Electromagnetic Theory
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Lecture No - 07
Electric Field – I

So here is my first example

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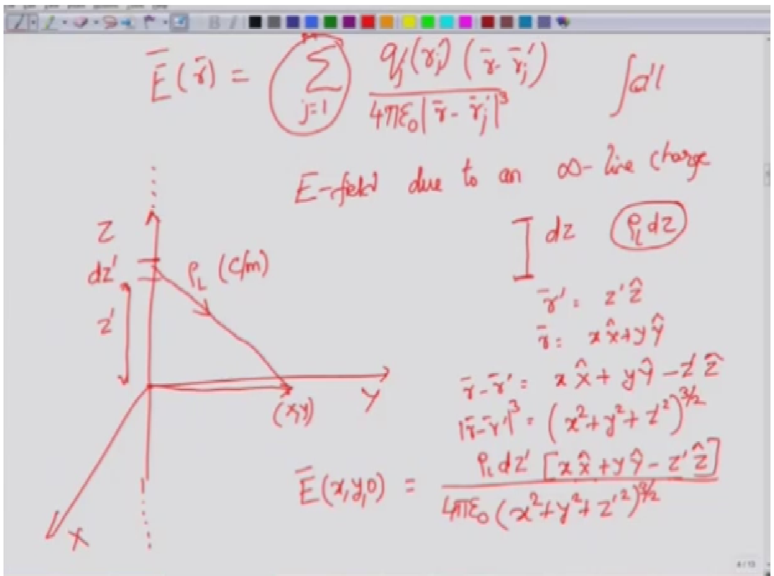

I want to calculate the electric field, remember this is electric field or electric field intensity. So, I want to calculate the electric field due to an infinite line charge. I have an infinite line charge that is simply meaning that there is a line charge of density rho L which is measured in Coulombs per meter located on a very, very narrow or a very, very thin wire like distribution and this wire extends all the way from minus infinity to plus infinity.

So, this line charge extends all the way from minus infinity to plus infinity. I can locate any coordinate system, to perform this particular calculation. So, let me say I am going to use the Cartesian coordinate system and I am going to line up my z axis along the line charge. So, the z axis is along the line charge and I have two other axis out here for me. Sorry, this is y axis and this is the x axis.

So, the line charge actually goes all the way towards from minus infinity along z to plus infinity along z. It has no other x or y dimensions. It is such narrow that it is fitting entirely on the line. I want to find the electric field. So, let me say I want to find the electric field at some point in the x y plane. So, I want to find the electric field at some point in the x y plane. So, on this plane the value of z will be equal to zero. How do I find the electric filed?

Well, we are going to use this super position rule that we have written down here except that this summation will now become an integration, integration over the line along which the charge is distributed. Now, remember that in Cartesian coordinate systems we showed that line integral along the z direction is simply dz along that particular direction. So, the line integral is actually along the z direction so, this d l z is actually along d z z prime.

Of course, in this case I don't really need the vector notations. I am not going to use the vector notation. This is how the line charges changes. Next, I need to first locate particular point on the source. So, I am going to locate a point at z prime height from the x y location. So, I have x y location out here and I am going to locate a point at z prime and I am going to consider a small incremental distance dz prime here.

What is the charge that is there in this particular small distribution, a small like differential length tz. What is the total charge that this differential line segment contains? It is simply the line charge density times dz and we are really thankful that, this line charge density is uniform in the sense that it does not depend on x and y coordinates in fact does not even depends on z coordinate. It is constant everywhere.

So, this is the amount of charge the differential charge that this particular line segment dz prime actually possesses. Now, this is the source point. So, source point is given by the position vector r prime which is z prime in the direction of z axis. Correct, what about the field point? Well to get the field point you have to find the posting vector at point x y in the z plane. So, the field point is x x hat plus y y hat.

There is no z component out there so you can clearly find out what is r minus r prime. This vector will be x x hat plus y y hat minus z z hat. So, this is r minus r prime vector which is actually a vector that is directed from the source to the field point. So, from the source to the field point you have vector which is this particular vector. So, what is the magnitude of this vector or magnitude cube of this vector?

It would be x square plus y square plus sorry this is z prime so z prime square to the power 3/2. The magnitude itself x square, plus y square, plus z prime square under root and then you have to simply raise it to the cubic power. So, it becomes x square + y square + z prime square to the power 3/2. Now, I can put down all these quantities inside the integral and I can find out what is the electric field at the position x y and zero, that is at the z plane at any point x and y.

So, this will be equal to what is the source charge value here? The incremental charge will be rho l dz. So, rho l dz, so dz prime is the incremental charge correct, divided by 4 pi epsilon zero this is the constant nothing to do with particular thing and I did not state that but I am assuming that this line charge is located in free space. So, I have 4 pi epsilon zero and this quantity x square plus y square plus y square plus z prime square to the power 3/2.

And then multiplied by the vector r minus r j prime. Remember so that vector will be x x hat plus y y hat minus z prime z hat. No, wonder people do not like electro magnetics as much as they would like to. Because there is - these differential integrals that are sitting over here and these are not very straight to evaluate all the times. But the beauty of electromagnetics is that if you perform this calculation you will realize that or if you substitute these calculations analytically with numerical methods you will realize that electromagnetics is actually in itself very simple.

Only thing is certain of these things will make it seem complicated but they are not really complicated. You can get hold of a very nice table of integrals books and then you can perform all these integrations or you can get hold of a computer and with a technic that I am going to describe to you in latter classes you can perform this entire calculation using the numerical technique. So, let us move on.

So, this electric field is of course not the total electric field because to get the total electric field I have to consider contributions of every segment of the line charge which is going all the way from minus infinity to plus infinity. In essence, what I am going to do is to integrate this whole thing from minus infinity to plus infinity. Now, my objective for the next few minutes is to evaluate this particular vary daunting looking integral. But we will see that this is not such a bad integral after all.

The important point to note here is that I have some integration

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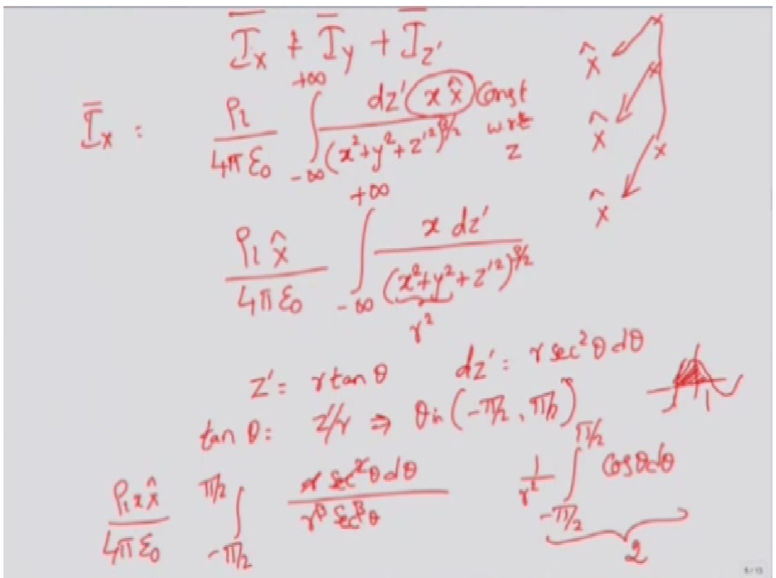

with respect to certain variable that is happening and there are certain factors in the denominator but there are factors in the numerator which are all vectors. So, we have probably not seen these types of integral in our earlier study. We had integrals in which the ingredient was simply a function of whatever the integral variable had. It had some functions, the functions could have some constants or something but that's all right.

That inside ingredients was always some function it was not a vector. Now, the question is what do I do with integrals when there are vectors? Thankfully, integral is a linear operation. Integration is a linear operation which means that the three integrals which I showed you in the last slide can actually be broken up into three different integrals. So, I actually can break up this into three different integrals and I will call them as Ix + Iy + Iz prime.

Where Ix will be the integral rho l/ 4 pi epsilon zero is a constant. So, I am going to take this out of the integral there is minus infinity plus infinity. The integration variable is z and there is x square plus y square plus z prime square to the power 3/2 and there is x into x hat. This is the vector. Similarly, you can find out or you can write down expression for I y bar and I z bar. I am not going to do all those calculations.

I am just going to show you how to do one integral and I am hoping that you can use the knowledge that you have already studied in terms of integration and differentiation to obtain the values of the other integrals. So, you have this particular integral you can solve this integral. First of all look at this, as z is changing in the sense that physically what is happening? I am moving from z is equal to minus infinity all the way to z is equal to plus infinity.

So, at each point as I move what is happening to the direction of the x vector? The direction of the x vector is still the same at every point along z axis the direction of the x vector is not changing. The x vector is remaining in the same direction as the x vector that would be there at the origin. So, the x vector is also not changing in magnitude that is the length of this arrow is not changing as I move up and down along the z axis.

So, in another words in this particular case we are fortunate that this entire term turns out to be constant with respect to z axis. So, I can take this entire constant out of the integral. Now, I will be left out with something that would look like this. Here itself, have minus infinity to plus infinity. There is x dz prime divided by x square plus y square plus z square to the power 3/2. Now using the well- known techniques of integration I am going to write down this square plus y square as sum r square.

This is constant because z is changing not x and y changing. If I have fixed x and y then the only way this is changing is because z is changing. So, I can call this x square plus y square as sum r square does not really matters what r is or you could call just as a square or s square whatever letter that you want to use you can use here and then write down z prime is equal to r tan theta. So, that tz prime will be equal to r sec square theta d theta and when I change the variable of integration from z prime to theta of course I have to change the limits of integration.

What is the new limit of integration for theta? Well, theta will be equal to z prime/r or rather tan theta will be equal to z prime/r and when z prime is equal to minus infinity tan theta will be equal to minus infinity and theta will be equal to minus pi/ 2 and on the other hand when z prime is equal to plus infinity then theta will be equal to plus pi/2. So, the integral variables are from minus pi/2 to plus pi/2. So the integral variables are from minus pi/2 to plus pi/2.

And all these constants I am not going to write at this point so you can actually take x also out does not really matter. So, I have rho L x x hat/ 4 pi epsilon zero and outside of the integral. I am not really bothered about that. Tz prime is r sec square theta d theta and in the denominator I have x square plus y square which is r square, r square plus z prime square, z prime square is r square tan square theta. So, r square is a common factor out there I can take that out.

One plus tan square theta is sec square theta and there is a root 3/2 power that you have to raise both of them. So, that leaves you r to the power three sec, to the power 3 theta. R cancels out with one of the r in the denominator sec square theta cancels out with sec theta in the denominator leaving the integral from minus pi/ 2 to plus pi/2 there is 1/r square here and 1/ sec theta is left out. Now, I know that sec theta is nothing but 1/ cos theta therefore 1/ sec theta is cos theta d theta.

Do you think this integral will be equal to zero? Well, see how the cos theta function is behaving so this is will be from minus pi/ 2 to plus pi/2. So, the area under this particular curve will not be equal to zero and you can actually found out what the area of the curve is. So if you carry out this integral cos theta becomes minus, cos theta becomes sin theta and then when you subtract this as two. So the result of all this for I x, let me write down the value of I x in the next slide.

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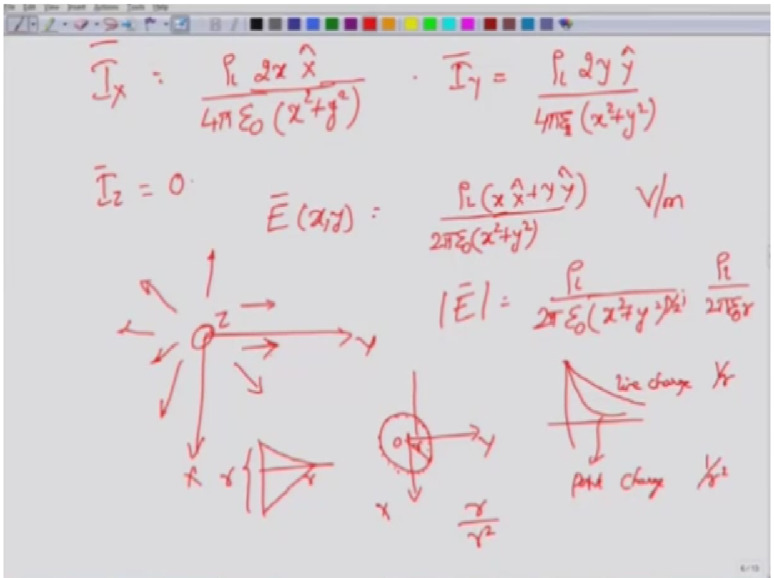

So, Ix is actually given by rho L/ 4 pi epsilon zero there is an r square in the denominator and on the numerator I get 2 x x hat. And what is r square? R square is nothing but square plus y square over here. Similarly, you can find out what is I y the integral over the y. You are going to see that this would be rho L 2y y hat divided by 4 pi epsilon zero square plus y square. You can also show that I z will be equal to zero.

So, that the total electric field at the point x and y is given by rho L/ 2 pi epsilon zero x square + y square x x hat plus y y hat volt/ meter. Don’t forget to use that volt/meter at the end because this is what the electric field is measured in. Before going on to the next part let us see what this equation is telling us. What is the direction of the electric field in the x y plane now? If you look at this equation and you have your x and y directions over here and the z axis is coming up this particular way.

So, in the z is equal to plane where we have calculated the electric field the direction of the electric field is always along that line which is at x x hat plus y y hat. So, if for example x is equal to 1, y is equal to 1 the direction will be along y hat plus x hat which would be making an angle of 45 degrees with respect to the x axis. So, you can actually see that the direction of the electric field is always what we call is radially away from the line.

What is happening to the magnitude of the electric field? Well, the magnitude of the electric field is obtained by looking at the magnitude of this particular vector. So, you have rho L/ 2 pi epsilon zero x square plus y square there is no change here. But the magnitude of this vector x x hat plus y y hat is x square plus y square under root, now there is square root fellow you can cancel that out with one of this terms in the denominator.

So, the magnitude is actually going as rho L/ 2 epsilon zero r where r is the –so, if this is my y and this is my x axis r is a set of all points which are at a distance r from the origin. So, the magnitude of the electric field is all constant over here in this particular at a distance r and more importantly the fields are going away as not as 1/r square but they are going as 1/r. This is the field of a line charge and this is a field of a point charge.

Point charge goes as 1/r square whereas line charge goes as 1/r, interesting. Why is the line charge going as 1/r? Well, what is happening as if you have the charge distribution over here so you are looking at the charge line at a distance r from this one so you have outmost a 90 degree field of view. So, if this is r then your field of view is approximately r. As you go away r the field goes as 1/ r square but the charge will multiply by r.

So, the numerator will increase as r denominator will decrease as r square and this r increases the numerator composite for the r decreasing the denominator or r square decreasing the denominator leaving you are with the field which is going as 1/r. Now, if you have followed this discussion so far you might have several questions one of the questions is why is this person doing this in the hard way?

This let us admit, is actually a hard way of doing this particular problem. Why was I doing this problem in a hard way? Because I wanted to illustrate that when you are faced with the problem in electromagnetics do not always rush to mathematics. Mathematic will always give you correct results there is no doubt about that one as long as your physical problem is correctly setup mathematic will always give you correct results.

However, if you start blindly applying mathematic unless you are mathematical wizard some of these problems are very, very hard to solve. If you neglect the physics side of the problem that is to say you forget how the charge is distributed. You forget how to choose the proper coordinate system. You neglect what is major aspect of these kind of problem is called symmetry. You will be doing lot of work which you could have done in a very short time.

If you had taken care of all these things so you never forget to use symmetry. Never forget to setup the appropriate coordinate system to reduce the amount of work that you are doing in order to get the same result. So, this result is correct only thing I did it the hard way to show you that never do these types of problems in this hard way. Look for symmetry look for an appropriate coordinate system and your work will be done in a much faster way.

So, the question is of course is this coordinate system not suited for this type of charge distribution? Answer is no. There are other coordinate systems and we are going to study the next coordinated system now which is much better suited for studying these types of problems. And that coordinate system is what is called as cylindrical coordinate system.

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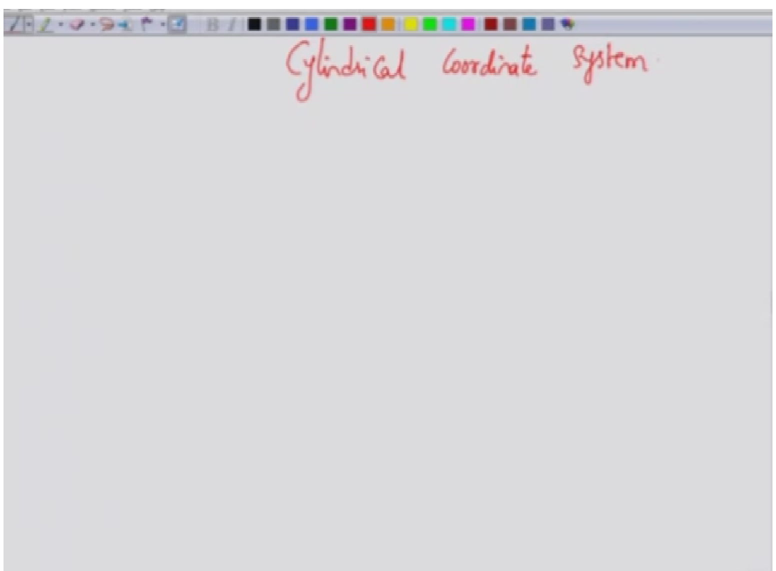

So, we are going to look at cylindrical coordinate system and some surprises will be there if you are coming from Cartesian coordinate system and hopefully keep these surprises in your mind.