Electromagnetic Theory
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Lecture - 75
Waveguide Losses

In this module we will sum up wave guides by calculating wave guide loses. I will not go into complete derivation of the loss calculation because this is mathematically kind of difficult and also involves lot of concepts from percolation theory which we are not ready to deal with in this course, but a preliminary understanding of the type of loses in a waveguide is required if you want to use these waveguides for any of your purposes.

Why would a waveguide exhibit loss? I mean why is that if we launch some power into the waveguide that entire power is not delivered from the source and to the load side. There could be many reasons.

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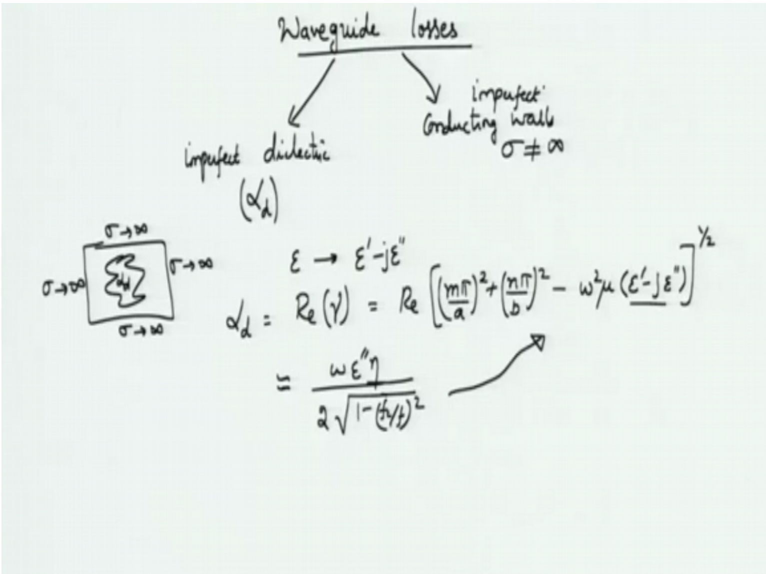

Two reasons which are main is that, the waveguide might be filled up with dielectric materials which are not quite okay, in the sense that they might have some conduction current to this loss tangent might be significant indicating that there would be some leakage current and the dielectric would then be carrying some power because of that. So these are called as dielectric losses and they are denoted by alpha d.

These dielectric losses are quite easy to model because all that would mean is that the waveguide wall is still assumed to be a perfect conductor. All this sigmas are going towards infinity. So there is no change in the boundary conditions at the wall. However, the dielectric is not perfect, but that is all right. All that we have to do is change epsilon to epsilon prime minus j epsilon double prime.

Remember this epsilon prime and epsilon double prime would model the loss tangent, one of them was giving you the conduction current and the other one was giving you the dielectric current. The ratio of this we called it as the loss tangent. So in all your formulas, all you have to do is to substitute epsilon by this particular complex number. Of course this complex number depends on frequency.

But you already know what frequency you are exerting the waveguide and because this is loss clearly this must mean that you have to find out the real part of gamma. So alpha d, the dielectric constant can be obtained by looking at the real part of gamma because gamma is alpha plus j beta. And this would be real part of m pi by a whole square plus n pi by b whole square minus omega square mu.

In place of epsilon that you would have had you need to write this as epsilon prime minus j epsilon double prime and then of course take the square root of this entire thing, okay? You can simply these formulas or of course if you have a nice calculator or an access to computer it is simple matter to write down a program for this and find out what would happen to alpha d, but in case we want to use certain approximations then you can use binomial theorem.

Ten takeout this epsilon prime here and then assume that epsilon double prime is small and then expand this one using binomial theorem and do some simple calculations. What you would see is that the dielectric constant would be approximated with an expression which says omega epsilon double prime eta divided by 2 multiplied by square root of 1 minus fz by f whole square, okay?

You do not have to remember this expression because this is not something that would be for us to be interesting. Here the better thing is if you have a calculator or an access to computer would be to go back to this expression itself and substitute epsilon prime minus j epsilon double prime for epsilon. So this is the dielectric loss and these are quite easy to model as I said. But where we get into difficult is when we try and find the losses because of the conducting walls.

So you have the conducting walls which make up the waveguide wall. But if we assume that sigma is not infinity. Sigma is not equal to infinity; in other words, we have imperfect conducting walls then we are in big soup because when this happens your boundary conditions must be changed.

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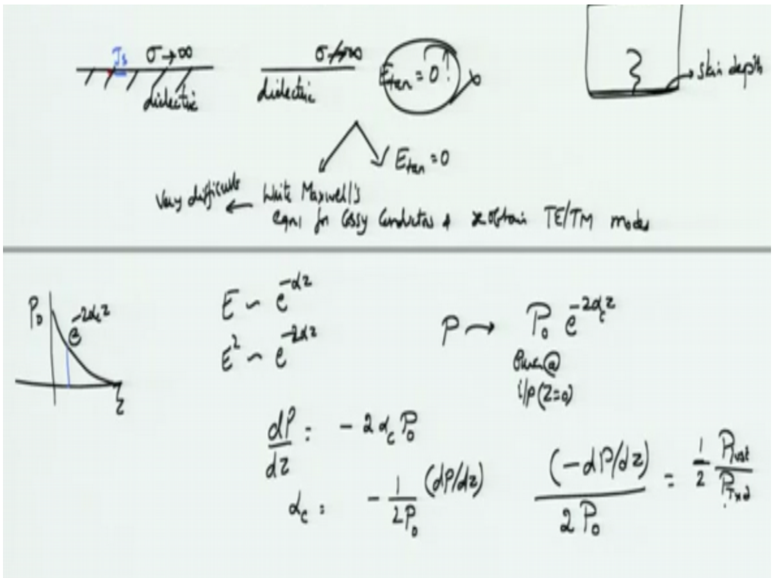

Remember when we calculate the boundary condition we assume that this would be a conductor and sigma going to infinity, this would be a dielectric, right? So the conductor was assumed to be perfectly conducting and for this one we said the horizontal e must go to zero, whereas the horizontal H or the tangential H must induce a certain Js here. it was easy for us to calculate this.

But the situation that we now have is that sigma is not going to infinity. Sigma is a finite thing and you have a dielectric but it is also not perfect dielectric, right? So this is kind of in between conductor and a dielectric, mostly towards conductor, but there are losses. In this case, I cannot say the tangential e component must be equal to zero, I cannot say this, right, because this is not true.

This condition is true only as long as the conductor was finite. Now we are trying to calculate the loses where the conductance is not finite for which I cannot use the original boundary condition. Now I seem to have two options or at least one option immediately is that write

Maxwell's equation for lossy conductors and reobtain all those expressions for TE and TM modes. This is something that I can try, but doing this would be very difficult.

This is actually very difficult procedure not that people have not tried it, there is lot of theory on that one and there are some very nice methods of finding out the Maxwell's equation for lossy conductors and obtain the TE and TM mode for that one. But that is something that is slightly beyond our scope of the course. Therefore, we will try and adopt a much simpler what might be called as a common sense approach.

The common sense approach would be that, well sigma may not be infinity but we are also not quite far from the skin depth operation that is the electric fields that could impinge from inside the waveguide on to the conductor walls. So you have this waveguide conductor walls, right? So the electric fields which are slashing in and inside the waveguide would meet but because of the lossiness they would actually start penetrating into the skin depth of the wall. So the propagation is still contained in the skin depth.

So we make a simplified assumption that the boundary condition still remains E tan equal to zero, but we will calculate the power assuming a perfect conductor and assuming this conductor and then equate how much power is getting lost. The idea here is very simple. The electric fields go as e to the power minus alpha z, therefore the power would go as, E square would go as e to the power minus 2 alpha z.

Now if you were to launch some power P into the waveguide, that power would decay as P0, this is the power at input that is at z equal to zero, times e to the power minus 2 alpha z, correct? So this alpha of the conducting walls we will label it as e power minus 2 alpha c z. So you launch a certain power and this power would basically go as e to the power minus 2 alpha c z, from the starting value of P0 along z direction.

Now what we do is, we find out at a particular distance for z, we find out how quickly the power is actually varying. So we say dp by dz, this is the rate at which power is decaying inside the waveguide, this would be equal to minus 2 alpha c times P, right? Now I will calculate what is alpha c, from this mathematical expression I will calculate.

This would be minus one by 2 p times dp by dz. If I am somehow able to measure this power loss per meter, dp by dz would simply indicate the change of power but if I want to include the loss, I can include this minus sign to that. So if I call this minus dp by dz as the power loss per unit length, divide that one by, sorry this has to be P0 here.

So this has to be the input power, then this would give me the attenuation constant alpha c, right? So how much power is getting lost to twice the transmitted power, okay? So you can express this fellow as minus or rather since we have already taken this minus sign into the loss thing, it would be equal to, sorry there is a two here, so it would be equal to half power lost divided by power transmitted.

We have already calculated the power transmitted for TE and TM cases earlier. So we have already calculated the power transmitted. That was some a b e max square divided by eta all those things we have already done that one. What we need to do is to simply calculate the power loss. Power loss is because of the skin depth of the waveguide and for the skin depth power loss we already know that it has to be proportional to transverse H square.

Because transverse h is the one that is relating to the surface current. So we calculate alpha c, to calculate alpha c I need to know what is the waveguide skin depth and how the power is being lost, okay?

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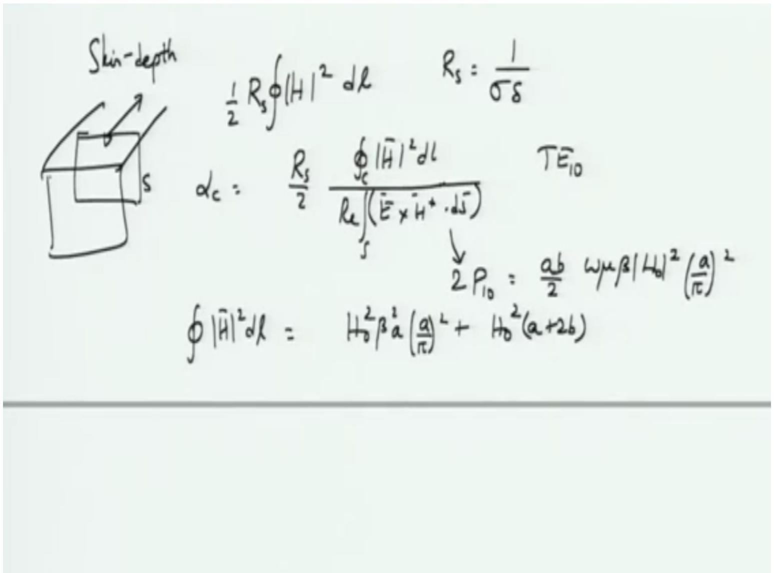

So consider the expression for the power loss for the skin depth, from the skin depth analysis we already know that power lost per unit area is given by Rs H mod square, where H would be the magnetic field, sorry, the power loss is actually half Rs multiplied by H square and this is the H that we need to evaluate on the surface, okay? So if you try doing this one and of course this is just a power density, so the power per unit area was this one.

So if you want to calculate the total power loss you need to appropriately integrate this one over the length of the propagation where Rs itself if the skin depth or the surface resistance that we have calculated given by 1 by sigma delta, right? So with this alpha c can be rewritten in terms of all these power calculations as Rs by 2 integral over H square, over one in the loop divided by real part of E cross H complex conjugate dot ds.

This is the power that is actually been transmitted, right? And this integration must be over whatever the loop that is formed by the surface S. So you have a waveguide here like this and on this surface, this is your surface and this would be your power that is lost over that loop, okay? So you can integrate over the four sides after calculating what is Js and you will be able to obtain an expression for the power loss.

As I said we are not interested in knowing the exact details of the calculation. You can see this one through the program and exercise that we are going to give you. These calculations are simple but they are quite tedious. So they are quite tedious over here, so we will not be able to solve those expressions. It just takes more time, it is just messy algebra.

So if you forget that messy algebra and note down the results you can see that alpha c, you can derive that alpha c to be equal to or rather you can first derive this transmitted power, the denominator part as 2 P10. For the TE 10 mode you can see that this 2 P10 is equal to a b by 2 omega mu beta H zero magnitude square, this was in fact e max square that we had calculated, times a by pi the whole square.

This is the power that has been calculated for the TE 10 case, this is what we calculated. This was a b by 4, but we need a 2 p 10 because there is a 2 here. So this times this power is given by this expression and for obtaining the power loss we need to go back to magnitude of H square dl, okay? Magnitude of H square dl, and if you calculate this one for the contour that is shown here, okay, for the contour that exists over here.

This would be equal to H0 square beta square a by pi whole square times a plus H0 square a plus 2 b. So there are 3 components here, one component comes from the x equal to zero and x equal to 1, H will be just H0 square, okay? Then there are two surface currents which are sitting at y equal to zero and y equal to b walls and those would contribute to the other two terms and this is all after performing the calculation, we can simplify the expression for alpha.

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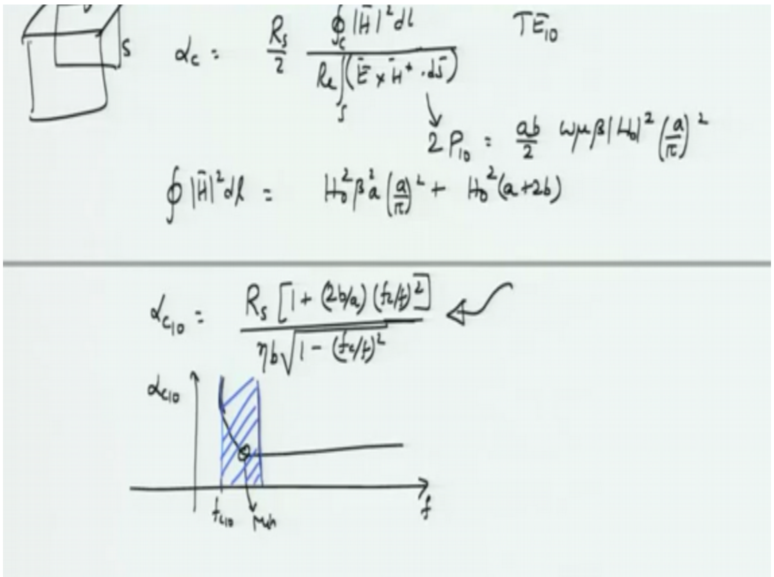

So you can substitute for these two expressions into alpha c and then simplify the relationship. What to see is that there is surface impedance of Rs that anyway comes in, divided by eta b 1 minus fc by f whole square under root and then you have in the numerator 1 plus 2 b by a fz by f whole square, okay? This is the expression for the conductive loses. You can rearrange these equations in whatever way you want.

But if you look at what is alpha c 10 which is the conduction loses for the TE 10 mode you will see that the power loss initially is quite high obviously before the waveguide is even propagated that will be very high power loss, infinite amount of power loss, so the mode will not propagate. But once the waveguide is actually propagating into beyond this fc 10 until let us say a certain dominant range, beyond this there would be higher order modes.

If you plot this one, the power actually goes like this. There is a minimum that exists here which you can obtain by differentiating this alpha c 10 with respect to f. Again this calculation would be quite tedious, the simple thing that you can try would be to just use Matlab expression, I mean Matlab program to solve for alpha c 10 for a given value of f.

In other words, you plot this function using Matlab and by trial and error you find out what is this alpha c 10, okay? So it would be not very difficult to do that one. The usual way of finding the minimum by differentiating this one is quite tedious. What you have to observe here is that for the dominant region, right, so if you operate the waveguide in the dominant region, the attenuation factor actually is less.

The attenuation factor for higher order modes which we have not calculated will be much higher than this dominant mode. This is the reason why we always want to operate all our waveguides such that the modes are always in the dominant mode, okay? So this brings us to the end of waveguides and we will begin with some other topics that are associated but not completely relevant to the waveguide, the wave phase velocity and group velocity that is concept which are quite general.

We will talk about that and then specialize those discussions to waveguide. We will then discuss dielectric waveguide and finally close the waveguide chapter. We then consider antennas and some numerical methods and we close the discussion of the course. Thank you.