Electromagnetic Theory
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Lecture - 73
Rectangular Waveguide: TE modes

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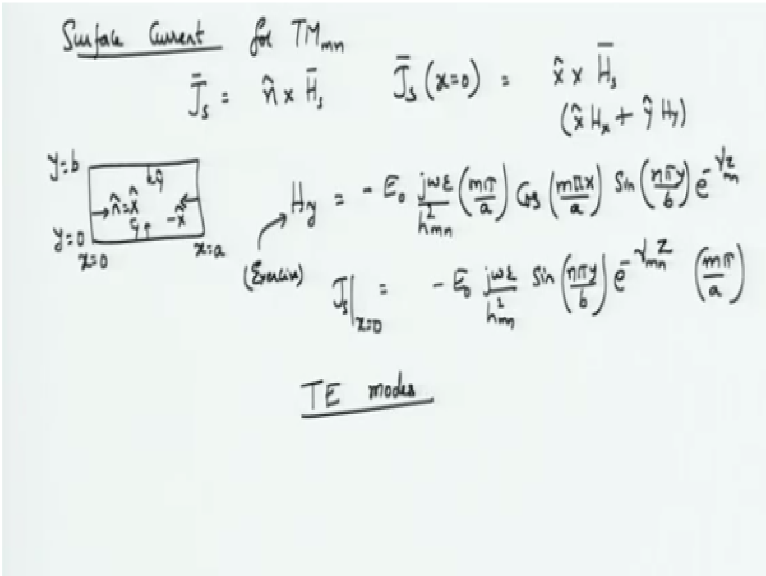

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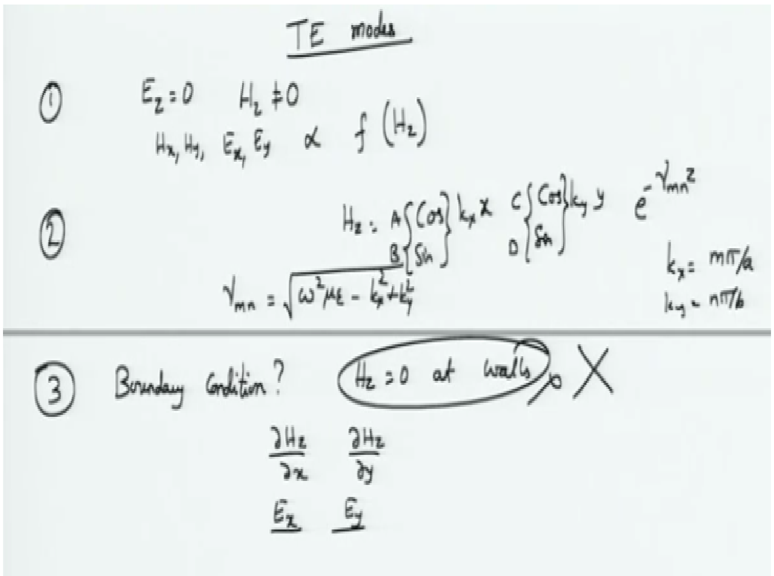

The development of TE modes is mathematically very similar to the development of TM mode. In the transverse electric case, you have EZ equal to zero, because the electric field has to be transverse to the direction of the propagation Z. So TE modes are characterised by having no component of EZ; however, this means that HZ must not be equal to zero and it will also mean that Hx, Hy, Ex and Ey must all be functions of Hz alone okay.

Now how do we obtain Hz well, this was first step in recognising what components exist and what components do not exist, the second step would be to apply Helmholtz equation and solve this Helmholtz equation for HZ okay. The equation form will be exactly similar, therefore HZ will have sin of something x cos of something y, we will not write down all those values okay. You already know what those values are.

However, what we need to write down is the form of HZ in terms of sin for x and y okay. HZ will have cos kxx sin of kxx, where both forms could essentially be there and then you have sin or rather you have cos kyy and then sin kyy right, multiplied of course by e power minus gamma mnz, you understand that gamma must also be having components m and n which will then be related to kx and ky, you already of course know what is kx, kx is m pi by a whereas ky is n pi b, I have not written down here, just for notational simplicity okay.

Also we know what is gamma mn, gamma mn will be equal to or this can be related to H itself. This would be equal to omega square mu epsilon minus kx square plus ky square right under square root. So we already know most of the things that we wanted to know, the amplitude for cos will be a, for sin it would be b, for cos here it would be c, and for sin it would be d okay.

So this is the Helmholtz equation for HZ, Helmholtz equation solutions for HZ in fact, so we have not written down the Helmholtz equation, but the equation would be exactly the same for what we wrote for EZ, you apply the variable separable criteria, you will then obtain this kind of an equation okay. So we obtained the expression for HZ.

Now, we are now into a small problem, what is the problem, well, let us try to apply the boundary condition for HZ. What should the boundary condition for HZ be, does HZ go to zero at walls, no, this is not true, this is wrong because HZ happens to be the tan longitudinal component for the walls and those components you know are the tangential components will not go to zero at the waveguide wall.

So what do we do, here we need to either resort to a formal solution of HZ, you know and then write down the appropriate Maxwell curl equation and then find out a new boundary condition for HZ or we might recognise that del HZ by del x and del HZ by del y must somehow be related to Ex or Ey correct. If they are related to Ex and Ey and for those conditions, we might want to obtain the boundary conditions right. So it would be corresponding tangential or of normal components which we can apply then.

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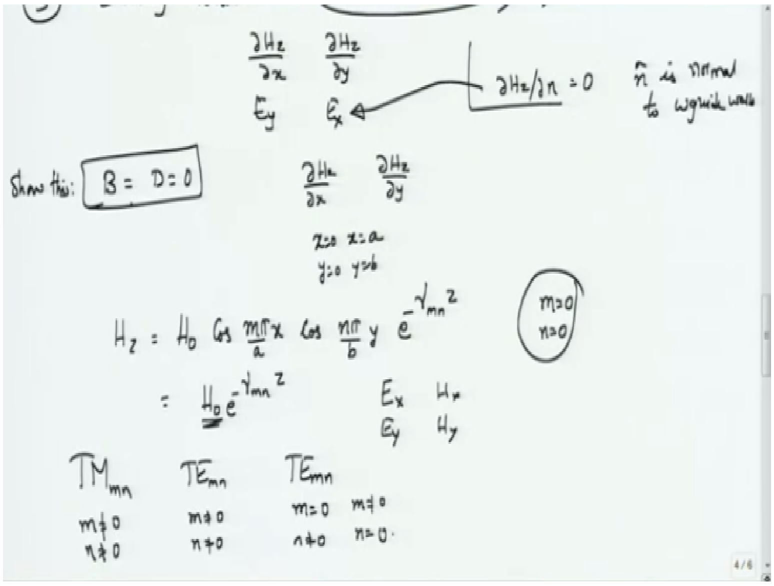

The up short of all this very hand waving analysis is that, at each of the waveguide walls we will apply the derivative condition okay or the normal condition. So del HZ by del n equal to zero where n is the normal to the wall okay, normal to waveguide wall. The philosophy behind the normal derivative of HZ being equal to zero is simply that these normal derivatives will determine the electric field components, which will be tangential to the appropriate waveguide walls.

If you really want to find what is this relationship, you have to go back to del HZ by del x and you can see that del HZ by del x actually corresponds to Ey and certainly this Ey component will be tangential at x equal to zero and at x equal to a walls and del HZ by del y corresponds to Ex components, which would be tangential at y equal to zero and y equal to b walls right.

So this would be the philosophy behind assuming that the boundary condition for HZ can be written not in terms of HZ itself, but in terms of its normal derivatives. This is called as normal derivative where n is the normal to the particular component okay. Of course, HZ itself happens to be the tangential component for the waveguide walls and that tangential component derivative of the tangential component with respect to the normal is called the normal derivative of HZ.

So I will not actually solve this now, the solution is kind of very simple once you know what is HZ, you need to actually differentiate this one to obtain del HZ by del x and del HZ by del y and appropriately substitute the conditions at x equal zero, x equal to a, y equal to zero and y equal to b walls. You do all those things and you will be able to show that b is equal to d is equal to zero okay.

I will leave this as an exercise to you, the solution is very simple, you take del HZ by del x, you take del HZ by del y okay and then once you have found out these, you need to apply the boundary conditions at x equal to zero, x equal to a, y equal to zero and y equal to b. So when you substitute all these values and especially when you apply the boundary conditions at x equal to zero and y equal to zero, you will find that b and d terms will go to zero okay.

A simple thing to see that one here is because del HZ by del x will turn cos into sin and that sin at x equal to zero will vanish this component, but here it would be cos of kxx that would vanish, so since this is vanishing the only condition that this can vanish is when b is equal to zero. Similarly, you can show that d must vanish in this expression for HZ okay. So simple exercise you try this out, if you do not get that one, we will give you the solution during the notes that we upload okay.

So now that we have applied this boundary conditions and obtained b equal to d equal to zero, we can proceed to write down what is HZ, HZ will also have some constant H zero, but the dependence on x and y will be cos m pi by a x cos n pi by b y. There would components of e power minus gamma m n z, that we may or we may not write down here. Look at this expression, now you try to find out what would be the corresponding values of m and n for the lowest order mode to occur.

Can we try the solution with m equal to zero, n equal to zero or any of these combinations. Certainly you can try because m equal to zero turns cos of some zero component into one, n equal to zero will turn the cos n pi by b into y term to one and both terms will ensure that HZ is still not zero right. So substitute m equal to zero, n equal to zero, the expression for HZ become quite simple, you just are left with H zero e power minus gamma m n into Z.

So you just have a constant H zero. Now the catch here is that this solution is also not alright because HZ component might exist, but nothing will exist for Ex, Ey or Hx and Hy okay. So because these other components are going to zero, you will not be able to get m and n components go to zero. In the TM case what we had was, TM m n, neither m nor n should have been equal to zero.

For T case, the modes will vanish only when both components are zero. However, one of them can be non-zero okay. So you cannot of course have for the TE m n case, m also equal to zero, n also equal to zero however, you can certainly have for the TE m n case, m equal to zero, n should not be zero or the other way around, m is not equal to zero, n is equal to zero. You can have this kind of solutions okay.

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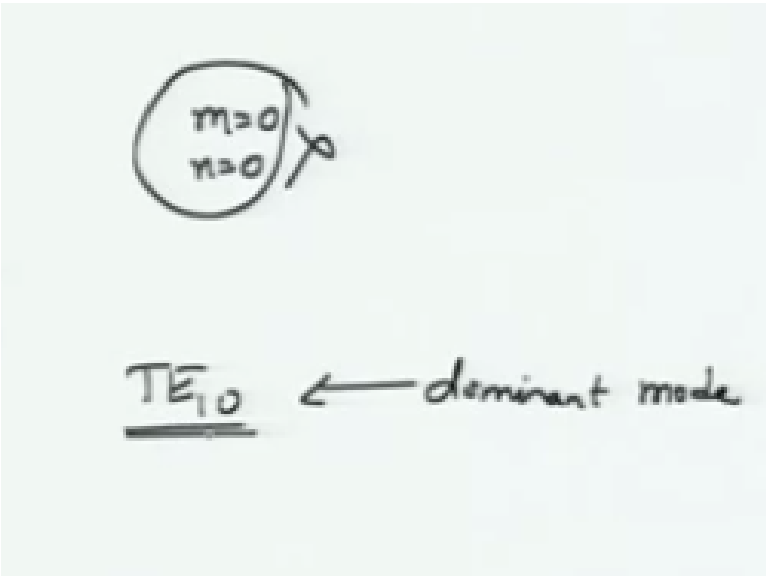

The fundamental mode for TE case turns out to be not zero zero as we thought it should be because the other components are not cooperating. The fundamental component for TE is that of one zero mode. This is the dominant mode and by far one of the most important choice for a mode to be propagated. So if you take a waveguide, your kind of assume that it is to be excited so as to launch a TE one zero mode.

Unless you are using these waveguides for filters, matching networks other things, most likely you are interested in trying to put all of your energy into TE one zero mode. There are very good reasons for it. First of all, it has a lowest cut off frequency, supports larger range of frequency operation and third would be that this has the least amount of dispersion okay.

So these are the different attributes of TE one zero mode, which is the dominant mode for a waveguide. So given a rectangular waveguide this is the dominant mode for the rectangular waveguide and for that mode it is interesting to just write down the expressions for electric and magnetic fields.

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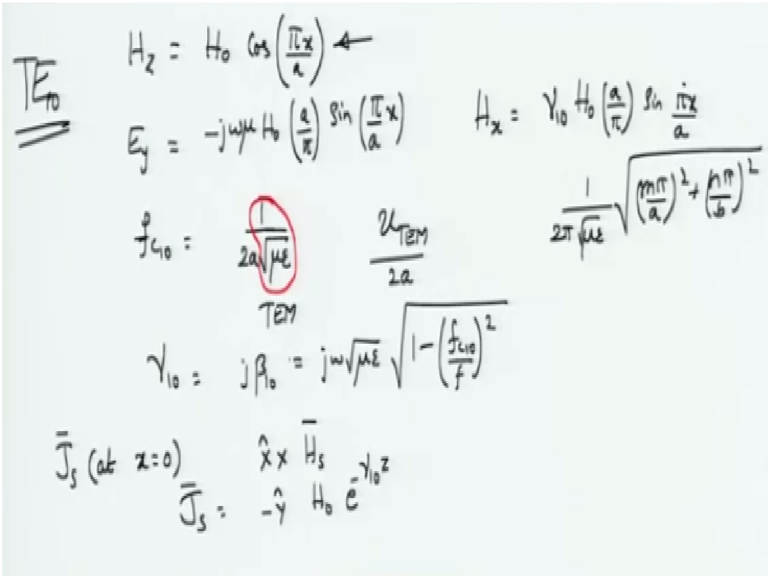

For the TE one zero case, the HZ component will be some constant H zero cos of pi x by a right. All the dependants on y is gone. There would e to the power minus gamma m n z that can be written as a common factor. If you go to Ex, you will see that Ex will be equal to zero because that would correspond to differential of HZ with respect to y whereas you will have Ey component that is non-zero.

Ey is given by minus j omega mu, these constants are really coming from the earlier module where we wrote down the longitudinal components. So if you do not get this constants, do not worry too much, just understand what the form of the solution should be since Ey should be del HZ by del x. Instead of cos it would become sin, the dependence on x would become sin and again the y dependence drops out okay.

So the y depends drops out and you get sin pi x by a, let me not write down the expression for how the dependence on z all the time. So what would be the other component that would exist, interesting, you have Ey component, therefore the component that should exist must be Hx and Hx component will be gamma one zero because you are really writing down the solutions for TE one zero mode.

So for the TE one zero mode, we can substitute m equal one, n equals zero and gamma is one zero and you will have component along sin. There are some additional components here so you have H zero a by pi okay sin pi x by a. These are the three components that would exist for TE one zero. So if you leave out this HZ, this is almost like a plane wave, wherein you have Ey and a minus Hx right.

So the ratio of these two should of course also give you the wave impedance for this particular waveguide. We will come to that one later. First what is the cut off frequency for this case. Cut off frequency expression is the same whether it is TM mode or the TE mode. Now substitute m equal to one, n equal to zero, it would be one by two a square root of mu epsilon right.

So you remember this one, it was actually one by two pi square mu epsilon m pi by a square plus n pi by b square under root. So now substitute m equals one and n equals zero, this b dependence goes away and m equal to one will pull out this pi by a here, and the numerator and denominator cancels and this is the expression that you are going to obtain.

We have already have said that one by mu epsilon, one by square root mu epsilon is the velocity of the wave inside the waveguide. So you can even think of this and in fact this velocity of the wave would actually be equal to the velocity of the TEM mode, not that is supported, but in the free space kind of a situation this would be the velocity right. So this can be written as u of the phase velocity or you can write down this as the TEM phase velocity divided by two a okay.

What would happen to gamma, gamma one zero will be equal to j beta one zero assuming that your frequency is larger than Fc one zero and this would be equal to j omega square root mu epsilon one minus Fc one zero by F square right. So this would be the expression for gamma okay. So we have written down the corresponding expressions for TE one zero.

The last thing that I would like to write down is the expression for the surface currents at the walls again as before I will write down the surface current at x equal to zero and in this case it would be x hat cross your H evaluated the surface. Now what are the H components, H components are x as well as z right. So for the x equal to zero component x cross x will go to zero, but x cross z will give you a current along minus y direction right.

So you will have x cross z will be along minus y direction and that would be H zero and that would be H zero and that is exactly equal to the corresponding surface current. So Js is equal to minus y hat H zero substituting x equal to zero will give you that one and of course along z it would still go as e power minus gamma one zero z for the case where F is greater than Fc, this gamma one zero can be written down as J beta one zero okay.

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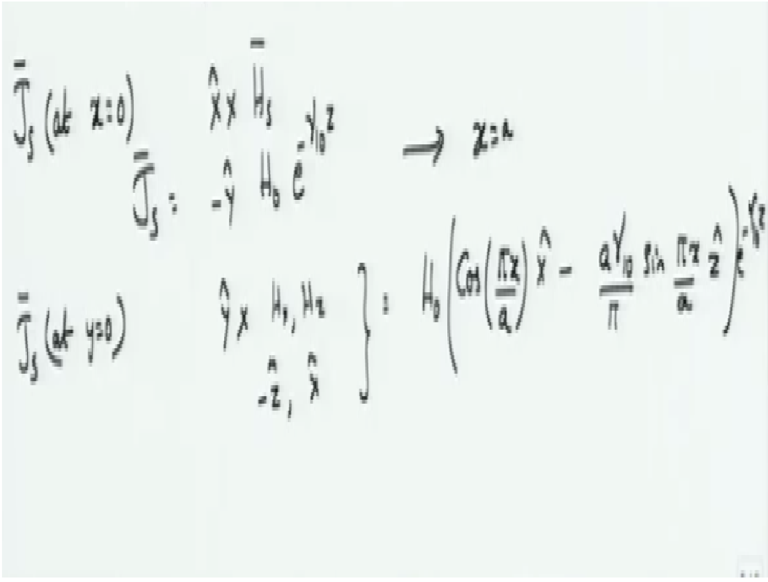

So this is your surface current at the x equal to zero wall but what is the surface current at y equal to zero wall, it would of course be the same surface current for Js it would be the same surface current at x equal to zero and at x equal to a because of the symmetry, but for y equal to zero it would be y hat that would the normal from the waveguide wall, so y hat cross Hx and Hz, y cross x will give you a component along minus z direction and then y cross z will give you a component along x direction.

So you will actually get two components for the surface current and you can evaluate this one by taking the appropriate cross product and then substituting y equal to zero in those expressions, the answer that you get is H zero cos pi x by a along x direction okay minus some constant a, of course a by pi is the constant, gamma one zero is the propagation constant sin pi x by a.

And this one would be along z direction, okay. So this would be along z direction e to the power minus gamma one zero z okay. So for the x equal to zero wall, there is only one component of the surface current whereas for y equal to zero, you have additional components for the current okay. You can also sketch the TE one zero mode or TE one, one mode, will not do those sketches, you can actually find out them from your textbook.

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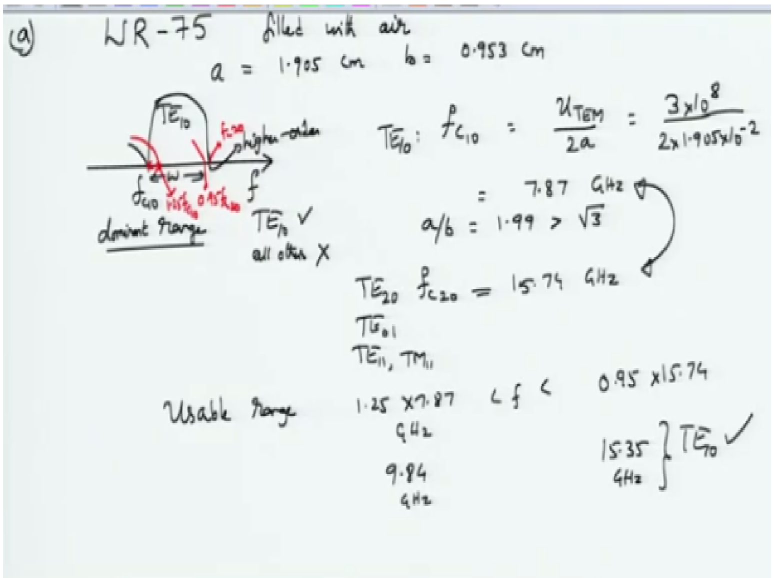

Let us try to solve one problem before we go on to the next topics in the waveguide, we want to consider a waveguide, which is widely used, which is called as WR-75 waveguide okay, this numbers are just a numbers, which would tell you the ratios of a and b. For this waveguide, which is filled with air, so the waveguide is filled with air, which means that mu is equal to mu zero and epsilon is equal to epsilon zero.

The dimensions of a and b are, a is one point nine zero five centimeters and be is point nine five three centimeters. Assuming that the waveguide itself is perfect, find the cut off frequency for the TE modes and also find out what is the range of frequencies over much only this mode exists. That is if you assume F, you know is increasing at some point you will reach the cut off frequency for one zero.

Here you will start having propagation and after a certain point you will start to have higher order modes okay. So that range or the bandwidth over, which there is only one particular mode that is you will have only TE one zero mode, it is called as the dominant mode range or dominant range. Dominant range is the one in which only TE one zero exist, all other modes are absent okay.

What is that particular dominant range for this waveguide? Well, the answer would be that you first find out what is the cut off frequency for TE one zero and then try to find out what is the next order mode okay. Once you find out the next order mode you can find out the cut off frequency there and that difference will tell you what is the dominant range. First of all, the cut off frequency for TE one zero mode, which is the lowest order is given by the phase velocity UTEM divided by two a.

I know that the waveguide is filled with air therefore it acts like a free space itself. So this fellow is c and c is three into ten to the power eight, two into one point nine zero five into ten to the power minus two and you will see that the cut off frequency is seven point eight seven gigahertz okay. Now what is the ratio of a by b.

A by b ratio is one point nine nine and once you find that the ratio of a to b is actually greater than square root of three okay. This is greater than square root of three, the dominant region would happen when the next higher order mode if you find out that would be TE two zero mode okay. For a waveguide in which this particular case, you know the ratio of a by b is greater than root three. The next higher order mode is TE two zero after that comes TE zero one, then comes TE one one okay, then comes TE one one and TM one one.

So the magnetic transfers magnetic mode does not begin to propagate until a few TE modes have been already propagating for the case of a by b greater than square root three okay. So for this case, the next cut off frequency would occur at Fc two zero and for that one can find out and that would simply be multiplying the original frequency Fc one zero by two, which would be equal to fifteen point seven four gigahertz.

So the range of frequencies over which you will have the dominant mode would be over the case where seven point eight seven to fifteen point seven four. Although, at seven point eight seven you really do not consider the waveguide to be propagating because you are really at the edge of the waveguide so you take some amount of frequency. So here you have this is Fc, but you will actually wait for some other you know, factor before you consider the waveguide to be operating right.

So if this frequency range at the end or at the band you would avoid because you do not want to have components, which are just getting cutting down over here. So you avoid some percent say let us say twenty-five percent higher than Fc one zero would be the range over which we operate. That is a very good thumb rule and you will also operate at the lower cut off frequency mode for the given wavelength to be around. So if this is your range, you would operate them at point nine five.

So you would actually operate anything between the range, the next higher order mode, so just, just about here. So if this is Fc two zero then you just go to point nine five Fc two zero and you start at one point two five Fc one zero. To avoid, I mean this is just an engineering thing, obviously there is no reason mathematically that you have to take this as the usable range. The usable range for this waveguide is chosen in such a way that you are just above the cut off frequency so that your wave component.

I mean frequency components are not getting attenuated at because of the lower order and you are not getting cut off because of the next higher order mode okay. So if you calculate the usable range according to that usable range for this example would be one point two five times seven point eight seven gigahertz okay to point nine five, five percent less than the next higher order mode, which is fifteen point seven four gigahertz.

And numerically these values turn out to be nine point eight four gigahertz to fifteen point three five gigahertz. So you have about five gigahertz of bandwidth roughly and this bandwidth is the bandwidth over which only TE one zero modes can be propagated or in the (()) (21:46) range only TE one zero modes exist and they would be propagated with whatever the characteristic propagation for TE one zero is concerned.

We will stop at this module. In the next module, we will consider some associated terms for waveguides and then quickly perform a simple calculation for the waveguide laws and we will close the chapter on waveguides. Thank you.