguide structure that propagates wave or carries electromagnetic energy not by having a metallic bonding wall but actually using the phenomenon of total internal reflection.

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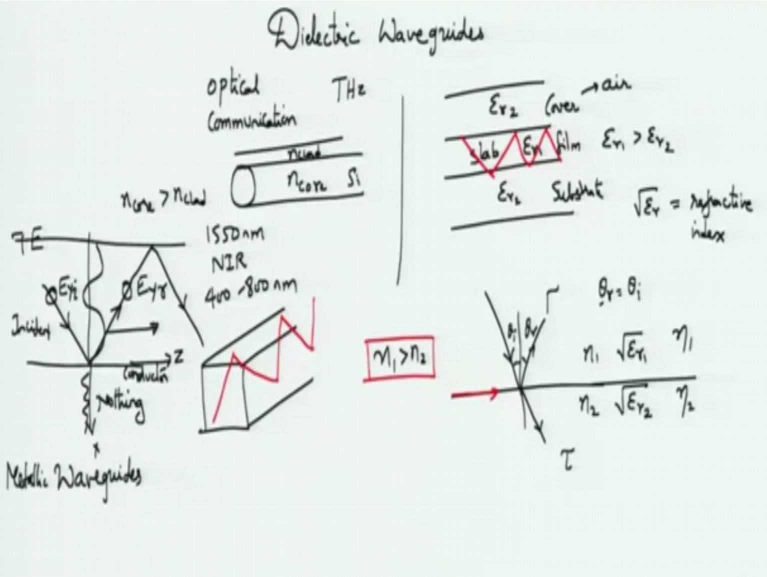

These are known as dielectric waveguides and these dielectric waveguides are very widely used to guide electromagnetic waves at very high frequencies. Specifically, these are used in optical communications where the frequencies are in tera hertz, in optical communication in the form of circular dielectric waveguide which would essentially consist of a rod, dielectric rod of some refractive index n core.

Let us say, surrounded by another rod of refractive index n clad, such that the core refractive index is greater than the refractive index of the cladding, okay? There could of course be some other jacket around this fiber but essential idea is that you actually have a dielectric medium or a dielectric rod in this case circular rod of refractive index which is higher than the refractive index of the surrounding medium.

And in fact, if you make this rod using silicon or combination of silicon and germanium we can show that the loses for this particular rod can be made very small and this is in fact the reason why optical communication has become wide spread now. You can actually carry electromagnetic energy or information in form of electromagnetic energy at frequencies which are around 1550 nanometer.

These are not visible range but the wavelength is around 1550 nanometer. These are called as near IR, that is near infrared region. There are of course fibers which can also carry electromagnetic energy in the form of light at visible wavelengths that is around 400-800 nanometers. These fibers although they are quite important the study of them requires us to go beyond this Cartesian co-ordinate system.

Because the symmetry of this fiber clearly indicates that one has to take a cylindrical co-ordinate system and something that we are not prepared to do in this course. So we are not going to discuss optical fibers although they are kind of the most popular dielectric waveguides but instead we will discuss those waveguides which are used on a chip or an optical device such as a laser or photo detector.

There are these different components which are now fabricated on chips, right? So these are the small area occupying devices and one has to carry optical energy from one part to another part on that and in those cases again one utilizes dielectric waveguides, okay? So the dimensions of those dielectric waveguides are chosen so as to get them or make them carry electromagnetic energy in the region, say 1550 nanometer region or some other region depending on what the wavelength you want.

But essential point is that they are not kilometers long. They are quite small. They occupy a certain small region of chip. These are called as dielectric waveguides and actually there are quite a different number of dielectric waveguides or different types of dielectric waveguides. The simplest one is called a slab waveguide which consists of certain material having refractive index.

Let us say epsilon r1 and surrounded by different medium of epsilon r2, the permittivity of epsilon r2 and if you want the wave to be confined inside this slab then one has to chose epsilon r1 to be greater than epsilon r2. Now in the previous case I discussed in terms of the refractive index, now I am discussing in terms of the relative permittivity, but you know that these two discussions are equivalent.

Because square root of epsilon r is equal to refractive index. So whatever I discussed for the optical fiber holds completely true for this dielectric slab waveguides as well. So this is kind of a slab but the slab is covered with two different refractive index material. There is a covering from the bottom as well as from the top. In optical interconnects or optical photonic devices these are named different.

They are named as substrate, film, usually the substrate is quite large, the width of the substrate is quite large and the film substrate is quite small and then the top one which covers the film is called as the cover. In certain cases, the cover will be air. In many cases the cover is essentially air, that is there is no actual cover on the top and the substrate is made up of another dielectric material.

So light of course will be confined inside this film and it would be propagating in whatever the direction that you have chosen to propagate. So this is something that is interesting. So if you compare t his waveguide with that of a metallic waveguide you had a nice metal hollow tube within which light was being conducted or electromagnetic waves were being conducted.

You could think of this conduction as a wave that is incident, hitting the top plate or the top wall of the waveguide getting reflected, getting again reflected from their bottom wall, getting reflected further and getting reflected, right? This reflection is no problem; you already know how to analyze this reflection. Take a conducting surface, so if you assume that this is a conducting surface and then send light or electromagnetic wave at a certain angle then we know that this wave would be reflected, nothing would be penetrating this one.

So there is actually nothing here except for the skin depth in non ideal conductors. But for an ideal conductor there is nothing below the conductors. So this is the conductor surface. So whatever the energy that you are transmitted or incident on the metal would be completely reflected off. In fact, if you write down the expression for say for the TE mode, this is one of the two modes that we have already discussed.

So for the TE mode, the electric field will be tangential to the interface but perpendicular to the direction of the propagation, correct? So if you write down the expression for Ey incident and Ey reflectant and then find out the total electric field in the region one in the incident region. This is the incident region, if you find out the total electric field here, you will actually see that there is essentially a standing wave along the direction that is perpendicular to the conductor.

Now to trap light or guide light you introduce one more conductor out here which means that the standing wave is now there along this particular direction but the way would actually propagate in the direction that is perpendicular to this. So there is essentially a standing wave between the slab but then the standing wave essentially propagates in the direction perpendicular to this.

For example, if this is the z direction and if this is the x direction, so if we assume that this is my x direction and therefore electric field is along the y direction, then there is a standing wave along x, but this standing wave itself will consists of a travelling component along z. Now the reason this happens is again multiple reflection. These waveguides hit on the top wall, again comes back and so on.

But the phenomenon of reflection is not total internal reflection. This is the straight forward reflection of a light or electromagnetic wave from a metallic boundary. So no matter what the light wavelength that whatever angle is, if you incident that on a metallic boundary or a metallic wall, that entire wave will be reflected at the same angle. There is no concept of critical angle. So this is for the metallic waveguide which we discussed.

Of course we discussed the 2D version of this or rather the 3-D version of this in the form of hollow tube which had bounding walls on all the four sides. So that is what we discussed. This is known as a parallel plate metallic waveguide but that is not really important for our course. Now a very similar thing happens for a dielectric waveguide except that the reflection phenomenon that is happening does depend on what is the angle of incidence.

Now why is that so? Recall we had discussed total internal reflection, right? What was the discussion there at that time? You had two medium of refractive index say n1 and n2 or equivalently the relative permittivity square root epsilon r1 and square root epsilon r2 or equivalently the impedances eta 1 and eta 2, correct? So this was the constitutive parameters of the two medium which was then nicely separated by this interface, okay?

Now light was incident and we looked at what would happen when light is incident on this at a certain angle, so this is the angle of incidence. You could have a TE mode or a TM mode that does not really matter. But what we have found was that there was reflection of the light at the same angle, right? So theta r the angle of reflection was exactly equal to the angle of incidence. But was there something in the second medium?

Yes, there was something in the second medium. There was something in the second medium and the amplitude of the wave in the second medium was related to the amplitude of the incident wave by a transmission co-efficient. Similarly, the reflected wave was related by the reflection co-efficient gamma. As long as this condition was that n1 is greater than n2, you would always find some transmission and some reflection.

So light that is incident would split into some reflected component and transmitted component and your angle of incidence really played no part into it unless you are actually taking theta equal to zero in which case you were simply propagating this along the interface. But that is not something that we considered or you considered incidence from normal side, even then you found that there was some transmitted component.

And there was a reflected component. So there was no specific angle at which this tau was actually equally to zero. Tau was non zero as long as n1 is greater than n2. Please note that in this dielectric waveguide module I will be reflecting only refractive index because that is the natural way of discussion of dielectric waveguides which are used by optical engineers. So for that reason we will abandon eta 1 and eta 2 or the relative permittivity.

And we will only talk about the refractive index. So as long as the incident medium had a higher refractive index compared to the second medium, the light was incident would split into some reflected component some transmitted component and clearly if your interest was not in the second medium but only in the first medium this transmitted component would represent a waste of energy.

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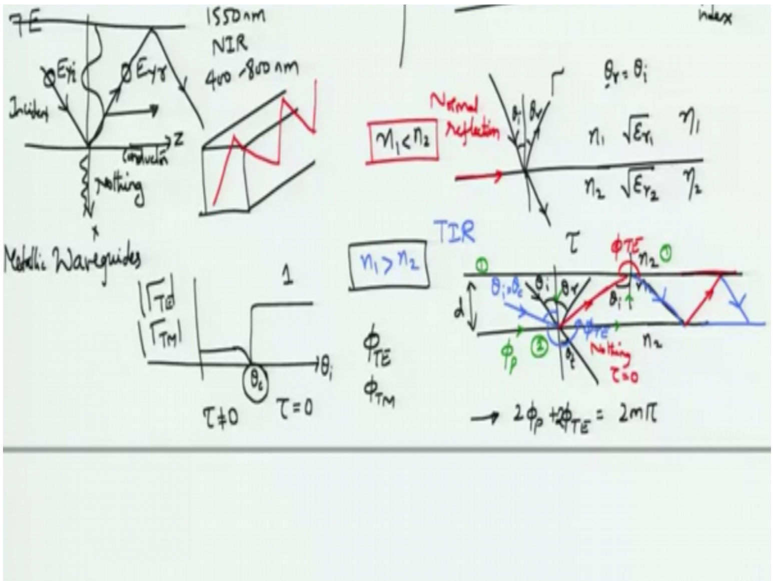

All right, now consider what happens to the case when n1 becomes less than n2, right? So assume that n1 is less than n2 what would happen to this particular scenario. We saw that if you were to plot r TE which was the reflection co-efficient of you can think of this as gamma TE with whatever the notation or gamma TM, right, and you plot the absolute value what you found was the reflective co-efficient had some value.

But after a certain time it went down and then it became one. So at a certain angle it became one. I might not be getting this plot of the diagram correctly. But if you go back to the discussion on reflection and refraction, there we saw that at a certain angle both TE reflection as well as TM reflection shot up to one. In fact, this was not even like this. It was kind of, this was essentially the way in which they would just shoot up.

So TE or TM I not really distinguishing between them. What I want to emphasize is that, there was a certain angle theta c below which your gamma TE, the reflection co-efficient or gamma TM for the reflection co-efficient, for the TE or TM modes was less than one. Which means that there was some transmission. So tau was actually nonzero over here. However, once you hit the critical angle.

So you are plotting this as the angle of incidence and once you have hit the critical angle the entire wave was actually reflected back to the first medium which means that there was no transmission of energy into the second medium. Now that is something that is interesting. Why is that so? So pictorially what was happening was, this is the normal, so this was one angle of incidence theta I, now we are assuming that n1 is actually greater than n2.

So light is incident there was some reflection, there was some transmission. So this was theta r and this was theta t. However, at a critical angle, when the angle of incidence was equal to critical angle theta c as measured from the normal, what happened was the entire wave got reflected. There was nothing here. So there was no transmission into the second medium. Now let us say light has been incident somehow, now light is getting reflected.

What happens is if I take another plane like this, right, so if I take another plane at a certain distance, whatever the distance let us say this is the distance d that I am calling which would be the slab thickness, what happens? Now this red line which represents the reflected ray of the lower interface would come and hit the interface at the top. Now if the refractive index of the top medium also happens to be n2.

This is called as a symmetric dielectric waveguide which is what we are going to consider in this example because of its mathematical simplicity to analyze. So if this was n2, whatever the critical angle that existed between the junction n1 and n2 would also be the same critical angle that existed at n1 and n2 and once you have exceeded the critical angle at the lower interface, you are coming in now with an angle that is actually greater than the critical angle.

So what should happen at the interface at the top here? Obviously there has to be one more total internal reflection which would then bend light again back to the first medium. So this is my incident or the first medium. Once again there would be reflection at this stage and then there would be one more reflection and eventually what you see is that the wave by following the zig zag path has actually managed to travel from one point on the film to the other point on the film.

So it seems that the electromagnetic energy is kind of trapped inside this slab and it is following in this way. There is one caveat which I would like to mention, we concentrated on the absolute value of gamma t but if you go back to the slide in which we discussed total internal reflection there was also two things which I mentioned which was phase upon reflection.

So whenever the light gets total internally reflected of a surface, it not only picks up the complete energy back but it will also pick up an additional phase. That additional phase depends on n2 and n1 as well as the angle of incidence. We do not really need to go into the details of how much this phase is one can actually do that one, but that is not really important. This phase that it picks up would get added. Now you look at the two blue lines.

I hope I have got the two blue lines correctly. So you have one blue line which you can think of the ray 1 or wave 1 which then hits this interface n1 and n2, gets total internally reflected because n1 is greater than n2. I wanted to show you n1 is greater than n2, sorry this is the case where n1 is less than n2. So this is n1 less than n2, this is n1 greater than n2. This is where you actually have total internal reflection.

This is the case where you have normal reflection. So the total internal reflection is n1 greater than n2. Now when your blue wave, the wave which I am representing by the blue line, hits the interface down here and gets totally internally reflected it picks up a certain phase phi TE or phi TM depending on what mode has been incident. Now let us say for argument sake that is just TE mode, so there is a TE mode which it gets picked up.

So there is a phi TE over here and then once the wave hits back, right, the top interface it will again pick up an additional phase of phi TE. In not only these two phases that it has picked up but while it has propagated it has actually picked up an additional phase. So if this is the length d and this is the angle theta i, you can find out how much distance it has propagated. So that is actually this line can be broken down into two pieces.

One is the horizontal and the other is the vertical and the blue one also can be broken down into horizontal as well as the vertical. You can see that the vertical ones are opposite in nature, they would be destroyed or they would be cancelled with each other, whereas the horizontal phase upon going from medium 1 to, from the top to the substrate or from substrate back to the top will add up. So let us call this phase as phi p.

Now you actually have to look at the total phase and then say that the total phase have to be 2 pi, otherwise or multiple of 2 pi, otherwise the rays that are incident over here which would have a certain wave front would then disruptively interfere with the next blue waves. So you can imagine that there are actually two wave fronts which we are incidenting in parallel. So one corresponding to this blue line over here.

And the other one corresponding to the next blue line. Only when the phase between these two wave fronts are in fact a multiple of 2 pi then these two wave fronts will interfere constructively and will be propagated down the waveguide. The condition for that to happen is that, 2 times phi p which is the phase that is picked up by the propagating wave plus phi TE which is also 2 times must be some multiple of pi.

In fact, you can relate phi p to the distance d, the angle of incidence theta i and phi TE can be calculated from the expression for the reflection co-efficient of TE or TM modes and use this equation to come up to different values of angle of incidence or in other words you can also talk about what is the d that needs to be chosen for a given angle of incidence. You can also discuss cut off wavelengths, cut off frequency, everything with this simple equation, okay?

Unfortunately, this equation based approach does not yield what kind of mode shapes are possible and what kind of mode shapes are sustainable. Moreover, this equation analysis assumes that the film thickness is quite large and therefore we can get away by describing light in terms of its ray picture. So whatever we discussed so far in this module is what is called as ray picture in which we are representing an electromagnetic wave by a ray.

So there is a corresponding wave front but we are not really bothered upon that wave front. So this equation although can be used to obtain very good numbers, the equation does not tell you what the mode shapes are and if you make that film thickness small then this analysis also not completely valid. So for these reason we will go back to Maxwell equations and then solve this problem from first principles.

We understand what would be the shapes of the TE mode and TM modes. For simplicity we will consider only one of them. So we will consider TE mode as the discussion that we are going to consider and we will leave the TM mode discussion to an exercise or just an extension of this theory, okay? So I hope you understand what we have discussed so far. Let me just very very briefly summarize here.

We started discussion dielectric waveguides which operate on a principle of total internal reflection. Total internal reflection is widely used in optical communication, photonic devices, wherever you want light to be trapped in a certain region of high refractive index.

You can do that by surrounding that high refractive index medium by low refractive index materials.

So one you do that and send light from the high density region, then light gets totally internally reflected by the interfaces and gets trapped inside the high density region provided your angle of incidence is greater than critical angle. The phenomenon by which a metallic wave guide would operate is completely different because there is no specification of the critical angle there.

Whereas for dielectric waveguides, your angle of incidence from the film or from the core of the fiber must exceed the critical angle. Moreover, it must also satisfy this phase criteria that the total propagation phase that it has picked up over the distance between consecutive rays plus any additional phase factor that it has picked up because of the TE or TM mode description. The total of that must be equal to an integral multiple of two pi.

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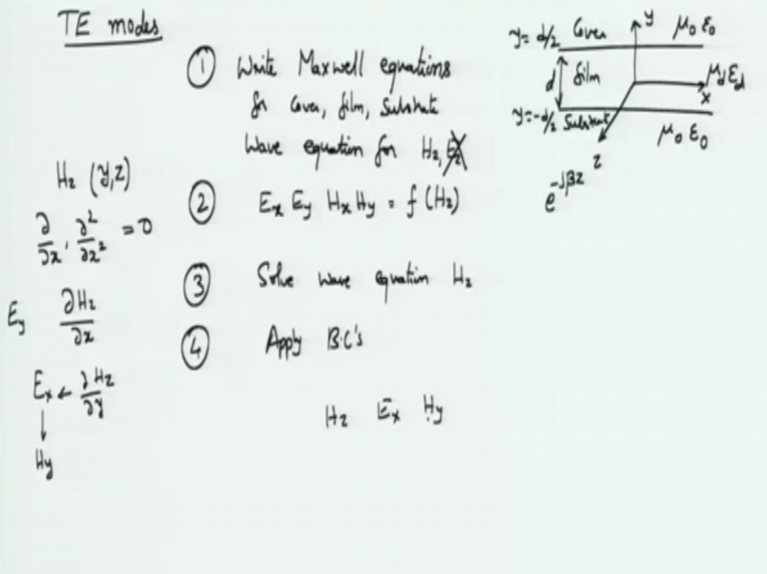

TE modes

① Write Maxwell equations for Cover, film, Substrate
Wave equation for H_z, ~~E_z~~

② $E_x\ E_y\ H_x\ H_y = f(H_z)$

③ Solve wave equation H_z

④ Apply B.C's

$H_z\ E_x\ H_y$

$H_z(y,z)$

$\frac{\partial}{\partial x}, \frac{\partial^2}{\partial z^2} = 0$

$E_y \quad \frac{\partial H_z}{\partial z}$

$E_x \leftarrow \frac{\partial H_z}{\partial y}$

$\downarrow$

H_y

So we start from this one and we will write Maxwell equation to understand the mode shapes. Let us start from the procedure. Let me just give you the brief idea of how to go about finding the modes. We will concentrate as I said on the TE modes. I will leave the TM mode discussions. Either we will consider it very briefly or I will leave it as an exercise. For the TE mode or the TM mode, does not matter, the dielectric waveguide.

The way, we will approach the problem would be to write Maxwell equations, okay? So write down what are the Maxwell equation as applicable to different regions or propagation. So the prototype problem that we are going to consider is a film having a certain thickness d and made up of some material of permeability mu d and epsilon d. Later we will replace this mu d and epsilon d b their appropriate refractive indexes.

So for now we will simply put that one. We will assume a symmetric waveguide in the sense that the cover as well as the substrate both are air. Of course you might ask how can such a slab be present, how can it hang in the middle of the air? The answer is that it cannot be really be done so, but we are going to use this idea of having in air just to simplify the mathematical analysis.

A more realistic case would have been a different epsilon over here. So maybe a different substrate material which would be more realistic, but the analysis of that is slightly difficult. So we are not going to do that thing. So for us the cover as well as the substrate both are air and having parameters mu zero and epsilon zero completely describing them. We also of course need to pick a particular co-ordinate system.

So let us whimsically pick z as this direction of propagation. The slab extends along x to a large length along x. So essentially meaning that waves would be kind of independent of x because we assume that the length of the waveguide is so large compared to x that there is essentially no variation of the waves inside x. We said the co-ordinate system at the center and call the axis along which you have the two interfaces as the y axis.

So you have the top surface at y equal to d by 2. The bottom surface at y equal to minus d by 2, okay and the way it is propagating along side, what sort of propagation should we choose? We already know that we have to choose e to the power gamma z propagation, but let us be one step e ahead of this gamma. Let us actually choose them to be e to the power minus j beta z. So we want a wave which is actually propagating as e power minus j beta z, okay?

So we write Maxwell equations in, so this is my cover, this is my film, this is my substrate, so I will have to write down Maxwell's equations for cover, film and substrate. Now instead of writing Maxwell equation you can write down wave equation, okay? So you can also write down wave equation for again we will choose Hz and Ez as the components. For the transverse electric modes there is no Ez component.

So everything will be written in terms of Hz and express Ex, Ey, Hx and Hy in terms of Hz. We will soon see that; you will not have all the terms over here for the TE modes in the parallel plate waveguide. You will have only 3 terms as we will see, okay? But you already know how to express this, right? So you start from Maxwell's equation, adjust them and then go back and write down every component in terms of Hz and Ez.

This is exactly similar to the waveguide analysis that we did for the metallic waveguides. There is no change in this procedure so far. Third we will solve the wave equation. We already know how to solve wave equation b variable separable method and then apply boundary condition. So these are the steps that we are going to follow. The steps look almost exactly the same as that of for the metallic waveguide as well, okay?

For this particular TE mode you will see that there is going to be a Hz or the Hz component, but this Hz can only be a function of y or z. Of course it will be function of time but we are looking at the phasor solutions. So it would be a function only of y or z. Why is that so? Again the reason is that del by del x terms are dropped out because the slab is essentially infinite along the x direction.

So all these terms like del by del x and del square by del x square would be equal to zero. So because of these fact, you will not find del Hz by del x which would have given you a component along Ey, rather what you find is that you will have Ex because you will have del Hz by del y. You will find Ex component corresponding to Ex we will find a Hy component, okay?

So these are the 3 different components that you are going to find. So for the TE modes the nonzero components are Hz, Ex and Hy. Of course Ex and Hy will be expressed in terms of Hz itself. The reason why I have Hy is because I have Ex and Ex and Hy must constitute the orthogonal components, right? So that ratio must correspond to the wave impedance as we have discussed earlier.